AN INTRODUCTION TO KNOT THEORY AND THE JONES POLYNOMIAL

Project report submitted to

COCHIN UNIVERSITY OF SCIENCE AND TECHNOLOGY

by

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28 June, 2022

CERTIFICATE

This is to certify that the project titled **"AN INTRODUCTION TO KNOT THEORY AND THE JONES POLYNOMIAL"** that is being submitted by Ms. SREELAKSHMI T to Cochin University of Science and Technology, Cochin-682022, is a bonafide record of study carried out by her under my supervision and guidance at the Department of Mathematics, Cochin University of Science and Technology during 2021-22. This project has not been submitted anywhere for any other degree of any other University or Board.

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Declaration

I, SREELAKSHMI T, hereby declare that this project report entitled "**AN INTRODUCTION TO KNOT THEORY AND THE JONES POLYNOMIAL**"that is being submitted to Cochin University of Science and Technology is a bonafide record of work done by me under the supervision of Dr. TATHAGATA BANERJEE. I also declare that this project has not been submitted for any other degree elsewhere.

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Acknowledgement

First and foremost, I thank GOD ALMIGHTY for all blessings he bestowed on me during the course of our work. I praise him for his great wisdom and guidance throughout the endeavour.

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Chapter 1

INTRODUCTION

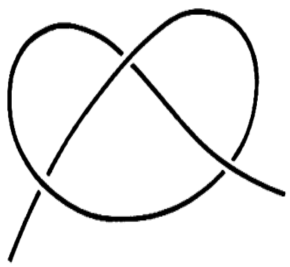
We have seen many knots in our day to day life which we use for different kind of purposes like tying, climbing, for making many things....The study of mathematical theory of knots as now referred to as Knot Theory can be traced back to the 19th century when the German Mathematician, Carl Friedrich Gauss created a method for tabulation of knots. Gauss applied the mathematical concept of knots for his work in electro dynamics. He wanted to know how much work was done on a magnetic pole along a closed curve in the presence of a loop of current.

Knot theory is a very interesting field of mathematics which combines many others, such as geometry, topology, and linear algebra.It is a study of mathematical knots.

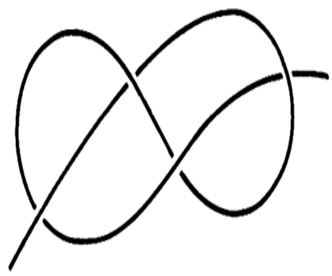
Chapter 2

BASIC CONCEPTS IN KNOT THEORY

We are familiar with commonly used knots like overhand knot and figure eight knot given below.



(a) Overhand knot



(b) Figure eight knot

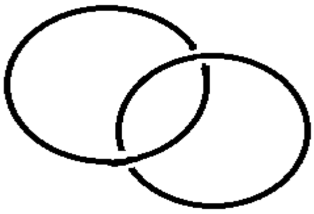
If we tie the ends of this string, it will become a loop.And such a knotted loop is a mathematical knot.And collection of such knots which do not intersect will give mathematical link.

2.1 Links and Knots

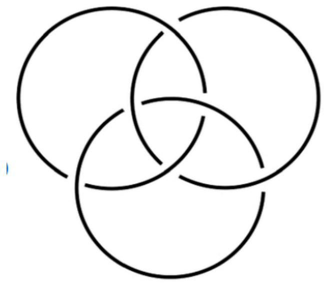
A link L of m components is a subset of S^3, or of $\mathbb{R}^3$, that consists of m disjoint, piecewise linear,simple closed curves.
A link of one component is a knot.
The piecewise linear condition means that the curves composing L are each made up of finite number of line segments placed end to end, straight being in linear structure of $\mathbb{R}^3 \subset \mathbb{R}^3 \cup \infty = S^3$.

- In other words , a knot is an embedding of S^1 into $\mathbb{R}^3$ or S^3.

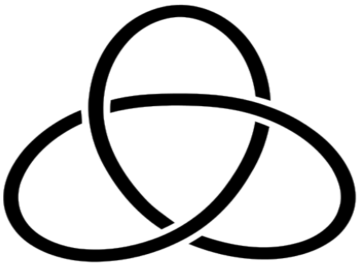
For example:



(a) Hopf link



(b) Borromean ring(link with two components)



(c) Trefoil knot(link with one component)

(d) Figure eight knot(listing knot)

→ A link (knot) can be represented using projection as in the above figures.Image of a link L in $\mathbb{R}^2$ together with “over and under” information at the crossings is called a **link diagram** of L.
.

→ We can assign orientation to a link (knot) and the orientation is denoted using arrow in the *link diagram*.Link with orientation is oriented link.A knot can have at most two orientations.

2.2 Equivalent links

Links L_1 and L_2 in S^3 are equivalent if there is an orientation preserving piecewise linear homeomorphism $h : S^3 \to S^3$ such that $h(L_1) = h(L_2)$.

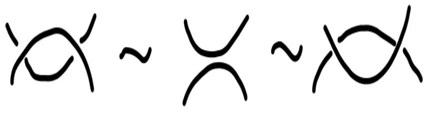
NOTE
Two links are equivalent if one can be transformed into other via deformation.There are different ways to change a link into an equivalent one.

2.2.1 Reidemeister moves

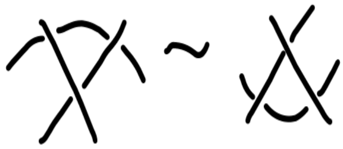
One knot can be transformed into an equivalent knot by Reidemeister moves.There are three type of Reidemeister moves.

- Type I
  It adds or removes a kink in the diagram.

- Type II
  Adding or removing two crossings.

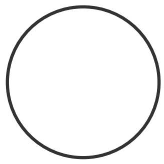


- Type III
  This preserves the number of crossings.



