

PINN Physics Layer Reference

Physics Laws & Equations Used in PINN Physics Layer

All physics equations are based on established rigid body dynamics and multirotor control literature [1, 2].

1. Rotational Dynamics (Euler's Equations)

Equation 1.1 - Roll Angular Acceleration:

$$\dot{p} = \frac{J_{yy} - J_{zz}}{J_{xx}} \cdot q \cdot r + \frac{\tau_x}{J_{xx}} \quad (1)$$

where:

- p = roll rate (rad/s)
- \dot{p} = roll angular acceleration (rad/s²)
- q = pitch rate (rad/s)
- r = yaw rate (rad/s)
- τ_x = roll torque (N · m)
- J_{xx} = moment of inertia about x-axis = 6.86×10^{-5} kg · m²
- J_{yy} = moment of inertia about y-axis = 9.2×10^{-5} kg · m²
- J_{zz} = moment of inertia about z-axis = 1.366×10^{-4} kg · m²

Equation 1.2 - Pitch Angular Acceleration:

$$\dot{q} = \frac{J_{zz} - J_{xx}}{J_{yy}} \cdot p \cdot r + \frac{\tau_y}{J_{yy}} \quad (2)$$

where:

- q = pitch rate (rad/s)
- \dot{q} = pitch angular acceleration (rad/s²)
- τ_y = pitch torque (N · m)

Equation 1.3 - Yaw Angular Acceleration:

$$\dot{r} = \frac{J_{xx} - J_{yy}}{J_{zz}} \cdot p \cdot q + \frac{\tau_z}{J_{zz}} \quad (3)$$

where:

- r = yaw rate (rad/s)
- \dot{r} = yaw angular acceleration (rad/s²)
- τ_z = yaw torque (N · m)

Key Feature: Real Euler equations with NO artificial damping terms. Pure physics-based rotational dynamics [1].

2. Simplified Euler Angle Integration

Equation 2.1 - Roll Angle Rate:

$$\dot{\phi} = p \quad (4)$$

where:

- ϕ = roll angle (rad)
- $\dot{\phi}$ = roll angle rate (rad/s)
- p = roll rate (rad/s)

Equation 2.2 - Pitch Angle Rate:

$$\dot{\theta} = q \quad (5)$$

where:

- θ = pitch angle (rad)
- $\dot{\theta}$ = pitch angle rate (rad/s)
- q = pitch rate (rad/s)

Equation 2.3 - Yaw Angle Rate:

$$\dot{\psi} = r \quad (6)$$

where:

- ψ = yaw angle (rad)
- $\dot{\psi}$ = yaw angle rate (rad/s)
- r = yaw rate (rad/s)

Note: These are simplified angular integrations valid for small angles, not the full nonlinear Euler kinematics [2].

3. Translational Dynamics (Vertical Motion)

Equation 3.1 - Vertical Acceleration:

$$\dot{v}_z = -g + \frac{T}{m \cdot \cos(\theta) \cdot \cos(\phi)} \quad (7)$$

where:

- v_z = vertical velocity (m/s, positive downward in NED frame)
- \dot{v}_z = vertical acceleration (m/s²)
- T = total thrust (N)
- θ = pitch angle (rad)
- ϕ = roll angle (rad)
- m = quadrotor mass = 0.068 kg
- g = gravitational acceleration = 9.81 m/s²

Equation 3.2 - Altitude Rate:

$$\dot{z} = v_z \quad (8)$$

where:

- z = altitude (m, positive downward in NED frame)
- \dot{z} = altitude rate (m/s)
- v_z = vertical velocity (m/s)

Note: No aerodynamic drag is included in the physics loss formulation. Translational dynamics based on Newton's second law [1, 2].

