EE22BTECH11029 - Komakula Sreeja

Question 9.3.16
Suppose that 90% of people are right-handed. What is the probability that atmost 6 of a random sample of 10 people are right-handed.
Solution: Given that 90% of the people are right-handed.

TABLE 1: Description of random variables

Parameters	Values	Description
X	$0 \leq X \leq 10$	Number of right-handed people
n	10	Sample space
p	0.9	Probability that the person is right-handed
μ	9	$n \times p$
σ	$\sqrt{0.9}$	$\sqrt{n \times p \times (1-p)}$

Gaussian Distribution
Central limit theorm:
Let, Z be a random variable

$$\bar{X} \sim \mathcal{N}\left(\mu, \frac{\sigma}{\sqrt{n}}\right) \tag{1}$$

$$Z \approx \frac{X-\mu}{\sigma}, \mathcal{N}(0,1) \tag{2}$$

The Q-function from the Normal-Distribution

$$Q(x) = \Pr(Z > x) \tag{3}$$
$$= 1 - \Pr(Z < x) \tag{4}$$
$$Q(-x) = \Pr(Z > -x) \tag{5}$$
$$= 1 - \Pr(Z > x) \tag{6}$$
$$\tag{7}$$

For:

$$X \leq 6 \tag{8}$$

With a 0.9 correction:

$$
\begin{aligned}
\Pr(X \leq 6) &= \Pr(X < 6.9) && (9)\\
\implies Z &< \frac{6.9-\mu}{\sigma} && (10)\\
Z &< \frac{-2.1}{\sqrt{0.9}} && (11)\\
Z &< -2.21 && (12)\\
\Pr(X \leq 6) &= \Pr(Z < -2.21) && (13)\\
&= 1 - \Pr(Z > -2.21) && (14)\\
&= 1 - 1 + \Pr(Z > 2.21) && (15)\\
&= \Pr(Z > 2.21) && (16)\\
&= 0.013553 && (17)\\
\implies \Pr(X \leq 6) &= 0.013553 && (18)
\end{aligned}
$$

Without correction:

$$
\begin{aligned}
X &\leq 6 && (19)\\
Z &\leq \frac{6-\mu}{\sigma} && (20)\\
Z &\leq \frac{-3}{\sqrt{0.9}} && (21)\\
Z &\leq -3.1622 && (22)\\
\Pr(X \leq 6) &= \Pr(Z \leq -3.1622) && (23)\\
&= 1 - \Pr(Z > -3.1622) && (24)\\
&= 1 - 1 + \Pr(Z > 3.1622) && (25)\\
&= 0.000783 && (26)
\end{aligned}
$$

Binomial Distribution

$$
\begin{aligned}
\Pr(X \leq 6) &= 1 - \Pr(X > 6) && (27)\\
&= 1 - \sum_{k=7}^{10} \binom{n}{k} p^k (1-p)^{n-k} && (28)\\
&= 1 - 0.9872 && (29)\\
&= 0.0128 && (30)
\end{aligned}
$$

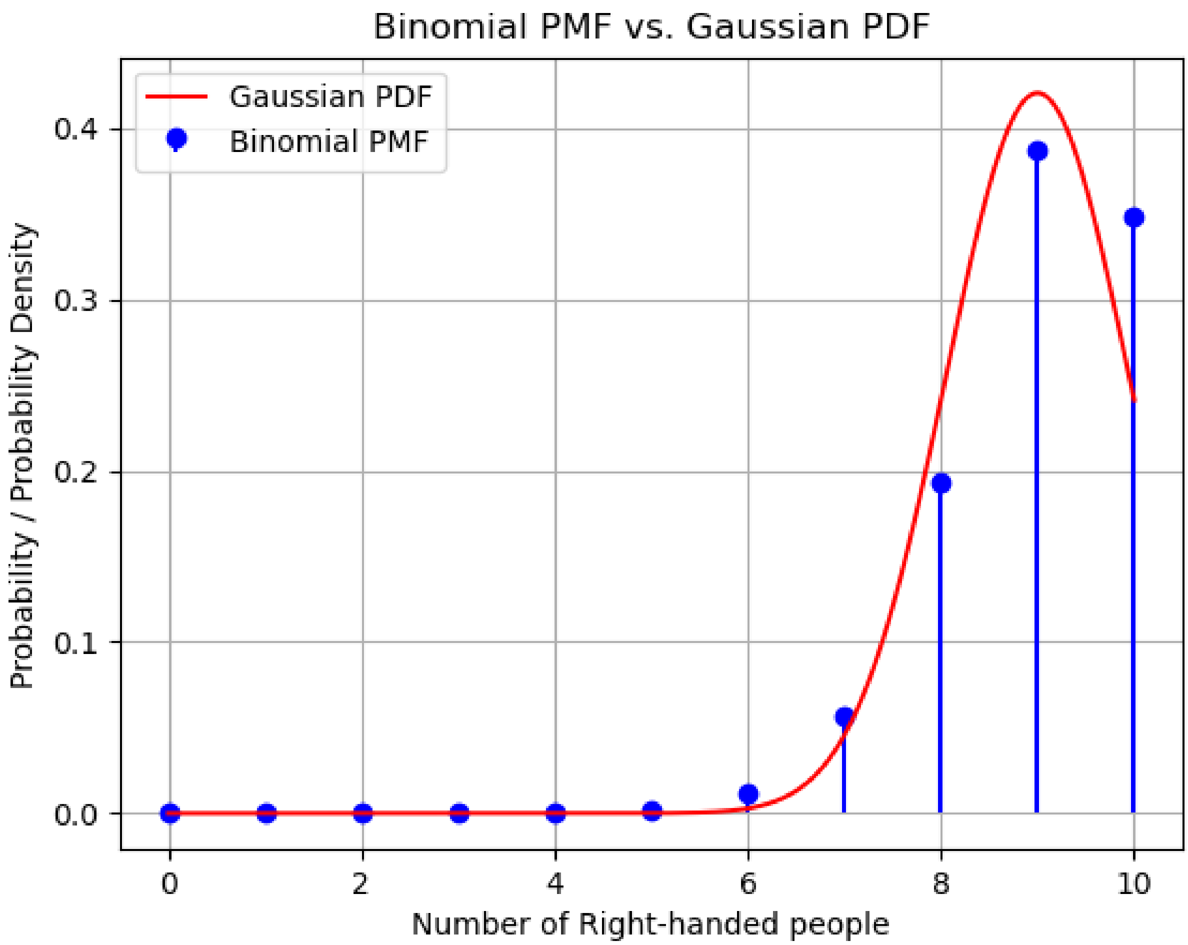

Fig. 1: Binomial vs Gaussian