

Computational Methods and Applications

Scribe II

Networks, Random Graphs and Percolation.

- A network is a data structure in which nodes are connected by edges
- Random graph is the general term to refer to probability distributions over graphs
Probability aspect comes since its generated in probabilistic manner.

Erdos-Renyi Random graphs.

Graph is characterised by n and p where n is the number of nodes/vertices and p is the probability with which an edge is included in a graph.

$G(n, p)$ can be thought of sampling a graph with n vertices and M edges with probability. $p^M (1-p)^{\binom{n}{2} - M}$.

→ If $p < 1/n$ then a graph in $G(n, p)$ will almost surely have no connected components of size larger than $O(\log n)$

→ If $p > 1/n$, then a graph in $G(n, p)$ will almost surely have a unique giant connected component containing a positive fraction of vertices and no other component will contain more than $O(\log n)$ vertices

→ If $p < \frac{(1-\epsilon) \ln n}{n}$, then a graph in $G(n, p)$ will almost surely contain isolated vertices and thus be disconnected

→ If $p > \frac{(1-\epsilon) \ln n}{n}$ then graph will almost surely be connected

Breadth First Search / Traversal.

- BFS is an algorithm for traversing or searching tree or graph data structures. It starts at an arbitrary node of a graph and explores all of the neighbor nodes at the present depth prior to moving on to the next level nodes.
- A queue data structure can be used for the implementation.
- Few applications are:
 - Shortest path and minimum spanning tree
 - Cycle detection in undirected graph.
 - Ford Fulkerson algorithm
 - Finding all nodes within one connected component

Percolation

- Percolation theory describes the behaviour of a network when nodes or links are removed.
- Bond percolation
 - A graph ~~usually~~ of $n \times n$ vertices (sites) in which edges between two neighbors may be open with probability p or closed with probability $1-p$.
 - For a given p , what is the probability that an open path exists from top to bottom.
- Square lattice \mathbb{Z}^2 in two dimensions exhibits a sharp phase transition at $p = 1/2$ (many paths will occur)