Unknot

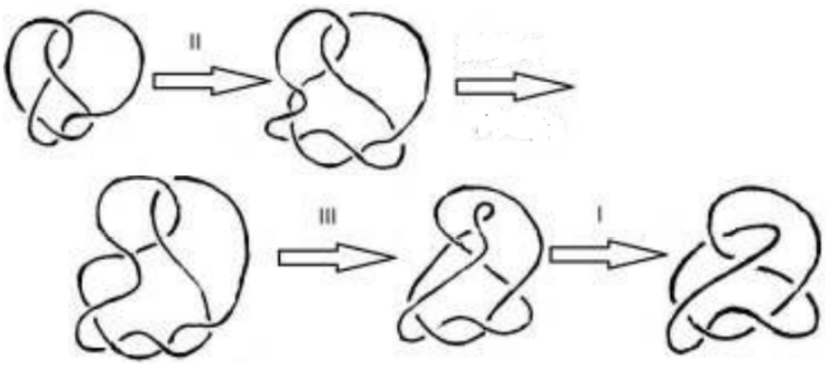
A knot is said to be unknot or trivial knot if it is boundary of an embedded disc in S^3.It is least knotted of all knots. It has a diagram with no crossings at all.



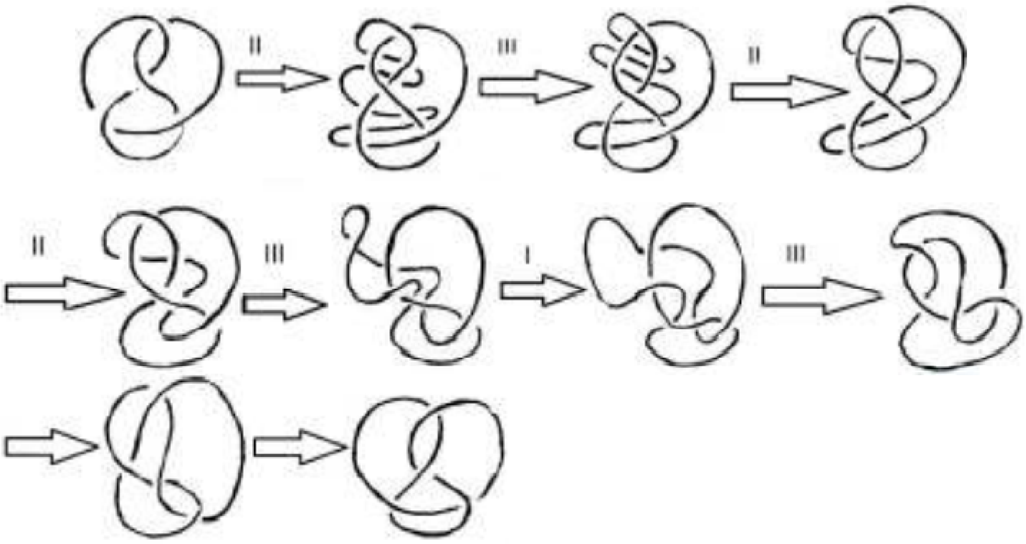
There are other projections equivalent to this figure of unknot.

Examples for equivalent knot projections

Example 1:



Example 2:



2.3 Regular Isotopy

Two unoriented link diagrams are said to be *regular isotopic* if they can be converted from one to another by Reidemeister moves of types II and III.

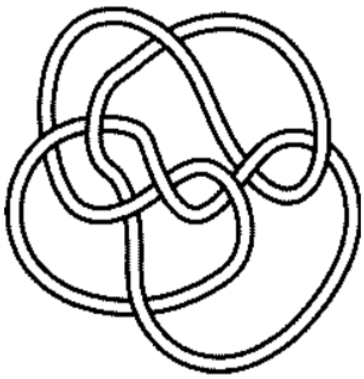
2.4 Reverse of a knot

If K is an oriented knot, then reverse of K is by denoted rK is the same knot as a set but with the other orientation.
Often K and rK are equivalent.

2.5 Reflection of L

If L is a link in S^3 and $\rho : S^3 \to S^3$ is an orientation-reversing piecewise linear homeomorphism, then $\rho(L)$ is called the *obverse* or *reflection* of L. $\rho(L)$ is denoted by $\overline{L}$.

- We can choose ρ to be the map $(x, y, z) \mapsto (x, y, -z)$. Then the diagram of $\overline{L}$ is same as that of L but all over-passes changed to under-passes. L and $\overline{L}$ may or may not be equivalent.
- There do exist oriented knots for which K, rK, $\overline{K}$ and $\overline{rK}$ are four distinct oriented knots.
  For example the given knot with 9 crossings.



2.6 Chiral knots

A knot is Chiral if it is not equivalent to its mirror image.
Otherwise it is called Amphichiral or Achiral.

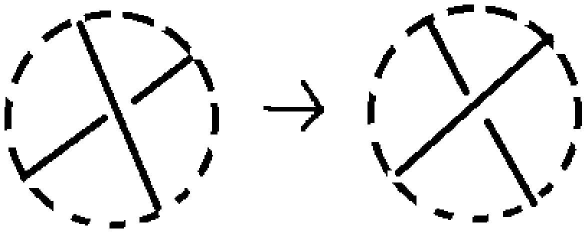
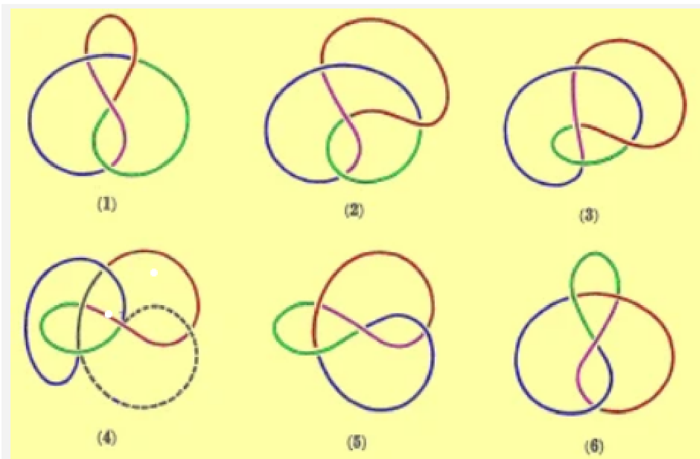


Figure 2.3: Mirror image

Trefoil knot is chiral whereas Figure eight knot and unknot are achiral.



