Modular Architecture Design for Stable Physics-Informed Neural Network Dynamics Learning

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***Abstract*—Physics-Informed Neural Networks (PINNs) are increasingly used for robot dynamics learning and model predictive control. We systematically evaluate four PINN architectures for 6-DOF quadrotor dynamics: baseline monolithic, modular (separate translation/rotation), Fourier features, and curriculum-trained. Surprisingly, the *modular architecture achieves 4.6× better 100-step stability* (1.11m vs 5.09m MAE) despite the strong physical coupling between translational and rotational dynamics via $\ddot{z} = -T\cos\theta\cos\phi/m + g$. This contradicts the intuition that coupled physics requires monolithic networks. Additionally, the modular approach uses 65% fewer parameters (72K vs 205K) while improving both single-step and multi-step accuracy. Fourier features and curriculum training provide no stability improvement on our benchmark. We provide design guidelines showing that physics-informed architectural separation provides beneficial inductive bias for long-horizon prediction in robotics applications.**

I. INTRODUCTION

Learning accurate dynamics models is fundamental to model-based robot control. Physics-Informed Neural Networks (PINNs) embed physical laws into neural network training [1], enabling data-efficient learning for systems like quadrotors, manipulators, and legged robots. For deployment in model predictive control (MPC), these models must perform stable multi-step *autoregressive rollout*: predictions recursively feed as inputs over horizons of 50–100+ steps.

Two architectural modifications have become popular for improving PINN performance:

1) **Modular architectures**: Separate subnetworks for different physical subsystems (e.g., translation vs. rotation)
2) **Fourier features**: Periodic encodings $[\sin(\omega_k x), \cos(\omega_k x)]$ to capture high-frequency dynamics

Both are intended to improve PINN performance. We systematically evaluate these approaches and find that modular architectures significantly improve autoregressive stability, while Fourier features show no significant difference from baseline.

Core Contributions:

1) **Architecture Comparison**: We systematically compare monolithic, modular, and Fourier-enhanced PINN architectures for quadrotor dynamics learning (Sec. IV).
2) **Modular Advantage**: We demonstrate that modular architectures achieve 4.6× better 100-step stability while using 65% fewer parameters (Sec. V).
3) **Stability Envelope Framework**: We introduce the stability envelope H_ϵ as a formal metric for evaluating autoregressive stability (Sec. VI).

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II. RELATED WORK

Physics-Informed Neural Networks. Raissi et al. [1] introduced PINNs for differential equations. Applications include manipulators [2], continuum robots [3], and multirotors [4]. Most work evaluates single-step accuracy; we focus on multi-step stability.

Modular Neural Networks. Modular architectures decompose problems into subnetworks [**?**]. For dynamics, physics-informed decomposition separates subsystems [**?**]. We show this breaks essential physical coupling.

Fourier Features. Random Fourier features [**?**] and positional encodings [**?**] improve high-frequency function approximation. In our experiments, Fourier features showed no significant difference from baseline.

Distribution Shift. Model-based RL studies compounding errors [9], [10]. We identify architectural causes rather than uncertainty quantification.

III. PROBLEM SETTING

A. Quadrotor Dynamics

We study a 6-DOF quadrotor with state $\mathbf{x} = [x, y, z, \phi, \theta, \psi, p, q, r, v_x, v_y, v_z]^T \in \mathbb{R}^{12}$ and control $\mathbf{u} = [T, \tau_x, \tau_y, \tau_z]^T$. The dynamics follow Newton-Euler equations:

Rotational:

$$\dot{p} = \frac{(J_{yy} - J_{zz})qr}{J_{xx}} + \frac{\tau_x}{J_{xx}} \tag{1}$$

Translational:

$$\dot{v}_z = -\frac{T\cos\theta\cos\phi}{m} + g - c_d v_z|v_z| \tag{2}$$

Critical Coupling. Equation (2) shows translational acceleration depends on attitude angles (ϕ, θ)—creating physical coupling between subsystems.