Equations Used in Data Generation but NOT in PINN

4. Full Nonlinear Euler Kinematics (Data Generation Only)

Equation 4.1 - Full Roll Angle Rate:

$$\dot{\phi} = p + \sin(\phi) \tan(\theta) \cdot q + \cos(\phi) \tan(\theta) \cdot r \quad (9)$$

Equation 4.2 - Full Pitch Angle Rate:

$$\dot{\theta} = \cos(\phi) \cdot q - \sin(\phi) \cdot r \quad (10)$$

Equation 4.3 - Full Yaw Angle Rate:

$$\dot{\psi} = \frac{\sin(\phi) \cdot q + \cos(\phi) \cdot r}{\cos(\theta)} \quad (11)$$

Why not in PINN: PINN uses simplified small-angle approximations ($\dot{\phi} = p$, $\dot{\theta} = q$, $\dot{\psi} = r$) which are valid for typical quadrotor maneuvers with angles $< 30^\circ$ [2].

5. Full 6DOF Translational Dynamics (Data Generation Only)

Equation 5.1 - X-Velocity Acceleration:

$$\dot{u} = r \cdot v - q \cdot w + \frac{f_x}{m} - g \sin(\theta) - c_d \cdot u \cdot |u| \quad (12)$$

Equation 5.2 - Y-Velocity Acceleration:

$$\dot{v} = p \cdot w - r \cdot u + \frac{f_y}{m} + g \cos(\theta) \sin(\phi) - c_d \cdot v \cdot |v| \quad (13)$$

Equation 5.3 - Z-Velocity Acceleration (with drag):

$$\dot{w} = q \cdot u - p \cdot v + \frac{f_z}{m} + g \cos(\theta) \cos(\phi) - c_d \cdot w \cdot |w| \quad (14)$$

where:

- u, v, w = body-frame velocities in x, y, z directions (m/s)
- f_x, f_y, f_z = body-frame forces (N)
- $c_d = 0.05 \text{ kg/m} = \text{quadratic drag coefficient}$
- Coriolis terms: $r \cdot v - q \cdot w$, $p \cdot w - r \cdot u$, $q \cdot u - p \cdot v$

Why not in PINN: PINN models only vertical (z-axis) dynamics for altitude control. Horizontal motion (u, v) is not controlled in the training data, and the simplified model assumes $f_x = f_y = 0$. Additionally, drag forces are omitted from physics loss. Full 6-DOF dynamics from [1, 2].

6. Body-to-World Frame Transformation (Data Generation Only)

Equation 6.1 - X-Position Rate:

$$\dot{x} = \cos(\psi) \cos(\theta) \cdot u + [\cos(\psi) \sin(\theta) \sin(\phi) - \sin(\psi) \cos(\phi)] \cdot v + [\sin(\psi) \sin(\phi) + \cos(\psi) \sin(\theta) \cos(\phi)] \cdot w \quad (15)$$

Equation 6.2 - Y-Position Rate:

$$\dot{y} = \sin(\psi) \cos(\theta) \cdot u + [\cos(\psi) \cos(\phi) + \sin(\psi) \sin(\theta) \sin(\phi)] \cdot v + [\sin(\psi) \sin(\theta) \cos(\phi) - \cos(\psi) \sin(\phi)] \cdot w \quad (16)$$

Equation 6.3 - Z-Position Rate:

$$\dot{z} = -[\sin(\theta) \cdot u - \cos(\theta) \sin(\phi) \cdot v - \cos(\theta) \cos(\phi) \cdot w] \quad (17)$$

Why not in PINN: PINN focuses on body-frame dynamics, not world-frame position tracking. Position is not required for learning the dynamics model. Rotation matrix formulation from [1].

7. Motor Thrust/Torque Mapping (Not Currently Used)

Equation 7.1 - Total Thrust from Motors:

$$T = k_t \cdot (\omega_1^2 + \omega_2^2 + \omega_3^2 + \omega_4^2) \quad (18)$$

Equation 7.2 - Roll Torque from Motors:

$$\tau_x = k_t \cdot L \cdot (\omega_2^2 - \omega_4^2) \quad (19)$$

Equation 7.3 - Pitch Torque from Motors:

$$\tau_y = k_t \cdot L \cdot (\omega_3^2 - \omega_1^2) \quad (20)$$

Equation 7.4 - Yaw Torque from Motors:

$$\tau_z = k_q \cdot (\omega_1^2 - \omega_2^2 + \omega_3^2 - \omega_4^2) \quad (21)$$

where:

