EE22BTECH11029 - Komakula Sreeja

Question 12.13.3.44

A letter is known to have come either from TATANAGAR or from CALCUTTA. On the envelope, just two consecutive letter TA are visible. What is the probability that the letter came from TATANAGAR.

Solution: Let X be two random variables choosing letters T and A respectively:

$$X = \begin{cases} 0, & \text{Choosing TA from TATANAGAR} \\ 1, & \text{Choosing TA from CALCUTTA} \end{cases} \tag{1}$$

Probability of choosing TA from either TATANAGAR or CALCUTTA:

$$p_X(k) = \frac{1}{2} \quad \{k = 0, 1\} \tag{2}$$

Let E_1 be an event of getting letters from TATANAGAR: Probability of choosing letters TA from TATANAGAR

$$= p(E_1) = \frac{2}{8} = \frac{1}{4} \tag{3}$$

Let E_2 be an event of getting letters from CALCUTTA: Probability of choosing letters TA from CALCUTTA

$$= p(E_2) = \frac{1}{7} \tag{4}$$

By using bayes theorem, we get the probability of getting two consecutive letters TA:

$$= p_X(0)\, p(E_1) \div \{p_X(0)\, p(E_1) + p_X(1)\, p(E_2)\} \tag{5}$$

$$= \frac{1}{2} \times \frac{1}{4} \div \{\frac{1}{2} \times \frac{1}{4} + \frac{1}{2} \times \frac{1}{7}\} \tag{6}$$

$$= \frac{1}{8} \div \frac{22}{112} \tag{7}$$

$$= \frac{7}{11} \tag{8}$$

TABLE 1: Description of random variables

Random Variable	Values	Description
X	0	TA is choosen from TATANAGAR
	1	TA is choosen from CALCUTTA