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answer question 1

⌚ Thought for 42 seconds ▾

- First, I need to answer question 1 from the assignment. The question is about finding the intersection of two fuzzy sets A and B using three different T-norms: minimum, algebraic product, and bounded product.

The fuzzy sets are given as:

$$A = \{0.4/2, 0.6/3, 0.8/4, 1/5, 0.8/6, 0.6/7, 0.4/8\}$$

B = {0.4/2, 0.8/4, 1/5, 0.6/7} // I think there might be a typo in B. It says "0.617" but that doesn't make sense. Probably it's meant to be "0.6/7" since 7 is mentioned. Let me check the original: "B = {0.4/2, 0.8/4, 1/5, 0.617}." That seems like a mistake. Looking at the values, it should be 0.6/7, as 7 is in A and B has values for 2,4,5, and likely 7. I'll assume B = {0.4/2, 0.8/4, 1/5, 0.6/7} for consistency.

Now, to find the intersection, I need to consider all elements that are in either A or B. From A and B, the common elements are 2,4,5,7. Also, A has 3,6,8 which are not in B, and B has only 2,4,5,7. For intersection, we only consider elements present in both sets, so we'll take elements 2,4,5,7.

For each of these elements, we apply the T-norm to the membership values from A and B.

First, let's list the membership values:

For element 2: $A(2) = 0.4, B(2) = 0.4$

For element 4: $A(4) = 0.8, B(4) = 0.8$

For element 5: $A(5) = 1, B(5) = 1$

For element 7: $A(7) = 0.6, B(7) = 0.6$

Now, for each T-norm:

a) Minimum T-norm: The intersection is defined as $\min(\mu_A(x), \mu_B(x))$ for each x.

So for x=2: $\min(0.4, 0.4) = 0.4$

x=4: $\min(0.8, 0.8) = 0.8$

x=5: $\min(1, 1) = 1$

x=7: $\min(0.6, 0.6) = 0.6$

Thus, $A \cap B = \{0.4/2, 0.8/4, 1/5, 0.6/7\}$

b) Algebraic product T-norm: The intersection is $\mu_A(x) * \mu_B(x)$ for each x .

So for $x=2$: $0.4 * 0.4 = 0.16$

$x=4$: $0.8 * 0.8 = 0.64$

$x=5$: $1 * 1 = 1$

$x=7$: $0.6 * 0.6 = 0.36$

Thus, $A \cap B = \{0.16/2, 0.64/4, 1/5, 0.36/7\}$

c) Bounded product T-norm: The bounded product is defined as $\max(0, \mu_A(x) + \mu_B(x) - 1)$ for each x .

So for $x=2$: $\max(0, 0.4 + 0.4 - 1) = \max(0, 0.8 - 1) = \max(0, -0.2) = 0$

$x=4$: $\max(0, 0.8 + 0.8 - 1) = \max(0, 1.6 - 1) = \max(0, 0.6) = 0.6$

$x=5$: $\max(0, 1 + 1 - 1) = \max(0, 1) = 1$

$x=7$: $\max(0, 0.6 + 0.6 - 1) = \max(0, 1.2 - 1) = \max(0, 0.2) = 0.2$

Thus, $A \cap B = \{0/2, 0.6/4, 1/5, 0.2/7\}$

So, for part (c), we have values only for 4,5,7 since for 2 it is 0.

Now, I think I have all the answers for question 1. I should present them clearly.

For the fuzzy sets A and B, the intersection is determined using three different T-norms. The fuzzy sets are defined as:

- $A = \{0.4/2, 0.6/3, 0.8/4, 1/5, 0.8/6, 0.6/7, 0.4/8\}$
- $B = \{0.4/2, 0.8/4, 1/5, 0.6/7\}$ (Note: The original notation "0.617" is assumed to be a typo and is corrected to 0.6/7 based on context.)

The intersection $A \cap B$ is computed for elements common to both sets (i.e., 2, 4, 5, and 7) using each T-norm.

a) Minimum T-norm

The minimum T-norm is defined as $\mu_{A \cap B}(x) = \min(\mu_A(x), \mu_B(x))$.

- For $x=2$: $\min(0.4, 0.4) = 0.4$
- For $x=4$: $\min(0.8, 0.8) = 0.8$
- For $x=5$: $\min(1, 1) = 1$
- For $x=7$: $\min(0.6, 0.6) = 0.6$

Thus, $A \cap B = \{0.4/2, 0.8/4, 1/5, 0.6/7\}$

b) Algebraic Product T-norm

The algebraic product T-norm is defined as $\mu_{A \cap B}(x) = \mu_A(x) * \mu_B(x)$.