- $\omega_1, \omega_2, \omega_3, \omega_4$ = motor speeds (rad/s)
- $k_t = 0.01 \text{ N}/(\text{rad/s})^2$ = thrust coefficient
- $k_q = 7.8263 \times 10^{-4} \text{ N} \cdot \text{m}/(\text{rad/s})^2$ = torque coefficient
- L = arm length from center to motor (m)

Why not in PINN: Currently, PINN receives thrust T and torques τ_x, τ_y, τ_z as direct inputs rather than computing them from motor speeds. These equations are only used to calculate physical limits during data generation. Motor thrust/torque coefficients from [1].

Complete Mapping: 18 State Outputs to Equations

8. State Variables and Their Physics Equations

#	Output	Equation Used	Status	Reason/Note
Rotational Dynamics				
1	\dot{p}	Eq 1.1: $\dot{p} = \frac{J_{yy}-J_{zz}}{J_{xx}} \cdot q \cdot r + \frac{\tau_x}{J_{xx}}$	USED	Full Euler equation
2	\dot{q}	Eq 1.2: $\dot{q} = \frac{J_{zz}-J_{xx}}{J_{yy}} \cdot p \cdot r + \frac{\tau_y}{J_{yy}}$	USED	Full Euler equation
3	\dot{r}	Eq 1.3: $\dot{r} = \frac{J_{xx}-J_{yy}}{J_{zz}} \cdot p \cdot q + \frac{\tau_z}{J_{zz}}$	USED	Full Euler equation
4	p	Integration: $p_{t+dt} = p_t + \dot{p} \cdot dt$	USED	Direct integration
5	q	Integration: $q_{t+dt} = q_t + \dot{q} \cdot dt$	USED	Direct integration
6	r	Integration: $r_{t+dt} = r_t + \dot{r} \cdot dt$	USED	Direct integration
Euler Angle Kinematics				
7	$\dot{\phi}$	Eq 2.1 (PINN): $\dot{\phi} = p$ Eq 4.1 (Data): Full non-linear	SIMPLIFIED	Small-angle approximation valid for $ \phi < 30^\circ$. Data uses full kinematics with sin, tan terms.
8	$\dot{\theta}$	Eq 2.2 (PINN): $\dot{\theta} = q$ Eq 4.2 (Data): Full non-linear	SIMPLIFIED	Small-angle approximation valid for $ \theta < 30^\circ$. Data uses full kinematics with cos, sin terms.
9	$\dot{\psi}$	Eq 2.3 (PINN): $\dot{\psi} = r$ Eq 4.3 (Data): Full non-linear	SIMPLIFIED	Small-angle approximation. Data uses full kinematics with gimbal lock at $\theta = \pm 90^\circ$.
10	ϕ	Integration: $\phi_{t+dt} = \phi_t + \dot{\phi} \cdot dt$	USED	Uses simplified $\dot{\phi}$
11	θ	Integration: $\theta_{t+dt} = \theta_t + \dot{\theta} \cdot dt$	USED	Uses simplified $\dot{\theta}$
12	ψ	Integration: $\psi_{t+dt} = \psi_t + \dot{\psi} \cdot dt$	USED	Uses simplified $\dot{\psi}$
Translational Dynamics				
13	\dot{u}	Eq 5.1: Full 6DOF with Coriolis and drag	NOT USED	PINN focuses on vertical-only control. No horizontal force inputs ($f_x = 0$) in training data.

#	Output	Equation Used	Status	Reason/Note
14	\dot{v}	Eq 5.2: Full 6DOF with Coriolis and drag	NOT USED	PINN focuses on vertical-only control. No horizontal force inputs ($f_y = 0$) in training data.
15	\dot{w}	Eq 3.1 (PINN): No drag Eq 5.3 (Data): With drag	SIMPLIFIED	PINN omits Coriolis terms (assumes $u = v = 0$) and drag ($c_d \cdot w \cdot w $) for simpler vertical dynamics.
16	u	Integration of \dot{u}	NOT USED	Not predicted since \dot{u} is not modeled. Passive/uncontrolled horizontal motion.
17	v	Integration of \dot{v}	NOT USED	Not predicted since \dot{v} is not modeled. Passive/uncontrolled horizontal motion.
18	w (v_z)	Integration: $w_{t+dt} = w_t + \dot{w} \cdot dt$	USED	Uses simplified \dot{w}

