

Numerical modeling of Radio jets from YSOs

```
object to mirror  
mirror_mod.mirror_object
```

```
operation == "MIRROR_X":  
    mirror_mod.use_x = True  
    mirror_mod.use_y = False  
    mirror_mod.use_z = False  
operation == "MIRROR_Y":  
    mirror_mod.use_x = False  
    mirror_mod.use_y = True  
    mirror_mod.use_z = False  
operation == "MIRROR_Z":  
    mirror_mod.use_x = False  
    mirror_mod.use_y = False  
    mirror_mod.use_z = True
```

```
selection at the end -add  
mirror_ob.select= 1  
modifier_ob.select= 1  
context.state.object.at  
("Selected" + str(modifier  
mirror_ob.select= 0  
by context.selected  
the objects one name)  
print("please select exactly
```

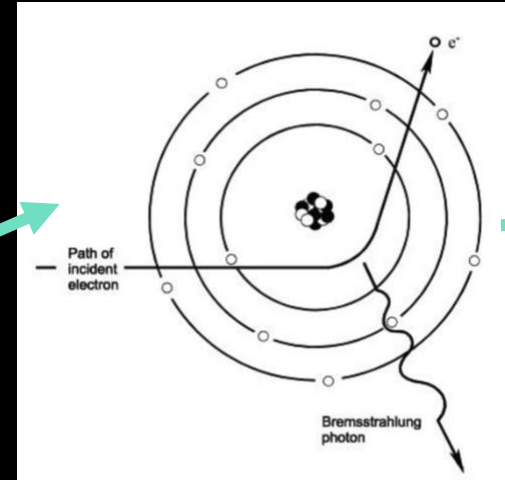
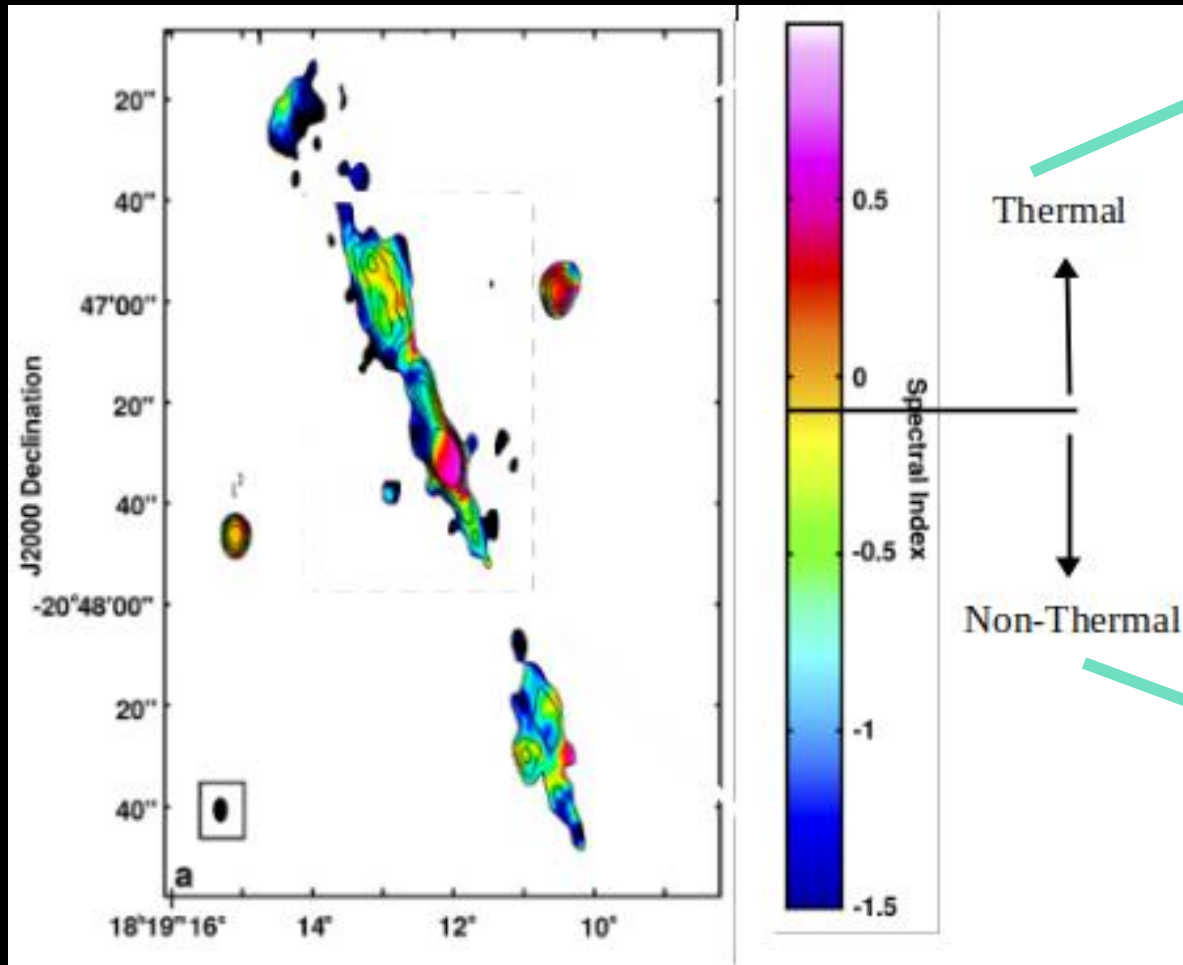
```
-- OPERATOR CLASSES --
```

```
types.Operator):  
    X mirror to the selected  
    object.mirror_mirror_x"  
    mirror X"
```

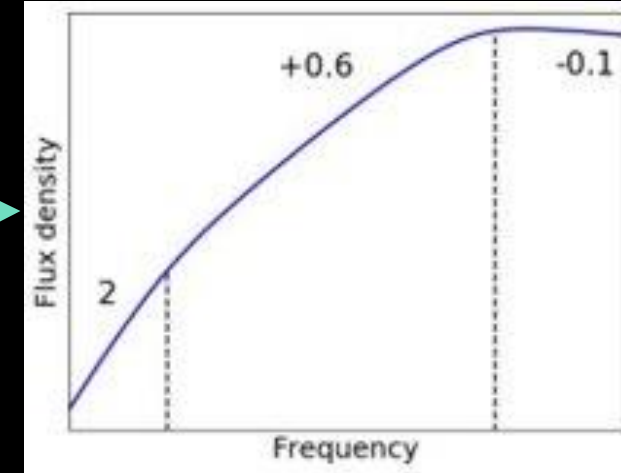
```
is not
```

Thermal + non-thermal emission from jets

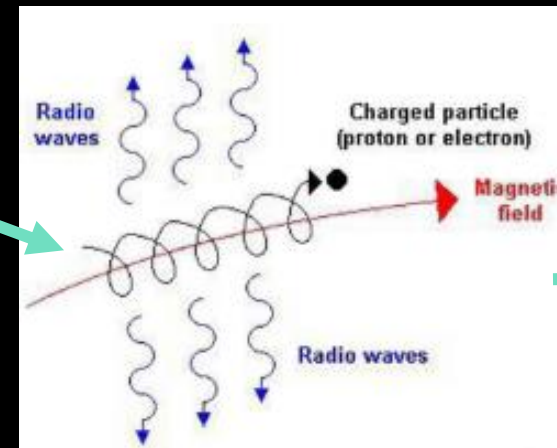
HH80-81



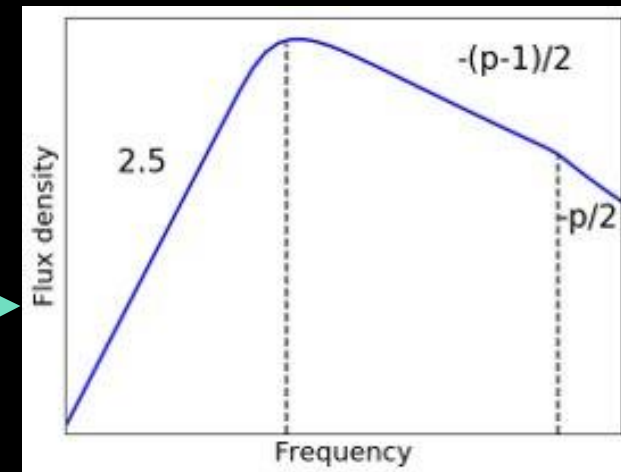
Credit: Ignacio Aviles



Free-free emission spectrum



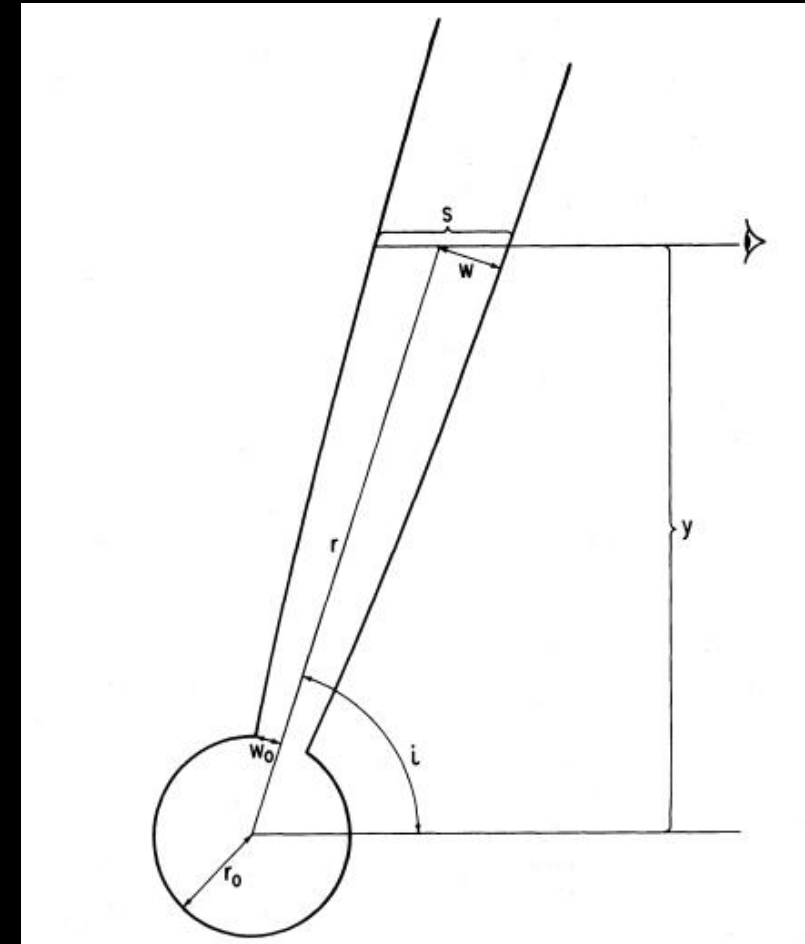
Credit: <http://nrumiano.free.fr>



Synchrotron emission spectrum

Motivation

- Develop a model to characterize the radio spectra of protostellar jets: **Thermal + non-thermal emission**
- **Reynolds (1986)** model: $S_\nu \propto v^\alpha$; $\alpha = +0.6$ for a spherical, isothermal, constant velocity thermal jet.
- Reynolds (1986) model limitations:
 - Only **free-free** emission is considered
 - Assumes that the jet is optically **thick** at the base and **thin** at farther radial distances.
 - Applicable only to **narrow** collimated jets
- Advantages of our model:
 - **Free-free + synchrotron** mechanisms
 - Incorporates generalized **geometry** (16–20%)
 - Incorporates **intermediate** optical depth (8 – 12%)
 - **Lateral variation** of ionization fraction



