Modular Physics-Informed Neural Networks for Stable Autoregressive Quadrotor Dynamics Prediction

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Abstract—**Physics-Informed Neural Networks (PINNs) embed physical laws into neural network training for dynamics learning. While PINNs achieve excellent single-step prediction accuracy, they suffer error accumulation during autoregressive rollout—the deployment regime required for model predictive control. We present a *modular PINN architecture* that separates translational and rotational dynamics into specialized subnetworks, achieving 4.6$\times$ improvement in 100-step stability (1.11m vs 5.09m MAE) while using 65% fewer parameters (72K vs 205K). On 6-DOF quadrotor dynamics, our modular approach outperforms both monolithic baselines and curriculum-based training methods. Surprisingly, physically decoupling the translation and rotation networks provides beneficial inductive bias that improves long-horizon prediction despite the strong physical coupling between these subsystems via $\ddot{z} = -T\cos\theta\cos\phi/m + g$. We provide complete architectural specifications and comparative experiments demonstrating that physics-informed architectural design is more effective than training-based approaches for autoregressive stability.**

Index Terms—**Physics-informed neural networks, curriculum learning, autoregressive prediction, robot dynamics, model predictive control.**

I. Introduction

PHYSICS-INFORMED Neural Networks (PINNs) [1] have emerged as a powerful paradigm for learning robot dynamics by embedding physical laws directly into neural network training. For model predictive control (MPC), learned dynamics models must perform stable *autoregressive rollout*: predictions recursively feed as inputs over horizons of 50–100+ steps. However, standard PINN training optimizes single-step accuracy, creating a fundamental mismatch with deployment requirements.

This mismatch causes performance degradation. A baseline PINN achieving 0.079m single-step error can accumulate 5.09m error over 100 steps—a 64$\times$ amplification. However, we show that modular architectures can dramatically reduce this error accumulation.

Core Contribution. We present *Curriculum Stability Training*, a training methodology that transforms unstable PINNs into stable dynamics predictors. Our approach achieves:

- **4.6$\times$ stability improvement**: 100-step MAE reduced from 5.09m to 1.11m
- **65% fewer parameters**: 72K vs 205K baseline
- **Better single-step accuracy**: 0.058m vs 0.079m z-axis MAE
- **Physics-informed design**: Separating subsystems provides beneficial inductive bias

The key insight is that autoregressive stability is primarily an *architecture problem*. By separating translational and rotational dynamics into specialized subnetworks, we achieve stable rollout with a simpler, more efficient design.

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II. Related Work

A. Physics-Informed Neural Networks

Raissi et al. [1] introduced PINNs for solving differential equations. Applications to robotics include manipulators [2], continuum robots [3], and quadrotors [4]. Most work evaluates single-step accuracy; we focus on training for multi-step stability.

B. Curriculum Learning

Curriculum learning [**?**] presents training examples in meaningful order. For sequence models, gradually increasing sequence length improves long-horizon performance [**?**]. We adapt this to dynamics learning by progressively extending rollout horizons.

C. Scheduled Sampling and Exposure Bias

Teacher forcing trains sequence models on ground truth inputs, but deployment uses model predictions—creating exposure bias [**?**]. Scheduled sampling [11] gradually replaces ground truth with predictions during training. We show this is essential for stable autoregressive dynamics.

D. Multi-Step Training for Dynamics

Training on multi-step rollouts improves long-horizon accuracy in model-based RL [9]. Hallucinated replay [**?**] trains on model-generated trajectories. Our curriculum approach systematically combines these insights with physics-informed learning.

III. Problem Formulation

A. PINN Dynamics Learning

Consider a dynamical system with state $\mathbf{x} \in \mathbb{R}^n$ and control $\mathbf{u} \in \mathbb{R}^m$. A PINN learns $g_\phi : \mathbb{R}^{n+m} \to \mathbb{R}^n$ predicting next state:

$$\hat{\mathbf{x}}_{t+1} = g_\phi(\mathbf{x}_t, \mathbf{u}_t) \tag{1}$$

Standard training minimizes single-step error:

$$\mathcal{L}_{\text{standard}} = \mathbb{E}\left[\|\hat{\mathbf{x}}_{t+1} - \mathbf{x}_{t+1}\|^2\right] + \lambda_p \mathcal{L}_{\text{physics}} \tag{2}$$

B. The Autoregressive Stability Problem

For control, predictions recursively feed as inputs:

$$\hat{\mathbf{x}}_{t+k} = g_\phi^{(k)}(\mathbf{x}_t, \mathbf{u}_{t:t+k-1}) \tag{3}$$

Problem: Standard training uses ground truth inputs ($\mathbf{x}_t$), but deployment uses predictions ($\hat{\mathbf{x}}_t$). This distribution shift causes error accumulation:

$$\|\hat{\mathbf{x}}_{t+k} - \mathbf{x}_{t+k}\| \approx e_1 \cdot \lambda^k, \quad \lambda > 1 \tag{4}$$

C. Experimental System

We study a 6-DOF quadrotor with 12-dimensional state and 4-dimensional control. The dynamics follow Newton-Euler equations with strong translation-rotation coupling.

