

EE22BTECH11029 - Komakula Sreeja

Question 12.13.3.44

A letter is known to have come either from TATANAGAR or from CALCUTTA. On the envelope, just two consecutive letter TA are visible. What is the probability that the letter came from TATANAGAR.

Solution: We know that the 52 playing cards contain 26 red cards and 26 black cards.

Let X_1 and X_2 be two random variables denoting the city and TA respectively:

$$X_1 = \begin{cases} 0, & \text{TATANAGAR} \\ 1, & \text{CALCUTTA} \end{cases} \quad (1)$$

$$X_2 = \begin{cases} 0, & \text{Choosing TA from TATANAGAR} \\ 1, & \text{Choosing TA from CALCUTTA} \end{cases} \quad (2)$$

Probability of choosing the city:

$$p_{X_1}(k) = \frac{1}{2} \quad \{k = 0, 1\} \quad (3)$$

Probability of choosing consecutive letters TA from TATANAGAR:

$$= p(X_2 = 0 | X_1 = 0) = \frac{1}{2} \times \frac{2}{8} \div \frac{1}{2} = \frac{1}{4} \quad (4)$$

Probability of choosing consecutive letters TA from CALCUTTA:

$$= p(X_2 = 1 | X_1 = 1) = \frac{1}{2} \times \frac{1}{7} \div \frac{1}{2} = \frac{1}{7} \quad (5)$$

By using bayes theorem, we get the probability of getting two consecutive letters TA from TATANAGAR:

$$= \{p_{X_1}(0) p(X_2 = 0 | X_1 = 0)\} \div \{p_{X_1}(0) p(X_2 = 0 | X_1 = 0) + p_{X_1}(1) p(X_2 = 1 | X_1 = 1)\} \quad (6)$$

$$= \frac{1}{2} \times \frac{1}{4} \div \left\{ \frac{1}{2} \times \frac{1}{4} + \frac{1}{2} \times \frac{1}{7} \right\} \quad (7)$$

$$= \frac{1}{8} \div \frac{22}{112} \quad (8)$$

$$= \frac{7}{11} \quad (9)$$

TABLE 1: Description of random variables

Random Variable	Values	Description
X_1	0	TATANAGAR
	1	CALCUTTA
X_2	0	TA is choosen from TATANAGAR
	1	TA is choosen from CALCUTTA