Reynolds (1986)

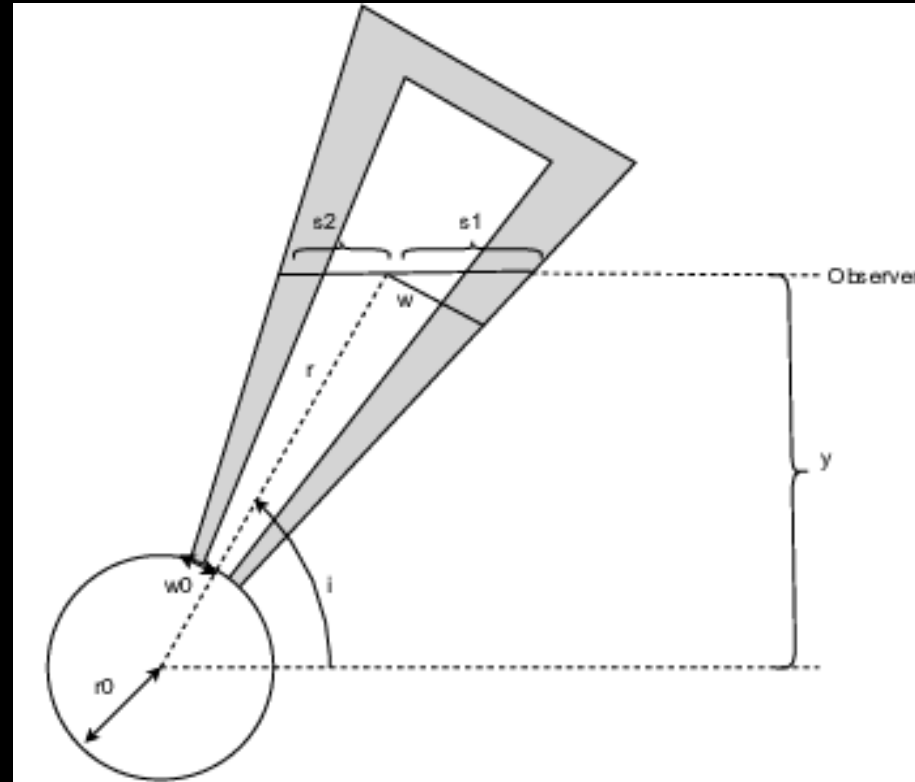
Model description

Model : Brief discussion

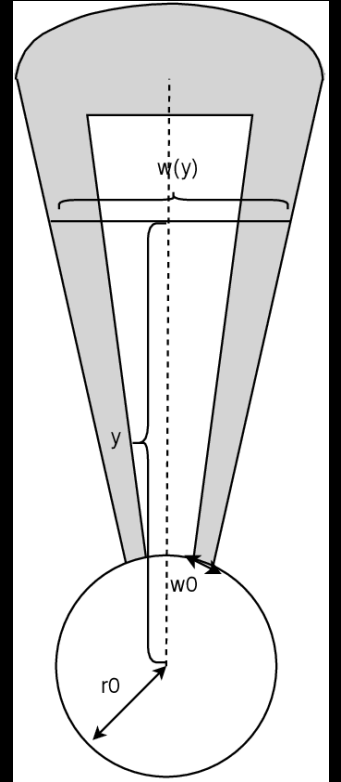
$$y = r \sin i$$
$$\omega = \omega_0 \left(\frac{r}{r_0} \right)^\epsilon = \omega_0 \left(\frac{y}{y_0} \right)^\epsilon$$

- **Radial Variation:**
 - ω , θ , x , n
- **Lateral Variation:**
 - x

Jet geometry : Inner thermal jet + thin shocked shell



Side view



Front view

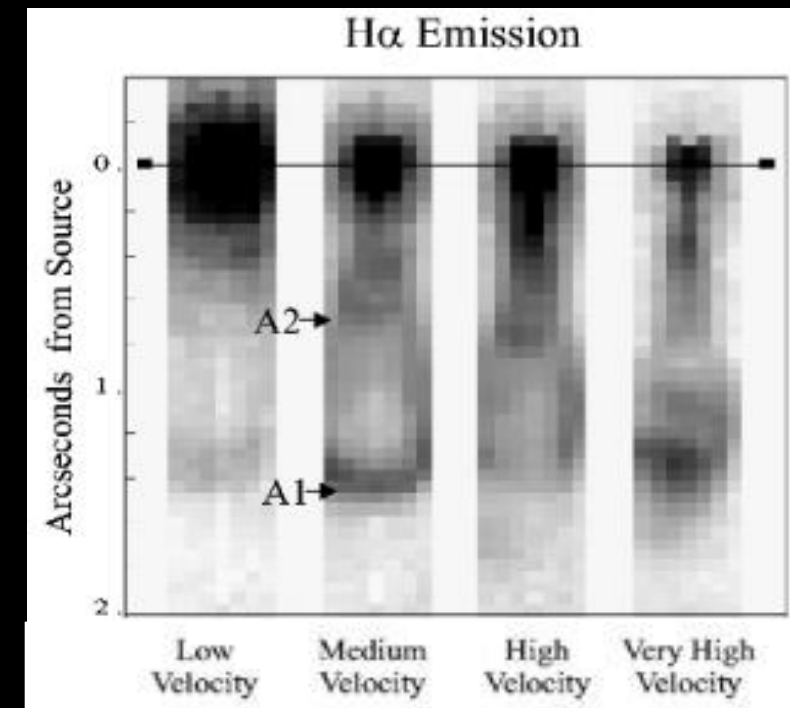
The shaded region represents the thin shocked material

ω : jet half-width, θ : opening angle, x : ionization fraction, n : number density

Model: Lateral variation in ionization fraction

Onion-like/nested velocity structure

- High velocity gas collimated into a small opening angle towards the axis of the jet/outflow
- Less collimated low velocity gas with a wide opening angle
- Assuming that ionization is a result of internal/external shocks, the strength of which depends on velocity
- Higher velocity regions creates stronger shocks resulting in higher ionization fraction



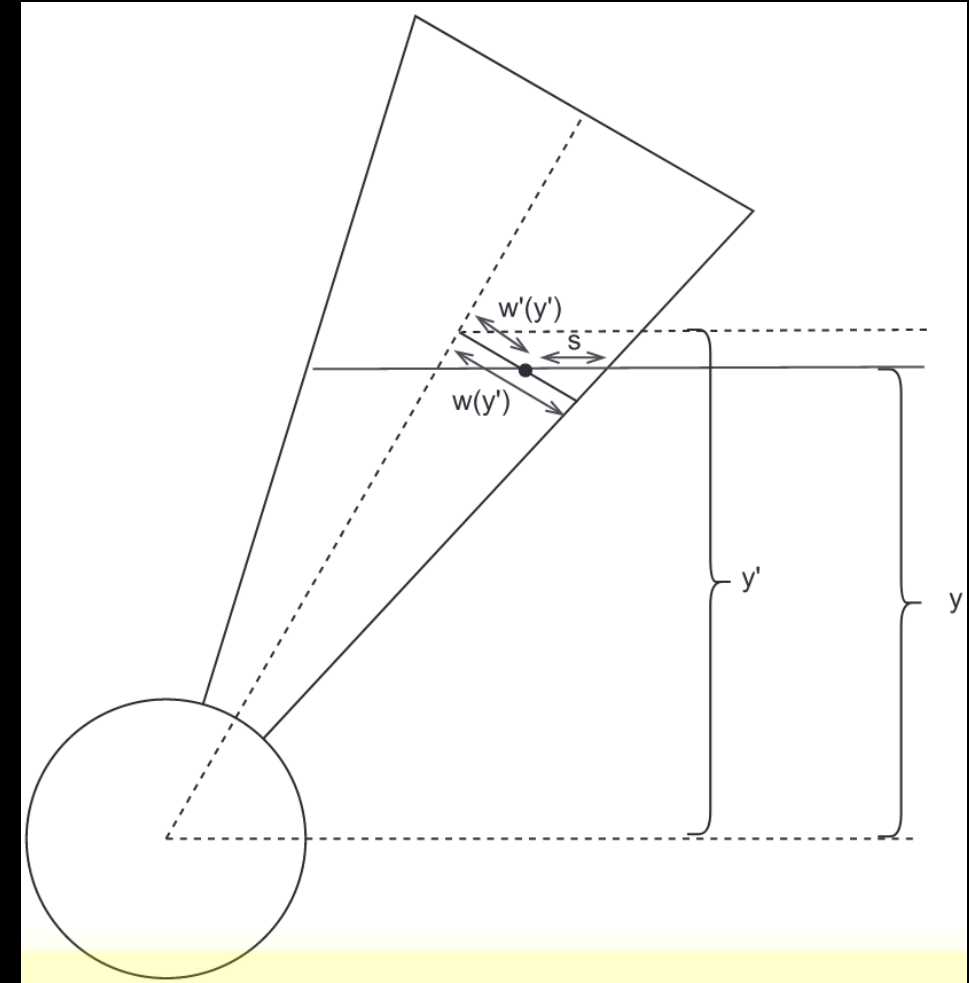
DG Tau jet, Bacciotti et al. 2002

Model: Lateral variation in ionization fraction

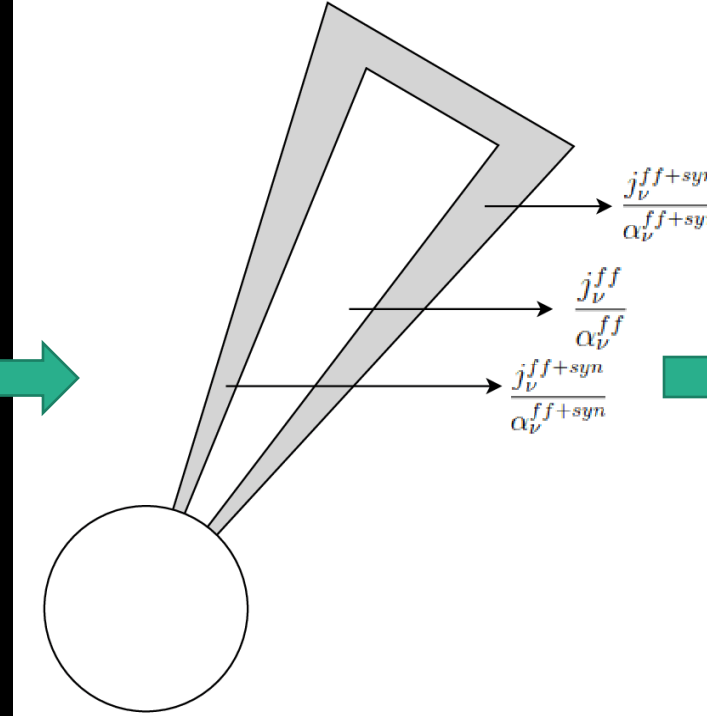
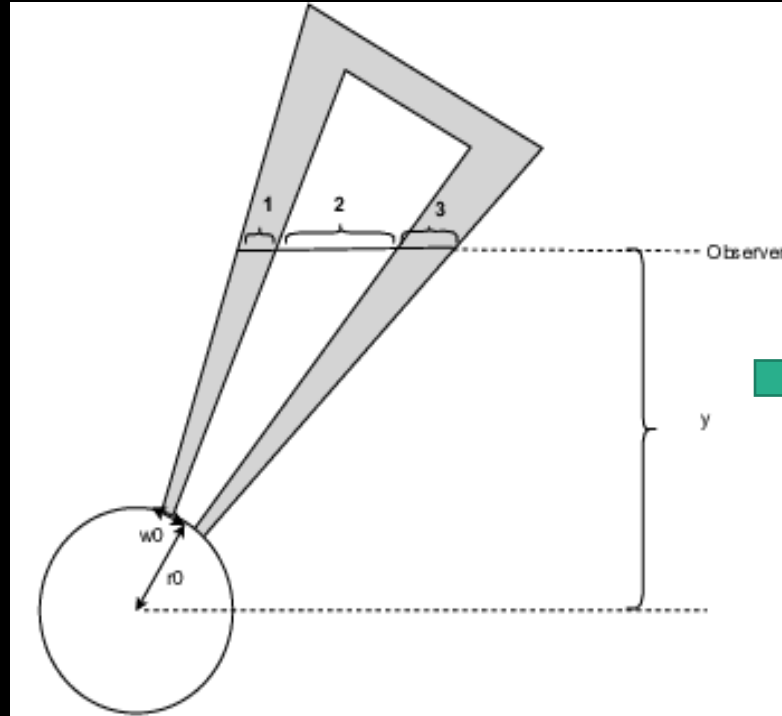
Onion-like/nested velocity structure

