ASSIGNMENT 1

EE22BTECH11029 - Komakula Sreeja

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Question 1.4.4 Verify that

$$OA = OB = OC$$

Solution: Given:

$$\mathbf{A} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} -4 \\ 6 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} -3 \\ -5 \end{pmatrix}, \mathbf{O} = \begin{pmatrix} \frac{-53}{12} \\ \frac{5}{12} \end{pmatrix}. \tag{1}$$

$$\tag{2}$$

The direction vector of OA:

$$\mathbf{OA} = \mathbf{A} - \mathbf{O} \tag{3}$$

$$= \begin{pmatrix} 1 \\ -1 \end{pmatrix} - \begin{pmatrix} \frac{-53}{12} \\ \frac{5}{12} \end{pmatrix} \tag{4}$$

$$= \begin{pmatrix} \frac{65}{12} \\ \frac{-17}{12} \end{pmatrix} \tag{5}$$

$$\tag{6}$$

The direction vector of OB:

$$\mathbf{OB} = \mathbf{B} - \mathbf{O} \tag{7}$$

$$= \begin{pmatrix} -4 \\ 6 \end{pmatrix} - \begin{pmatrix} \frac{-53}{12} \\ \frac{5}{12} \end{pmatrix} \tag{8}$$

$$= \begin{pmatrix} \frac{5}{12} \\ \frac{67}{12} \end{pmatrix} \tag{9}$$

$$\tag{10}$$

The direction vector of OC:

$$\mathbf{OC} = \mathbf{C} - \mathbf{O} \tag{11}$$

$$= \begin{pmatrix} -3 \\ -5 \end{pmatrix} - \begin{pmatrix} \frac{-53}{12} \\ \frac{5}{12} \end{pmatrix} \tag{12}$$

$$= \begin{pmatrix} \frac{17}{12} \\ \frac{-65}{12} \end{pmatrix} \tag{13}$$

$$\tag{14}$$

We know that, length of AB is given as:

$$\|B - A\| = \sqrt{(B-A)^\top (B-A)} \tag{15}$$

Similarly, Length of OA:$\|A - O\|$

$$= \sqrt{\begin{pmatrix} \frac{65}{12} \\ \frac{-17}{12} \end{pmatrix}^\top \begin{pmatrix} \frac{65}{12} \\ \frac{-17}{12} \end{pmatrix}} \tag{16}$$

$$= \sqrt{\left[\frac{65}{12}\right]^2 + \left[\frac{-17}{12}\right]^2} \tag{17}$$

$$= \sqrt{\frac{4225}{144} + \frac{289}{144}} \tag{18}$$

$$= \sqrt{\frac{4514}{144}} \tag{19}$$

Length of OB:$\|B - O\|$

$$= \sqrt{\begin{pmatrix} \frac{5}{12} \\ \frac{67}{12} \end{pmatrix}^\top \begin{pmatrix} \frac{5}{12} \\ \frac{67}{12} \end{pmatrix}} \tag{20}$$

$$= \sqrt{\left[\frac{5}{12}\right]^2 + \left[\frac{67}{12}\right]^2} \tag{21}$$

$$= \sqrt{\frac{25}{144} + \frac{4489}{144}} \tag{22}$$

$$= \sqrt{\frac{4514}{144}} \tag{23}$$

Length of OC:$\|C-0\|$

$$= \sqrt{\begin{pmatrix} \frac{17}{12} \\ \frac{-65}{12} \end{pmatrix}^\top \begin{pmatrix} \frac{17}{12} \\ \frac{-65}{12} \end{pmatrix}} \tag{24}$$

$$= \sqrt{\left[\frac{17}{12}\right]^2 + \left[\frac{-65}{12}\right]^2} \tag{25}$$

$$= \sqrt{\frac{289}{144} + \frac{4225}{144}} \tag{26}$$

$$= \sqrt{\frac{4514}{144}} \tag{27}$$

$$\implies OA = OB = OC \tag{28}$$

Hence verified.