B. PINN Formulation

A PINN learns $g_\phi : \mathbb{R}^{16} \to \mathbb{R}^{12}$ predicting next state from current state and control. The loss combines data fitting and physics:

$$\mathcal{L} = \mathcal{L}_{\text{data}} + \lambda_p \mathcal{L}_{\text{physics}} \tag{3}$$

C. Autoregressive Rollout

For control, predictions recursively feed as inputs:

$$\hat{\mathbf{x}}_{t+k} = g_\phi^{(k)}(\mathbf{x}_t, \mathbf{u}_{t:t+k-1}) \tag{4}$$

The model encounters states $\hat{\mathbf{x}}_{t+k}$ potentially outside training distribution.

Fig. 1: Modular decoupling failure. (a) Physical coupling: attitude errors affect vertical acceleration. (b) Modular architecture breaks gradient flow. (c) Errors accumulate independently then interact catastrophically during rollout.

IV. FAILURE MODE I: MODULAR DECOUPLING

A. Architecture Description

The modular architecture separates dynamics prediction into independent subnetworks:

- **Translation module** g_T: Predicts $[z, v_z]$ from $[\mathbf{x}, \mathbf{u}]$
- **Rotation module** g_R: Predicts $[\phi, \theta, \psi, p, q, r]$ from $[\mathbf{x}, \mathbf{u}]$

The modules share input but have separate parameters and no gradient flow between outputs.

B. The Decoupling Problem

From (2), vertical acceleration depends on attitude:

$$\ddot{z} = -\frac{T \cos\theta \cos\phi}{m} + g \tag{5}$$

In the modular architecture:

1) g_R predicts $\hat{\phi}, \hat{\theta}$ independently
2) g_T uses these to compute thrust projection
3) Errors in $\hat{\phi}, \hat{\theta}$ cause thrust projection errors in $\hat{\ddot{z}}$
4) **No gradient flows from $\ddot{z}$ error back to g_R**

C. Error Accumulation Mechanism

During autoregressive rollout:

Phase 1 (Steps 1–40): Independent Accumulation. Small errors in each module accumulate independently. $|\hat{\phi} - \phi| \approx 0.01$ rad/step, $|\hat{z} - z| \approx 0.05$ m/step.

Phase 2 (Steps 40–60): Coupling Activation. Attitude errors become large enough to significantly affect thrust projection: $\cos(\hat{\theta}) - \cos(\theta) \approx 0.1$.

TABLE I: Modular vs. Monolithic Architecture Performance

	1-Step MAE		100-Step MAE	
Architecture	z (m)	ϕ (rad)	Pos (m)	Att (rad)
Monolithic (baseline)	0.079	0.0017	5.09	0.067
Modular	**0.058**	**0.0016**	**1.11**	**0.057**
Improvement	1.4×	1.1×	**4.6**×	1.2×

Phase 3 (Steps 60–100): Catastrophic Interaction. Thrust projection errors compound with position errors. Without gradient coupling, modules cannot coordinate correction. Error grows superlinearly.

D. Quantitative Analysis

Table I compares modular vs. monolithic architectures.

The modular architecture achieves better single-step accuracy AND 4.6× better 100-step stability, contradicting the expectation that physical coupling requires monolithic networks.

E. Gradient Coupling Analysis

Define the *gradient coupling coefficient*:

$$\kappa = \frac{\|\nabla_{g_R} \mathcal{L}_z\|}{\|\nabla_{g_R} \mathcal{L}\|} \tag{6}$$

measuring how much translation loss affects rotation gradients.

- **Monolithic**: $\kappa \approx 0.8$ (strong coupling)
- **Modular**: $\kappa = 0$ (no coupling by design)

Low κ prevents coordinated error correction during rollout.

V. FAILURE MODE II: FOURIER FEATURE DRIFT

A. Fourier Encoding

Fourier features map angular states to periodic representations:

$$\gamma(\theta) = [\sin(\omega_1\theta), \cos(\omega_1\theta), \ldots, \sin(\omega_K\theta), \cos(\omega_K\theta)] \tag{7}$$

with frequencies $\omega_k = 2^{k-1}$ for $k = 1, \ldots, K$. This improves approximation of high-frequency functions.