- **High velocity gas collimated into a small opening angle towards the axis of the jet/outflow**
- **Less collimated low velocity gas with a wide opening angle**
- **Assuming that ionization is a result of internal/external shocks, the strength of which depends on velocity**
- **Higher velocity regions creates stronger shocks resulting in higher ionization fraction**

$$x(s) = x_a(y') \left[\frac{w(y')}{w(y') - w'(y')} \right]^{q'_x}$$



Model : LOS flux density



$$I_{\nu_1}^{ff+syn} = \int_0^{\tau_{\nu_1}^{ff+syn}} \frac{j_{\nu_1}^{ff+syn}(s)}{\alpha_{\nu_1}^{ff+syn}(s)} e^{-(\tau_{\nu_1}^{ff+syn} - \tau(s))} d\tau$$

$$C_{1,\nu} n^{ff}(s)^2 + C_{2,\nu} n^{syn}(s)$$

$$I_{\nu_2}^{ff+syn} = \frac{j_{\nu_2}^{ff}}{\alpha_{\nu_2}^{ff}} (1 - e^{-\tau_{\nu_2}^{ff}})$$

$$\propto (n^{ff})^2$$

$$I_{\nu_3}^{ff+syn} = \int_0^{\tau_{\nu_3}^{ff+syn}} \frac{j_{\nu_3}^{ff+syn}(s)}{\alpha_{\nu_3}^{ff+syn}(s)} e^{-(\tau_{\nu_3}^{ff+syn} - \tau(s))} d\tau$$

Region 1 & 3: Free-Free + Synchrotron
Region 2 : purely Free-Free

$$I_{\nu}(y) = I_{\nu_1}^{ff+syn} e^{-(\tau_{\nu_2}^{ff} + \tau_{\nu_3}^{ff+syn})} + I_{\nu_2}^{ff+syn} e^{-\tau_{\nu_3}^{ff+syn}} + I_{\nu_3}^{ff+syn}$$

Two-fold application of the model

- YSO jets have been seen to appear as chains of well-defined **knots/lobes** powered by the central object, instead of being a continuous outflow
- Knots have been identified in a wide range of bands from **radio to X-rays**

Knot formation

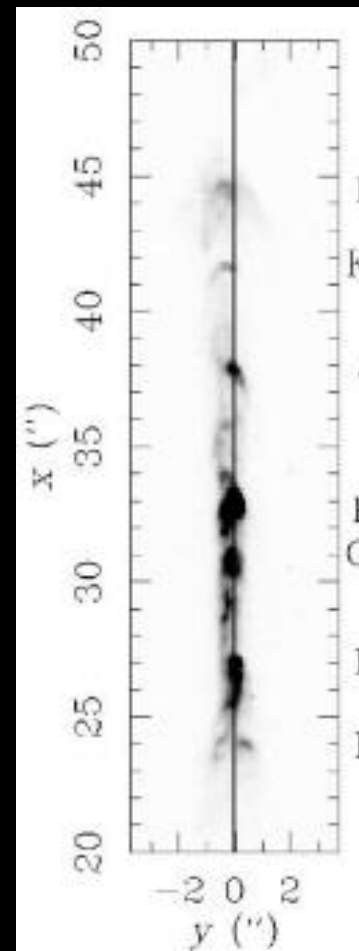
MHD/thermal
instabilities

*Huarte-Espinosa et al.
2012; Lebedev et al. 2005*

Time variability
in accretion disk

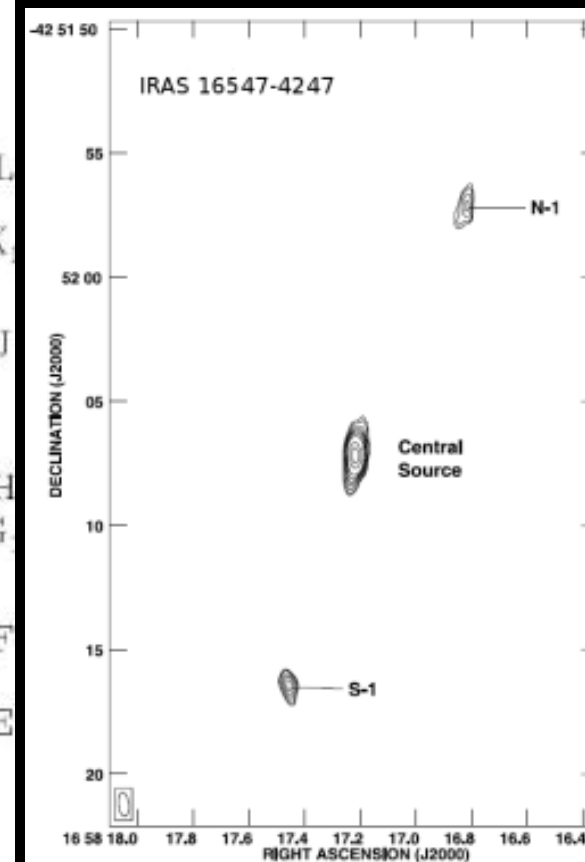
Bonito et al. 2010

[SII] HST



HH 111, Raga et al. 2002

2cm VLA



Rodriguez et al. 2005

Two-fold application of the model

- Observations indicate presence of non-thermal emission from knots/lobes of few YSO jets, while in most cases the emission can be explained due to thermal free-free mechanism.

Inner thermal jet

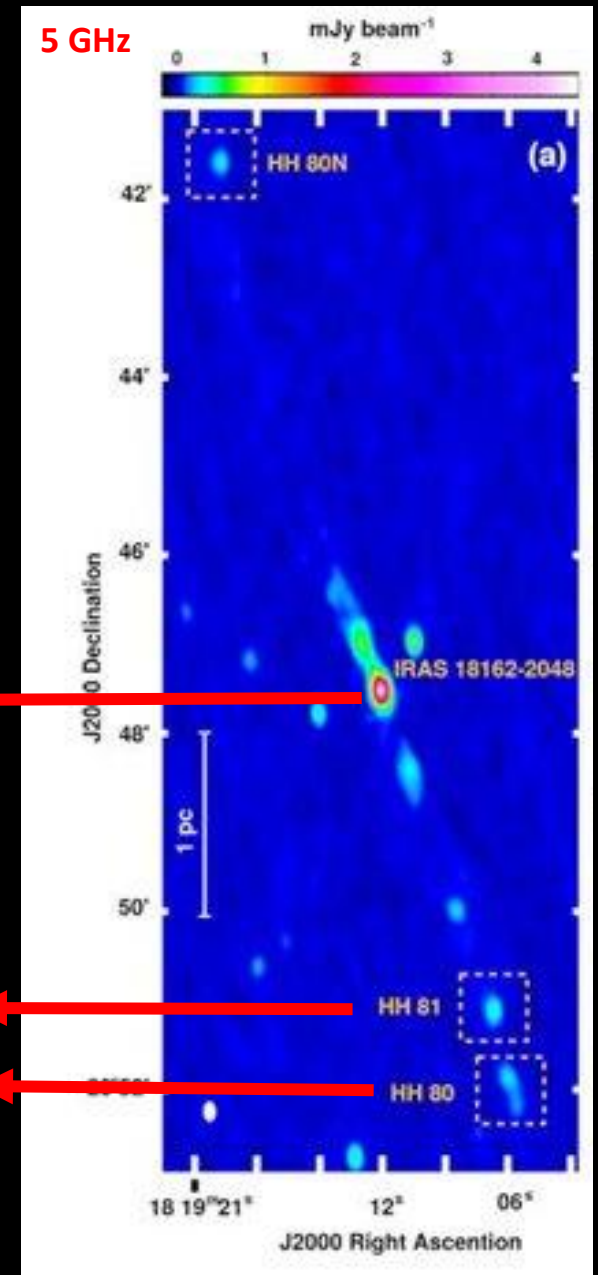
Outer knots/lobes

Spectral indices
 $+0.2 \pm 0.01$

-0.26 ± 0.01

-0.63 ± 0.19

(Vig et al. 2018)



