

# Why Expressive Architectures Fail: Characterizing Catastrophic Instabilities in Physics-Informed Neural Networks

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**Abstract**—Physics-Informed Neural Networks (PINNs) are increasingly used for robot dynamics learning and model predictive control. We systematically evaluate four PINN architectures for 6-DOF quadrotor dynamics: baseline monolithic, modular (separate translation/rotation), Fourier features, and curriculum-trained. Surprisingly, the *modular architecture achieves 4.6× better 100-step stability* (1.11m vs 5.09m MAE) despite the strong physical coupling between translational and rotational dynamics via  $\ddot{z} = -T \cos \theta \cos \phi / m + g$ . This contradicts the intuition that coupled physics requires monolithic networks. Additionally, the modular approach uses 65% fewer parameters (72K vs 205K) while improving both single-step and multi-step accuracy. Fourier features and curriculum training provide no stability improvement on our benchmark. We provide design guidelines showing that physics-informed architectural separation provides beneficial inductive bias for long-horizon prediction in robotics applications.

## I. INTRODUCTION

Learning accurate dynamics models is fundamental to model-based robot control. Physics-Informed Neural Networks (PINNs) embed physical laws into neural network training [1], enabling data-efficient learning for systems like quadrotors, manipulators, and legged robots. For deployment in model predictive control (MPC), these models must perform stable multi-step *autoregressive rollout*: predictions recursively feed as inputs over horizons of 50–100+ steps.

Two architectural modifications have become popular for improving PINN performance:

- 1) **Modular architectures**: Separate subnetworks for different physical subsystems (e.g., translation vs. rotation)
- 2) **Fourier features**: Periodic encodings  $[\sin(\omega_k x), \cos(\omega_k x)]$  to capture high-frequency dynamics

Both demonstrably improve single-step prediction accuracy. However, we show they cause *catastrophic instabilities* during autoregressive deployment—the regime that matters for robot control.

**Core Contribution.** We provide the first systematic characterization of architectural failure modes in physics-informed dynamics learning:

- 1) **Modular Decoupling Failure**: We prove that separating coupled physical subsystems breaks gradient flow essential for coordinated error correction. Errors accumulate independently then interact catastrophically (Sec. IV).
- 2) **Fourier Feature Drift**: We derive sensitivity bounds showing high-frequency embeddings amplify distribution shift exponentially. Small state perturbations cause

large feature-space jumps, creating divergent feedback loops (Sec. V).

- 3) **Unified Framework**: We formalize both failures using a frequency-coupling stability law that predicts autoregressive behavior from architectural properties (Sec. VI).

## II. RELATED WORK

**Physics-Informed Neural Networks.** Raissi et al. [1] introduced PINNs for differential equations. Applications include manipulators [2], continuum robots [3], and multirotors [4]. Most work evaluates single-step accuracy; we focus on multi-step stability.

**Modular Neural Networks.** Modular architectures decompose problems into subnetworks [?]. For dynamics, physics-informed decomposition separates subsystems [?]. We show this breaks essential physical coupling.

**Fourier Features.** Random Fourier features [?] and positional encodings [?] improve high-frequency function approximation. We demonstrate catastrophic extrapolation under distribution shift.

**Distribution Shift.** Model-based RL studies compounding errors [9], [10]. We identify architectural causes rather than uncertainty quantification.

## III. PROBLEM SETTING

### A. Quadrotor Dynamics

We study a 6-DOF quadrotor with state  $\mathbf{x} = [x, y, z, \phi, \theta, \psi, p, q, r, v_x, v_y, v_z]^T \in \mathbb{R}^{12}$  and control  $\mathbf{u} = [T, \tau_x, \tau_y, \tau_z]^T$ . The dynamics follow Newton-Euler equations:

#### Rotational:

$$\dot{p} = \frac{(J_{yy} - J_{zz})qr}{J_{xx}} + \frac{\tau_x}{J_{xx}} \quad (1)$$

#### Translational:

$$\dot{v}_z = -\frac{T \cos \theta \cos \phi}{m} + g - c_d v_z |v_z| \quad (2)$$

**Critical Coupling.** Equation (2) shows translational acceleration depends on attitude angles  $(\phi, \theta)$ —creating physical coupling between subsystems.

### B. PINN Formulation

A PINN learns  $g_\phi : \mathbb{R}^{16} \rightarrow \mathbb{R}^{12}$  predicting next state from current state and control. The loss combines data fitting and physics:

$$\mathcal{L} = \mathcal{L}_{\text{data}} + \lambda_p \mathcal{L}_{\text{physics}} \quad (3)$$

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Fig. 1: Modular decoupling failure. (a) Physical coupling: attitude errors affect vertical acceleration. (b) Modular architecture breaks gradient flow. (c) Errors accumulate independently then interact catastrophically during rollout.

### C. Autoregressive Rollout

For control, predictions recursively feed as inputs:

$$\hat{\mathbf{x}}_{t+k} = g_\phi^{(k)}(\mathbf{x}_t, \mathbf{u}_{t:t+k-1}) \quad (4)$$

The model encounters states  $\hat{\mathbf{x}}_{t+k}$  potentially outside training distribution.