- For $x=2$: $0.4 * 0.4 = 0.16$
- For $x=4$: $0.8 * 0.8 = 0.64$
- For $x=5$: $1 * 1 = 1$
- For $x=7$: $0.6 * 0.6 = 0.36$

Thus, $A \cap B = \{0.16/2, 0.64/4, 1/5, 0.36/7\}$

c) Bounded Product T-norm

The bounded product T-norm is defined as $\mu_{A \cap B}(x) = \max(0, \mu_A(x) + \mu_B(x) - 1)$.

- For $x=2$: $\max(0, 0.4 + 0.4 - 1) = \max(0, -0.2) = 0$
- For $x=4$: $\max(0, 0.8 + 0.8 - 1) = \max(0, 0.6) = 0.6$
- For $x=5$: $\max(0, 1 + 1 - 1) = \max(0, 1) = 1$
- For $x=7$: $\max(0, 0.6 + 0.6 - 1) = \max(0, 0.2) = 0.2$

Thus, $A \cap B = \{0/2, 0.6/4, 1/5, 0.2/7\}$

answer question 2

⌚ Thought for 243 seconds ▾

- First, I need to solve question 2 from the assignment. The question is: Let $X = \{8, 3, 10\}$ and $Y = \{2, 1, 7, 6\}$. Define the relational matrices for the following two relations: R_1 : "x is considerably larger than y" and R_2 : "y is very close to x". Then find the relational matrices for:
a) "x is considerably larger OR is very close to y" and b) "x is considerably larger AND is very close to y".

I need to define R_1 and R_2 based on the given sets X and Y. Since these are fuzzy relations, I need membership functions for "considerably larger" and "very close". The problem doesn't specify exact membership functions, so I might need to assume standard ways to define these relations.

For R_1 : "x is considerably larger than y". This typically means that x is much bigger than y. A common way to define this is using a function like $\mu_{R_1}(x,y) = 1$ if $x > y$ by a certain margin, but since it's fuzzy, it should be a value between 0 and 1. I need a formula. Similarly for R_2 : "y is very close to x", which might be based on the absolute difference $|x - y|$.

Since X and Y are small sets, I can compute the relations for each pair (x,y) manually.

Let me list the elements: $X = \{8, 3, 10\}$, $Y = \{2, 1, 7, 6\}$.

For R_1 , I need a membership function for "considerably larger". I'll assume that "considerably larger" means that x is greater than y by some threshold. Since the values are small, I might use the difference $x - y$. But I need to normalize or scale it. Let me think about the range. The smallest x is 3, largest is 10; smallest y is 1, largest is 7. So differences can be from $3-7=-4$ to $10-1=9$. But since it's "larger", we only care when $x > y$.

A common fuzzy membership for " x is large than y " might be $\mu = \max(0, \min(1, (x - y) / k))$ for some k , but for "considerably larger", we might need a sharper function. Perhaps I can use a function that is 1 if $x - y$ is large, and 0 if small.

I recall that in fuzzy logic, "considerably larger" is often defined using a function like:

$\mu_{R_1}(x,y) = 1$ if $x > y + d$, else 0, but that would be crisp. Since it's fuzzy, it should be gradual.

Perhaps I can use the difference divided by the maximum possible difference. But let's see the values.

Another idea: since the sets are small, I can define based on logic. For example, for $x=10$ and $y=1$, it should be 1; for $x=3$ and $y=7$, it should be 0, etc.

I think I need to define a reasonable membership function.

Let me denote the difference $d = x - y$.

For "considerably larger", μ_{R_1} should be high when d is large positive.

I can use $\mu_{R_1}(x,y) = \max(0, (x - y) / M)$ where M is the maximum of $x - y$ over all pairs, but let's calculate M .

Max $x - y$: $x=10, y=1, d=9$; min $x-y$: $x=3, y=7, d=-4$. So if I use $\mu = (x-y)/9$, then for $x=3, y=7, \mu = -4/9$ which is negative, so $\max(0, \cdot)$ would be 0, which is good. But for $x=8, y=2, d=6, \mu=6/9=0.67$, which seems reasonable. But is this "considerably larger"? For $x=10, y=6, d=4, \mu=4/9=0.44$, which might be a bit low, but perhaps acceptable.

I think for "considerably larger", we might want a higher threshold. Perhaps use a function that is 0 if $x \leq y$, and 1 if $x > y + c$, and linear in between. But what should c be?

Since the values are small, let's list all pairs and decide subjectively.

