

Consider a triangle with vertices

$$\mathbf{A} = \begin{pmatrix} -1 \\ -3 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} -4 \\ 2 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} -6 \\ 3 \end{pmatrix} \quad (1)$$

TABLE 1: Vectors.

Parameters	Values	Description
\mathbf{m}_1	$\begin{pmatrix} -3 \\ 5 \end{pmatrix}$	AB
\mathbf{m}_2	$\begin{pmatrix} -2 \\ 1 \end{pmatrix}$	BC
\mathbf{m}_3	$\begin{pmatrix} 5 \\ -6 \end{pmatrix}$	CA
$\ A - B\ $	5.83	Length of AB
$\ B - C\ $	2.24	Length of BC
$\ C - A\ $	7.81	Length of CA
$\text{rank}\begin{pmatrix} 1 & 1 & 1 \\ \mathbf{A} & \mathbf{B} & \mathbf{C} \end{pmatrix}$	3	Non-collinear
\mathbf{n}_1	$\begin{pmatrix} 5 \\ 3 \end{pmatrix}$	AB
c_1	-14	
\mathbf{n}_2	$\begin{pmatrix} 1 \\ 2 \end{pmatrix}$	BC
c_2	0	
\mathbf{n}_3	$\begin{pmatrix} -6 \\ -5 \end{pmatrix}$	CA
c_3	21	
Area	3.5	Area of triangle
$\angle A$	8.84°	Angles
$\angle B$	147.53°	
$\angle C$	23.63°	

TABLE 2: Median.

Parameters	Values	Description
\mathbf{D}	$\begin{pmatrix} -5 \\ 2.5 \end{pmatrix}$	Midpoint of BC
\mathbf{E}	$\begin{pmatrix} -3.5 \\ 0 \end{pmatrix}$	Midpoint of CA
\mathbf{F}	$\begin{pmatrix} -2.5 \\ -0.5 \end{pmatrix}$	Midpoint of AB
\mathbf{m}_4	$\begin{pmatrix} -4 \\ 5.5 \end{pmatrix}$	AD
\mathbf{n}_4	$\begin{pmatrix} 5.5 \\ 4 \end{pmatrix}$	
c_4	-17.5	
\mathbf{m}_5	$\begin{pmatrix} 0.5 \\ -2 \end{pmatrix}$	BE
\mathbf{n}_5	$\begin{pmatrix} -2 \\ -0.5 \end{pmatrix}$	
c_5	7	
\mathbf{m}_6	$\begin{pmatrix} 3.5 \\ -3.5 \end{pmatrix}$	CF
\mathbf{n}_6	$\begin{pmatrix} -3.5 \\ -3.5 \end{pmatrix}$	
c_6	10.5	
\mathbf{G}	$\begin{pmatrix} -3.67 \\ 0.67 \end{pmatrix}$	Centroid
$\frac{BG}{GE}$	2	Division ratio by \mathbf{G}
$\frac{CG}{GF}$		
$\frac{AG}{GD}$		
$\text{rank}\begin{pmatrix} 1 & 1 & 1 \\ \mathbf{A} & \mathbf{D} & \mathbf{G} \end{pmatrix}$	2	Collinear
$\text{rank}\begin{pmatrix} 1 & 1 & 1 \\ \mathbf{B} & \mathbf{E} & \mathbf{G} \end{pmatrix}$		
$\text{rank}\begin{pmatrix} 1 & 1 & 1 \\ \mathbf{C} & \mathbf{F} & \mathbf{G} \end{pmatrix}$		

