

EE22BTECH11029 - Komakula Sreeja

Question 12.13.3.44

A letter is known to have come either from TATANAGAR or from CALCUTTA. On the envelope, just two consecutive letter TA are visible. What is the probability that the letter came from TATANAGAR.

Solution: Let X_1 and X_2 be two random variables choosing letters T and A respectively:

$$X_i = \begin{cases} 0, & \text{Choosing from TATANAGAR} \\ 1, & \text{Choosing from CALCUTTA} \end{cases} \quad \{i = 1, 2\} \quad (1)$$

Probability of choosing T:

$$p_{X_1}(k) = \frac{1}{2} \quad \{k = 0, 1\} \quad (2)$$

Probability that A is choosen from TATANAGAR after selecting T from TATANAGAR:

$$= p(X_2 = 0|X_1 = 0) = \frac{2}{8} \times \frac{2}{9} \div \frac{2}{9} = \frac{1}{4} \quad (3)$$

Probability that A is choosen from TATANAGAR after selecting T from CALCUTTA:

$$= p(X_2 = 0|X_1 = 1) = 0 \quad (4)$$

Probability that A is choosen from CALCUTTA after selecting T from TATANAGAR :

$$= p(X_2 = 1|X_1 = 0) = 0 \quad (5)$$

Probability that A is choosen from CALCUTTA after selecting T from CALCUTTA:

$$= p(X_2 = 1|X_1 = 1) = \frac{1}{7} \times \frac{2}{8} \div \frac{2}{8} = \frac{1}{7} \quad (6)$$

By using bayes theorem, we get the probability of getting two consecutive letters TA

$$= p_{X_1}(0) p(X_2 = 0|X_1 = 0) \div \{p_{X_1}(0) \{p(X_2|X_1 = 0) + p_{X_1}(1) p(X_2|X_1 = 1)\} \quad (7)$$

$$= \frac{1}{2} \times \frac{1}{4} \div \left\{ \frac{1}{2} \times \frac{1}{4} + 0 + 0 + \frac{1}{2} \times \frac{1}{7} \right\} \quad (8)$$

$$= \frac{1}{8} \div \frac{22}{112} \quad (9)$$

$$= \frac{7}{11} \quad (10)$$

TABLE 1: Description of random variables

Random Variable	Values	Description
X_1	0	T is choosen from TATANAGAR
	1	T is choosen from CALCUTTA
X_2	0	A is choosen from TATANAGAR
	1	A is choosen from CALCUTTA