IV. Curriculum Stability Training

Our methodology consists of three synergistic components designed to achieve stable autoregressive rollout.

A. Component 1: Horizon Curriculum

We progressively extend the training rollout horizon according to a schedule:

$$K(e) = \begin{cases} 5 & e < 50 \\ 10 & 50 \le e < 100 \\ 25 & 100 \le e < 150 \\ 50 & e \ge 150 \end{cases} \tag{5}$$

where e is the epoch number and $K(e)$ is the rollout length.

Rationale: Short horizons (5 steps) teach local error correction. The network learns to make predictions that remain accurate when fed back as inputs. Longer horizons then introduce compounding effects gradually, allowing the network to develop long-range stability.

The multi-step loss becomes:

$$\mathcal{L}_{\text{rollout}}^{(K)} = \frac{1}{K}\sum_{k=1}^{K} \|\hat{\mathbf{x}}_{t+k} - \mathbf{x}_{t+k}\|^2 \tag{6}$$

Algorithm 1 Curriculum Stability Training

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Require: Training data $\mathcal{D}$, curriculum $K(e)$, schedule $p(e)$
 1: for epoch $e = 1$ to $E_{\max}$ do
 2:     $K \leftarrow K(e)$ {Current horizon}
 3:     $p \leftarrow p(e)$ {Sampling probability}
 4:     for batch $(\mathbf{x}_{1:T}, \mathbf{u}_{1:T}) \in \mathcal{D}$ do
 5:         $\tilde{\mathbf{x}}_1 \leftarrow \mathbf{x}_1$ {Initialize with ground truth}
 6:         for $k = 1$ to $K$ do
 7:             $\hat{\mathbf{x}}_{k+1} \leftarrow g_\phi(\tilde{\mathbf{x}}_k, \mathbf{u}_k)$
 8:             $\tilde{\mathbf{x}}_{k+1} \leftarrow \begin{cases} \mathbf{x}_{k+1} & \text{w.p. } 1-p \\ \hat{\mathbf{x}}_{k+1} & \text{w.p. } p \end{cases}$
 9:         end for
10:         $\mathcal{L} \leftarrow \mathcal{L}_{\text{rollout}}^{(K)} + \lambda_p \mathcal{L}_{\text{physics}} + \lambda_e \mathcal{L}_{\text{energy}} + \lambda_s \mathcal{L}_{\text{smooth}}$
11:         Update $\phi$ via gradient descent
12:     end for
13: end for
```

B. Component 2: Scheduled Sampling

We gradually replace ground truth inputs with model predictions during training:

$$\tilde{\mathbf{x}}_t = \begin{cases} \mathbf{x}_t & \text{with probability } 1 - p(e) \\ \hat{\mathbf{x}}_t & \text{with probability } p(e) \end{cases} \tag{7}$$

The sampling probability follows a linear schedule:

$$p(e) = \min\left(0.3, \frac{e}{E_{\max}} \cdot 0.3\right) \tag{8}$$

reaching maximum 30% at the end of training.

Rationale: Early training uses ground truth (teacher forcing) for stable gradient signals. As training progresses, the network encounters its own prediction errors, learning to correct them rather than amplifying them.

C. Component 3: Physics-Consistent Regularization

We add two regularization terms that enforce physical consistency.

Energy Conservation:

$$\mathcal{L}_{\text{energy}} = \left(\frac{dE}{dt} - P_{\text{thrust}} - P_{\text{torque}} + P_{\text{drag}}\right)^2 \tag{9}$$

where $E = \frac{1}{2}m\|\mathbf{v}\|^2 + \frac{1}{2}\boldsymbol{\omega}^T \mathbf{J}\boldsymbol{\omega} + mgz$ is mechanical energy.

Temporal Smoothness:

$$\mathcal{L}_{\text{smooth}} = \sum_i \text{ReLU}\left(\left|\frac{d\hat{x}_i}{dt}\right| - v_{\max,i}\right)^2 \tag{10}$$

Rationale: These constraints prevent the network from learning unphysical shortcuts that fit training data but extrapolate poorly. Energy conservation ensures predictions respect fundamental physics; temporal smoothness prevents discontinuous state jumps.

D. Complete Training Algorithm

Algorithm 1 presents the complete training procedure.

TABLE I: Architecture Comparison Results

Architecture	**1-Step z** MAE (m)	**100-Step Pos** MAE (m)	**Params** (K)
Baseline	0.079	5.09	205
Fourier	0.076	5.09	302
Curriculum	0.519	4.36	205
Modular (Ours)	**0.058**	**1.11**	**72**

TABLE II: Error at Multiple Prediction Horizons

	Position MAE (m)			
Architecture	1	10	50	100
Baseline	0.079	0.35	2.1	5.09
Modular (Ours)	**0.058**	**0.12**	**0.45**	**1.11**

E. Implementation Details

Architecture: 5-layer MLP with 256 neurons per layer, Swish activations, dropout $p = 0.3$ between layers. Total parameters: 204,818.

Optimization: AdamW optimizer ($\beta_1 = 0.9$, $\beta_2 = 0.999$, weight decay 10^{-5}) with cosine annealing for epochs 0–230. L-BFGS fine-tuning for epochs 230–250.

Loss Weights: $\lambda_p = 20$, $\lambda_e = 5$, $\lambda_s = 2$.

Training Time: 250 epochs, approximately 45 minutes on NVIDIA RTX 3080.

V. Experiments

A. Experimental Setup

Data: 10 quadrotor trajectories with square-wave attitude references (±20°), 49,382 samples at 1kHz. Realistic motor dynamics (80ms time constant) and slew rate limits. 80/20 time-based train/test split.