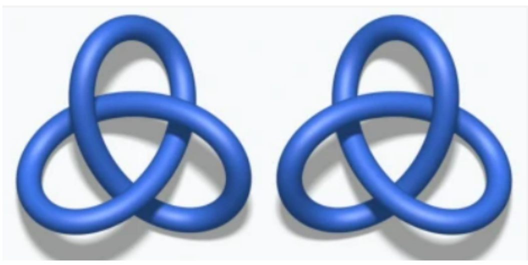
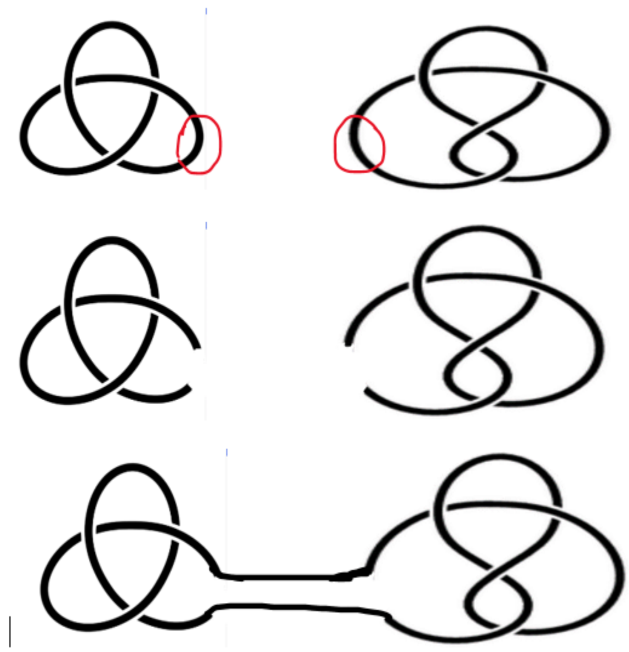


Figure 2.4: left-handed trefoil and right-handed trefoil

2.7 Sum of knots

Two oriented knots K_1 and K_2 can be added together to get their sum K_1+ K_2. We compose knots by tying one and the other to same piece of string.Their sum is also known as composition of knots or connected sum.



→The Unknot is a zero for this addition

2.8 Prime knot

A knot K is a *prime knot* if it is not the unknot, and K=K_1 + K_2 implies that K_1 or K_2 is the unknot.

- If it is not prime, then it is called composite knot.
- Trefoil knot is a prime knot.

Chapter 3

KNOT INVARIANTS

To differentiate knots we need invariants.
Parameters of knots which are not affected by Reidemeister moves are ***knot invariants***(Unchanging characters of knots).Knot invariant is same for equivalent knots.
For example:

- Tricolorability
- Crossing number
- Unknotting number
- Linking number etc..

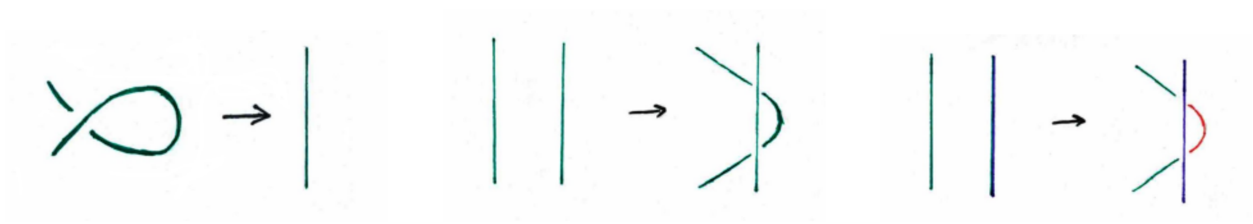
3.1 Tricolorability

A knot is tricolorable if each of the strands of the knot can be colored in one of three different colors such that:

1. Atleast two colors must be used
2. Incident crossing strands are all of either same color or different color.

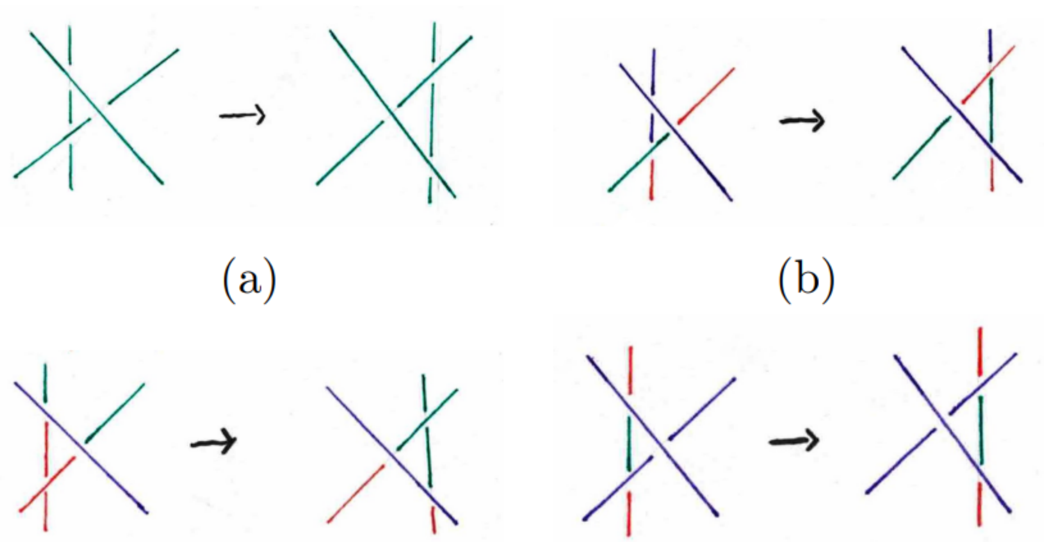
This definition is strengthened as follows:
That is, a knot diagram D is tricolored if each string is colored in one of three colors and at each crossing either all three colors or only one color meet.