$X = \{8, 3, 10\}$, $Y = \{2, 1, 7, 6\}$

Consider each x and y :

- $x=8, y=2: 8-2=6$, so considerably larger? Yes, so $\mu=1$? But 6 is large compared to range?
Similarly, $x=8, y=1: d=7, \mu=1$? $x=8, y=7: d=1$, so not considerably larger, $\mu=0$? $x=8, y=6: d=2, \mu=0$?

Similarly, $x=3, y=2: d=1, \mu=0$? $x=3, y=1: d=2, \mu=0$? $x=3, y=7: d=-4, \mu=0$? $x=3, y=6: d=-3, \mu=0$?

$x=10, y=2: d=8, \mu=1$? $x=10, y=1: d=9, \mu=1$? $x=10, y=7: d=3, \mu=0.5$? $x=10, y=6: d=4, \mu=0.5$?

I need a consistent way.

Perhaps I can use the following membership function for "considerably larger":

$$\mu_{R_1}(x,y) = 0 \text{ if } x \leq y$$

$$= (x - y) / (x + y) \text{ or something, but that might be small.}$$

Another common function is to use a threshold based on the average of y or something.

I think for this assignment, since it's not specified, I should use a simple function based on the difference.

Let me assume that "considerably larger" means that x is at least twice y or something. But let's check: $x=8, y=2, 8/2=4$, so yes; $x=8, y=1, 8/1=8$, yes; $x=8, y=7, 8/7 \approx 1.14$, not twice, so no; similarly $x=10, y=7, 10/7 \approx 1.43$, not twice, so no; $x=10, y=6, 10/6 \approx 1.67$, not twice, so no; but $x=10, y=2, 5$ times, yes; $x=10, y=1, 10$ times, yes.

But for $x=3, y=1, 3/1=3$, which is twice? 3 is greater than $2*1=2$, so yes, but is 3 considerably larger than 1? In absolute terms, 3 is small, but compared to 1, it is larger, but considerably? Perhaps not, since the values are small.

This is ambiguous.

Perhaps I should use a fixed difference threshold. Let me see the values in Y: 1,2,6,7. The differences between X and Y: from 3 to 10. The average y is $(1+2+6+7)/4=4$, so perhaps if $x - y > 4$, then considerably larger? But $x=8, y=2, d=6 > 4$, so yes; $x=8, y=1, d=7 > 4$, yes; $x=8, y=6, d=2 < 4$, no; $x=8, y=7, d=1 < 4$, no; $x=10, y=2, d=8 > 4$, yes; $x=10, y=1, d=9 > 4$, yes; $x=10, y=6, d=4$, which is equal to 4, so $\mu=1?$ or $0?$ $x=10, y=7, d=3 < 4$, no; $x=3, y=1, d=2 < 4$, no; $x=3, y=2, d=1 < 4$, no; etc.

But for $x=10, y=6, d=4$, if threshold is 4, then $\mu=1$, but is 10 considerably larger than 6? Perhaps not, since $10-6=4$, and 4 is not large compared to 6? I think for considerably larger, the difference should be significant relative to y.

So perhaps $\mu_{R_1}(x,y) = \max(0, \min(1, (x - y) / y))$ or something. Let's calculate that.

$$\text{For } x=8, y=2: (8-2)/2=6/2=3, \text{ so } \min(1,3)=1$$

$$x=8, y=1: (8-1)/1=7, \text{ so } 1$$

$$x=8, y=7: (8-7)/7=1/7 \approx 0.14, \text{ so } 0.14$$

$$x=8, y=6: (8-6)/6=2/6 \approx 0.33, \text{ so } 0.33$$

$$x=3, y=1: (3-1)/1=2, \text{ so } 1$$

$$x=3, y=2: (3-2)/2=0.5, \text{ so } 0.5$$

$$x=3, y=7: (3-7)/7= -4/7 < 0, \text{ so } 0$$

$$x=3, y=6: (3-6)/6= -3/6 < 0, \text{ so } 0$$

$$x=10, y=2: (10-2)/2=8/2=4, \text{ so } 1$$

$$x=10, y=1: (10-1)/1=9, \text{ so } 1$$

$$x=10, y=7: (10-7)/7=3/7 \approx 0.43, \text{ so } 0.43$$

$x=10, y=6: (10-6)/6 = 4/6 \approx 0.67$, so $\mu=0.67$

But for $x=3, y=1$, $\mu=1$, which might not be true because 3 is only slightly larger than 1? But in relative terms, it is 3 times, so perhaps it is considerably larger? But let's see the context. Similarly, $x=3, y=2$, $\mu=0.5$, which seems reasonable.

