EE22BTECH11029 - Komakula Sreeja

Question 9.3.16

Suppose that 90% of people are right-handed. What is the probability that atmost 6 of a random sample of 10 people are right-handed.

Solution: Given that 90% of the people are right-handed. Let p and q be probability that people are right-handed and left-handed respectively.

$$p = \frac{9}{10} \tag{1}$$

$$q = 1 - p = \frac{1}{10} \tag{2}$$

Using the gaussian approximation method:

$$\mu = np = 10 \times \frac{9}{10} = 9 \tag{3}$$

$$\sigma = \sqrt{npq} = \sqrt{10 \times \frac{9}{10} \times \frac{1}{10}} = \sqrt{0.9} \tag{4}$$

We know that Q-function is given as

$$Q(x) = \Pr(X > x) \tag{5}$$

$$= \int_{x}^{\infty} \frac{1}{\sqrt{2\pi}} \times e^{-\frac{x^2}{2}} dx \tag{6}$$

Now, we want to find the probability that at most 6 people are right-handed:

$$\Pr(X \leq 6) = 1 - \Pr(X > 6) \tag{7}$$

$$= 1 - \Pr\left(\frac{X - \mu}{\sigma} > \frac{6 - 9}{\sqrt{0.9}}\right) \tag{8}$$

$$= 1 - \Pr\left(Z > \frac{-3}{0.9487}\right) \tag{9}$$

$$= 1 - \Pr(Z > -3.1622) \tag{10}$$

$$= 1 - Q(-3.1622) \tag{11}$$

$$= 1 - \int_{-3.1622}^{\infty} \frac{1}{\sqrt{2\pi}} \times e^{-\frac{x^2}{2}} dx \tag{12}$$

$$= 0.000783 \tag{13}$$

Therefore, the probability that atmost 6 out of 10 people in the random sample are right-handed is 0.000783 which is approximately 0.078%.