Legend

- **USED** - Equation actively used in PINN physics loss (perfect match with data generation)
- **SIMPLIFIED** - PINN uses a simplified approximation; data generation uses full complex equation
- **NOT USED** - Variable exists in training data but completely absent from PINN physics loss

Detailed Explanations

Why SIMPLIFIED (4 variables):

- **Outputs 7-9 (Euler angle rates):** PINN uses small-angle approximations ($\dot{\phi} = p$, $\dot{\theta} = q$, $\dot{\psi} = r$) which are valid for typical quadrotor maneuvers with angles $< 30^\circ$. Data generation uses full nonlinear Euler kinematics with trigonometric coupling terms (\sin, \cos, \tan).
- **Output 15 (vertical acceleration):** PINN omits Coriolis coupling terms (assumes horizontal velocities $u = v = 0$) and aerodynamic drag ($c_d \cdot w \cdot |w|$) for a simpler vertical-only dynamics model.

Why NOT USED (4 variables):

- **Outputs 13-14 (horizontal accelerations):** PINN focuses on vertical-only quadrotor control. Training data has no horizontal control inputs ($f_x = f_y = 0$), making

horizontal dynamics passive/uncontrolled. Not needed for altitude and attitude stabilization task.

- **Outputs 16-17 (horizontal velocities):** Cannot be predicted since their derivatives (\dot{u}, \dot{v}) are not modeled in PINN. These are byproducts of the full 6DOF simulation but not required for the vertical control objective.

Summary of PINN Model Scope

What PINN Models:

- Full 3D rotational dynamics (Euler's equations)
- Simplified Euler angle integration (small-angle approximation)
- Vertical translational dynamics (without drag)
- Altitude tracking

What PINN Does NOT Model:

- Full nonlinear Euler kinematics
- Horizontal motion (x, y velocities and positions)
- Aerodynamic drag forces
- Body-to-world frame transformations
- Motor speed to thrust/torque mapping

Justification: The PINN focuses on a simplified vertical flight dynamics model suitable for altitude control and small-angle attitude stabilization, which covers the majority of typical quadrotor flight scenarios.

References

All physics equations in this document are sourced from the following established literature on quadrotor dynamics and control:

References

- [1] Mahony, R., Kumar, V., & Corke, P. (2012). *Multirotor Aerial Vehicles: Modeling, Estimation, and Control of Quadrotor*. IEEE Robotics & Automation Magazine, 19(3), 20-32. DOI: 10.1109/MRA.2012.2206474

Coverage: Complete Newton-Euler formulation for quadrotor 6-DOF dynamics including:

- Euler equations for rotational dynamics (Equations 1.1-1.3)
- Full nonlinear Euler angle kinematics (Equations 4.1-4.3)
- Translational dynamics in body frame (Equations 5.1-5.3)
- Body-to-world frame rotation matrix transformations (Equations 6.1-6.3)
- Motor thrust and torque coefficients (Equations 7.1-7.4)

- [2] Beard, R. W., & McLain, T. W. (2012). *Small Unmanned Aircraft: Theory and Practice*. Princeton University Press. Chapter 4: Forces and Moments.

Coverage: Forces and moments formulation for small UAVs including:

- Simplified Euler angle integration (Equations 2.1-2.3)
- Small-angle approximation validity ranges
- Vertical translational dynamics (Equation 3.1-3.2)
- Aerodynamic drag modeling in body frame

Reference Coverage

These two authoritative sources provide complete coverage of all physics equations used in both:

- **PINN Physics Loss:** Euler rotational dynamics, simplified kinematics, vertical dynamics
- **Data Generation:** Full 6-DOF dynamics, nonlinear kinematics, frame transformations, motor mapping

The PINN implementation uses a subset of these equations (simplified model) while the data generation uses the complete formulation (full model), as documented in Section 8.