Matrix theory Assignment 1

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***Abstract*—This document contains the procedure to get a equation of a line that is equidistant from the two parallel lines.**

Download the python code from the below link. Go through the README file in the reposotory.

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https://github.com/Sreeakanth/EE5609/
    Assignment_1/code.py
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1 Problem

Find the equation of the line which is equidistant from two parallel lines.

$$\begin{pmatrix}9 & 7\end{pmatrix}\mathbf{x} = 7 \tag{1.0.1}$$

$$\begin{pmatrix}3 & 2\end{pmatrix}\mathbf{x} = -6 \tag{1.0.2}$$

I think there is a typo in the problem , I think they are non parallel lines, However the procedure is similar.

Assume that an arbitrary point $\begin{pmatrix}x' & y'\end{pmatrix}$ is on the resultant line. Then the Idea of computing the line that is equidistant from two other lines is compute the distant between an arbitrary point $\begin{pmatrix}x' & y'\end{pmatrix}$ to both lines and equate then with opposite signs.

the distance from arbitrary point $\begin{pmatrix}x' & y'\end{pmatrix}$ to line L1 is same as L2 with opposite sign. Let's $ax+by+c = 0$ and $dx + ey + f = 0$ be the two given lines then

$$\frac{ax' + by' + c}{\sqrt{a^2 + b^2}} = -\frac{dx' + ey' + f}{\sqrt{d^2 + e^2}} \tag{1.0.3}$$

When we reformulate the above equation to the general form of linear equation we will get.

$$\mathbf{n} = \begin{pmatrix}-4\\ 3\end{pmatrix}$$

$$\begin{pmatrix}x' & y'\end{pmatrix}\begin{pmatrix}(\frac{a}{\sqrt{a^2+b^2}} + \frac{d}{\sqrt{d^2+e^2}})\\ (\frac{b}{\sqrt{a^2+b^2}} + \frac{e}{\sqrt{d^2+e^2}}\end{pmatrix} = -(\frac{c}{\sqrt{a^2 + b^2}} + \frac{f}{\sqrt{d^2 + e^2}}) \tag{1.0.4}$$

So the coefficients of the results line $\begin{pmatrix}g & h\end{pmatrix}$**x'**=-i are

$$\begin{pmatrix}g\\ h\\ i\end{pmatrix} = \begin{pmatrix}\frac{a}{\sqrt{a^2+b^2}} + \frac{d}{\sqrt{d^2+e^2}}\\ \frac{b}{\sqrt{a^2+b^2}} + \frac{e}{\sqrt{d^2+e^2}}\\ \frac{c}{\sqrt{a^2+b^2}} + \frac{f}{\sqrt{d^2+e^2}}\end{pmatrix} \tag{1.0.5}$$

When we substitute the given data in the above equations we will get the answer. given data is $a = 9$,$b = 7$,$c = -7$ and $d = 3$,$e = 2$,$f = 6$ so the answer is

$$\begin{pmatrix}g\\ h\\ i\end{pmatrix} = \begin{pmatrix}1.62\\ 1.168\\ 1.05\end{pmatrix} \tag{1.0.6}$$

I also took parallel lines and then compute the line which is equidistant from the two parallel lines

$$\begin{pmatrix}2 & 4\end{pmatrix}\mathbf{x} = 8 \tag{1.0.7}$$

$$\begin{pmatrix}2 & 4\end{pmatrix}\mathbf{x} = 16 \tag{1.0.8}$$

Now the coefficient of the line gx+hy+i=0 as mentioned by the above procedure is.

$$\begin{pmatrix}g\\ h\\ i\end{pmatrix} = \begin{pmatrix}0.89\\ 1.78\\ -5.36\end{pmatrix} \tag{1.0.9}$$

You can see the slopes of all the three lines in the above examples are similar (-0.5).

2 One more method for parallel lines

Let two lines

$$ax + by + c1 = 0 \tag{2.0.1}$$

$$ax + by + c2 = 0 \tag{2.0.2}$$

Then the distance between two lines is

$$d = \frac{\mid c1 - c2 \mid}{\sqrt{a^2 + b^2}} \tag{2.0.3}$$

Let the equation of the line which is parallel to both the equation is

$$ax + by + k = 0 \tag{2.0.4}$$

As the line is equidistant from the two lines

$$\frac{\mid k - c1 \mid}{\sqrt{a^2 + b^2}} = \frac{\mid k - c2 \mid}{\sqrt{a^2 + b^2}} \tag{2.0.5}$$

$$\mid k - c1 \mid = \pm \mid k - c2 \mid \tag{2.0.6}$$

By solving the above equation we can get the k value and corresponding equation.

2.1 Example

$$\begin{pmatrix} 2 & 4 \end{pmatrix} \mathbf{x} = 8 \tag{2.1.1}$$

$$\begin{pmatrix} 2 & 4 \end{pmatrix} \mathbf{x} = 16 \tag{2.1.2}$$

So c1=-8 and c2=-16 then k value from the above procedure is

$$k + 8 = \pm k + 16 \tag{2.1.3}$$

$$k + 8 = -k - 16 \tag{2.1.4}$$

$$2k = -24 \tag{2.1.5}$$

$$k = -12 \tag{2.1.6}$$

So the line which is equidistant from the above two lines is

$$\begin{pmatrix} 2 & 4 \end{pmatrix} \mathbf{x} = 12 \tag{2.1.7}$$