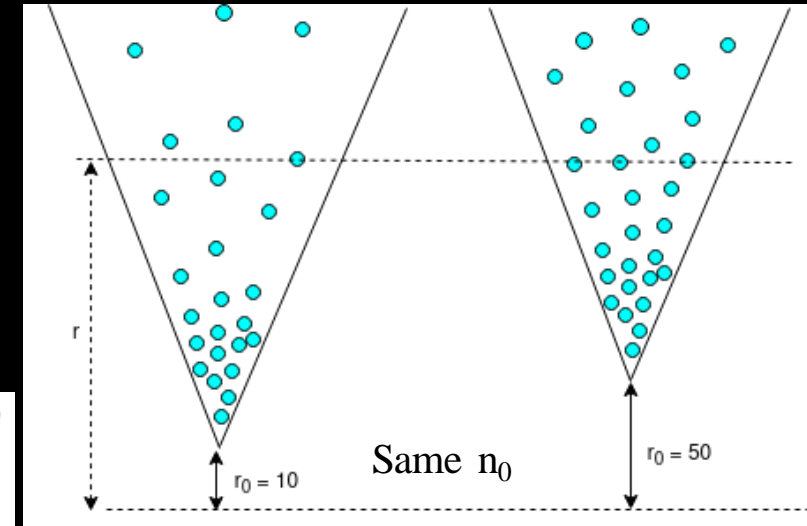
HH80-81 jet, Carrasco-Gonzalez 2010

Results

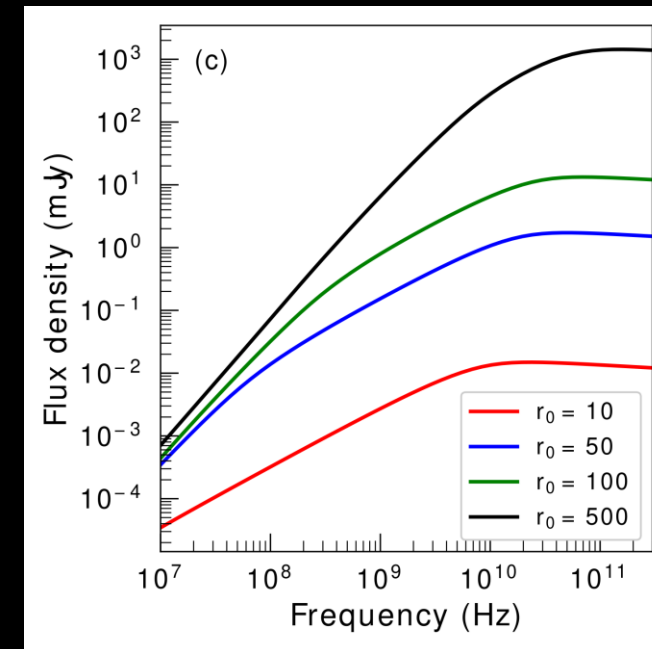
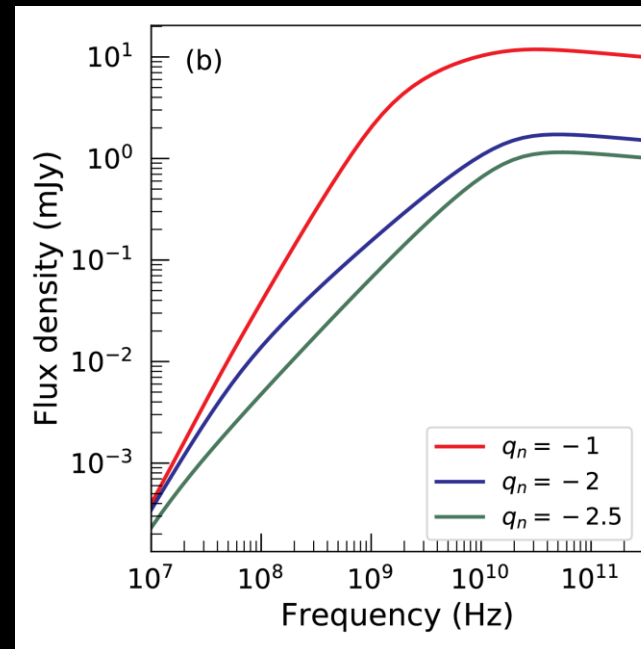
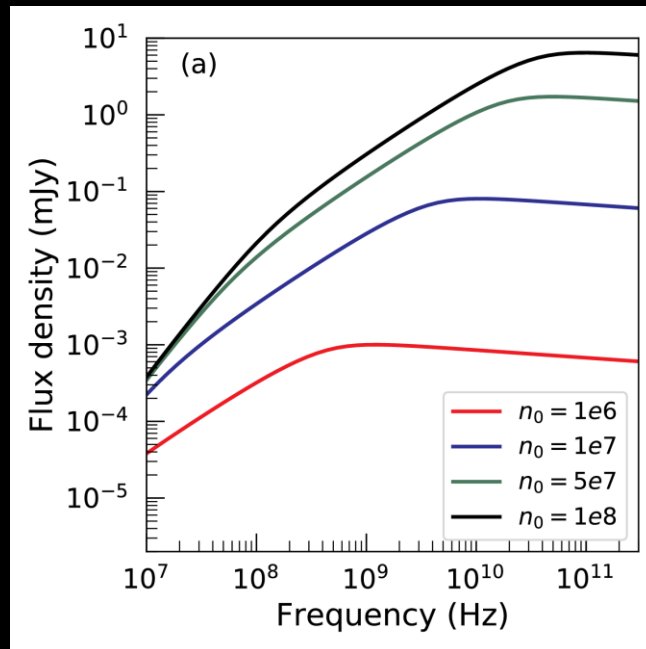
Effect of jet parameters on free-free emission

- Case of a fully **thermal** jet
- Located close to the central YSO ($r_0 = 10\text{-}50$ au)
 - r_0 : Jet injection radius
 - n_0 : Jet number density at r_0
 - q_n : Power-law index of n profile

$$n = n_0 \left(\frac{r}{r_0} \right)^{q_n} = n_0 \left(\frac{y}{y_0} \right)^{q_n}$$



n_0, q_n, r_0

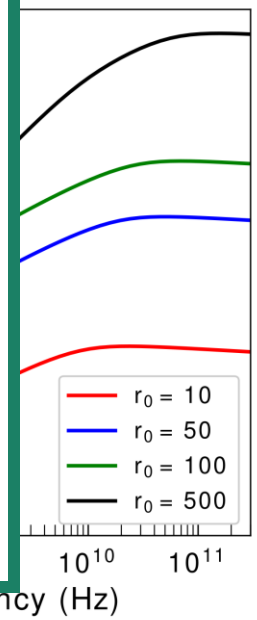
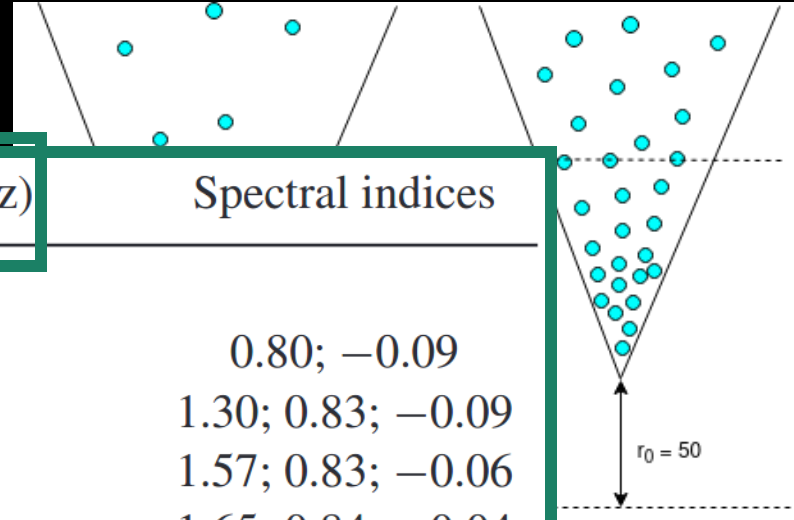


Effect of jet parameters on free-free emission

•

Variable parameter	Value	Turnover frequencies (GHz)	Spectral indices
n_0 (cm^{-3})	10^6	0.76	0.80; -0.09
	10^7	0.04; 6.87	1.30; 0.83; -0.09
	5×10^7	0.18; 31.17	1.57; 0.83; -0.06
	10^8	0.35; 60.59	1.65; 0.84; -0.04
q_n	-1	1.67; 32.70	1.86; 0.36; -0.08
	-2	0.18; 31.17	1.57; 0.83; -0.06
	-2.5	0.03; 30.58	1.43; 1.04; -0.06
r_0 (au)	10	14.92	0.88; -0.07
	50	0.18; 31.17	1.57; 0.83; -0.06
	100	0.86; 45.52	1.72; 0.76; -0.06
	500	101.35	1.71; -0.01

r_0, r_{LOSmax}



$n_0,$

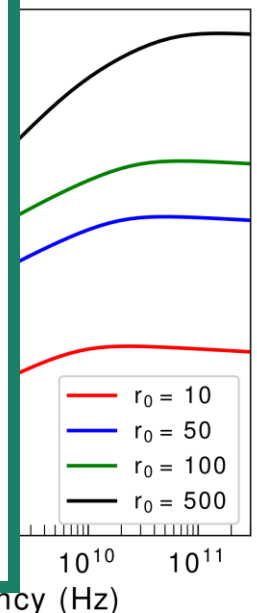
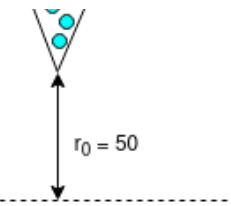
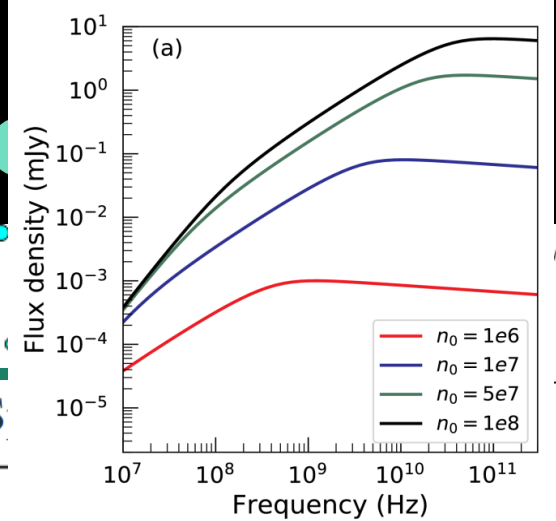
Effect of jet parameters on frequency emission

•
•

Variable parameter	Value	Turnover frequencies (GHz)	Spectral index
n_0 (cm^{-3})	10^6	0.76	0.80; -0.09
	10^7	0.04; 6.87	1.30; 0.83; -0.09
	5×10^7	0.18; 31.17	1.57; 0.83; -0.06
	10^8	0.35; 60.59	1.65; 0.84; -0.04
q_n	-1	1.67; 32.70	1.86; 0.36; -0.08
	-2	0.18; 31.17	1.57; 0.83; -0.06
	-2.5	0.03; 30.58	1.43; 1.04; -0.06
r_0 (au)	10	14.92	0.88; -0.07
	50	0.18; 31.17	1.57; 0.83; -0.06
	100	0.86; 45.52	1.72; 0.76; -0.06
	500	101.35	1.71; -0.01

r_0, r_{LOSmax}

Turnover frequencies (GHz)



Frequency (Hz)

Frequency (Hz)

Frequency (Hz)

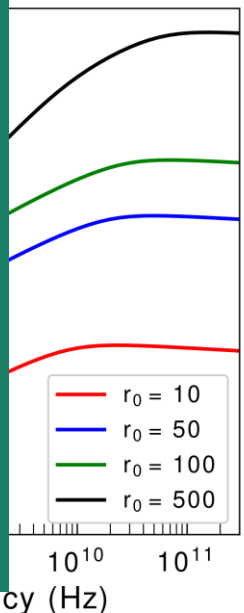
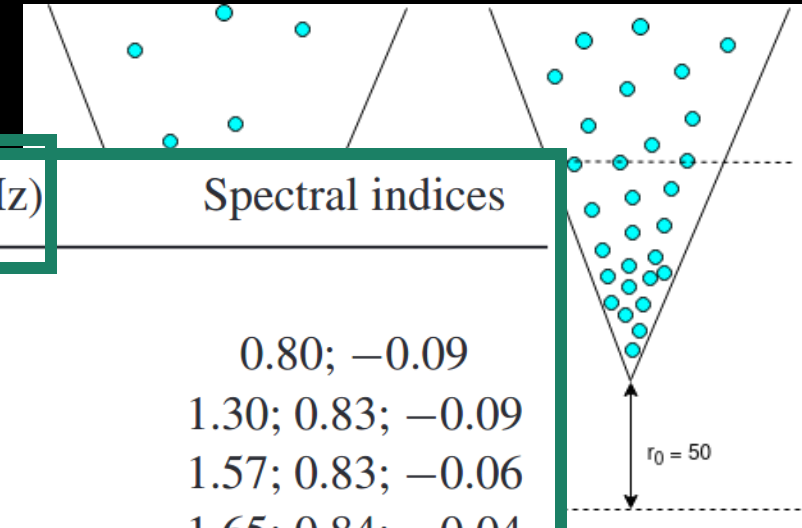
$n_0,$