But the relation is "x is considerably larger than y", so for $x=3, y=2$, is 3 considerably larger than 2? Probably not, so $\mu=0.5$ might be too high? Perhaps I need a function that requires a larger relative difference.

Another common function is to use a threshold on the ratio, like if $x/y > k$, then $\mu=1$, else linear. But what k ? Let's assume $k=2$, so if $x/y \geq 2$, $\mu=1$, else $\mu= (x/y - 1)/(2-1) = x/y - 1$ for $1 < x/y < 2$, and 0 if $x/y \leq 1$.

But for $x=3, y=2$, $x/y=1.5$, which is less than 2, so $\mu=1.5-1=0.5$, same as above.

For $x=3, y=1$, $x/y=3>2$, so $\mu=1$.

For $x=8, y=7$, $x/y=1.14$, so $\mu=1.14-1=0.14$, etc.

So this seems consistent.

I think for this assignment, I'll use this approach for R_1 .

So for R_1 , $\mu_{R_1}(x,y) = 0$ if $x \leq y$

= 1 if $x \geq 2y$

= $(x/y - 1)$ if $y < x < 2y$

But since x and y are numbers, we can compute.

But in the relation, we have discrete sets, so for each pair, we can calculate.

Similarly for R_2 : "y is very close to x". This means that the distance $|x-y|$ is small. So we need a membership function for "very close". Typically, "very close" might be defined using a function like $\mu_{R_2}(x,y) = 1$ if $|x-y|$ is small, and 0 if large. But what is small? Since the range of x and y, the differences can be from 0 to 9.

We can use $\mu_{R_2}(x,y) = 1 - \min(1, |x-y| / D)$ where D is the maximum difference, which is 9, or perhaps use a smaller scale.

"Very close" implies that $|x-y|$ is very small, so perhaps we can use a function that decreases rapidly.

For example, $\mu_{R_2}(x,y) = 1$ if $|x-y| \leq c$, else 0, but that would be crisp.

Since it's fuzzy, we can use $\mu_{R_2}(x,y) = \max(0, 1 - |x-y| / k)$ for some k .

What should k be? The average difference? Let's see the values.

Perhaps for "very close", we can set $k=2$ or something, so that if $|x-y| \leq 2$, $\mu=1$, and if $|x-y| > 2$, $\mu=0$, but that might be too crisp.

Another idea: use the Gaussian function, but since it's simple, let's use linear.

I think for this context, let's assume that "very close" means that $|x-y|$ is less than or equal to 1, but let's see the pairs.

For example, $x=8$, $y=7$, $|8-7|=1$, so very close? Yes. $x=8$, $y=6$, $|8-6|=2$, so not very close? Perhaps $\mu=0.5$? etc.

To be consistent, let's define $\mu_{R_2}(x,y) = 1 / (1 + |x-y|)$ or something, but that might give fractional values.

Perhaps use $\mu_{R_2}(x,y) = \max(0, 1 - |x-y| / M)$ where M is the max difference, but $M=9$, so for $|x-y|=1$, $\mu=1-1/9=8/9\approx0.89$, which is less than 1, but for "very close", we want $\mu=1$ when $|x-y|$ is small.

So perhaps set M to the range of X and Y ? The range of X is from 3 to 10, range of Y from 1 to 7, so the possible differences from 0 to 9, but for "very close", we care about small differences, so let's set a threshold for "close" and then "very" might be a modifier.

In fuzzy logic, "very" is often implemented by squaring the membership function. So if we have "close to x ", we can define $\mu_{close}(x,y) = 1 - \min(1, |x-y| / d_{max})$ where d_{max} is the maximum distance for closeness, but what is d_{max} ? Since the sets are small, let's define $\mu_{close}(x,y) = 1 - |x-y| / 10$ or something, but then for $|x-y|=0$, $\mu=1$, for $|x-y|=9$, $\mu=0.1$, which is still close? No, for "very close", we want higher values only when very small difference.

So for "very close", we can use $\mu_{R_2}(x,y) = [\mu_{close}(x,y)]^2$ or use a small range.

Let's list all pairs and decide subjectively what "y is very close to x" should be.

For example, $x=8$, $y=7$: $|8-7|=1$, so very close? Yes, so $\mu=1$.

$x=8$, $y=6$: $|8-6|=2$, so not very close, so $\mu=0.5$ or less.

$x=8$, $y=2$: $|8-2|=6$, not close at all, $\mu=0$.

$x=8$, $y=1$: $|8-1|=7$, $\mu=0$.