We call it a trivial coloring if only one color is used.
This is a knot invariant.To see that consider the coloring of reidemeister moves.
In Type I move only one color is used in both twist and untwist.
In Type II move tricolorability is an inavriant.



(a) (b) (c)

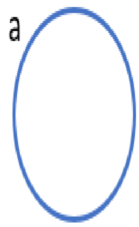
In Type III move, as we can see in the figure tricolorability is an invariant.

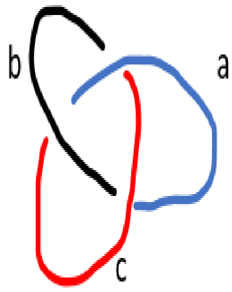


(a) (b)

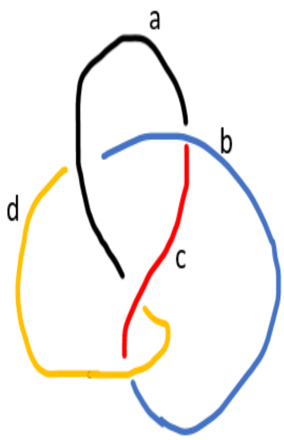
(c) (d)

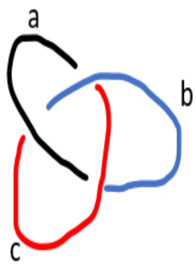
Example 1:
Trefoil knot is tricolorable but unknot is not. Therefore, Trefoil is a non-trivial knot.
That is; we cannot deform trefoil knot into an unknot unless we cut it.





Example 2:





Trefoil knot and Figure eight knot are not equivalent.

3.2 Crossing number

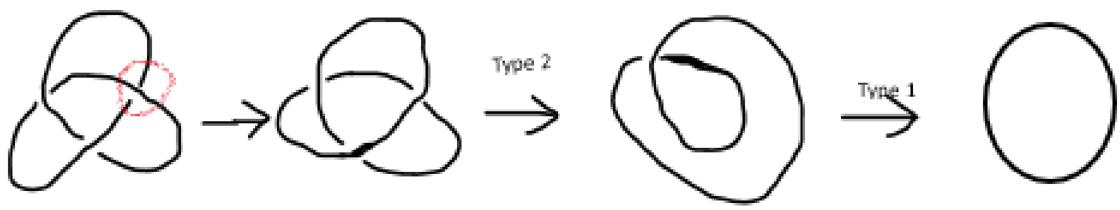
Crossing number of a knot is the minimal number of crossings needed for a diagram of the knot.This is a knot invariant.
That is; knots with different crossing numbers cannot be equivalent because it is defined in terms of a minimum taken over the infinity of all possible diagrams of a knot.

- There is a table containing diagrams of all prime knots by neglecting orientation with crossing number atmost 8.There are 35 such knots omitting unknot.
- A notation like 5_2 means it is the second knot in the traditional ordering of knots with crossing number 5.
- Crossing number is very difficult to calculate because we have to consider every posssible diagrams of the knot.

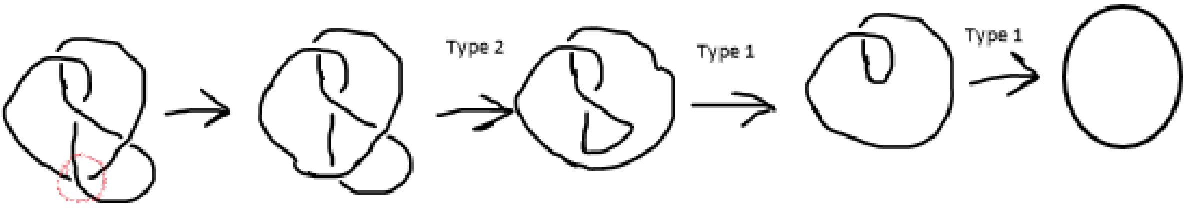
3.3 Unknotting number

Unknotting number of a knot K is denoted by $u(K)$.It is the minimum number of crossing changes (from 'over'to 'under'and vice versa) needed to change K to the unknot , where the minimum is taken over all possible sets of crossing changes in all possible diagrams of K.
$\star$This is very difficult to calculate.
$\star$ If a given knot K is not an unknot, but that one crossing changes on some diagram does give the unknot, then definitely $(u)K) = 1$.

Example 1:
For torus knot one cross change in this figure gives the unknot.
Thus u(torus knot) $= 1$



Example 2:
Similarly, one cross change in the given figure gives unknot.
Therefore, u(figure eight knot) = 1



3.4 Alternating knot

Suppose the "over" and "under" nature of crossings alternates as one travels along a knot diagram.If a knot has such diagram, then it is called *alternating knot*.