Effect of jet parameters on free-free emission

•

Variable parameter	Value	Turnover frequencies (GHz)	Spectral indices
n_0 (cm^{-3})	10^6	0.76	0.80; -0.09
	10^7	0.04; 6.87	1.30; 0.83; -0.09
	5×10^7	0.18; 31.17	1.57; 0.83; -0.06
	10^8	0.35; 60.59	1.65; 0.84; -0.04
q_n	-1	1.67; 32.70	1.86; 0.36; -0.08
	-2	0.18; 31.17	1.57; 0.83; -0.06
	-2.5	0.03; 30.58	1.43; 1.04; -0.06
r_0 (au)	10	14.92	0.88; -0.07
	50	0.18; 31.17	1.57; 0.83; -0.06
	100	0.86; 45.52	1.72; 0.76; -0.06
	500	101.35	1.71; -0.01

r_0, r_{LOSmax}



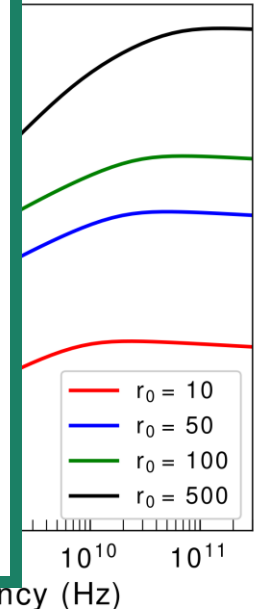
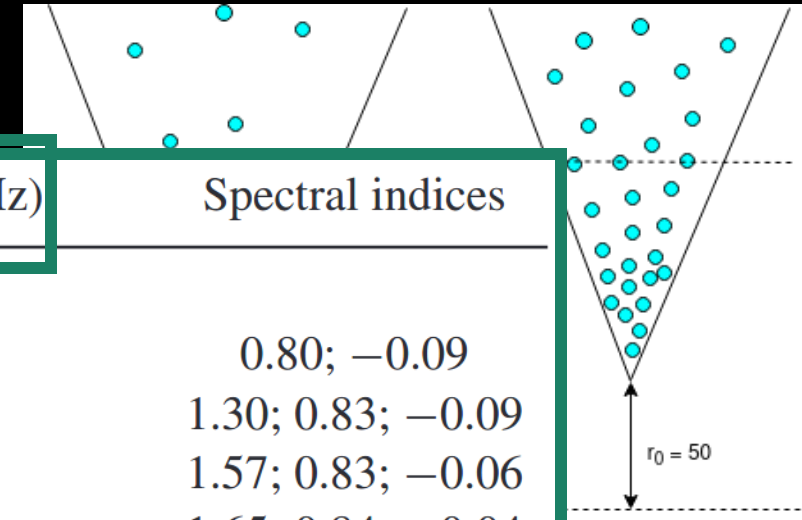
$n_0,$

Effect of jet parameters on free-free emission

•

Variable parameter	Value	Turnover frequencies (GHz)	Spectral indices
n_0 (cm^{-3})	10^6	0.76	0.80; -0.09
	10^7	0.04; 6.87	1.30; 0.83; -0.09
	5×10^7	0.18; 31.17	1.57; 0.83; -0.06
	10^8	0.35; 60.59	1.65; 0.84; -0.04
q_n	-1	1.67; 32.70	1.86; 0.36; -0.08
	-2	0.18; 31.17	1.57; 0.83; -0.06
	-2.5	0.03; 30.58	1.43; 1.04; -0.06
r_0 (au)	10	14.92	0.88; -0.07
	50	0.18; 31.17	1.57; 0.83; -0.06
	100	0.86; 45.52	1.72; 0.76; -0.06
	500	101.35	1.71; -0.01

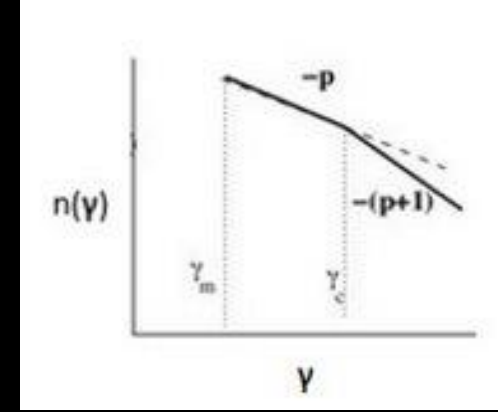
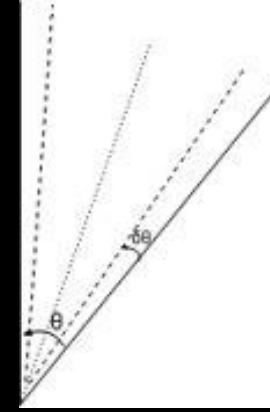
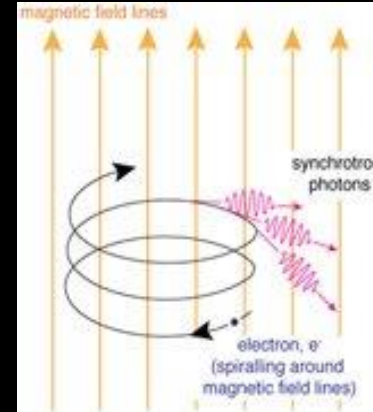
r_0, r_{LOSmax}



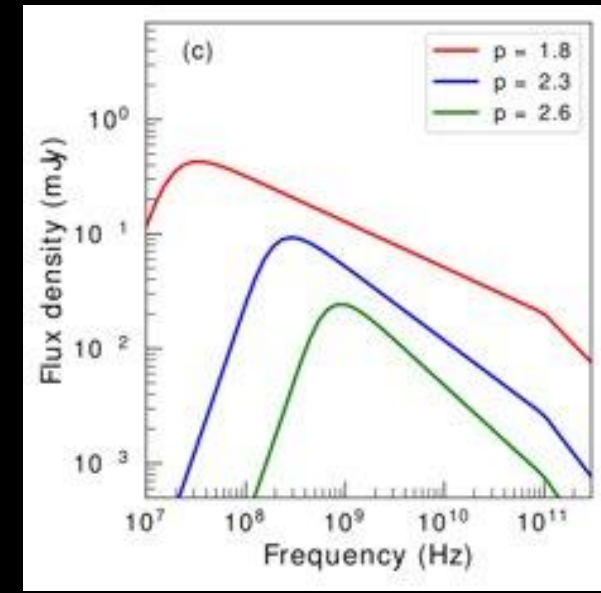
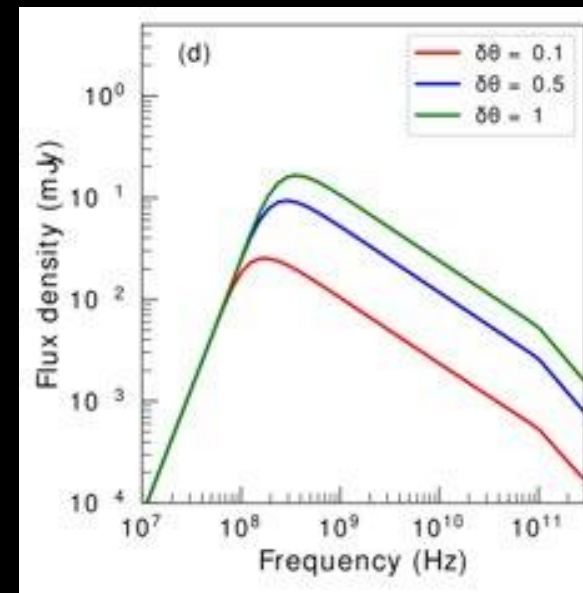
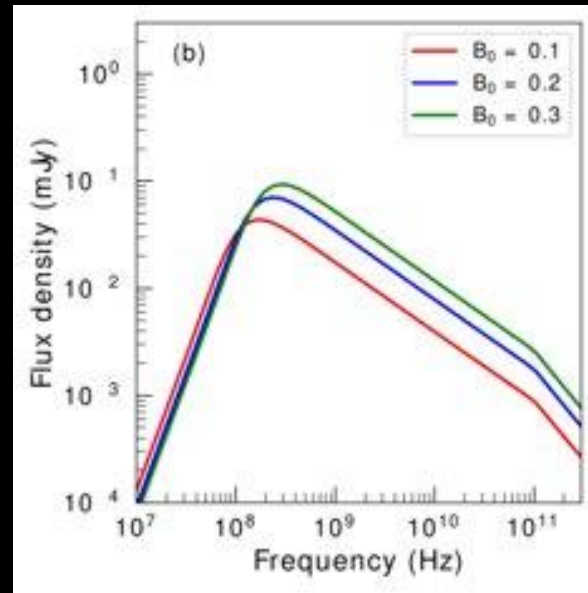
$n_0,$

Effect of jet parameters on Synchrotron emission

- Jet with both **thermal** and **non-thermal** emission
- Located farther from the YSO ($r_0 = 3000$ au)
 - B_0 : Magnetic field strength at r_0
 - $\delta\theta$: Angular thickness of shocked shell
 - p : Power-law index of number density distribution of non-thermal electrons in energy space



$B_0, \delta\theta, p$



Effect of jet parameters on Synchrotron emission

$\propto \tau_{\text{ff+syn}} = 1, \nu_c$ (Govoni F., Feretti L., 2004)

- Jet with both thermal and non-thermal emission

Variable parameter	Value	Turnover frequencies (GHz)	Spectral indices
B_0 (mG)	0.1	0.17; 100	2.20; -0.64; -1.10
	0.2	0.24; 100	2.20; -0.64; -1.1
	0.3	0.29; 100	2.31; -0.63; -1.13
p	1.8	0.03; 100	1.18; -0.40; -0.90
	2.3	0.29; 100	2.31; -0.63; -1.13
	2.6	0.90; 100	2.39; -0.77; -1.23
$\delta\theta$ ($^\circ$)	0.1	0.17; 100	2.19; -0.63; -1.06
	0.5	0.29; 100	2.31; -0.63; -1.13
	1	0.37; 100	2.26; -0.64; -1.14



Frequency (Hz) Frequency (Hz) Frequency (Hz)

