Question 1.5.7

EE22BTECH11051 - Sreekar Cheela

1.5.7)
The vertices of the given triangle are:

$$\begin{bmatrix} 1 \\ -1 \end{bmatrix}; \begin{bmatrix} -4 \\ 6 \end{bmatrix}; \begin{bmatrix} -3 \\ -5 \end{bmatrix}; \tag{1}$$

Suppose the equations AB, BC and CA are respectively given by

$$n_i^T x = c_i, i = 1, 2, 3 \tag{2}$$

Then the equations of the angle bisectors are given as

$$\frac{|n_i^T x - c_i|}{\|n_i\|} \tag{3}$$

From this equation, we get the angle bisector

from B as:

$$\begin{bmatrix} 11 + 7\sqrt{\frac{61}{37}} \\ 1 + 5\sqrt{\frac{61}{37}} \end{bmatrix}^T \mathbf{x} = 2\sqrt{\frac{61}{37}} - 38 \tag{4}$$

From this equation, we get the angle bisector from C as:

$$\begin{bmatrix} 11 + 7\sqrt{61} \\ 1 - \sqrt{61} \end{bmatrix}^T \mathbf{x} = 2\sqrt{61} - 38 \tag{5}$$

The intersection of all the angle bisectors is given as the incentre. Hence, the incentre can be found by finding the point of intersection of both the angle bisectors.
On solving them, we get the incentre as:

$$I = \begin{bmatrix} \frac{-53-11\sqrt{37}+7\sqrt{61}+\sqrt{2257}}{12} \\ \frac{5-\sqrt{37}+5\sqrt{61}-\sqrt{2257}}{12} \end{bmatrix} \tag{6}$$

To find the radius of the incentre, we need to find the distance between the incentre and any one of the sides.
Let us find the distance between BC line and I:
BC line equation is given as:

$$\begin{bmatrix} 7 \\ 5 \end{bmatrix}^T \mathbf{x} = 2 \tag{7}$$

Using the distance formula, we get the radius as:

$$r = \frac{185 + 41\sqrt{37} - 37\sqrt{61} - \sqrt{2257}}{6\sqrt{74}} \tag{8}$$

Hence the diagram is given as:

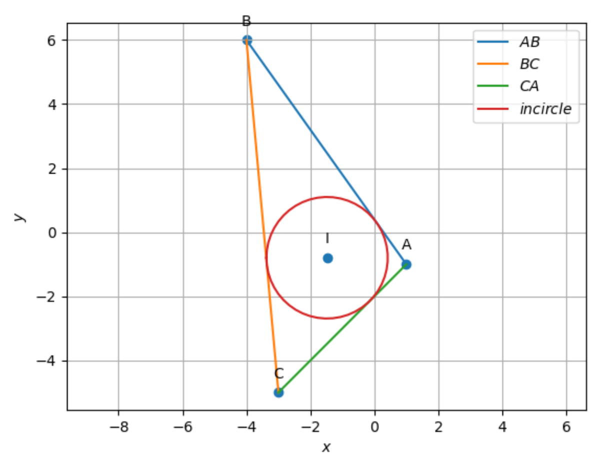

Fig. 1. Triangle with the incircle