EE22BTECH11029 - Komakula Sreeja

Question 9.3.16
Suppose that 90% of people are right-handed. What is the probability that atmost 6 of a random sample of 10 people are right-handed.
Solution: Given that 90% of the people are right-handed.

TABLE 1: Description of random variables

Parameters	Values	Description
n	10	Sample space
p	0.9	Probability that the person is right-handed
Y	$0 \le Y \le 10$	Number of people thatare right-handed
$\mu = np$	9	Mean
$\sigma = \sqrt{np(1-p)}$	0.9	Standard deviation
$\Pr(Y \le 6) = \Pr(Y < 6)$	0.00078	Gaussian (without correction)
$\Pr(Y \le 6) = \Pr(Y < 6.9)$	0.0136	Gaussian (with correction)
$\Pr(Y \le 6)$	0.0128	Binomial

Gaussian Distribution
Central limit theorm:

$$Y \sim \mathcal{N}\left(\mu, \frac{\sigma}{\sqrt{n}}\right) \tag{1}$$

$$\tag{2}$$

The Q-function from the Normal-Distribution

$$\begin{aligned} Q(y) &= \Pr(Y > y) && (3)\\ &= 1 - \Pr(Y < y) && (4)\\ Q(-y) &= \Pr(Y > -y) && (5)\\ &= 1 - \Pr(Y > y) && (6) \end{aligned}$$

CDF of Y:

$$\begin{aligned} p_Y(y) &= \Pr(Y < y) && (7)\\ &= \Pr\left(\frac{Y-\mu}{\sigma} < \frac{y-\mu}{\sigma}\right) && (8)\\ &= \Pr\left(X < \frac{y-\mu}{\sigma}\right) && (9)\\ &= 1 - Q\left(\frac{y-\mu}{\sigma}\right) && (10) \end{aligned}$$

With a 0.9 correction:

$$\Pr(Y \leq 6) = \Pr(Y < 6.9) \tag{11}$$
$$= 1 - Q\left(\frac{6.9 - 9}{\sqrt{0.9}}\right) \tag{12}$$
$$= 1 - Q\left(\frac{-2.1}{0.9487}\right) \tag{13}$$
$$= 1 - Q(-2.21) \tag{14}$$
$$= Q(2.21) \tag{15}$$
$$= 0.013553 \tag{16}$$

Without correction:

$$\Pr(Y \leq 6) = \Pr(Y < 6) \tag{17}$$
$$= 1 - Q\left(\frac{6 - 9}{\sqrt{0.9}}\right) \tag{18}$$
$$= 1 - Q\left(\frac{-3}{0.9487}\right) \tag{19}$$
$$= 1 - Q(-3.1622) \tag{20}$$
$$= Q(3.1622) \tag{21}$$
$$= 0.000783 \tag{22}$$

Binomial Distribution

$$\Pr(Y \leq 6) = 1 - \Pr(Y > 6) \tag{23}$$
$$= 1 - \sum_{k=7}^{10} \binom{n}{k} p^k (1-p)^{n-k} \tag{24}$$
$$= 1 - 0.9872 \tag{25}$$
$$= 0.0128 \tag{26}$$

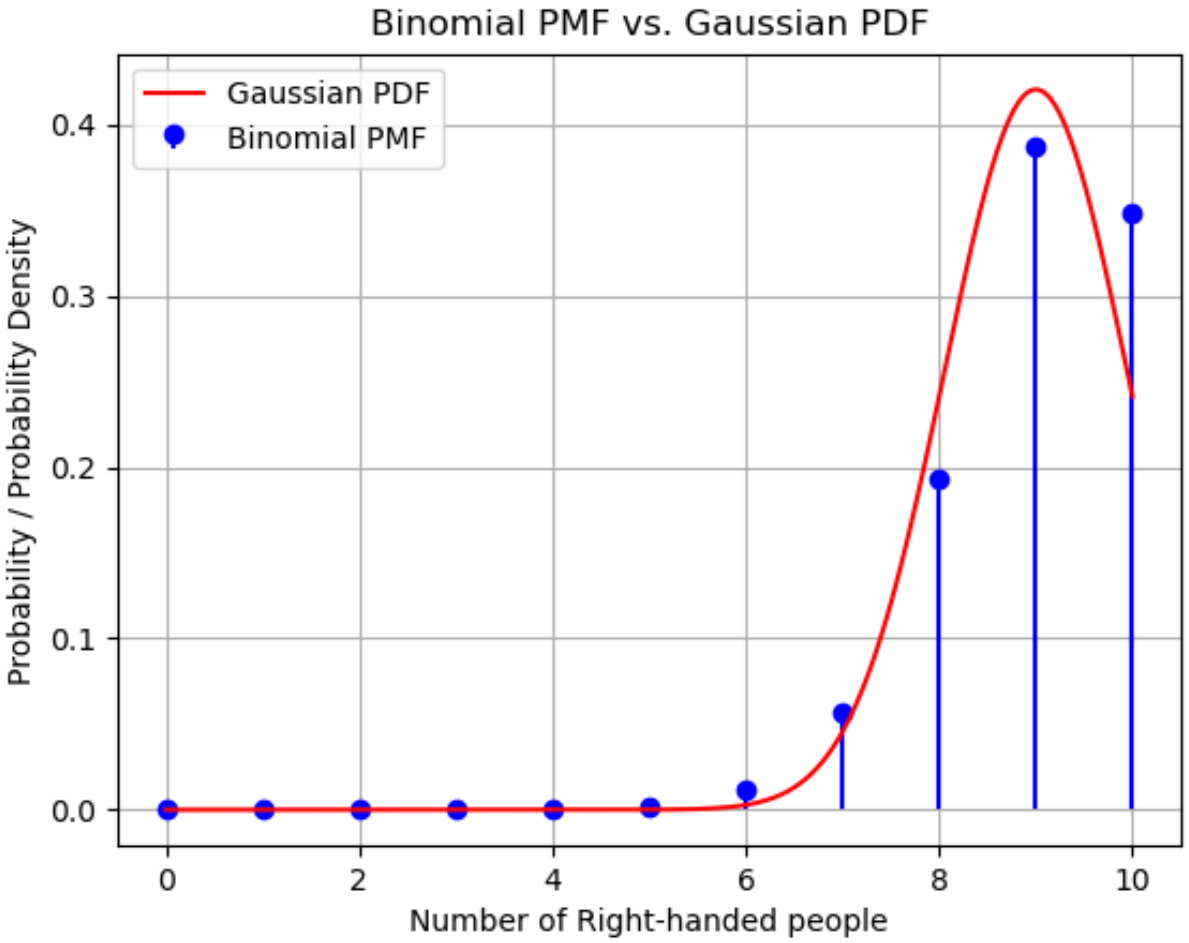

Fig. 1: Binomial vs Gaussian