Effect of jet parameters on Synchrotron emission

- Jet with both thermal and non-thermal emission

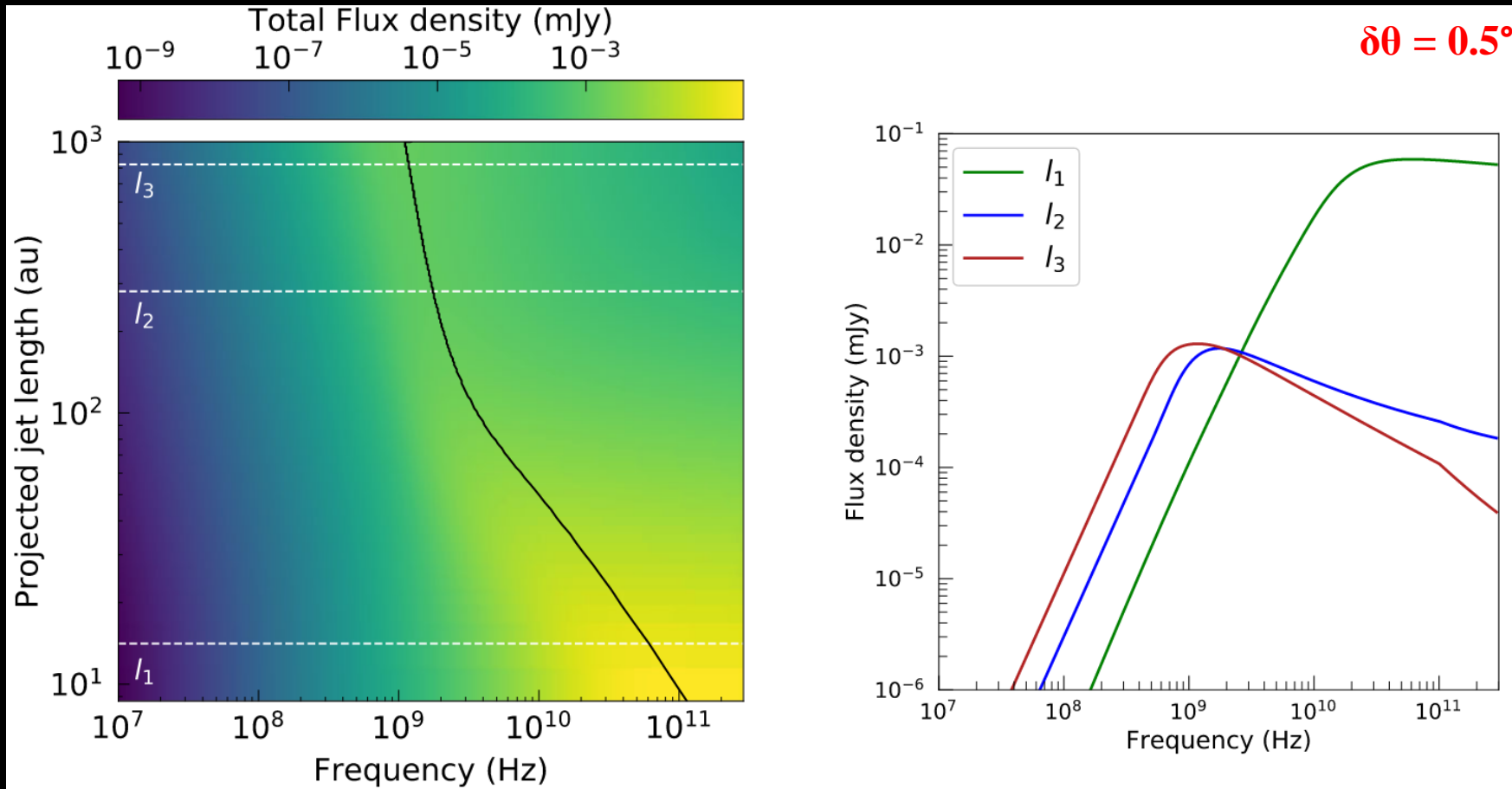
$\propto \tau_{\text{ff+syn}} = 1, \nu_c$ (Govoni F., Feretti L., 2004)

Variable parameter	Value	Turnover frequencies (GHz)	Spectral indices
B_0 (mG)	0.1	0.17; 100	2.20; -0.64; -1.10
	0.2	0.24; 100	2.20; -0.64; -1.1
	0.3	0.29; 100	2.31; -0.63; -1.13
p	1.8	0.03; 100	1.18; -0.40; -0.90
	2.3	0.29; 100	2.31; -0.63; -1.13
	2.6	0.90; 100	2.39; -0.77; -1.23
$\delta\theta$ ($^\circ$)	0.1	0.17; 100	2.19; -0.63; -1.06
	0.5	0.29; 100	2.31; -0.63; -1.13
	1	0.37; 100	2.26; -0.64; -1.14

$\tau \propto B_0, p, \delta\theta$

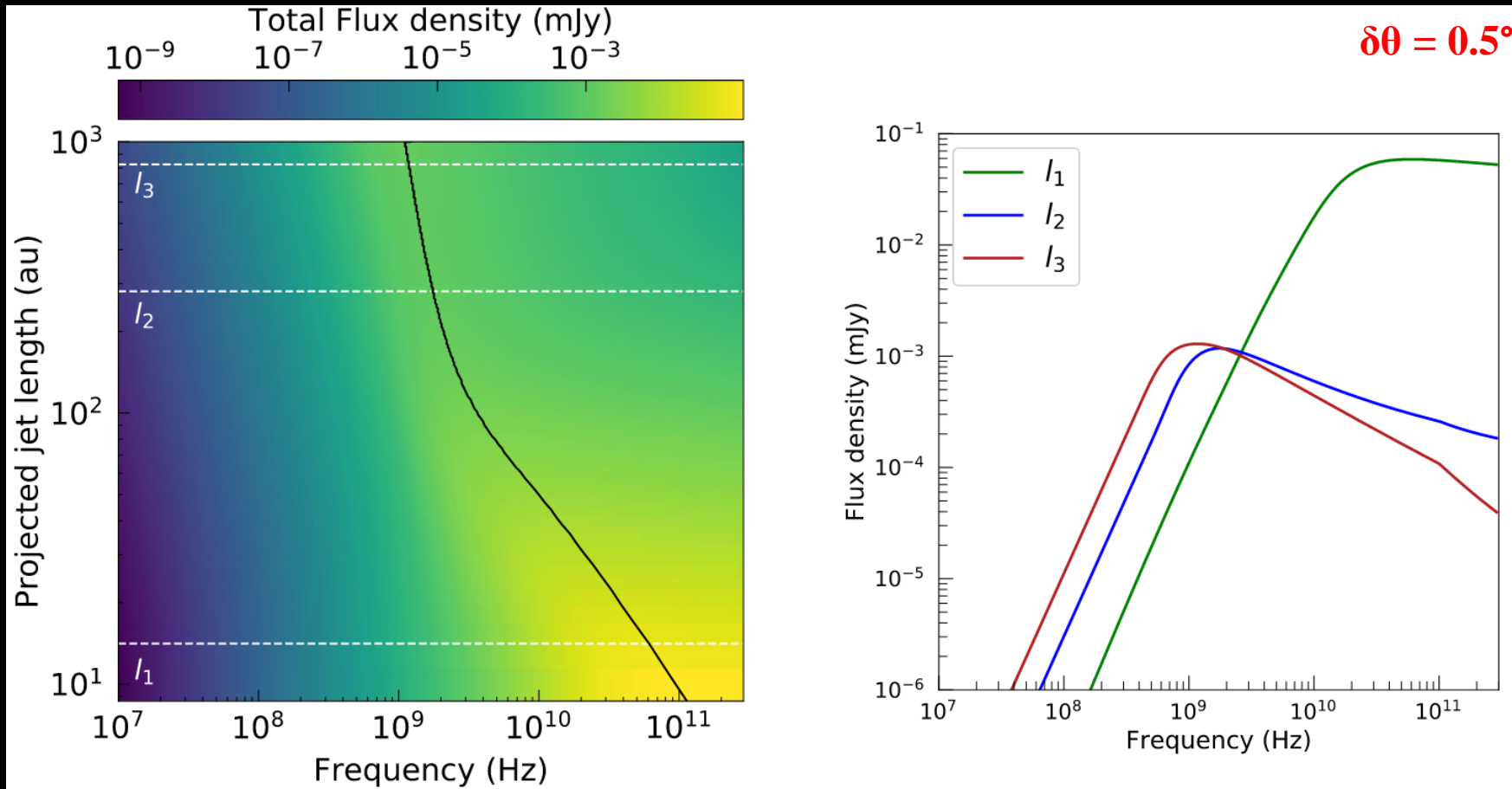


Flux variation across jet length



- $r_0 = 3000$ au
- $\theta_0 = 30^\circ$
- $\eta_e = 10^{-5}$
- $p = 2.3$
- $B = 0.3$ mG
- $n_0 = 500$ cm $^{-3}$
- $x_0 = 0.2$
- $q_x = -0.5$
- $q_x = -1$

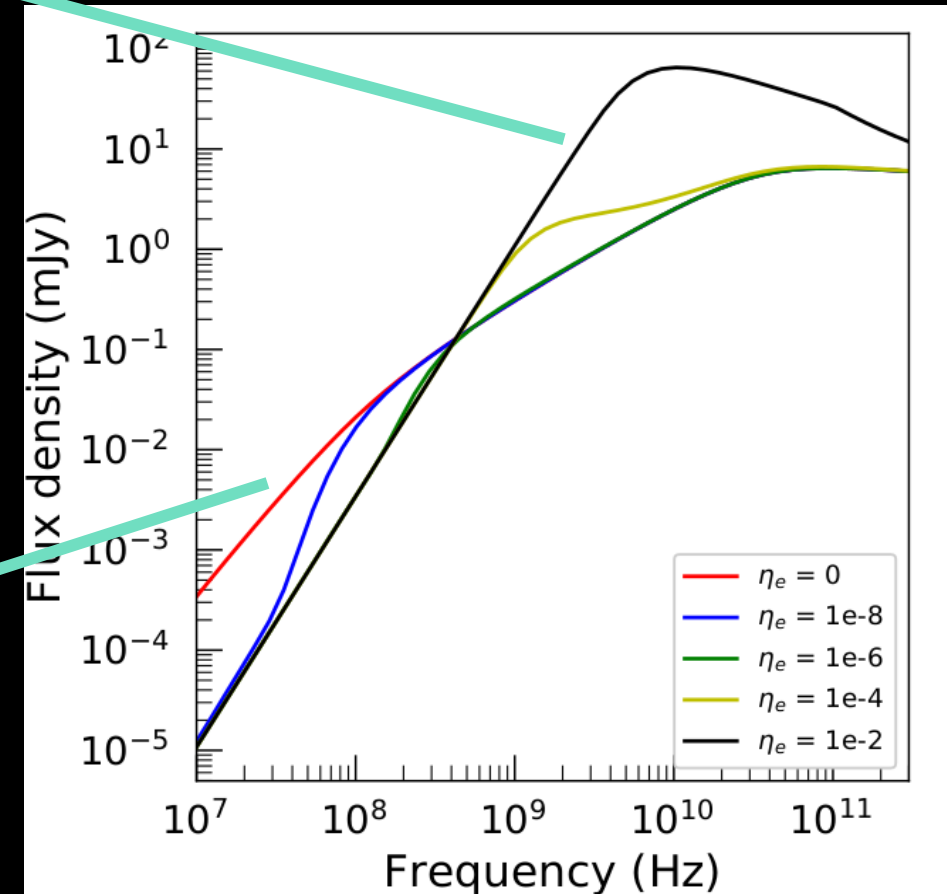
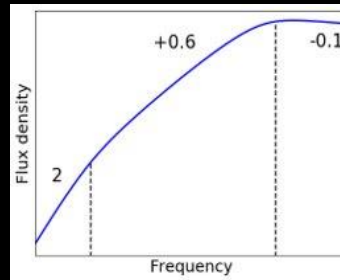
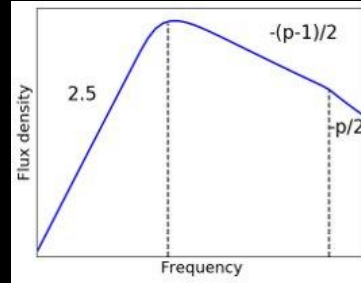
Flux variation across jet length



$$\begin{aligned} j_{v,\text{syn}}, \alpha_{v,\text{syn}}, \tau_{v,\text{syn}} &\propto n \, x \\ j_{v,\text{ff}}, \alpha_{v,\text{ff}}, \tau_{v,\text{ff}} &\propto n^2 \, x^2 \end{aligned}$$