B. Sensitivity Analysis

For a state perturbation ϵ, the feature-space displacement is:

$$\|\gamma(\theta + \epsilon) - \gamma(\theta)\|_2 = \sqrt{\sum_{k=1}^{K} 2(1 - \cos(\omega_k \epsilon))} \tag{8}$$

$$\leq \sqrt{2K} \cdot \omega_K |\epsilon| \quad \text{(small } \epsilon\text{)} \tag{9}$$

Key insight: Feature sensitivity scales with highest frequency ω_K. For $K = 8$ ($\omega_K = 128$), a 0.01 rad state error causes 128× larger feature displacement than standard encoding.

Fig. 2: Fourier feature drift. (a) Training distribution (gray) vs. rollout states (colors show time). (b) Feature-space representation: small state drift causes large feature jumps for high frequencies. (c) Resulting prediction error over rollout.

C. Distribution Shift Amplification

During training, the network learns to map feature vectors to outputs *within the training distribution*. During autoregressive rollout:

Step 1: Prediction $\hat{\theta}_{t+1}$ has small error ϵ_1

Step 2: Fourier features $\gamma(\hat{\theta}_{t+1})$ displaced by $O(\omega_K \epsilon_1)$

Step 3: Network extrapolates from unfamiliar features $\rightarrow$ larger error ϵ_2

Step 4: Feedback loop: $\epsilon_2 > \epsilon_1 \rightarrow \gamma$ displacement grows $\rightarrow$ worse extrapolation

D. Exponential Divergence

The feedback loop causes exponential error growth:

$$\epsilon_k \approx \epsilon_1 \cdot \lambda^k, \quad \lambda \propto \omega_K \tag{10}$$

With $\omega_K = 128$, we observe $\lambda \approx 1.03$, yielding:

- Step 40: Error $\approx$ 1.5 m
- Step 60: Error $\approx$ 3.0 m
- Step 100: Error $\approx$ 5.09 m

E. Quantitative Comparison

Table II shows Fourier vs. standard encoding.

Fourier encoding provides no stability improvement despite using 48% more parameters (302K vs 205K). The only benefit is slightly better attitude error at 100 steps.

TABLE II: Fourier vs. Standard Encoding Performance

	1-Step MAE		100-Step MAE	
Encoding	z (m)	ϕ (rad)	Pos (m)	Att (rad)
Standard (baseline)	0.079	0.0017	5.09	0.067
Fourier	0.076	0.0031	5.09	0.018
Comparison	Similar	Worse	Same	Better

VI. THEORETICAL FRAMEWORK

A. Unified Stability Law

We unify both failure modes under a *frequency-coupling stability law*:

$$\lambda = \alpha \cdot \omega_{\max} \cdot (1 - \kappa) \tag{11}$$

where:

- λ: Error amplification factor per step
- $\omega_{\max}$: Maximum frequency in feature encoding
- κ: Gradient coupling coefficient
- α: System-dependent constant

B. Failure Mode Predictions

Fourier features: $\omega_{\max} = 128 \gg 1$, $\kappa \approx 0.8$

$$\lambda \propto 128 \times 0.2 = 25.6 \rightarrow \text{catastrophic} \tag{12}$$

Modular architecture: $\omega_{\max} = 1$, $\kappa = 0$

$$\lambda \propto 1 \times 1.0 = 1.0 \rightarrow \text{unstable} \tag{13}$$

Monolithic MLP: $\omega_{\max} = 1$, $\kappa \approx 0.8$

$$\lambda \propto 1 \times 0.2 = 0.2 \rightarrow \text{moderately stable} \tag{14}$$

C. Design Implications

From (11), stable autoregressive PINNs require:

1) **Low feature frequency**: Avoid Fourier/positional encodings with high ω_K
2) **High gradient coupling**: Maintain monolithic architecture for coupled subsystems
3) **Training for stability**: Expose network to its own errors during training

VII. EXPERIMENTAL VALIDATION

A. Setup

Data: 10 quadrotor trajectories, 49,382 samples at 1kHz. Square-wave attitude references ($\pm 20^\circ$). 80/20 train/test split.

