Equation Mapping for All 18 State Outputs

Comprehensive Equation Mapping

This document maps each of the 18 state outputs to the specific physics equation used to compute it.

Rotational Dynamics Outputs (6 variables)

#	Output	Equation Used	Location
1	$\dot{p}$ (roll acceleration)	$\dot{p} = \frac{J_{yy}-J_{zz}}{J_{xx}} \cdot q \cdot r + \frac{\tau_x}{J_{xx}}$	Data gen: line 243 PINN: line 260
2	$\dot{q}$ (pitch acceleration)	$\dot{q} = \frac{J_{zz}-J_{xx}}{J_{yy}} \cdot p \cdot r + \frac{\tau_y}{J_{yy}}$	Data gen: line 244 PINN: line 261
3	$\dot{r}$ (yaw acceleration)	$\dot{r} = \frac{J_{xx}-J_{yy}}{J_{zz}} \cdot p \cdot q + \frac{\tau_z}{J_{zz}}$	Data gen: line 245 PINN: line 262
4	p (roll rate)	$p_{t+dt} = p_t + \dot{p} \cdot dt$ (integration)	Data gen: line 247 PINN: line 264
5	q (pitch rate)	$q_{t+dt} = q_t + \dot{q} \cdot dt$ (integration)	Data gen: line 248 PINN: line 265
6	r (yaw rate)	$r_{t+dt} = r_t + \dot{r} \cdot dt$ (integration)	Data gen: line 249 PINN: line 266

Euler Angle Kinematics Outputs (6 variables)

#	Output	Equation Used	Location
7	$\dot{\phi}$ (roll angle rate)	Data gen: $\dot{\phi} = p + \sin(\phi)\tan(\theta) \cdot q + \cos(\phi)\tan(\theta) \cdot r$ PINN: $\dot{\phi} = p$ (simplified)	Data gen: line 251 PINN: line 269
8	$\dot{\theta}$ (pitch angle rate)	Data gen: $\dot{\theta} = \cos(\phi) \cdot q - \sin(\phi) \cdot r$ PINN: $\dot{\theta} = q$ (simplified)	Data gen: line 252 PINN: line 270
9	$\dot{\psi}$ (yaw angle rate)	Data gen: $\dot{\psi} = \frac{\sin(\phi) \cdot q + \cos(\phi) \cdot r}{\cos(\theta)}$ PINN: $\dot{\psi} = r$ (simplified)	Data gen: line 253 PINN: line 271
10	ϕ (roll angle)	$\phi_{t+dt} = \phi_t + \dot{\phi} \cdot dt$ (integration)	Data gen: line 255 PINN: line 269
11	θ (pitch angle)	$\theta_{t+dt} = \theta_t + \dot{\theta} \cdot dt$ (integration)	Data gen: line 256 PINN: line 270
12	ψ (yaw angle)	$\psi_{t+dt} = \psi_t + \dot{\psi} \cdot dt$ (integration)	Data gen: line 257 PINN: line 271

Note: Red equations show data generation (full nonlinear). Blue equations show PINN physics loss (simplified for small angles).

Translational Dynamics Outputs (6 variables)

#	Output	Equation Used	Location
13	$\dot{u}$ (x-velocity acceleration)	Data gen: $\dot{u} = r \cdot v - q \cdot w + \frac{f_x}{m} - g\sin(\theta) - c_d \cdot u \cdot \lvert u \rvert$ PINN: NOT USED	Data gen: line 272 PINN: N/A
14	$\dot{v}$ (y-velocity acceleration)	Data gen: $\dot{v} = p \cdot w - r \cdot u + \frac{f_y}{m} + g\cos(\theta)\sin(\phi) - c_d \cdot v \cdot \lvert v \rvert$ PINN: NOT USED	Data gen: line 273 PINN: N/A
15	$\dot{w}$ (z-velocity acceleration)	Data gen: $\dot{w} = q \cdot u - p \cdot v + \frac{f_z}{m} + g\cos(\theta)\cos(\phi) - c_d \cdot w \cdot \lvert w \rvert$ PINN: $\dot{v}_z = -g + \frac{T}{m \cdot \cos(\theta) \cdot \cos(\phi)}$ (NO drag)	Data gen: line 274 PINN: line 252
16	u (x-velocity)	$u_{t+dt} = u_t + \dot{u} \cdot dt$ (integration)	Data gen: line 276 PINN: N/A
17	v (y-velocity)	$v_{t+dt} = v_t + \dot{v} \cdot dt$ (integration)	Data gen: line 277 PINN: N/A
18	w (z-velocity, v_z)	$w_{t+dt} = w_t + \dot{w} \cdot dt$ (integration)	Data gen: line 278 PINN: line 253

Note: PINN only models vertical (z-axis) translational dynamics, not x/y dynamics.

Additional Position Outputs (NOT in 18 core states)

Output	Equation Used	Why Not in PINN
$\dot{x}$ (x-position rate)	Body-to-world rotation matrix transformation: $\dot{x} = \cos(\psi)\cos(\theta) \cdot u + [\cos(\psi)\sin(\theta)\sin(\phi) - \sin(\psi)\cos(\phi)] \cdot v + [\sin(\psi)\sin(\phi) + \cos(\psi)\sin(\theta)\cos(\phi)] \cdot w$	PINN focuses on body-frame dynamics, not world-frame position
$\dot{y}$ (y-position rate)	Body-to-world rotation matrix transformation: $\dot{y} = \sin(\psi)\cos(\theta) \cdot u + [\cos(\psi)\cos(\phi) + \sin(\psi)\sin(\theta)\sin(\phi)] \cdot v + [\sin(\psi)\sin(\theta)\cos(\phi) - \cos(\psi)\sin(\phi)] \cdot w$	PINN focuses on body-frame dynamics, not world-frame position
$\dot{z}$ (z-position rate)	$\dot{z} = -[\sin(\theta) \cdot u - \cos(\theta)\sin(\phi) \cdot v - \cos(\theta)\cos(\phi) \cdot w]$	Already captured through $v_z = w$ in body frame
x, y, z (positions)	Integration of $\dot{x}, \dot{y}, \dot{z}$	Position tracking not required for dynamics learning

Summary of Discrepancies

Variable	Data Generation	PINN Physics Loss
Euler angle rates $(\dot{\phi}, \dot{\theta}, \dot{\psi})$	Full nonlinear Euler kinematics with trigonometric coupling	Simplified: $\dot{\phi} = p$, $\dot{\theta} = q$, $\dot{\psi} = r$ (small angle approximation)

Variable	Data Generation	PINN Physics Loss
Vertical acceleration ($\dot{w}$)	Includes quadratic aerodynamic drag: $-c_d{\cdot}w{\cdot}\|w\|$ where $c_d = 0.05$ kg/m	NO drag term: $\dot{v}_z = -g + \frac{T}{m\cdot\cos\theta\cdot\cos\phi}$
Horizontal dynamics ($\dot{u}, \dot{v}$)	Full 6DOF translational equations with Coriolis terms and drag	NOT MODELED in PINN
Position (x, y, z)	Computed via body-to-world transformation	NOT MODELED in PINN (only altitude z tracked)