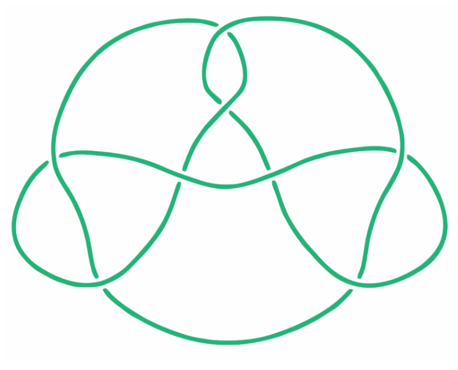


Figure 3.1: Simplest non alternating prime knot has 8 crossings

NOTE
A crossing in a diagram of an oriented link can be allocated a sign.The crossing is said to be positive or negative or to have sign $+1$ or -1. The standard convention is given in the figure :

The convention uses orientations of both strands appearing at the crossing and also the orientation of space.A positive crossing shows one strand (either strand) passing the other in the manner of a "right-hand screw".For a knot, the sign of a crossing does not depend on the knot orientation chosen,for reversing orientations of both strands at a crossing leaves the sign unchanged.

3.5 Linking number

Suppose that L is a two- component oriented link with components L_1 and L_2. The linking number $lk(L_1, L_2)$ of L_1 and L_2 is half the sum of the signs, in the diagram for L, of the crossings at which one strand is from L_1 and the other is from L_2.

→ Finding Linking number of some Links

(1) Linking number $= (1+1)/2 = 1$

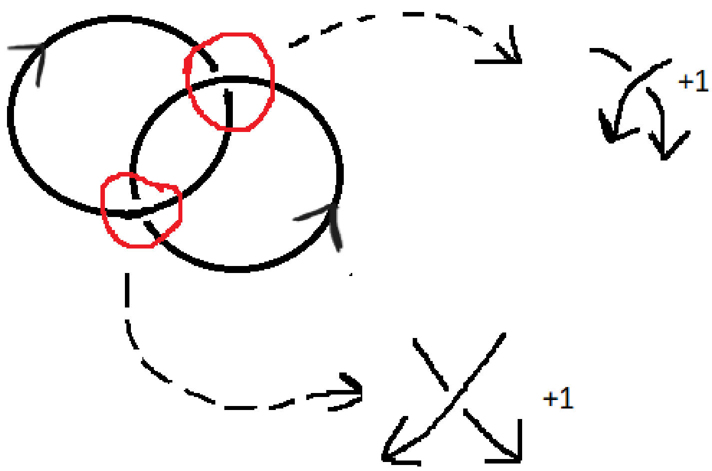


Figure 3.2

(2) Linking number $= (-1-1)/2 = -1$

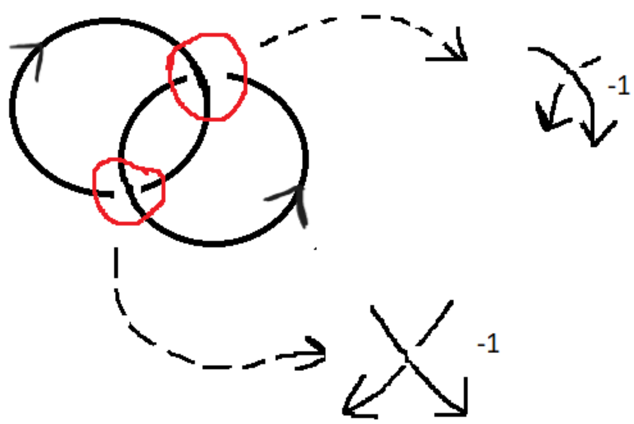


Figure 3.3

(3) Linking number $= (-1-1-1-1)/2 = -2$

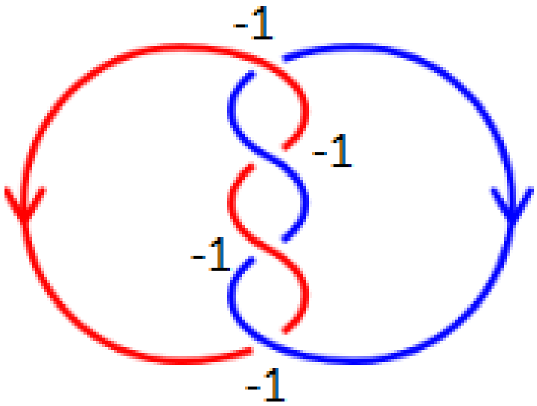


Figure 3.4

(4) Linking number $= 0$

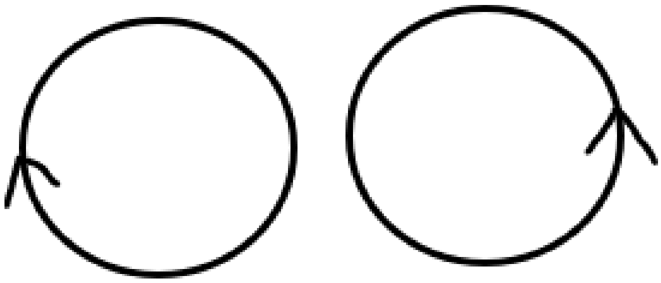


Figure 3.5

Now we consider links as boundary of some surface embedded in S^3.And such surfaces are used for studying links in different ways such that they can be used to decompose knots into sum of prime knots.

3.6 Pretzel link

Pretzel link consists of a finite number tangles made of two intertwined circular helices and is denoted by $P(a_1, a_2, a_3, ...a_n)$ where each a_i is the integer indicating the number of crossings in the various tangles of the diagram

- a_i is positive if they are counter-clockwise or left-handed
- a_i is negative if if clockwise or right-handed

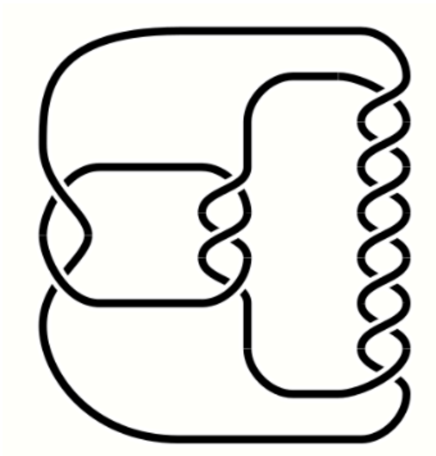


Figure 3.6: (-2,3,7)

If pretzel link is also a knot, then it is pretzel knot.

