MAJLIS ARTS AND SCIENCE COLLEGE PURAMANNUR

THIRD SEMESTER STUDY CAMP

Probability Distributions and Sampling Theory

MODULE-1

MAJLIS ARTS AND SCIENCE COLLEGE

Affiliated to the University of Calicut,

Approved by the Government of Kerala,

Probability Distributions And Sampling Theory

Module-1 - Standard Distributions

Short Answer

1. Define rectangular distribution?

a) A continuous random variable X is said to have a rectangular distribution if its pdf is given by

$$f(x) = \frac{1}{b-a}, \quad a \leq x \leq b$$
$$= 0, \quad \text{elsewhere}.$$

2. Define Pareto distribution?

a) Let X be a continuous random variable. If the pdf of X is given by

$$f(x) = \frac{\theta}{x_0}\left(\frac{x_0}{x}\right)^{\theta+1}, \quad \theta>0, x_0>0$$

Then X is said to follow Pareto distribution.

3. Define negative binomial distribution?

A) Let X be a discrete r.v assuming values

0, 1, 2, 3, ...

If the pmf of X is given by

$$P(X=x) = P(x) = (x+k-1)C_{(k-1)}\, p^k q^x$$

$$x = 0, 1, 2, \ldots$$
$$p+q=1$$

$$= 0 \quad \text{otherwise}$$

then X is said to follow a negative Binomial distribution with parameters k & p.

4. If a random variable $X \to N(40, 5^2)$, find $P(45 \le X \le 50)$?

A) Given $X \to N(40, 5^2)$

$\therefore \mu = 40$ and $\sigma = 5$

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Then

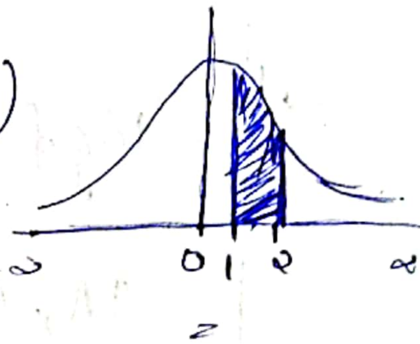
$$z = \frac{x-\mu}{\sigma} \to N(0,1)$$

$$P(45 \le x \le 50)$$

$$= P\left(\frac{45-40}{5} \le \frac{x-40}{5} \le \frac{50-40}{5}\right)$$

$$= P(1 \le z \le 2)$$

$$= P(0 \le z \le 2) - P(0 \le z \le 1)$$



$$= 0.4772 - 0.3413$$

$$= \underline{0.1359}$$

5. Write the mean and variance of a binomial distribution with parameters (15, 0.3)

A Mean $= np$ and variance $= npq$

a. Here $n = 15$ and $p = 0.3$

$\therefore q = 1 - 0.3 = 0.7$

$\Rightarrow$ Mean $= np = 15 \times 0.3 = 4.5$ //

Variance $= npq = 15 \times 0.3 \times 0.7 = 3.15$ //

Short Essay

6. Obtain the mgf of x following Poisson distribution with parameter λ?

Ans

Mgf

$$M_x(t) = E(e^{tx})$$

$$f(x) = \frac{e^{-\lambda}\lambda^x}{x!}, \quad x = 0, 1, 2, \ldots$$

$$= \sum_0^\infty e^{tx} f(x)$$

$$= \sum_0^\infty e^{tx} \cdot \frac{e^{-\lambda}\lambda^x}{x!}$$

$$= \sum_0^\infty \frac{(\lambda e^t)^x \cdot e^{-\lambda}}{x!}$$

$$= e^{-\lambda} \sum_0^\infty \frac{(\lambda e^t)^x}{x!}$$

$$= e^{-\lambda} e^{\lambda e^t}$$

$$= e^{\lambda(e^t - 1)}$$

7. The repair time of a machine follows exponential distribution with an average of 2 hours. What is the probability that the repair time will be more than 2 hours?

A In Exponential distribution the pdf is

$$f(x) = \theta e^{-\theta x}, \quad x > 0, \theta > 0$$

Its mean $\quad E(x) = 1/\theta = 2$

$$\therefore \theta = 1/2.$$

we have to find the probability that the repair time will be more than 2 hours.

$$\Rightarrow P(X > 2) = \int_2^{\infty} f(x)\, dx$$

$$= \int_2^{\infty} \tfrac{1}{2} e^{-x/2}\, dx$$

$$= \tfrac{1}{2} \left[\frac{e^{-x/2}}{-1/2} \right]_2^{\infty}$$

$$= \tfrac{1}{2} \left[0 - \frac{e^{-1}}{-1/2} \right] = \underline{\underline{e^{-1}}}$$

