EE22BTECH11029 - Komakula Sreeja

Question 11.16.3.20

While shuffling a pack of 52 playing cards, 2 cards are dropped. Find the probabilty that the missing cards to be of different colours.

Solution: We know that the 52 playing cards contain 26 red cards and 26 black cards.

Let X_1 and X_2 be two random variables denoting the colour of first card and second card respectively:

$$X_i = \begin{cases} 0, & \text{red card} \\ 1, & \text{black card} \end{cases} \quad \{i = 1, 2\} \tag{1}$$

Probability of choosing the first card:

$$p_{X_1}(k) = \frac{26}{52} \quad \{k = 0, 1\} \tag{2}$$

$$= \frac{1}{2} \tag{3}$$

Probability that second card choosen is red after selecting the first card as red

$$= p(X_2 = 0|X_1 = 0) = \frac{25}{51} \times \frac{26}{52} \div \frac{26}{52} = \frac{25}{51} \tag{4}$$

Probability that second card choosen is black after selecting the first card as red

$$= p(X_2 = 1|X_1 = 0) = \frac{26}{51} \times \frac{26}{52} \div \frac{26}{52} = \frac{26}{51} \tag{5}$$

Probability that second card choosen is red after selecting the first card as black

$$= p(X_2 = 0|X_1 = 1) = \frac{26}{51} \times \frac{26}{52} \div \frac{26}{52} = \frac{26}{51} \tag{6}$$

Probability that second card choosen is black after selecting the first card as black

$$= p(X_2 = 1|X_1 = 1) = \frac{25}{51} \times \frac{26}{52} \div \frac{26}{52} = \frac{25}{51} \tag{7}$$

Probabilty that both cards have different colour:

$$pr(X_1 \neq X_2) = p_{X_1}(0)\, p(X_2 = 1|X_1 = 0) + p_{X_1}(1)\, p(X_2 = 0|X_1 = 1) \tag{8}$$

$$= \frac{26}{52} \times \frac{26}{51} + \frac{26}{52} \times \frac{26}{51} \tag{9}$$

$$= 2 \times \frac{26}{52} \times \frac{13}{51} \tag{10}$$

$$= \frac{26}{51} \tag{11}$$

TABLE 1: Description of random variables

Random Variable	Values	Description
X_1	0	First card is red
	1	First card is black
X_2	0	Second card is red
	1	Second card is black