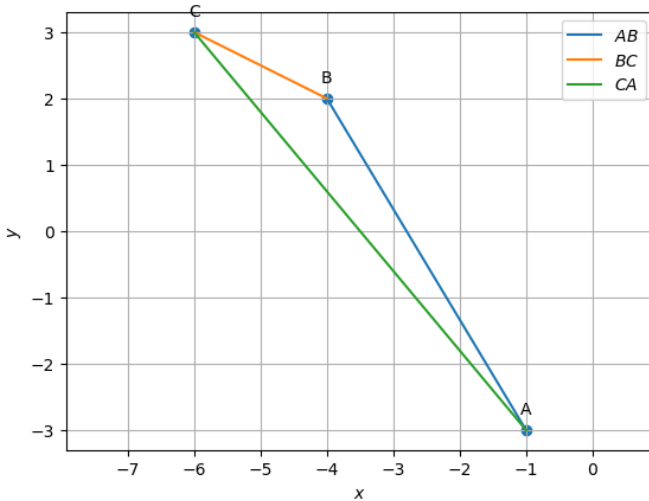


Fig. 1: Triangle plotted using python

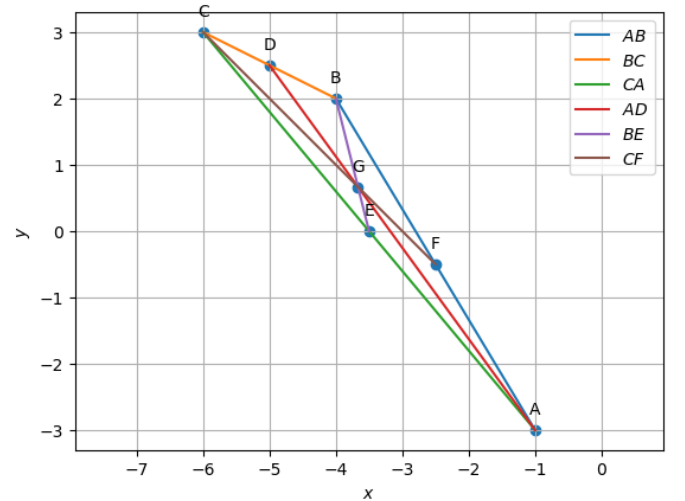


Fig. 2: Medians plotted using python

TABLE 3: Altitude.

Parameters	Values	Description
D_1	$\begin{pmatrix} 0.4 \\ -0.2 \end{pmatrix}$	Foot of altitude from A
E_1	$\begin{pmatrix} -4.69 \\ 1.43 \end{pmatrix}$	Foot of altitude from B
F_1	$\begin{pmatrix} -4.97 \\ 3.62 \end{pmatrix}$	Foot of altitude from C
m_7	$\begin{pmatrix} 1.4 \\ 2.8 \end{pmatrix}$	AD_1
n_7	$\begin{pmatrix} 2.8 \\ -1.4 \end{pmatrix}$	
c_7	1.4	
m_8	$(-0.69 // -0.57)$	BE_1
n_8	$\begin{pmatrix} -0.57 \\ 0.69 \end{pmatrix}$	
c_8	3.67	
m_9	$\begin{pmatrix} 1.03 \\ 0.62 \end{pmatrix}$	CF_1
n_9	$\begin{pmatrix} 0.62 \\ 1.03 \end{pmatrix}$	
c_9	-6.8	
H	$\begin{pmatrix} 5.43 \\ 9.86 \end{pmatrix}$	Orthocentre

TABLE 4: Perpendicular Bisector.

Parameters	Values	Description
m_{10}	$\begin{pmatrix} 1 \\ 2 \end{pmatrix}$	AD_2
n_{10}	$\begin{pmatrix} -2 \\ 1 \end{pmatrix}$	
c_{10}	12.5	
m_{11}	$\begin{pmatrix} 6 \\ 5 \end{pmatrix}$	BE_2
n_{11}	$\begin{pmatrix} -5 \\ 6 \end{pmatrix}$	
c_{11}	17.5	
m_{12}	$\begin{pmatrix} -5 \\ -3 \end{pmatrix}$	CF_2
n_{12}	$\begin{pmatrix} 3 \\ -5 \end{pmatrix}$	
c_{12}	-5	
O	$\begin{pmatrix} -8.21 \\ -3.93 \end{pmatrix}$	Circumcentre
$\ O - A\ $	7.27	$OA = OB = OC = R$
$\ O - B\ $	7.27	
$\ O - C\ $	7.27	
R	7.27	
$\angle BOC$	17.68°	$\angle BOC = 2\angle BAC$
$\angle BAC$	8.84°	
$\angle AOC$	64.94°	$\angle AOC = 2\angle ABC$
$\angle ABC$	147.53°	
$\angle AOB$	312.74°	$\angle AOB = 2\angle BCA$
$\angle BCA$	23.63°	