8 If $X \to N(12, 4)$. Obtain (i) $P(X \leq 20)$

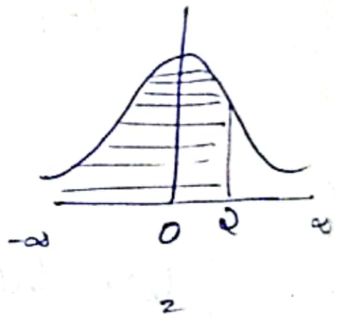
(ii) $P(0 \leq X \leq 24)$ and (iii) $P(|X-12| \geq 8)$

a. Given $X \to N(12, 4)$

$\therefore \mu = 12$ and $\sigma = 4$

(i) $P(X \leq 20) = P\left(\frac{X-12}{4} \leq \frac{20-12}{4}\right)$

$= P(Z \leq 2)$



$= P(-\infty \leq Z \leq 0) + P(0 \leq Z \leq 2)$

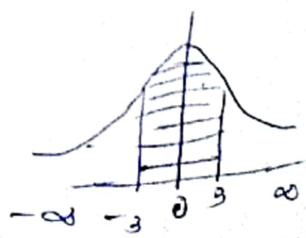
$= 0.5 + 0.4772$

$= \underline{\underline{0.9772}}$

(ii) $P(0 \leq X \leq 24) = P\left(\frac{0-12}{4} \leq \frac{X-12}{4} \leq \frac{24-12}{4}\right)$

$= P(-3 \leq Z \leq 3)$

$= P(-3 \leq Z \leq 0) + P(0 \leq Z \leq 3)$



$$= P(0 \le Z \le 3) + P(0 \le Z \le 3) \quad (\because \text{Symmetry})$$

$$= 2 \times P(0 \le Z \le 3)$$

$$= 2 \times 0.49865$$

$$= \underline{\underline{0.9973}}$$

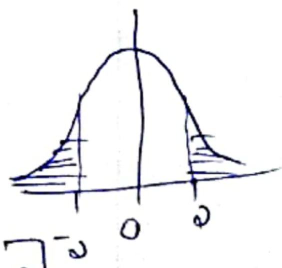
(iii) $P(|X-12| \ge 8)$

$$\Rightarrow P(X-12 \ge 8 \text{ or } -(X-12) \ge 8)$$

$$\Rightarrow P(X-12 \ge 8) + P(X-12 \le -8)$$

$$\Rightarrow P\left(\frac{X-12}{4} \ge \frac{8}{4}\right) + P\left(\frac{X-12}{4} \le -8/4\right)$$

$$\Rightarrow P(Z \ge 2) + P(Z \le -2)$$



$$\Rightarrow \left[P(0 \le Z \le \infty) - P(0 \le Z \le 2)\right]$$

$$+ \left[P(-\infty \le Z \le 0) - P(-2 \le Z \le 0)\right]$$

$$= 2\left[P(0 \le Z \le \infty) - P(0 \le Z \le 2)\right]$$

$$= 2 \times (0.5 - 0.4772) = \underline{0.0456}$$

9\. If x follows gamma distribution with one parameter p, obtain $V(x)$?

a) pdf

$$f(x) = \frac{m^p}{\Gamma p} e^{-mx} x^{p-1} \quad , x>0, p>0, m>0$$

$$V(x) = E(x^2) - \{E(x)\}^2$$

$$E(x) = \int_0^\infty x f(x)\,dx = \int_0^\infty x \cdot \frac{m^p}{\Gamma p} e^{-mx} x^{p-1}\,dx$$

$$= \frac{m^p}{\Gamma p} \int_0^\infty e^{-mx} x^{(p+1)-1}\,dx$$

$$= \frac{m^p}{\Gamma p} \cdot \frac{\Gamma(p+1)}{m^{p+1}}$$

using gamma integral $\int_0^\infty e^{-mx} x^{p-1}\,dx = \frac{\Gamma p}{m^p}$

$$= \frac{p\,\Gamma p}{m \cdot \Gamma p} = \frac{p}{m}$$

$\Gamma(p+1) = p\,\Gamma p$

$$E(x^2) = \int_0^\infty x^2 f(x)\,dx$$

$$= \int_0^\infty x^2 \frac{m^p}{\Gamma p} e^{-mx} x^{p-1}\,dx$$

$$= \frac{m^p}{\Gamma p} \int_0^\infty e^{-mx} \, x^{(p+2)-1} \, dx$$

$$= \frac{m^p}{\Gamma p} \cdot \frac{\Gamma(p+2)}{m^{p+2}}$$

$$= \frac{p+1 \; \Gamma(p+1)}{m^2 \, \Gamma p} = \frac{(p+1) \, p \, \cancel{\Gamma p}}{m^2 \, \cancel{\Gamma p}}$$

$$= \frac{p(p+1)}{m^2}$$

$$= \frac{p^2 + p}{m^2}$$

$$\therefore V(x) = E(x^2) - (E(x))^2$$

$$= \frac{p^2+p}{m^2} - \left(\frac{p}{m}\right)^2$$

$$= \frac{p^2+p-p^2}{m^2} = \underline{p/m^2}$$

10. Give the properties of Normal distribution?

A * The normal curve is symmetrical about the ordinate at $x = \mu$.

* The mean = median = Mode.

* The curve extends from $-\infty$ to $+\infty$

* For a normal distribution $\beta_1 = 0$ and $\beta_2 = 3$

* All odd order central moments vanish

ie $\mu_{2r+1} = 0$, $r = 0, 1, 2, \ldots$

* The point of ~~inflection~~ inflexion of the curve are $x = \mu \pm \sigma$.