Gaussian 9.3.2

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Question: There are 5% defective items in a large bulk of items. What is the probability that a sample of 10 items will include not more than one defective item?
Solution: Let X be random variable defined as

Parameter	Values	Description
n	10	Number of articles
p	0.05	Probability of being defective
X	$1 \leq X \leq 10$	X defective elements out of 10
Y	$1 \leq Y \leq 10$	gaussian variable
$\mu = np$	0.5	mean
$\sigma = \sqrt{np(1-p)}$	0.475	standard deviation

1) Binomial Distribution

$$n = 10; p = \frac{1}{20} \tag{1}$$

Pmf of X for $0 \leq k \leq 10$ is

$$p_X(k) = {}^nC_k p^k (1-p)^{n-k} \tag{2}$$

Then the probability that in our sample space of 10 items not more than one are defective is given as:

$$p_X(0) + p_X(1) = {}^{10}C_0 \left(\frac{1}{20}\right)^0 \left(1 - \frac{1}{20}\right)^{10} + {}^{10}C_1 \left(\frac{1}{20}\right)^1 \left(1 - \frac{1}{20}\right)^9 \tag{3}$$

Hence we get;

$$p_X(0) + p_X(1) = 29 \left(\frac{19^9}{20^{10}}\right) = 0.91386 \tag{4}$$

2) Gaussian Distribution
Let Y be our gaussian variable. Now using Central Limit Theroem, we can use the gaussian distribution function, which is given as:

$$p_Y(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \tag{5}$$

Hence the probability of the sample not including more than one defective item is given as;

$$\int_{-\infty}^{1} \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} dx = 0.765917 \tag{6}$$

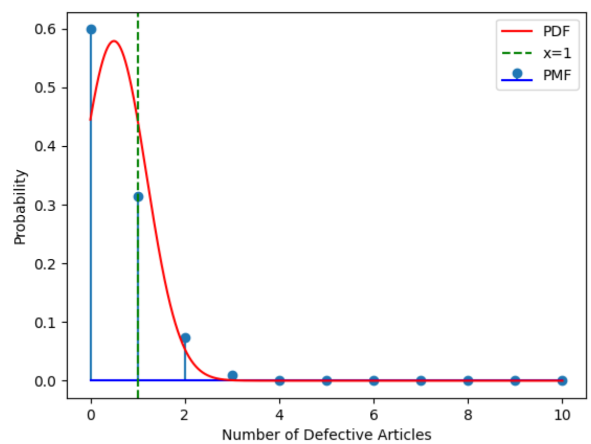

Fig. 2. Binomial-PMF and Gaussian-PDFof X