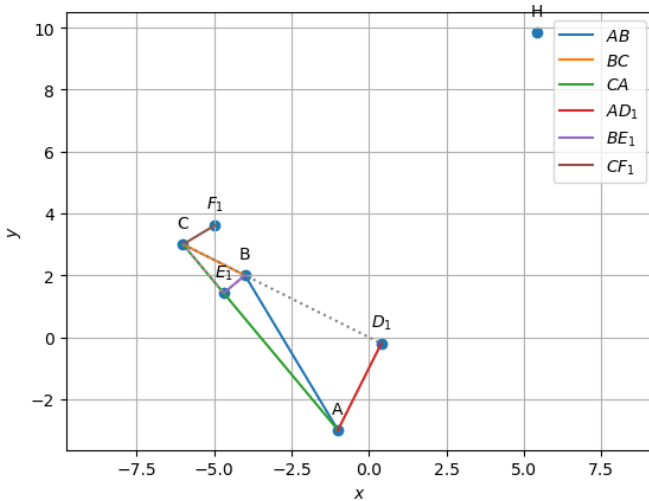


Fig. 3: Altitudes plotted using python

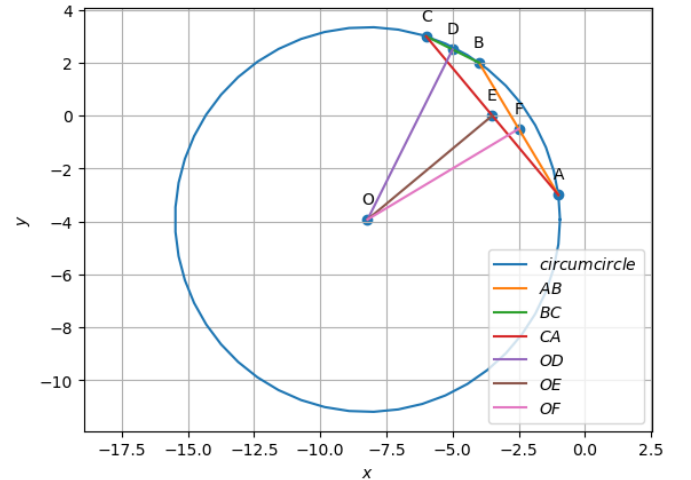


Fig. 4: Perpendicular bisectors plotted using python

TABLE 5: Angular Bisector.

Parameters	Values	Description
\mathbf{m}_{13}	$\begin{pmatrix} 1.15 \\ -1.63 \end{pmatrix}$	AI
\mathbf{n}_{13}	$\begin{pmatrix} -1.63 \\ -1.15 \end{pmatrix}$	
c_{13}	5.09	
\mathbf{m}_{14}	$\begin{pmatrix} -0.38 \\ -0.41 \end{pmatrix}$	BI
\mathbf{n}_{14}	$\begin{pmatrix} 0.41 \\ -0.38 \end{pmatrix}$	
c_{14}	-2.40	
\mathbf{m}_{15}	$\begin{pmatrix} -1.53 \\ 1.22 \end{pmatrix}$	CI
\mathbf{n}_{15}	$\begin{pmatrix} -1.22 \\ -1.53 \end{pmatrix}$	
c_{15}	2.69	
\mathbf{I}	$\begin{pmatrix} -4.31 \\ 1.66 \end{pmatrix}$	Incentre
\mathbf{D}_3	$\begin{pmatrix} -4.11 \\ 2.06 \end{pmatrix}$	Point of contact with BC
\mathbf{E}_3	$\begin{pmatrix} -4.65 \\ 1.38 \end{pmatrix}$	Point of contact with AC
\mathbf{F}_3	$\begin{pmatrix} -3.93 \\ 1.89 \end{pmatrix}$	Point of contact with AB
$\ \mathbf{I} - \mathbf{D}_3\ $	0.44	$ID_3 = IE_3 = IF_3 = r$
$\ \mathbf{I} - \mathbf{E}_3\ $	0.44	
$\ \mathbf{I} - \mathbf{F}_3\ $	0.44	
r	0.44	
$\angle BAI$	4.42°	$\angle BAI = \angle CAI$
$\angle CAI$	4.42°	
$\angle ABI$	73.76°	$\angle ABI = \angle CBI$
$\angle CBI$	73.76°	
$\angle ACI$	11.81°	$\angle ACI = \angle BCI$
$\angle BCI$	11.81°	

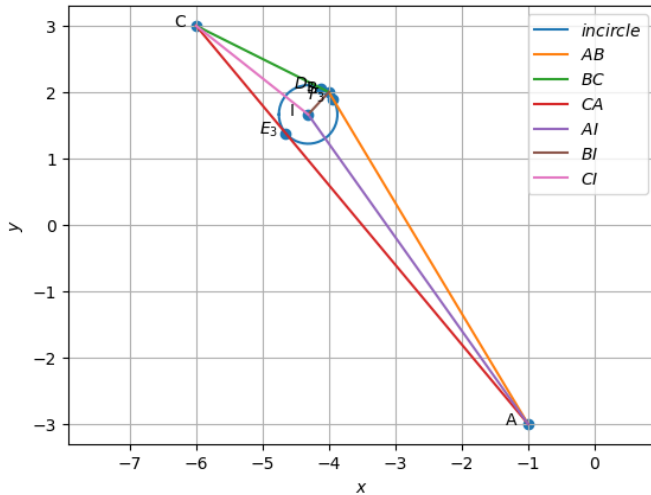


Fig. 5: Angle bisectors plotted using python