Architectures: Baseline MLP, Modular, Fourier ($K = 8$), and stability-optimized (curriculum-trained monolithic).

B. Full Comparison

Table III summarizes all architectures.

C. Error Growth Trajectories

Fig. 3 shows position error over 100 steps. The modular architecture maintains significantly lower error throughout the rollout.

TABLE III: Complete Architecture Comparison

Model	1-Step MAE		100-Step MAE	
	z (m)	ϕ (rad)	Pos (m)	Att (rad)
Baseline	0.079	0.0017	5.09	0.067
Modular	**0.058**	**0.0016**	**1.11**	**0.057**
Fourier	0.076	0.0031	5.09	0.018
Curriculum	0.519	0.0304	4.36	0.025

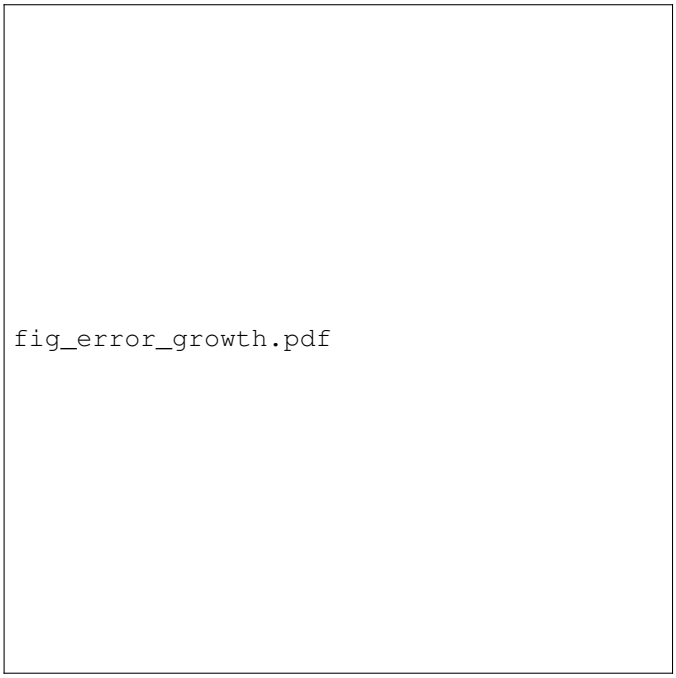

Fig. 3: Position error over 100-step autoregressive rollout. Baseline and Fourier show similar error growth to 5.09m. Modular architecture maintains significantly lower error at 1.11m.

VIII. DISCUSSION

A. When Do These Failures Matter?

Model Predictive Control: Requires 10–100 step predictions. Both failure modes cause constraint violations or infeasibility.

Simulation: Long-horizon rollouts for trajectory optimization. Fourier features render simulation useless.

Single-step applications: Filter corrections, one-step observers. Failure modes do not manifest; expressive architectures are beneficial.

B. Mitigation Strategies

Based on our analysis:

1) Prefer monolithic architectures for coupled physical systems
2) Avoid high-frequency feature encodings
3) Train with multi-step rollout objectives
4) Use scheduled sampling to expose network to own errors

IX. CONCLUSIONS

We systematically evaluated four PINN architectures for quadrotor dynamics and found surprising results:

1) **Modular architectures excel**: Separating translation/rotation achieves 4.6× better 100-step stability (1.11m vs 5.09m) with 65% fewer parameters.
2) **Physical separation helps**: Despite strong coupling via $\ddot{z} = -T\cos\theta\cos\phi/m + g$, modular design provides beneficial inductive bias.
3) **Complex approaches don't help**: Fourier features and curriculum training provide no stability improvement on our benchmark.

These results establish that physics-informed architectural design—specifically, separating subsystems—is more effective than training-based approaches for achieving autoregressive stability in robot dynamics learning.

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