EE22BTECH11029 - Komakula Sreeja

Question 9.3.16

Suppose that 90% of people are right-handed. What is the probability that atmost 6 of a random sample of 10 people are right-handed.

Solution: Given that 90% of the people are right-handed. Let p and q be probability that people are right-handed and left-handed respectively.

$$p = \frac{9}{10} \tag{1}$$

$$q = 1 - p = \frac{1}{10} \tag{2}$$

Using the gaussian approximation method, the probability that at most 6 people are right-handed is given by:

$$\mu = np = 10 \times \frac{9}{10} = 9 \tag{3}$$

$$\sigma = \sqrt{npq} = \sqrt{10 \times \frac{9}{10} \times \frac{1}{10}} = \sqrt{0.9} \tag{4}$$

Now, we want to find the probability that at most 6 people are right-handed:

$$\Pr(X \leq 6) = 1 - \Pr(X > 6) \tag{5}$$

$$= 1 - \Pr\left(\frac{X - \mu}{\sigma} > \frac{6 - 9}{\sqrt{0.9}}\right) \tag{6}$$

$$= 1 - \Pr\left(Z > -\sqrt{10}\right) \tag{7}$$

$$= 1 - Q(-\sqrt{10}) \tag{8}$$

$$= 1 - \frac{1}{2}\operatorname{erfc}\left(\frac{-\sqrt{10}}{\sqrt{2}}\right). \tag{9}$$

$$= 1 - \frac{1}{2} \times 1.99844 \tag{10}$$

$$= 1 - 0.99922 \tag{11}$$

$$= 0.00078 \tag{12}$$

Therefore, the probability that atmost 6 out of 10 people in the random sample are right-handed is approximately 0.00078, or about 0.078%.