EE22BTECH11029 - Komakula Sreeja

Question 9.3.16
Suppose that 90% of people are right-handed. What is the probability that atmost 6 of a random sample of 10 people are right-handed.
Solution: Given that 90% of the people are right-handed.

TABLE 1: Description of random variables

Parameters	Values	Description
Y	$0 \le Y \le 10$	Number of right-handed people
n	10	Sample space
p	0.9	Probability that the person is right-handed
μ	9	$n \times p$
σ	$\sqrt{0.9}$	$\sqrt{n \times p \times (1-p)}$

Gaussian Distribution
Central limit theorm:

$$Y \sim \mathcal{N}\left(\mu, \frac{\sigma}{\sqrt{n}}\right) \tag{1}$$

$$X \approx \frac{Y-\mu}{\sigma} \tag{2}$$

The Q-function from the Normal-Distribution

$$Q(x) = \Pr(X > x) \tag{3}$$

$$= 1 - \Pr(X < x) \tag{4}$$

$$Q(-x) = \Pr(X > -x) \tag{5}$$

$$= 1 - \Pr(X > x) \tag{6}$$

CDF of Y:

$$p_Y(y) = \Pr(Y < y) \tag{7}$$

$$= \Pr\left(\frac{Y-\mu}{\sigma} < \frac{y-\mu}{\sigma}\right) \tag{8}$$

$$= \Pr\left(X < \frac{y-\mu}{\sigma}\right) \tag{9}$$

$$= 1 - Q\left(\frac{y-\mu}{\sigma}\right) \tag{10}$$

$$= 1 - Q(x) \tag{11}$$

With a 0.9 correction:

$$\Pr(Y \leq 6) = \Pr(Y < 6.9) \tag{12}$$
$$= 1 - Q\left(\frac{6.9 - 9}{\sqrt{0.9}}\right) \tag{13}$$
$$= 1 - Q\left(\frac{-2.1}{0.9487}\right) \tag{14}$$
$$= 1 - Q(-2.21) \tag{15}$$
$$= Q(2.21) \tag{16}$$
$$= 0.013553 \tag{17}$$

Without correction:

$$\Pr(Y \leq 6) = \Pr(Y < 6) \tag{18}$$
$$= 1 - Q\left(\frac{6 - 9}{\sqrt{0.9}}\right) \tag{19}$$
$$= 1 - Q\left(\frac{-3}{0.9487}\right) \tag{20}$$
$$= 1 - Q(-3.1622) \tag{21}$$
$$= Q(3.1622) \tag{22}$$
$$= 0.000783 \tag{23}$$

Binomial Distribution

$$\Pr(Y \leq 6) = 1 - \Pr(Y > 6) \tag{24}$$
$$= 1 - \sum_{k=7}^{10} \binom{n}{k} p^k (1-p)^{n-k} \tag{25}$$
$$= 1 - 0.9872 \tag{26}$$
$$= 0.0128 \tag{27}$$

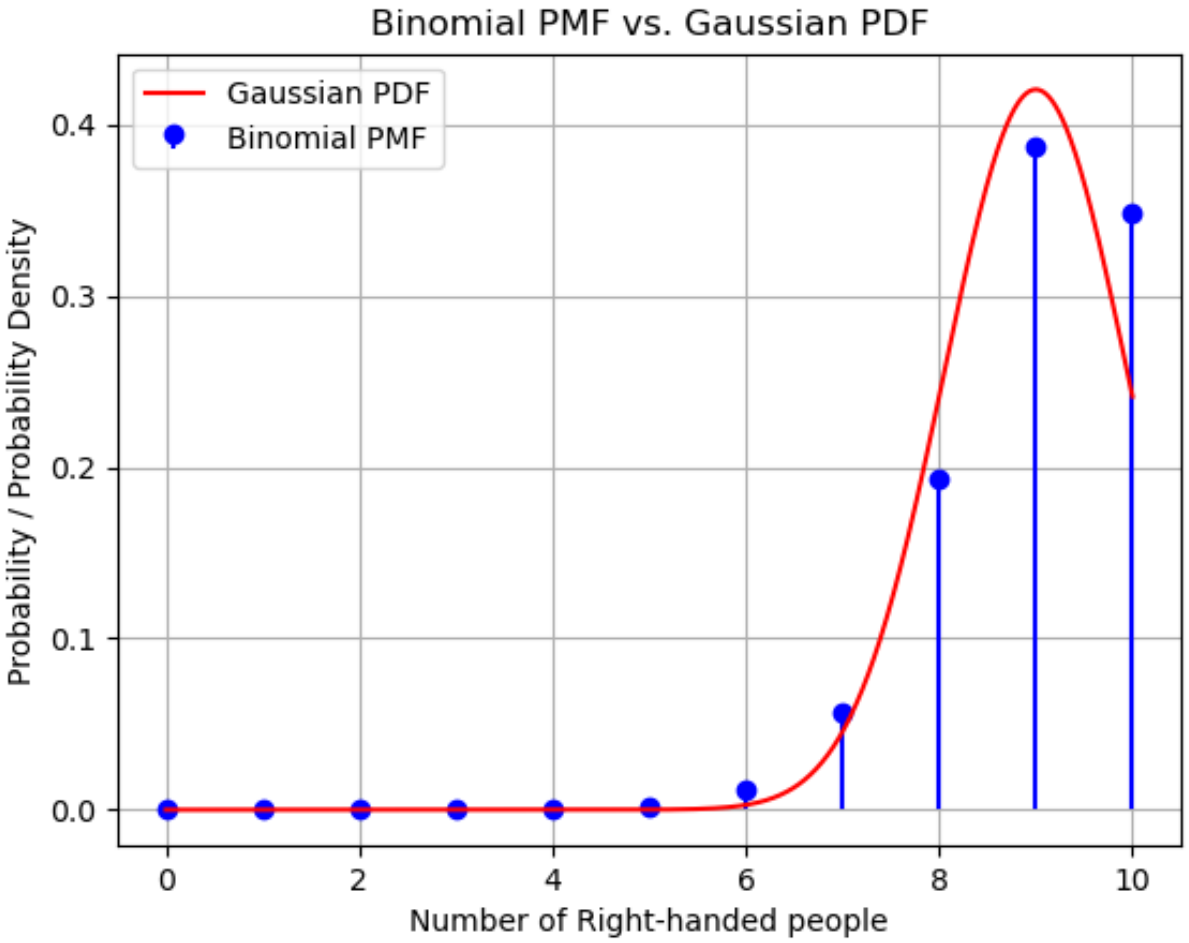

Fig. 1: Binomial vs Gaussian