Jet radio spectrum

- To study the **transition from free-free to synchrotron** dominated spectrum
- Case of a jet with a combination of both free-free and synchrotron emission
- η_e : Fraction of relativistic electrons capable of emitting synchrotron
- η_e is varied to regulate the contribution of synchrotron on the overall jet spectrum



Application of the model

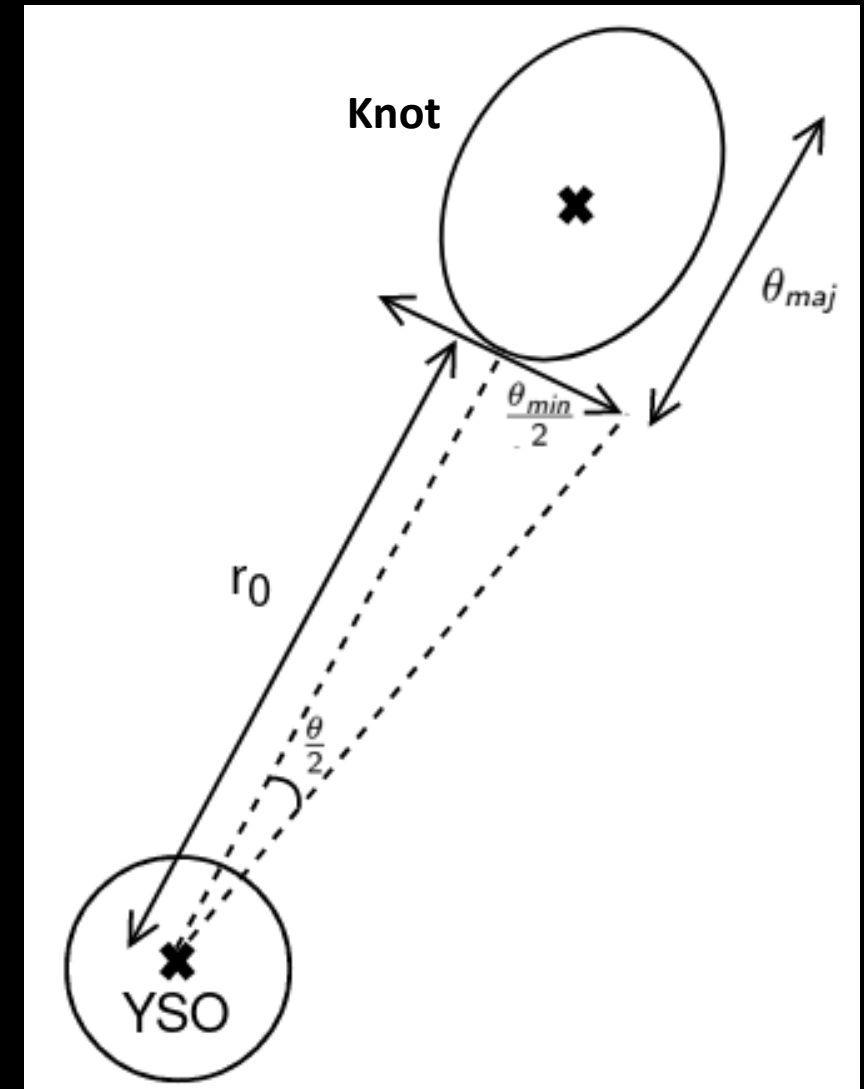
Comparison of model with Observations

Determination of knot parameters from observations

$$\text{size} = \theta_{maj}$$

$$r_0 = \text{radial distance of the knot center} - \frac{\text{size}}{2}$$

$$\theta_0 = 2 \tan^{-1} \left(\frac{\theta_{min}}{2r_0} \right)$$

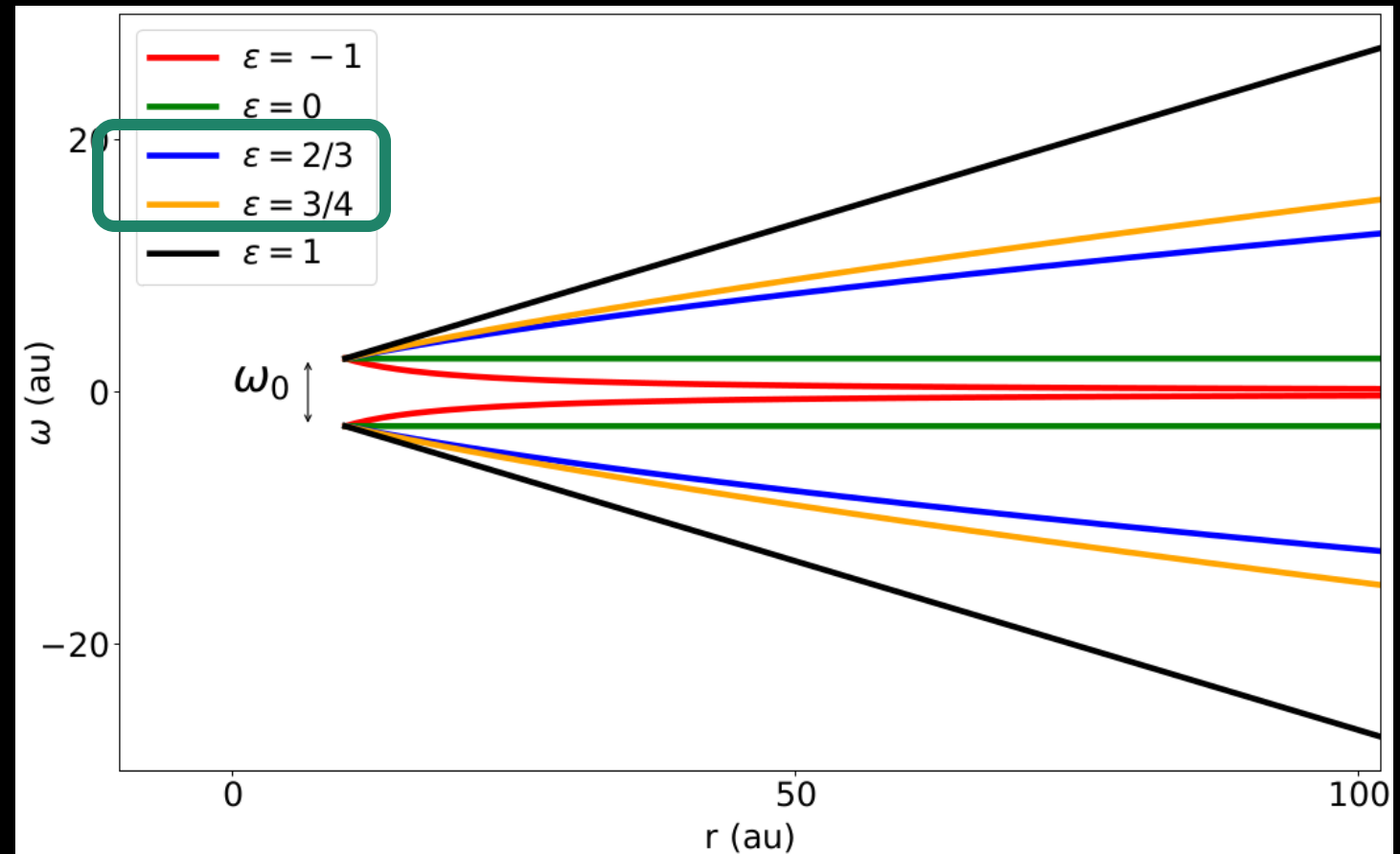


Comparison of model with Observations

Assumed knot parameters based on knowledge from observations and models:

- $T = 10^4 \text{ K}$
- $\epsilon = 2/3$ and $3/4$ (Reynolds 1986)

$$w(y) = w_0 \left(\frac{r_a}{r_0} \right)^\epsilon = w_0 \left(\frac{y}{y_0} \right)^\epsilon$$



Best-fit model: Model grids generated

- $q_n, q_x, q'_x, \delta\theta, p, \eta_e$: Physical parameters that are difficult to be constrained by observations.
- For few jets, n_0 measurement is not available: $n_0 * x_0$

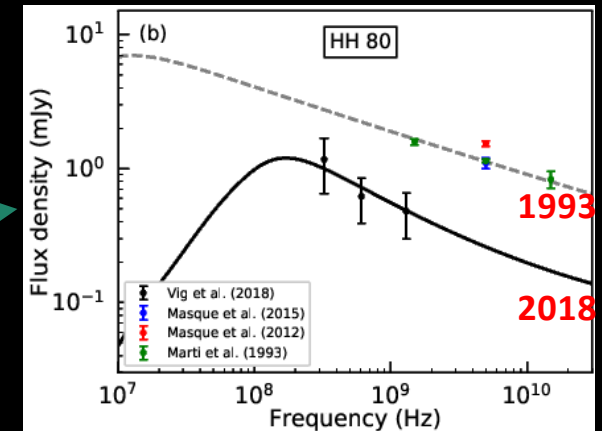
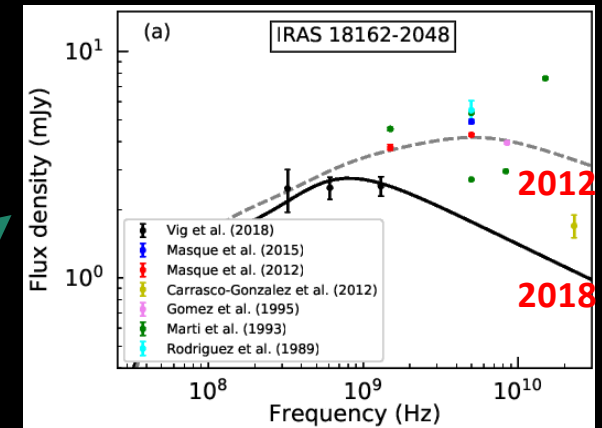
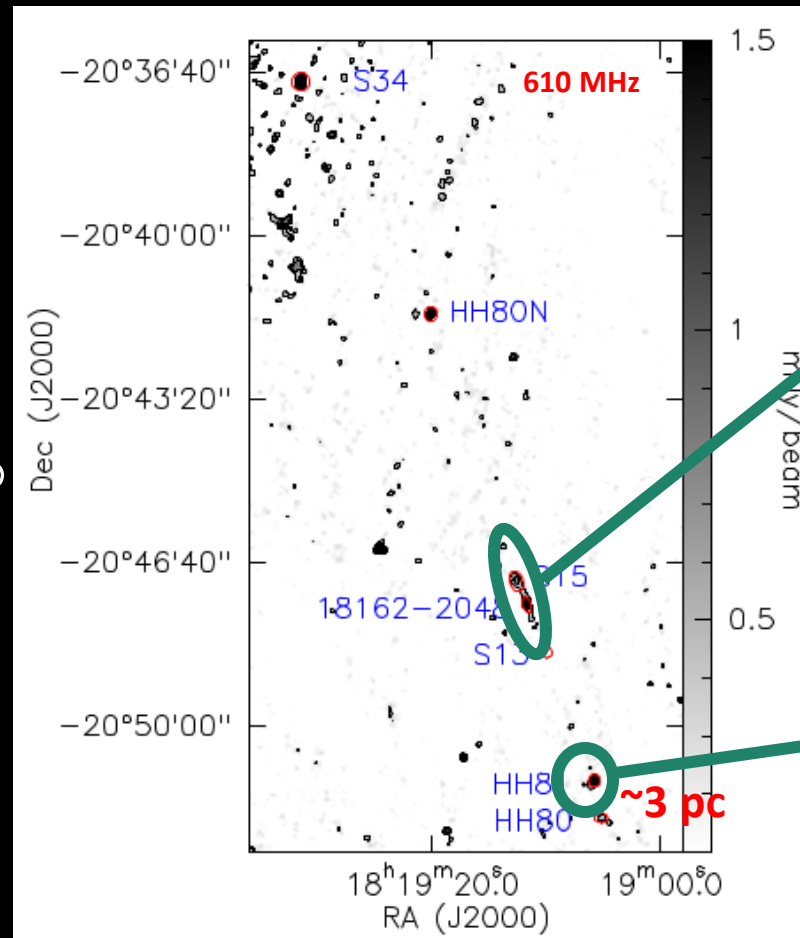
Parameters	Range Explored and Grid spacing
Number density at jet base (n_0 [cm ⁻³])	10 ² to 10 ⁶ by a factor of 10
Power-law index of radial number density variation (q_n)	-4 to -1 in steps of 0.5
Power-law index of radial ionization fraction variation (q_x)	-5 to 0 in steps of 0.5
Power-law index of lateral ionization fraction variation (q'_x)	-5 to 0 in steps of 0.5
Angular thickness of shocked region ($\delta\theta$ [°])	0.01 to 0.5 in steps of 0.1
Power-law index of non-thermal electron population (p)	1.7 to 2.6 in steps of 0.1
Fraction of relativistic electrons (η_e^{rel})	10 ⁻⁷ to 10 ⁻⁴ by a factor of 10

