Computational Methods and Applications

Scribe - I

Monte Carlo Methods

Random Variable : is a mathematical formalization of a quantity or object which depends on random events

→ A random variable is a variable whose value is unknown or a function that assigns values to each of an experiment's outcomes

→ It is called so because the value of this variable cannot be defined with certainity

A discrete random variable is one which may take only a countable number of distinct values.

→ Discrete distribution describes the probability of occurance of each value of a discrete random variable.

Probability that the top side of a 6 faced die shows 3 dots when thrown is 1/6. (Assuming unbiased die with 1, 2, 3 ... 6 dots on each face)

→ Sampling from a discrete distribution

To simulate this experiment, we make use of a continuous random variable uniformly distributed in the interval $[0, 1]$

Probability mass function (PMF) $p(x) = P[X = x]$ for any $x \in \mathcal{X}$

Cumulative distribution function (CDF)

$$F(x_j) = P[X \leq x_j] = \sum_{i \leq j} p(x_i)$$

Let X be discrete random variable taking values in set $\mathcal{X} = \{x_1, x_2 \dots x_k\}$

We consider the partition of interval $(0,1)$

$$[(0, F(x_1)), (F(x_1), F(x_2)), \dots (F(x_{k-1}), 1)]$$

Let U be a uniformly distributed random variables in the interval $(0,1)$

Let $I_j \in \{0,1\}$ take the value 1 only if $U \in (F(x_j), F(x_{j+1}))$

Then we have $P(I_j = 1) = P[X = x_j]$

and this can be used to generate from the distribution of X

So: from CDF we take intervals and then map them

~~like~~ eg: 0 ~~to~~ 1/6 mapped to 1.

$(1/6, 2/6) \rightarrow 2$ - - - etc

If the partitions are shuffled it will ~~of~~ not change the probabilities unless the association between intervals and faces are maintained

Monte carlo method rely on repeated random sampling to obtain numerical results.

Estimating π

- Consider a unit square centered at origin of a 2D plane.
- Inscribe a circle within this square
- Generate (x, y) uniformly and independently from interval $[-0.5, 0.5]$ and place them on the plane.
- for large # of points

$$\frac{\text{\# of points within circle}}{\text{\# of points within the square}} = \frac{\text{area of circle}}{\text{area of square}} = \frac{\pi}{4}$$

$$\therefore \pi = \frac{4 \times \text{no. of points within circle}}{\text{no. of points within the square}}$$