3.7 Seifert Surfaces

A Seifert surface for an oriented link L in S^3 is a connected compact oriented surface contained in S^3 that has L as its oriented boundary.

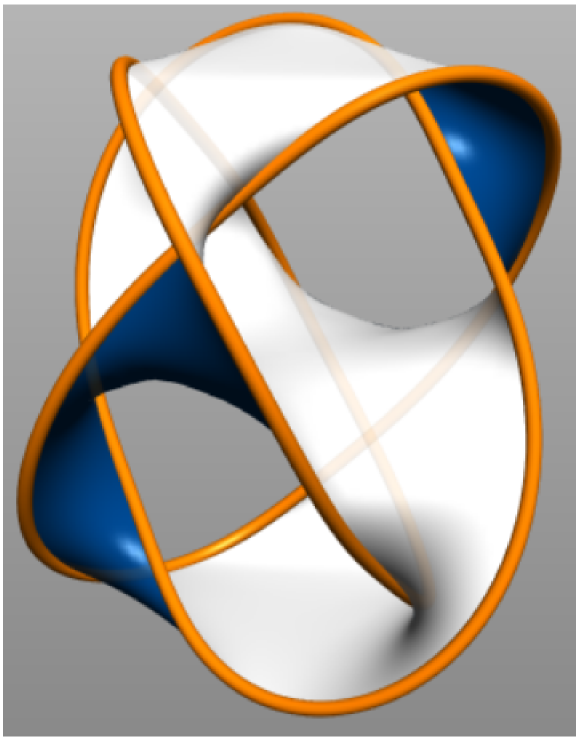


Figure 3.7: Seifert surface of Borromean ring

Theorem 3.7.1. *Any oriented link in S^3 has a Seifert surface*

Chapter 4

THE JONES POLYNOMIAL

4.1 Kauffman Bracket

The Kauffman bracket is a function from unoriented link diagrams in the oriented plane to laurent polynomials with integer coefficients in an indeterminate A. It maps a diagram to D to $\langle \mathrm{D} \rangle \in \mathbb{Z}[A^{-1}, A]$ and is characterised by

(i) $\langle \bigcirc \rangle = 1$

(ii) $\langle D \ \sqcup \ \bigcirc \rangle = (-A^{-2} - A^{2})\langle D \rangle$

(iii) $\langle$  $\rangle = A\langle$  $\rangle + A^{-1}\langle$  $\rangle$.

- Bracket polynomial of a diagram with n crossings can be calculated by expressing it as linear sum of 2^n diagrams with no crossings using (iii)
- Any diagram with C components and no crossing has (i) and (ii)

$$
\begin{aligned}
\langle\underbrace{\bigcirc\ \bigcirc\ \bigcirc.....\bigcirc}_{c}\rangle &= \langle\underbrace{(\bigcirc\ \bigcirc\ \bigcirc.....\bigcirc)}_{c-1}\rangle \sqcup \langle\bigcirc\rangle \\
&= (-A^{-2}-A^{2})\langle\underbrace{(\bigcirc\ \bigcirc\ \bigcirc.....\bigcirc)}_{c-1}\rangle \\
&= (-A^{-2}-A^{2})^{2}\langle\underbrace{(\bigcirc\ \bigcirc\ \bigcirc.....\bigcirc)}_{c-2}\rangle
\end{aligned}
$$

$$
\Longrightarrow \langle\underbrace{\bigcirc\ \bigcirc\ \bigcirc.....\bigcirc}_{c}\rangle = (-A^{-2}-A^{2})^{c-1}
$$

- D and $\bar{D}$ be the diagram of an oriented link and its reflection. Then $\langle\bar{D}\rangle = \overline{\langle D\rangle}$

Lemma 4.1.1. *If a diagram is changed by a Type 1 Reidemeister move, its bracket polynomial changes in the following way:*

$\langle$  $\rangle = -A^{3}\langle$  $\rangle$

$\langle$  $\rangle = -A^{-3}\langle$  $\rangle$

Proof.

$\langle$  $\rangle = A\langle$  $\rangle + A^{-1}\langle$  $\rangle$

$= A[(-A^{-2}-A^{2})]\langle$  $\rangle + A^{-1}\langle$  $\rangle$

$= (-A^{-1}-A^{3}+A^{-1})\langle$ $\rangle$

$= -A^{3}\langle$ $\rangle$

$\langle$  $\rangle = A\langle$  $\rangle + A^{-1}\langle$  $\rangle$

$= [A + A^{-1}(-A^{-2} - A^{2})]\langle$ $\rangle$

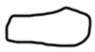
$= (A - A^{-3} - A)\langle$ $\rangle$

$= -A^{-3}\langle$ $\rangle$

$\square$

Examples of finding Kauffman bracket of links

Example 1:

$\langle$  $\rangle = A\langle$  $\rangle + A^{-1}\langle \bigcirc\bigcirc \rangle$

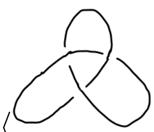
$$\begin{aligned} &= A + A^{-1}(-A^{-2} - A^{2}) \\ &= A - A^{-3} - A \\ &= -A^{-3} \end{aligned}$$

Example 2:

$\langle$  $\rangle = A\langle$  $\rangle + A^{-1}\langle$  $\rangle$

$$\begin{aligned} &= A(-A^{3}\langle \bigcirc \rangle) + A^{-1}(-A^{-3}\langle \bigcirc \rangle) \\ &= (-A^{4} - A^{-4})\langle \bigcirc \rangle \\ &= -A^{4} - A^{-4} \end{aligned}$$

Example 3:

$\langle$  $\rangle = A\langle$  $\rangle + A^{-1}\langle$  $\rangle$

$= A(-A^{4} - A^{-4}) + A^{-1}(-A^{-3}\langle$  $\rangle)$

$$\begin{aligned} &= A(-A^{4} - A^{-4}) + -A^{-4}(-A^{-3}\langle \bigcirc \rangle) \\ &= (-A^{5} - A^{-3} + A^{-7})\langle \bigcirc \rangle \\ &= A^{-7} - A^{5} - A^{-3} \end{aligned}$$

Lemma 4.1.2. *If a diagram D is changed by a Type 2 or Type 3 Reidemeister move, then $\langle D \rangle$ does not change. That is,*

(i) $\langle$  $\rangle = \langle$  $\rangle$

(ii) $\langle$  $\rangle = \langle$  $\rangle$

Hence $\langle D \rangle$ is invariant under regular isotopy of D.

Proof. (i)

$\langle$  $\rangle = A\langle$  $\rangle + A^{-1}\langle$  $\rangle$

$= -A^{-2}\langle$  $\rangle + \langle$  $\rangle + A^{-2}\langle$  $\rangle$

$= \langle$  $\rangle$

(ii)

$\langle$  $\rangle = A\langle$  $\rangle + A^{-1}\langle$  $\rangle$

$= A\langle$  $\rangle + A^{-1}\langle$  $\rangle$

$= A\langle$  $\rangle + A^{-1}\langle$  $\rangle$

$= \langle$  $\rangle$

Here the second line follows from the first by using (i)

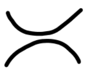
□

4.2 Writhe

The writhe $w(D)$ of an *oriented* link is the sum of the signs of the crossings of D, where each crossing has a sign $+1$ or -1.

NOTE

- $w(D)$ does not change if D is changed under a Type II or Type III Reidemeister move.

  $w($  $) = 0 = -1 + 1 = w($  $)$

- $w(D)$ does change by +1 or -1 if D is changed by a Type I Reidemeister move.
  Writhe is not a knot invariant.

Theorem 4.2.1. *Let D be diagram of an oriented link L. Then the expression*

$$(-A)^{-3w(D)}\langle D\rangle$$

is an invariant of an oriented link L.

4.3 The Jones Polynomial

The Jones Polynomial $V(L)$ of an oriented link L is the Laurent Polynomial in $t^{1/2}$, with integer coefficients, defined by

$$V(L) = \big((-A)^{-3w(D)}\langle D\rangle\big)_{t^{1/2}=A^{-2}} \in \mathbb{Z}[t^{-1/2}, t^{1/2}],$$

where D is any oriented diagram for L

- Here $t^{1/2}$ is just an indeterminate the square of which is t.
- From Theorem 4.2.1, it is known that $(-A)^{-3w(D)}\langle D\rangle$ is an invariant. This shows Jones polynomial is a knot invariant.
- For unknot, since there is no crossing at all $w(D) = 1$
  Therefore,
  $$\begin{aligned} V(unknot) &= V(\bigcirc) \\ &= (-A^0\langle\bigcirc\rangle) \\ &= 1 \end{aligned}$$
- If the orientation of every component of a link is changed, then the sign of each crossing does not change. Thus, Jones polynomial of a knot does not depend upon the orientation chosen for the knot.
- If $\bar{L}$ is the reflection of the oriented link L, then $V(\bar{L})$ is obtained from L by interchanging $t^{-1/2}$ and $t^{1/2}$
- $V(K_1 + K_2) = V(K_1)V(K_2)$
- Jones Polynomial is same for equivalent links.

The Jones Polynomial is characterised by the following proposition, which follows easily from the above definition (though historically it preceeded the definition).

Proposition. *The Jones Polynomial invariant is a function*

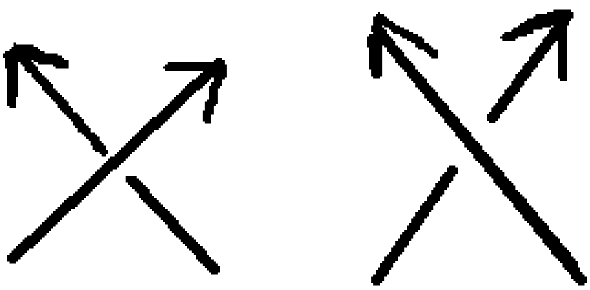
$$V : \{\textit{Oriented links in } S^3\} \to \mathbb{Z}[t^{-1/2}, t^{1/2}]$$

such that

(i) $V(unknot) = 1$

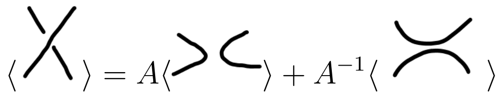
(ii) whenever three oriented links $L_+, L_-, andL_0$ *are the same, except in the neighbourhood of a point where they are as shown in the Figure , then*

$$t^{-1}V(L_+) - tV(L_-) + (t^{-1/2} - t^{1/2})V(L_0) = 0$$

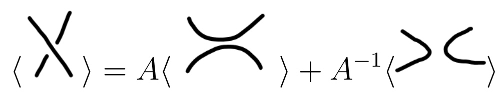


(a) L_+ (b) L_- (c) L_0

Proof.