Case I: HH80-81

- High mass protostar: 11-15 M_{\odot}
- Distance: 1.4 kpc

Fernandez-Lopez et al. 2011, Anez-Lopez et al. 2020

Vig et al. 2018



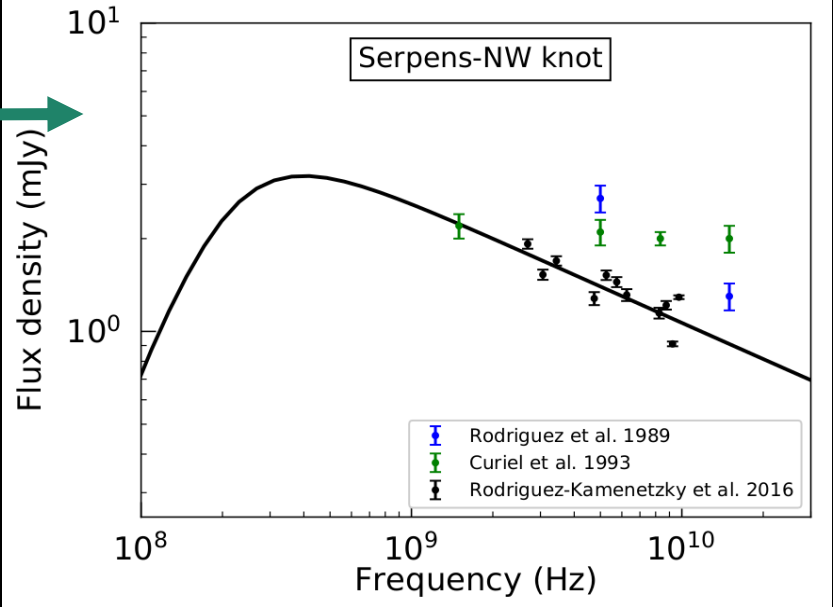
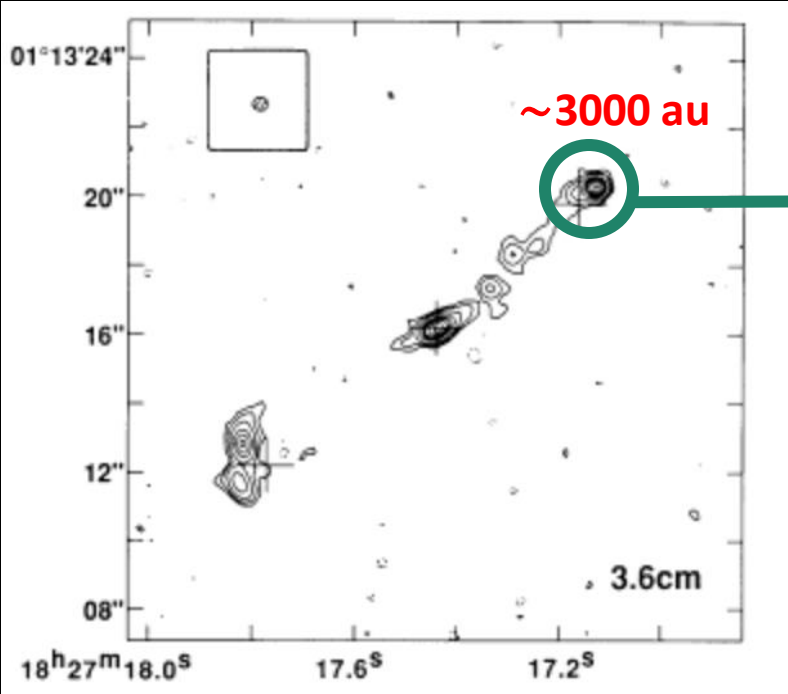
Model	ϵ	$n_0 * x_0$	q_n	q_x	$q_{x'}$	$\delta\theta$	p	η_e	χ^2
18162/2016	2/3	4.5×10^4	-1	-0.2	0.0	0.10	1.7	1×10^{-4}	0.87
18162/2009	2/3	3×10^5	-2	-0.5	0.0	0.10	1.7	1×10^{-4}	0.60
HH80/2016	2/3	200	-2	-2.5	-3.5	0.01	2.3	1×10^{-6}	0.30
HH80/1989	2/3	200	-3	-0.2	-3.0	0.01	1.7	1×10^{-6}	1.07

Case II: Serpens Triple radio source

- Intermediate mass protostar:
 3 M_{\odot}
- Distance: 436 pc

Hull et al. 2016, Ortiz-Leon et al. 2017

Curiel et al. 1993



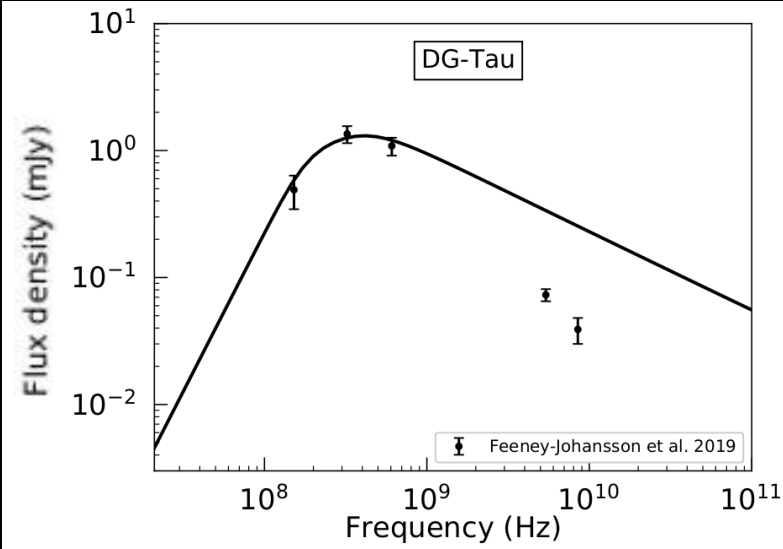
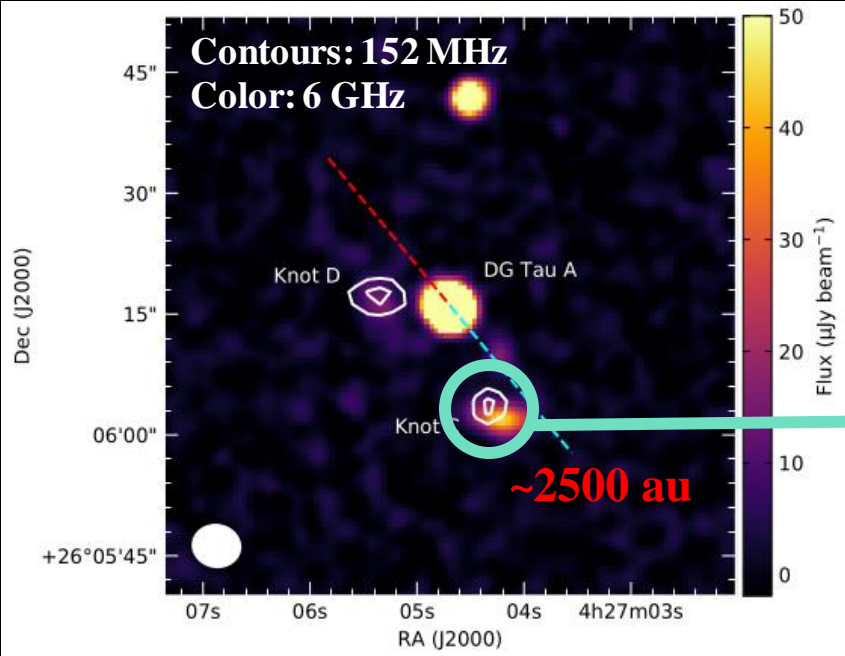
ε	$n_0 \cdot x_0$	q_n	q_x	$q_{x'}$	$\delta\theta$	p	η_e	χ^2
2/3	1.2×10^4	-1	-1	0	0.5	1.8	1×10^{-6}	319.3

Case III: DG Tau A

- Low mass protostar: $0.67 M_{\odot}$
- Distance: 120 pc

Lynch et al. 2013, Bailer-Jones et al. 2018

Feeney Johansson et al. 2019



ϵ	$n_0 * x_0$	q_n	q_x	$q_{x'}$	$\delta\theta$	p	η_e	χ^2
3/4	500	-3	-1	0	0.2	2.25	1×10^{-5}	47.8

Summary

- Features of our new model developed for protostellar jets:
 - Thermal **free-free** + non-thermal **synchrotron** emission
 - **Lateral** variation in ionization fraction
 - Generalized **geometry and optical depth** calculations
- Model can be employed to **estimate physical parameters** that are difficult to be constrained from observations.
- Model has been **applied** to few protostellar jets (HH80-81, Serpens triple radio source and DG-Tau) to estimate jet parameters that were unknown due to observational limitations.
- The estimated parameters (q_n, p, η_e) are reasonable and consistent with expected values from jet shock models etc.