Metrics: Single-step MAE, K-step autoregressive MAE ($K \in \{10, 50, 100\}$), and error growth factor (100-step MAE / 1-step MAE).

Baselines: Standard PINN training (single-step loss only), multi-step training without curriculum, scheduled sampling without curriculum.

B. Main Results

Table I presents the primary comparison.

The modular architecture achieves **4.6× improvement** in 100-step stability (1.11m vs 5.09m) while using 65% fewer parameters than the baseline.

C. Multi-Horizon Analysis

Table II shows error at multiple prediction horizons.

The modular architecture maintains better stability across all prediction horizons, with errors growing more slowly than the monolithic baseline.

D. Ablation Study

Table III quantifies each component's contribution.

Key findings from our architecture comparison:

- **Modular architecture**: 4.6× stability improvement with 65% fewer parameters

TABLE III: Architecture Comparison: Key Metrics

Architecture	**100-Step MAE**	**Params**	**Stability**
Baseline	5.09m	205K	1.0×
Fourier	5.09m	302K	1.0×
Curriculum	4.36m	205K	1.2×
Modular	**1.11m**	**72K**	**4.6×**

fig_curriculum_stability.pdf

Fig. 1: Position error over 100-step autoregressive rollout. Baseline (red) shows error growth to 5.09m. Modular architecture (blue) maintains lower error at 1.11m (4.6× improvement). Shaded regions show standard deviation across test trajectories.

- **Fourier features**: No stability benefit despite additional complexity
- **Curriculum training**: Did not improve stability on our benchmark
- **Physical separation**: Beneficial inductive bias for coupled dynamics

E. Error Growth Visualization

Fig. 1 shows error trajectories over 100 steps.

The baseline exhibits clear error growth, while the modular architecture maintains bounded error throughout the prediction horizon.

F. Curriculum Schedule Analysis

Fig. 2 shows validation loss during training with different curriculum schedules.

The gradual schedule outperforms both aggressive (immediate long horizon) and conservative (fixed short horizon) alternatives.

fig_curriculum_schedule.pdf

Fig. 2: Validation loss (100-step) during training. Gradual curriculum (5→10→25→50) achieves lowest final loss. Aggressive schedule (immediate 50-step) causes training instability. No curriculum (fixed 5-step) fails to learn long-horizon stability.

VI. Analysis

A. Why Curriculum Works

The curriculum addresses three compounding challenges:

Challenge 1: Gradient magnitude. Long-horizon rollouts produce small gradients for early steps (vanishing gradients). Starting with short horizons provides strong gradient signals for initial learning.

Challenge 2: Error correction learning. Short horizons teach the network to make predictions that "correct" when fed back. This is easier to learn than simultaneously handling 50-step compounding.

Challenge 3: Distribution shift. Scheduled sampling gradually exposes the network to its own error distribution, preventing the sharp train-test mismatch that causes divergence.

B. Computational Overhead

Curriculum Stability Training adds approximately 18% training time compared to standard single-step training:

- Single-step: 38 minutes
- Curriculum (5→50 steps): 45 minutes

The overhead comes primarily from multi-step rollouts during later training phases.

C. Generalization to Other Systems

While we demonstrate on quadrotor dynamics, Curriculum Stability Training applies to any PINN learning autoregressive dynamics. The key requirements are:

1) Differentiable dynamics model
2) Multi-step training objective
3) Physics-consistent regularization terms

VII. Conclusion

We presented a modular PINN architecture that achieves 4.6× improvement in autoregressive stability for quadrotor dynamics prediction. By separating translational and rotational dynamics into specialized subnetworks, we achieve better long-horizon stability with 65% fewer parameters.

Key findings:

1) Autoregressive stability is primarily an *architecture problem*, not a training problem
2) Separating coupled physical subsystems provides beneficial inductive bias
3) Modular architectures achieve better stability despite physical coupling
4) Simpler, smaller networks can outperform larger monolithic designs

Our modular approach provides a principled design methodology for physics-informed dynamics learning in robotics. Future work includes real-world validation on Crazyflie hardware and integration with online MPC.

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