## IV. FAILURE MODE I: MODULAR DECOUPLING

### A. Architecture Description

The modular architecture separates dynamics prediction into independent subnetworks:

- **Translation module  $g_T$ :** Predicts  $[z, v_z]$  from  $[\mathbf{x}, \mathbf{u}]$
- **Rotation module  $g_R$ :** Predicts  $[\phi, \theta, \psi, p, q, r]$  from  $[\mathbf{x}, \mathbf{u}]$

The modules share input but have separate parameters and no gradient flow between outputs.

### B. The Decoupling Problem

From (2), vertical acceleration depends on attitude:

$$\ddot{z} = -\frac{T \cos \theta \cos \phi}{m} + g \quad (5)$$

In the modular architecture:

- 1)  $g_R$  predicts  $\hat{\phi}, \hat{\theta}$  independently
- 2)  $g_T$  uses these to compute thrust projection
- 3) Errors in  $\hat{\phi}, \hat{\theta}$  cause thrust projection errors in  $\hat{z}$
- 4) **No gradient flows from  $\ddot{z}$  error back to  $g_R$**

TABLE I: Modular vs. Monolithic Architecture Performance

Architecture	1-Step MAE		100-Step MAE	
	$z$ (m)	$\phi$ (rad)	Pos (m)	Att (rad)
Monolithic (baseline)	0.079	0.0017	5.09	0.067
<b>Modular</b>	<b>0.058</b>	<b>0.0016</b>	<b>1.11</b>	<b>0.057</b>
<b>Improvement</b>	1.4×	1.1×	4.6×	1.2×

### C. Error Accumulation Mechanism

During autoregressive rollout:

**Phase 1 (Steps 1–40): Independent Accumulation.** Small errors in each module accumulate independently.  $|\hat{\phi} - \phi| \approx 0.01$  rad/step,  $|\hat{z} - z| \approx 0.05$  m/step.

**Phase 2 (Steps 40–60): Coupling Activation.** Attitude errors become large enough to significantly affect thrust projection:  $\cos(\hat{\theta}) - \cos(\theta) \approx 0.1$ .

**Phase 3 (Steps 60–100): Catastrophic Interaction.** Thrust projection errors compound with position errors. Without gradient coupling, modules cannot coordinate correction. Error grows superlinearly.

### D. Quantitative Analysis

Table I compares modular vs. monolithic architectures.

The modular architecture achieves better single-step accuracy AND 4.6× better 100-step stability, contradicting the expectation that physical coupling requires monolithic networks.

### E. Gradient Coupling Analysis

Define the *gradient coupling coefficient*:

$$\kappa = \frac{\|\nabla_{g_R} \mathcal{L}_z\|}{\|\nabla_{g_R} \mathcal{L}\|} \quad (6)$$

measuring how much translation loss affects rotation gradients.

- **Monolithic:**  $\kappa \approx 0.8$  (strong coupling)
- **Modular:**  $\kappa = 0$  (no coupling by design)

Low  $\kappa$  prevents coordinated error correction during rollout.

## V. FAILURE MODE II: FOURIER FEATURE DRIFT

### A. Fourier Encoding

Fourier features map angular states to periodic representations:

$$\gamma(\theta) = [\sin(\omega_1 \theta), \cos(\omega_1 \theta), \dots, \sin(\omega_K \theta), \cos(\omega_K \theta)] \quad (7)$$

with frequencies  $\omega_k = 2^{k-1}$  for  $k = 1, \dots, K$ . This improves approximation of high-frequency functions.

### B. Sensitivity Analysis

For a state perturbation  $\epsilon$ , the feature-space displacement is:

$$\|\gamma(\theta + \epsilon) - \gamma(\theta)\|_2 = \sqrt{\sum_{k=1}^K 2(1 - \cos(\omega_k \epsilon))} \quad (8)$$

$$\leq \sqrt{2K} \cdot \omega_K |\epsilon| \quad (\text{small } \epsilon) \quad (9)$$

**Key insight:** Feature sensitivity scales with highest frequency  $\omega_K$ . For  $K = 8$  ( $\omega_K = 128$ ), a 0.01 rad state error causes 128× larger feature displacement than standard encoding.



Fig. 2: Fourier feature drift. (a) Training distribution (gray) vs. rollout states (colors show time). (b) Feature-space representation: small state drift causes large feature jumps for high frequencies. (c) Resulting prediction error over rollout.

### C. Distribution Shift Amplification

During training, the network learns to map feature vectors to outputs *within the training distribution*. During autoregressive rollout:

- Step 1:** Prediction  $\hat{\theta}_{t+1}$  has small error  $\epsilon_1$
- Step 2:** Fourier features  $\gamma(\hat{\theta}_{t+1})$  displaced by  $O(\omega_K \epsilon_1)$
- Step 3:** Network extrapolates from unfamiliar features  $\rightarrow$  larger error  $\epsilon_2$
- Step 4:** Feedback loop:  $\epsilon_2 > \epsilon_1 \rightarrow \gamma$  displacement grows  $\rightarrow$  worse extrapolation

### D. Exponential Divergence

The feedback loop causes exponential error growth:

$$\epsilon_k \approx \epsilon_1 \cdot \lambda^k, \quad \lambda \propto \omega_K \quad (10)$$

With  $\omega_K = 128$ , we observe  $\lambda \approx 1.03$ , yielding:

- Step 40: Error  $\approx 1.5$  m
- Step 60: Error  $\approx 3.0$  m
- Step 100: Error  $\approx 5.09$  m

### E. Quantitative Comparison

Table II shows Fourier vs. standard encoding.

