D 92976 (Pages : 2) Name..............................

Reg. No..............................

THIRD SEMESTER (CBCSS–UG) DEGREE EXAMINATION, NOVEMBER 2020

Statistics

STA 3C 03—PROBABILITY DISTRIBUTIONS AND SAMPLING THEORY

Time : Two Hours Maximum : 60 Marks

Use of Calculator and Statistical table are permitted.

Section A (Short Answer Type Questions)

Answer at least **eight** *questions.*
Each question carries 3 *marks.*
All questions can be attended.
Overall Ceiling 24.

1. Obtain the m.g.f. of the random variable X following Bernoulli distribution with parameter p.
2. If the mean and variance of a binomial random variable X are 5 and 4, find P (X = 2).
3. Obtain the variance of X following uniform distribution over $[a, b]$.
4. Find the standard deviation of X following normal distribution with fourth central moment 243.
5. Define parameter and statistic.
6. Define convergence in probability.
7. State Bernoulli's Law of Large Numbers.
8. Define primary and secondary data.
9. In simple random sampling of n units from a population of N units, prove that the probability for each of the items to be included in the sample is n/N.
10. Identify the probability distribution of the mean of a sample of size 36 taken from a normal population with mean 10 and variance 9. Find the probability that the sample mean is greater than 10.5.
11. If X and Y are independent standard normal random variables, identify the probability distributions of (i) X / Y ; (ii) $[X / Y]^2$.
12. Define F-distribution.

(8 × 3 = 24 marks)

Turn over

Section B (Short Essay/Paragraph Type Questions)

Answer at least **five** *questions.*
Each question carries 5 *marks.*
All questions can be attended.
Overall Ceiling 25.

13. If the $(r-1)^{th}$, r^{th} and $(r+1)^{th}$ central moments of X following Poisson distribution with parameter λ are, μ_{r-1}, μ_r and μ_{r+1}, show that $\mu_{r+1} = r\lambda\mu_{r-1} + \lambda \frac{d}{d\lambda}\mu_r$.

14. If the r.v., X follow uniform distribution over [0,1], show that $Y = -2 \log_e X$ follow exponential distribution.

15. State Chebyshev's inequality. Let X denotes the number shown, when an unbiased die is thrown. Use Chebyshev's inequality to show that $P(|X-3.5|<2) > \frac{13}{48}$.

16. State and prove Weak Law of Large numbers.

17. Using Central limit theorem, prove that binomial distribution with parameters (n, p) tends to normal distribution for large n.

18. Explain stratified random sampling.

19. A random sample of size 12 is taken from a normal population with variance σ^2. If the sample variance is 9, identify two numbers a and b such that $P(a < \sigma^2 < b) = 0.80$.

(5 × 5 = 25 marks)

Section C (Essay Type Questions)

Answer any **one** *question.*
The question carries 11 *marks.*

20. (i) Find the m.g.f. of the random variable X following $N(\mu, \sigma^2)$.

(ii) If $X_1 \sim N(\mu_1, \sigma_1^2), X_2 \sim N(\mu_2, \sigma_2^2)$, where X_1 and X_2 are independent, obtain the probability distribution of $Y = aX_1 + bX_2$ for any two constants a and b.

21. (i) Define t-distribution. If t follow $t_{(n)}$, show that $E(t) = 0$.

(ii) If t follow $t_{(n)}$, prove that t^2 follow $F_{(1,n)}$.

(1 × 11 = 11 marks)