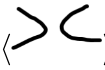
(1)



(2)

Multiplying equation (1) by A and (2) by A^{-1} gives

$A\langle$  $\rangle = A^2\langle$  $\rangle + \langle$  $\rangle$ (3)

$A^{-1}\langle$  $\rangle = \langle$  $\rangle + A^{-2}\langle$  $\rangle$ (4)

$(3) - (4) \implies$

$A\langle$  $\rangle - A^{-1}\langle$  $\rangle = (A^2 - A^{-2})\langle$  $\rangle$

We are given three oriented links L_+, L_-, and L_0, which are same except in the neighbourhood of a point where they are shown in the figure.So while calculating the writhe every other crossings are same.
So we obtain

$$w(L_+) - 1 = w(L_-) + 1 = w(L_0)$$

$$V(L_+) = \big((-A)^{-3w(L_+)}\langle L_+\rangle\big)_{t^{1/2}=A^{-2}}$$

$$= (-A)^{-3(w(L_0)+1)}\langle L_+\rangle$$

$$V(L_-) = \big((-A)^{-3w(L_-)}\langle L_-\rangle\big)_{t^{1/2}=A^{-2}}$$

$$= (-A)^{-3(w(L_0)-1)}\langle L_-\rangle$$

$$
\begin{aligned}
-A^4V(L_+) + A^{-4}V(L_-) &= (-A^4)(-A)^{-3(w(L_0)+1)}\langle L_+\rangle + (A^{-4})(-A)^{-3(w(L_0)-1)}\langle L_-\rangle \\
&= (-A)^{-3w(L_0)+1}\langle L_+\rangle + (-A)^{-3w(L_0)-1}\langle L_-\rangle \\
&= (-A)^{-3w(L_0)+1}(A\langle)(\rangle + A^{-1}\langle \asymp \rangle)+ \\
&\qquad (-A)^{-3w(L_0)-1}(A\langle \asymp \rangle + A^{-1}\langle)(\rangle) \\
&= \big((-A)^{-3w(L_0)+2} - (-A)^{-3w(L_0)-2}\big)\langle)(\rangle + \\
&\qquad \big((-A)^{-3w(L_0)} - (-A)^{-3w(L_0)}\big)\langle \asymp \rangle \\
&= (-A)^{-3w(L_0)} \quad [A^2 - A^{-2}]\langle)(\rangle \\
&= [A^2 - A^{-2}] \quad (-A)^{-3w(L_0)}\langle)(\rangle \\
&= [A^2 - A^{-2}]V(L_0)
\end{aligned}
$$

Substitute $A^{-2} = t^{1/2}$

Then

$$-A^4V(L_+) + A^{-4}V(L_-) = [A^2 - A^{-2}]V(L_0) \implies$$

$$-t^{-1}V(L_+) + tV(L_-) = (t^{-1/2} - t^{1/2})V(L_0) \implies$$

$$-t^{-1}V(L_+) + tV(L_-) - (t^{-1/2} - t^{1/2})V(L_0) = 0 \implies$$

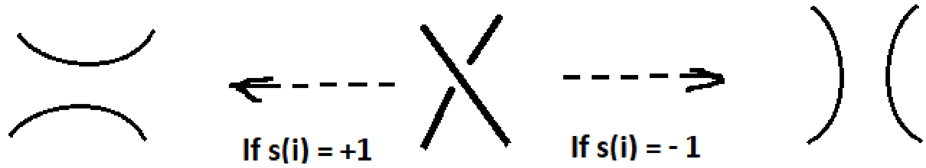
$$t^{-1}V(L_+) - tV(L_-) + (t^{-1/2} - t^{1/2})V(L_0) = 0$$

□

4.4 state

Let D be an n-crossing link diagram with its crossings labelled 1,2,3,...n.
A *state* for D is a funtion $s : \{1, 2, 3, ...n\} \longrightarrow \{-1, +1\}$

- There are 2^n such states
- Given D and a state s for D, let sD be a diagram constructed from D by replacing each crossing by two segments that do not cross.
- There are two ways to do this. At i^{th} crossing one way (the positive way) is if $s(i) = 1$, and the other way (the negative way) is used if $s(i) = -1$



- The diagram sD having no crossing at all, is just a set of disjoint simple closed curves. Let there be $|sD|$ such curves.

Proposition. *If D is a link diagram with n crossings, the Kauffman bracket of D is given by*

$$\langle D\rangle = \sum_{s}\Big(A^{\Sigma_{i=1}^{n}s(i)}(-A^{-2}-A^{2})^{|sD|-1}\Big)$$

where the summation is over all functions $s:\{1,2,3,...,n\}\longrightarrow\{+1,-1\}$.

- Let s_+ and s_-be the constant states. That is, for every i, $s_+(i)=1$ and $s_-(i)=-1$.
  s_+ is the only state for which $\Sigma_{i=1}^{n}s(i)=n$ and s_- is the only one for which $\Sigma_{i=1}^{n}s(i)=-n$.

Adequate
The diagram D is plus-adequate if $|s_+D|>|sD|$ for all s with $\Sigma_{i=1}^{n}s(i)=n-2$ and is minus-adequate if $|s_-D|>|sD|$ for all s with $\Sigma_{i=1}^{n}s(i)=2-n$.If both conditions hold, D is called adequate.

Proposition. *A reduced alternating link diagram is adequate.*

- *Reduced alternating diagram of a knot is a diagram with the minimal number of crossings for the knot. This was inherently a conjecture of Tait's when he was compiling the first knot tables.*

4.5 CONCLUSION

The project is a humble attempt to explore the basics of Knot Theory and The Jones Polynomial.Interest in Knot Theory from general mathematical community grew signifiacantly after Vaughan Jones discovery of Jones Polynomial.And further developments in Topology gave rise to Reidemeister moves, Alexander Polynomial etc.In the last several decades of the 20th century, scientists became interested in studying physical knots in order to understand knotting phenomena in DNA and other polymers. Knot theory can be used to determine if a molecule is chiral or not.It also has applications in many other fields.

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