EE22BTECH11029 - Komakula Sreeja

Question 9.3.16
Suppose that 90% of people are right-handed. What is the probability that atmost 6 of a random sample of 10 people are right-handed.
Solution: Given that 90% of the people are right-handed.

TABLE 1: Description of random variables

Parameters	Values	Description
n	10	Sample space
p	0.9	Probability that the person is right-handed
Y	$0 \leq Y \leq 10$	Number of people thatare right-handed
$\mu = np$	9	Mean
$\sigma = \sqrt{n}p(1-p)$	0.9	Standard deviation

Gaussian Distribution
Central limit theorm:

$$Y \sim \mathcal{N}\left(\mu, \frac{\sigma}{\sqrt{n}}\right) \tag{1}$$

$$\tag{2}$$

CDF of Y is

$$f_Y(y) = \Pr(Y \leq 6) \tag{3}$$

We know that

$$Q(x) = \Pr(X > x), x > 0, X \sim N(0,1) \tag{4}$$

$$Q(-x) = \Pr(X > -x), x < 0, X \sim N(0,1) \tag{5}$$

$$= 1 - Q(x) \tag{6}$$

Hence,
CDF for $y > \mu$:

$$f_Y(y) = 1 - Q\left(\frac{y-\mu}{\sigma}\right) \tag{7}$$

CDF for $y < \mu$:

$$f_Y(y) = 1 - Q\left(\frac{y-\mu}{\sigma}\right) = Q\left(\frac{\mu-y}{\sigma}\right) \text{from definition of Q-function} \tag{8}$$

With a 0.9 correction:

$$
\begin{aligned}
f_Y(6) &= \Pr(Y < 6.9) && (9)\\
&= 1 - Q\left(\frac{6.9-9}{\sqrt{0.9}}\right) && (10)\\
&= Q\left(\frac{2.1}{0.9487}\right) && (11)\\
&= Q(2.21) && (12)\\
&= 0.013553 && (13)
\end{aligned}
$$

Without correction:

$$
\begin{aligned}
f_Y(6) &= \Pr(Y < 6) && (14)\\
&= 1 - Q\left(\frac{6-9}{\sqrt{0.9}}\right) && (15)\\
&= Q\left(\frac{3}{0.9487}\right) && (16)\\
&= Q(3.1622) && (17)\\
&= 0.000783 && (18)
\end{aligned}
$$

TABLE 2: Comparision

Number of right-handed people	Binomial	Gaussian	Gaussian (0.9)	Error(%)	Error (0.9)(%)
Atmost 6	0.012795	0.000783	0.013553	-93.88	55.92

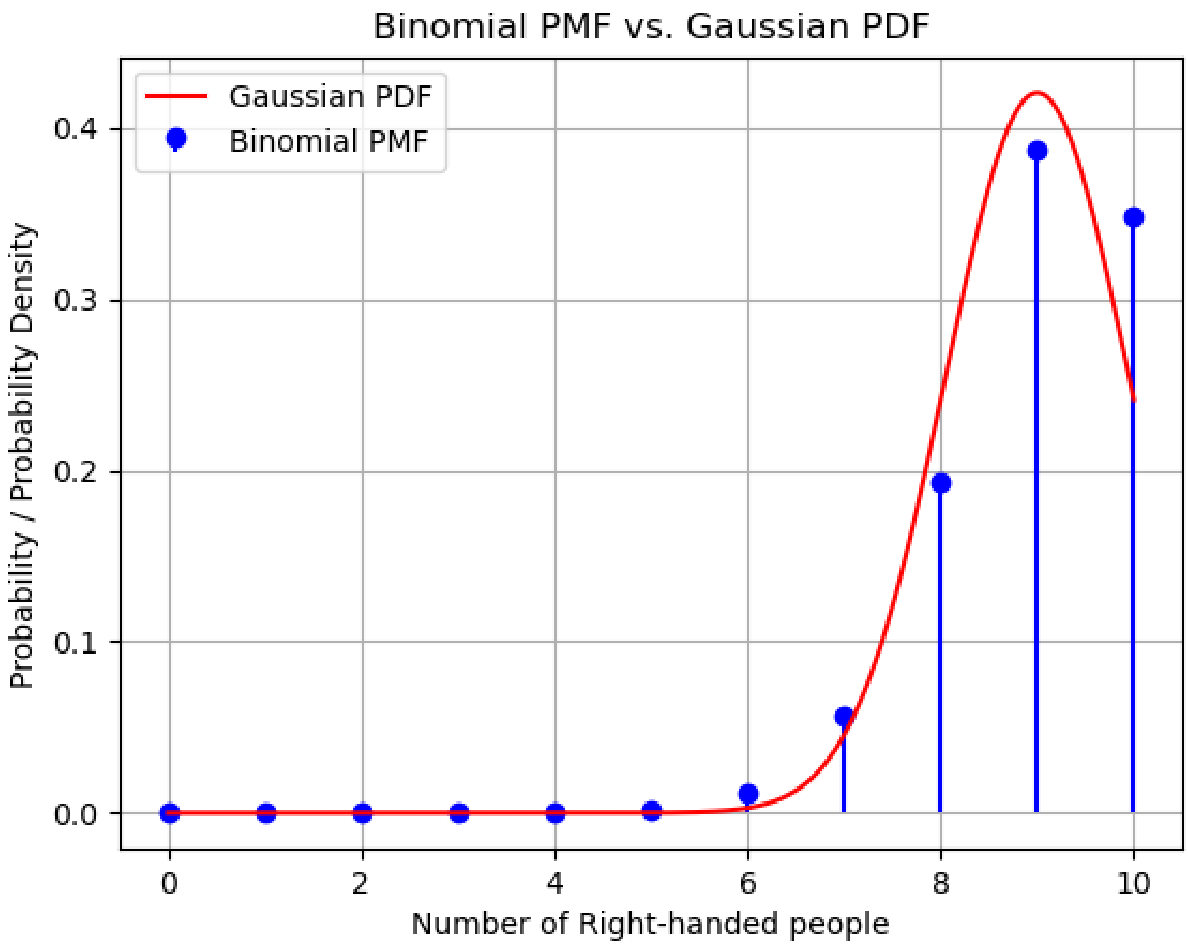

Fig. 1: Binomial vs Gaussian