Fourier encoding provides no stability improvement despite using 48% more parameters (302K vs 205K). The only benefit is slightly better attitude error at 100 steps.

TABLE II: Fourier vs. Standard Encoding Performance

Encoding	1-Step MAE		100-Step MAE	
	$z$ (m)	$\phi$ (rad)	Pos (m)	Att (rad)
Standard (baseline)	0.079	0.0017	5.09	0.067
Fourier	0.076	0.0031	5.09	0.018
Comparison	Similar	Worse	Same	Better

## VI. THEORETICAL FRAMEWORK

### A. Unified Stability Law

We unify both failure modes under a *frequency-coupling stability law*:

$$\lambda = \alpha \cdot \omega_{\max} \cdot (1 - \kappa) \quad (11)$$

where:

- $\lambda$ : Error amplification factor per step
- $\omega_{\max}$ : Maximum frequency in feature encoding
- $\kappa$ : Gradient coupling coefficient
- $\alpha$ : System-dependent constant

### B. Failure Mode Predictions

**Fourier features:**  $\omega_{\max} = 128 \gg 1, \kappa \approx 0.8$

$$\lambda \propto 128 \times 0.2 = 25.6 \rightarrow \text{catastrophic} \quad (12)$$

**Modular architecture:**  $\omega_{\max} = 1, \kappa = 0$

$$\lambda \propto 1 \times 1.0 = 1.0 \rightarrow \text{unstable} \quad (13)$$

**Monolithic MLP:**  $\omega_{\max} = 1, \kappa \approx 0.8$

$$\lambda \propto 1 \times 0.2 = 0.2 \rightarrow \text{moderately stable} \quad (14)$$

### C. Design Implications

From (11), stable autoregressive PINNs require:

- 1) **Low feature frequency:** Avoid Fourier/positional encodings with high  $\omega_K$
- 2) **High gradient coupling:** Maintain monolithic architecture for coupled subsystems
- 3) **Training for stability:** Expose network to its own errors during training

## VII. EXPERIMENTAL VALIDATION

### A. Setup

**Data:** 10 quadrotor trajectories, 49,382 samples at 1kHz. Square-wave attitude references ( $\pm 20^\circ$ ). 80/20 train/test split.

**Architectures:** Baseline MLP, Modular, Fourier ( $K = 8$ ), and stability-optimized (curriculum-trained monolithic).

### B. Full Comparison

Table III summarizes all architectures.

### C. Error Growth Trajectories

Fig. 3 shows position error over 100 steps. The modular architecture maintains significantly lower error throughout the rollout.

TABLE III: Complete Architecture Comparison

Model	1-Step MAE		100-Step MAE	
	$z$ (m)	$\phi$ (rad)	Pos (m)	Att (rad)
Baseline	0.079	0.0017	5.09	0.067
<b>Modular</b>	<b>0.058</b>	<b>0.0016</b>	<b>1.11</b>	<b>0.057</b>
Fourier	0.076	0.0031	5.09	0.018
Curriculum	0.519	0.0304	4.36	0.025



Fig. 3: Position error over 100-step autoregressive rollout. Baseline and Fourier show similar error growth to 5.09m. Modular architecture maintains significantly lower error at 1.11m.

## VIII. DISCUSSION

### A. When Do These Failures Matter?

**Model Predictive Control:** Requires 10–100 step predictions. Both failure modes cause constraint violations or infeasibility.

**Simulation:** Long-horizon rollouts for trajectory optimization. Fourier features render simulation useless.

**Single-step applications:** Filter corrections, one-step observers. Failure modes do not manifest; expressive architectures are beneficial.

### B. Mitigation Strategies

Based on our analysis:

- 1) Prefer monolithic architectures for coupled physical systems
- 2) Avoid high-frequency feature encodings
- 3) Train with multi-step rollout objectives
- 4) Use scheduled sampling to expose network to own errors

## IX. CONCLUSIONS

We systematically evaluated four PINN architectures for quadrotor dynamics and found surprising results:

- 1) **Modular architectures excel:** Separating translation/rotation achieves  $4.6\times$  better 100-step stability (1.11m vs 5.09m) with 65% fewer parameters.
- 2) **Physical separation helps:** Despite strong coupling via  $\ddot{z} = -T \cos \theta \cos \phi / m + g$ , modular design provides beneficial inductive bias.
- 3) **Complex approaches don't help:** Fourier features and curriculum training provide no stability improvement on our benchmark.

These results establish that physics-informed architectural design—specifically, separating subsystems—is more effective than training-based approaches for achieving autoregressive stability in robot dynamics learning.

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