EE22BTECH11029 - Komakula Sreeja

Question 9.3.16

Suppose that 90% of people are right-handed. What is the probability that atmost 6 of a random sample of 10 people are right-handed.

Solution: Given that 90% of the people are right-handed.

TABLE 1: Description of random variables

Parameters	Values	Description
n	10	Sample space
p	0.9	Probability that the person is right-handed
Y	$0 \leq Y \leq 10$	Number of people thatare right-handed
$\mu = np$	9	Mean
$\sigma = \sqrt{np(1-p)}$	0.9	Standard deviation

Gaussian Distribution

Central limit theorm:

$$Y \sim \mathcal{N}\left(\mu, \frac{\sigma}{\sqrt{n}}\right) \tag{1}$$

$$\tag{2}$$

CDF of Y is

$$F_Y(y) = \Pr(Y \leq y) \tag{3}$$

We know that

$$Q(x) = \Pr(X > x), x > 0, X \sim N(0,1) \tag{4}$$

$$Q(-x) = \Pr(X > -x), x < 0, X \sim N(0,1) \tag{5}$$

$$= 1 - Q(x) \tag{6}$$

Hence,
CDF :

$$F_Y(y) = \begin{cases} 1 - Q\left(\frac{y-\mu}{\sigma}\right), & \text{if } y > \mu \\ 1 - Q\left(\frac{y-\mu}{\sigma}\right) = Q\left(\frac{\mu-y}{\sigma}\right), & \text{if } y < \mu \end{cases} \tag{7}$$

With a 0.9 correction:

$$F_Y(6) = \Pr(Y < 6.9) \tag{8}$$

$$= 1 - Q\left(\frac{6.9-9}{\sqrt{0.9}}\right) \tag{9}$$

$$= Q\left(\frac{2.1}{0.9487}\right) \tag{10}$$

$$= Q(2.21) \tag{11}$$

$$= 0.013553 \tag{12}$$

Without correction:

$$F_Y(6) = \Pr(Y \le 6) \tag{13}$$
$$= 1 - Q\left(\frac{6-9}{\sqrt{0.9}}\right) \tag{14}$$
$$= Q\left(\frac{3}{0.9487}\right) \tag{15}$$
$$= Q(3.1622) \tag{16}$$
$$= 0.000783 \tag{17}$$

TABLE 2: Comparision

Number of right-handed people	Binomial	Gaussian	Gaussian (0.9)	Error(%)	Error (0.9)(%)
Atmost 6	0.012795	0.000783	0.013553	-93.88	55.92

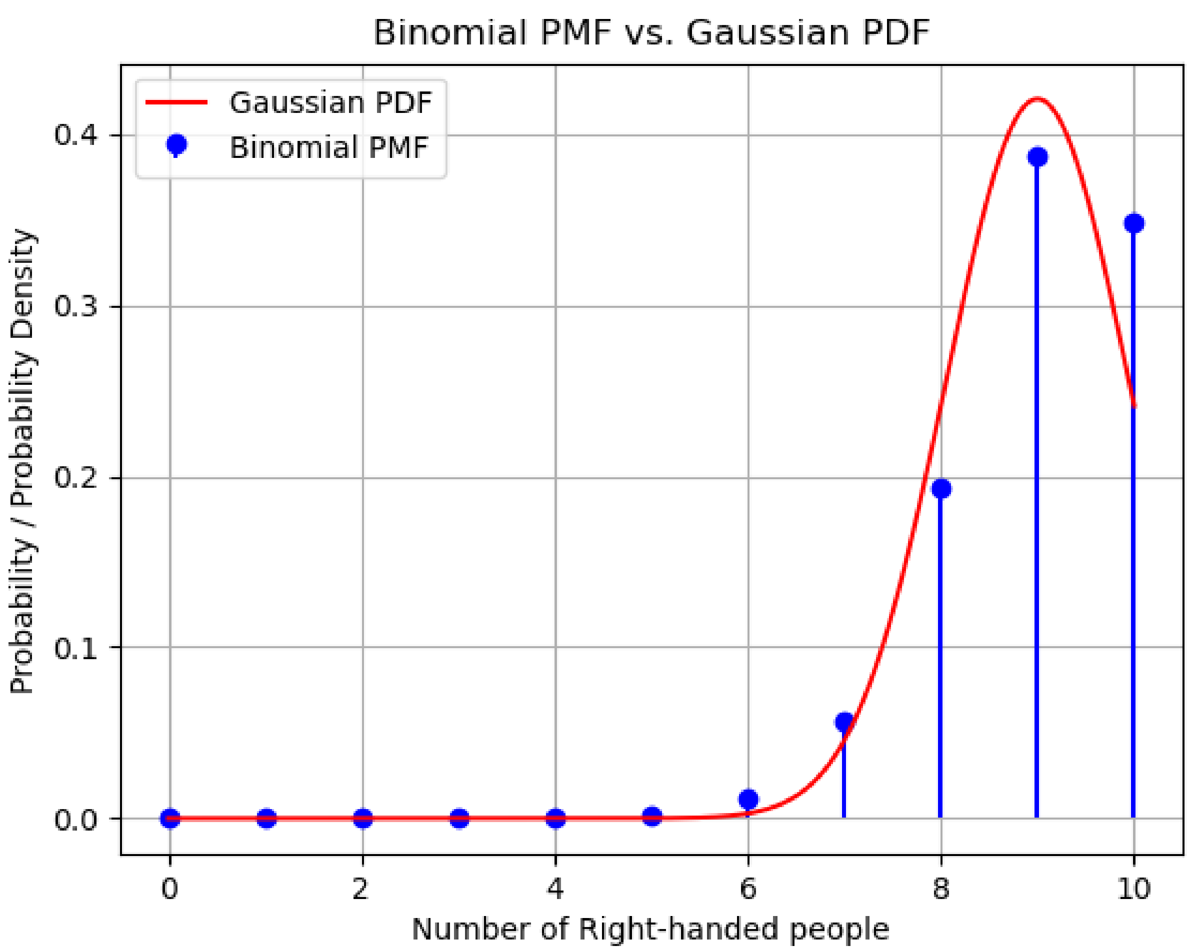

Fig. 1: Binomial vs Gaussian