

# Question 41.2023

EE22BTECH11051

## Question:

Suppose that  $X_1, X_2, \dots, X_{10}$  are independent and identically distributed random vectors each having  $N_2(\mu, \Sigma)$  distribution, where  $\Sigma$  is non-singular. If

$$U = \frac{1}{1 + (\bar{X} - \mu)^T \Sigma^{-1} (\bar{X} - \mu)}, \quad (1)$$

where  $\bar{X} = \frac{1}{10} \sum_{i=1}^{10} X_i$  then the value of  $\log_e P(U \leq \frac{1}{2})$  equals

- 1) -5
- 2) -10
- 3) -2
- 4) -1

## Solution:

It has been given that:

$$X_i \sim N_2(\mu, \Sigma) \quad (2)$$

$$= \bar{X} \sim N_2\left(\mu, \frac{\Sigma}{n}\right) \quad (3)$$

Hence;

$$\frac{(\bar{X} - \mu)^2}{\left(\frac{\Sigma}{n}\right)} \sim \chi_2^2 \quad (4)$$

we can now write this as;

$$n(\bar{X} - \mu)^T \Sigma^{-1} (\bar{X} - \mu) \sim \chi_2^2 \quad (5)$$

Let Y be;

$$Y = n(\bar{X} - \mu)^T \Sigma^{-1} (\bar{X} - \mu) \quad (6)$$

Now;

$$P\left(U \leq \frac{1}{2}\right) = P\left(\frac{1}{1 + \frac{Y}{n}} \leq \frac{1}{2}\right) \quad (7)$$

$$= P\left(\frac{Y}{n} \geq 1\right) \quad (8)$$

$$= P(Y \geq 10) \quad (9)$$

$$= \int_{10}^{\infty} \frac{1}{2} e^{-\frac{y}{2}} dy \quad (10)$$

$$= e^{-5} \quad (11)$$

Hence;

$$\log_e P(U \leq \frac{1}{2}) = -5 \quad (12)$$