

EE22BTECH11029 - Komakula Sreeja

Question 11.16.3.20

While shuffling a pack of 52 playing cards, 2 cards are dropped. Find the probability that the missing cards to be of different colours.

Solution: We know that the 52 playing cards contain 26 red cards and 26 black cards.

Let X_1 be a random variable denoting the colour of first card:

$$X_1 = \begin{cases} 0, & \text{red card} \\ 1, & \text{black card} \end{cases} \quad (1)$$

Probability of choosing the first card:

$$p_{X_1}(k) = \frac{26}{52} \quad \{k = 0, 1\} \quad (2)$$

$$= \frac{1}{2} \quad (3)$$

Let X_2 be a random variable denoting the colour of second card:

$$X_2 = \begin{cases} 0, & \text{red card} \\ 1, & \text{black card} \end{cases} \quad (4)$$

Using conditional probability:

$$p(X_2 = 0|X_1 = 0) = \left(\frac{25}{51}\right)\left(\frac{26}{52}\right) \quad (5)$$

$$= \frac{25}{102} \quad (6)$$

$$p(X_2 = 1|X_1 = 0) = \left(\frac{26}{51}\right)\left(\frac{26}{52}\right) \quad (7)$$

$$= \frac{13}{51} \quad (8)$$

$$p(X_2 = 0|X_1 = 1) = \left(\frac{26}{51}\right)\left(\frac{26}{52}\right) \quad (9)$$

$$= \frac{13}{51} \quad (10)$$

$$p(X_2 = 1|X_1 = 1) = \left(\frac{25}{51}\right)\left(\frac{26}{52}\right) \quad (11)$$

$$= \frac{25}{102} \quad (12)$$

Probability that both cards have different colour:

$$p(\text{different colours}) = p(X_2 = 0|X_1 = 1) + p(X_2 = 1|X_1 = 0) \quad (13)$$

$$= \frac{13}{51} + \frac{13}{51} \quad (14)$$

$$= 2\left(\frac{13}{51}\right) \quad (15)$$

$$= \frac{26}{51} \quad (16)$$

TABLE 1: Description of random variables

Random Variable	Values	Description
X_1	0	First card is red
	1	First card is black
X_2	0	Second card is red
	1	Second card is black