

EE22BTECH11029 - Komakula Sreeja

Question 9.3.16

Suppose that 90% of people are right-handed. What is the probability that atmost 6 of a random sample of 10 people are right-handed.

Solution: Given that 90% of the people are right-handed.

TABLE 1: Description of random variables

Parameters	Values	Description
n	10	Sample space
p	0.9	Probability that the person is right-handed
Y	$0 \leq Y \leq 10$	Number of people thatare right-handed
$\mu = np$	9	Mean
$\sigma = \sqrt{np(1-p)}$	0.9	Standard deviation
$\Pr(Y \leq 6) = \Pr(Y < 6)$	0.00078	Gaussian (without correction)
$\Pr(Y \leq 6) = \Pr(Y < 6.9)$	0.0136	Gaussian (with correction)
$\Pr(Y \leq 6)$	0.0128	Binomial

Gaussian Distribution

Central limit theorem:

$$Y \sim \mathcal{N}\left(\mu, \frac{\sigma}{\sqrt{n}}\right) \quad (1)$$

(2)

CDF of Y:

$$p_Y(y) = \Pr(Y < y) \quad (3)$$

$$= \Pr\left(\frac{Y - \mu}{\sigma} < \frac{y - \mu}{\sigma}\right) \quad (4)$$

$$= \Pr\left(X < \frac{y - \mu}{\sigma}\right) \quad (5)$$

$$= 1 - Q\left(\frac{y - \mu}{\sigma}\right) \quad (6)$$

With a 0.9 correction:

$$\Pr(Y \leq 6) = \Pr(Y < 6.9) \quad (7)$$

$$= 1 - Q\left(\frac{6.9 - 9}{\sqrt{0.9}}\right) \quad (8)$$

$$= 1 - Q\left(\frac{-2.1}{0.9487}\right) \quad (9)$$

$$= 1 - Q(-2.21) \quad (10)$$

$$= Q(2.21) \quad (11)$$

$$= 0.013553 \quad (12)$$

Without correction:

$$\Pr(Y \leq 6) = \Pr(Y < 6) \quad (13)$$

$$= 1 - Q\left(\frac{6-9}{\sqrt{0.9}}\right) \quad (14)$$

$$= 1 - Q\left(\frac{-3}{0.9487}\right) \quad (15)$$

$$= 1 - Q(-3.1622) \quad (16)$$

$$= Q(3.1622) \quad (17)$$

$$= 0.000783 \quad (18)$$

Binomial Distribution

$$\Pr(Y \leq 6) = 1 - \Pr(Y > 6) \quad (19)$$

$$= 1 - \sum_{k=7}^{10} \binom{n}{k} p^k (1-p)^{n-k} \quad (20)$$

$$= 1 - 0.9872 \quad (21)$$

$$= 0.0128 \quad (22)$$

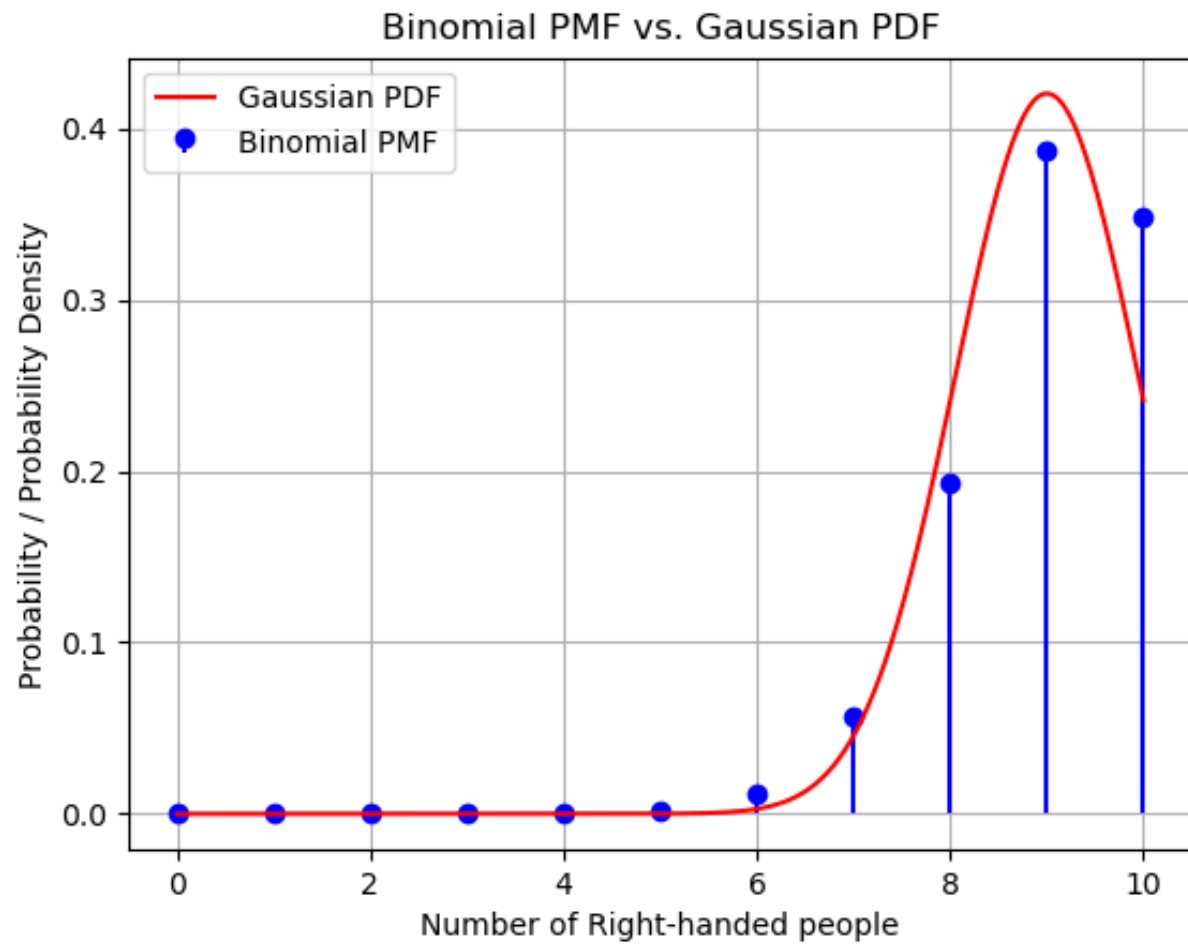


Fig. 1: Binomial vs Gaussian