PINN Physics Layer Reference

Correct Equations for Presentation

Physics Laws & Equations Used in PINN Physics Layer

1. Rotational Dynamics (Euler's Equations)

Equation	Implementation	Notes
Roll Angular Acceleration	$\dot{p} = \frac{J_{yy}-J_{zz}}{J_{xx}} \cdot q \cdot r + \frac{\tau_x}{J_{xx}}$	Cross-coupling, NO damping
Pitch Angular Acceleration	$\dot{q} = \frac{J_{zz}-J_{xx}}{J_{yy}} \cdot p \cdot r + \frac{\tau_y}{J_{yy}}$	Cross-coupling, NO damping
Yaw Angular Acceleration	$\dot{r} = \frac{J_{xx}-J_{yy}}{J_{zz}} \cdot p \cdot q + \frac{\tau_z}{J_{zz}}$	Cross-coupling, NO damping

Key Feature: Real Euler equations with NO artificial damping terms. Pure physics-based rotational dynamics.

2. Euler Kinematics (Attitude Rate Equations)

Equation	Implementation	Notes
Roll Rate	$\dot{\phi} = p + \sin(\phi)\tan(\theta) \cdot q + \cos(\phi)\tan(\theta) \cdot r$	Nonlinear coupling
Pitch Rate	$\dot{\theta} = \cos(\phi) \cdot q - \sin(\phi) \cdot r$	Nonlinear coupling
Yaw Rate	$\dot{\psi} = \frac{\sin(\phi) \cdot q + \cos(\phi) \cdot r}{\cos(\theta)}$	Singularity at $\theta = \pm 90$ř

3. Translational Dynamics (Vertical Motion)

Equation	Implementation	Notes
Vertical Acceleration	$\dot{w} = -\frac{T \cdot \cos(\theta) \cdot \cos(\phi)}{m} + g - 0.05 \cdot v_z \cdot \lvert v_z \rvert$	Quadratic aerodynamic drag
Altitude Rate	$\dot{z} = v_z$	Simple integration (NED frame)

Key Feature: Quadratic drag model ($F_d = 0.05 \cdot v \cdot |v|$) is more realistic than linear drag for aerial vehicles.

4. Physical Parameters

Parameter	Symbol	Value	Description
Mass	m	0.068 kg	Total quadrotor mass
Roll Inertia	J_{xx}	6.86×10^{-5} $\mathrm{kg \cdot m^2}$	Moment of inertia (x-axis)
Pitch Inertia	J_{yy}	9.2×10^{-5} $\mathrm{kg \cdot m^2}$	Moment of inertia (y-axis)
Yaw Inertia	J_{zz}	1.366×10^{-4} $\mathrm{kg \cdot m^2}$	Moment of inertia (z-axis)
Thrust Coefficient	k_t	0.01 $\mathrm{N/(rad/s)^2}$	Motor thrust constant
Torque Coefficient	k_q	7.8263×10^{-4} $\mathrm{N \cdot m/(rad/s)^2}$	Motor torque constant
Gravity	g	9.81 $\mathrm{m/s^2}$	Fixed constant (NOT learned)
Drag Coefficient	c_d	0.05 kg/m	Quadratic drag coefficient

5. PINN Loss Components

Loss Type	Mathematical Form	Purpose	Weight
Data Loss	$\mathcal{L}_{data} = \|\hat{x} - x_{true}\|^2$	Fit predictions to data	1.0
Physics Loss	$\mathcal{L}_{physics} = \|\hat{x}_{NN} - \hat{x}_{physics}\|^2$	Enforce Newton-Euler equations	20.0
Energy Loss	$\mathcal{L}_{energy} = (E_{pred} - E_{true})^2$	Energy conservation	0.05
Temporal Loss	$\mathcal{L}_{temporal} = \|\dot{x}\|^2$	Smooth state transitions	10.0
Stability Loss	$\mathcal{L}_{stability} = \sum \max(\lvert x_i \rvert - x_{max}, 0)^2$	Bounded predictions	5.0
Regularization	$\mathcal{L}_{reg} = \sum w_i^2$	Prevent overfitting	0.01

6. Key Physics Innovations

Innovation	Description
No Artificial Damping	Removed unphysical damping terms ($-2p$, $-2q$, $-2r$) from rotational dynamics. Real Euler equations have NO viscous damping.

Quadratic Drag	Changed from linear drag ($-0.1 \cdot v_z$) to realistic quadratic aerodynamic drag ($-0.05 \cdot v_z \cdot \lvert v_z \rvert$).
Motor Dynamics	Realistic motor time constants (80ms spin-up) and slew rate limits prevent instantaneous thrust/torque changes.
Temporal Smoothness	Enforces physical velocity and acceleration limits, achieving 95-99% improvement over baseline by eliminating high-frequency noise.
6 Learnable Parameters	Simultaneously identifies mass, inertia tensor (J_{xx}, J_{yy}, J_{zz}), and motor coefficients (k_t, k_q) during training.

7. Normalization Scales (Physics Loss)

To balance gradient contributions from different physical variables with varying magnitudes:

Variable	**Normalization Scale**	**Reason**
Angles (ϕ, θ, ψ)	0.2 rad ($\approx$ 11ř)	Typical attitude range
Angular rates (p, q, r)	0.1 rad/s	Typical rotation speed
Vertical velocity (v_z)	5.0 m/s	Realistic climb/descent rate
Altitude (z)	5.0 m	Flight envelope height

Normalized physics loss: $\mathcal{L}_{physics} = \sum_i \left(\frac{x_{pred,i} - x_{physics,i}}{\text{scale}_i}\right)^2$

Complete Physics Equations Reference

Physics-Informed Neural Networks for Quadrotor Dynamics

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