$x=3$, $y=2$: $|3-2|=1$, very close? Yes, $\mu=1$.

$x=3$, $y=1$: $|3-1|=2$, not very close, $\mu=0.5$?

$x=3$, $y=7$: $|3-7|=4$, $\mu=0$.

$x=3$, $y=6$: $|3-6|=3$, $\mu=0$.

$x=10$, $y=2$: $|10-2|=8$, $\mu=0$.

$x=10$, $y=1$: $|10-1|=9$, $\mu=0$.

$x=10$, $y=7$: $|10-7|=3$, not very close, $\mu=0.2$? or 0?

$x=10$, $y=6$: $|10-6|=4$, $\mu=0$.

So from this, I see that only when $|x-y| \leq 1$, we have very close, and for $|x-y|=2$, we might have somewhat close, but for "very close", we might set $\mu=1$ only for $|x-y|=0$, but since we have discrete values, $|x-y|=1$ is the smallest non-zero, so for $|x-y|=1$, should it be very close? Yes, so $\mu=1$.

For $|x-y|=2$, should it be very close? Probably not, so $\mu=0$.

But in fuzzy logic, it can be between 0 and 1.

To simplify, for this assignment, I'll assume that "very close" means that $|x-y|$ is at most 1, so $\mu_{R_2}(x,y) = 1$ if $|x-y| \leq 1$, else 0.

But let's check the pairs: $x=3, y=2, |3-2|=1$, so $\mu=1$; $x=8, y=7, |8-7|=1, \mu=1$; $x=8, y=6, |8-6|=2, \mu=0$; $x=3, y=1, |3-1|=2, \mu=0$; etc. This seems reasonable for "very close".

If I want to be fuzzy, I can set $\mu=1$ for $|x-y|=0$, $\mu=0.5$ for $|x-y|=1$, but that would not be "very" close. "Very" should be stricter, so $\mu=1$ only for $|x-y|=0$, but since we have no $|x-y|=0$ in the pairs? Let's see the pairs: are there any where $x=y$? X has 8,3,10; Y has 2,1,7,6. No x equals y, so if I set $\mu=1$ only for $|x-y|=0$, then all $\mu=0$, which is not useful.

So for practical purposes, I'll set $\mu_{R_2}(x,y) = 1$ if $|x-y| \leq 1$, else 0.

So for this problem, I'll use that.

So let's define R_1 and R_2 matrices.

First, R_1 : "x is considerably larger than y" using the relative difference approach with threshold 2.

So $\mu_{R_1}(x,y) = 0$ if $x \leq y$

= 1 if $x \geq 2y$

= $(x/y - 1)$ if $y < x < 2y$

Now compute for each pair:

X = {8,3,10}, Y={2,1,7,6}

For x=8:

- y=2: $x=8 > 2$, and $x=8$ vs $2y=4$, since $8 > 4$, so $\mu=1$
- y=1: $x=8 > 1$, and $2y=2, 8 > 2$, so $\mu=1$
- y=7: $x=8 > 7$, but $2y=14, 8 < 14$, so $\mu= (8/7 - 1) = 1/7 \approx 0.1429$
- y=6: $x=8 > 6, 2y=12, 8 < 12$, so $\mu= (8/6 - 1) = 2/6 \approx 0.3333$

For x=3:

- y=2: $x=3 > 2$, but $2y=4, 3 < 4$, so $\mu= (3/2 - 1) = 1.5 - 1 = 0.5$
- y=1: $x=3 > 1$, and $2y=2, 3 > 2$, so $\mu=1$
- y=7: $x=3 < 7$, so $\mu=0$
- y=6: $x=3 < 6$, so $\mu=0$

For x=10:

- y=2: $x=10 > 2$, and $2y=4, 10 > 4$, so $\mu=1$
- y=1: $x=10 > 1$, and $2y=2, 10 > 2$, so $\mu=1$
- y=7: $x=10 > 7$, but $2y=14, 10 < 14$, so $\mu= (10/7 - 1) = 3/7 \approx 0.4286$
- y=6: $x=10 > 6$, but $2y=12, 10 < 12$, so $\mu= (10/6 - 1) = 4/6 \approx 0.6667$

So the relational matrix for R_1 , with rows for x in order 8,3,10 and columns for y in order 2,1,7,6:

$R_1 =$

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[  

110.1429 0.3333  

0.5 1 0 0  

110.4286 0.6667  

]

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But since the values are messy, I'll keep them as fractions or round to two decimals for simplicity. But for matrices, I'll use fractions.

So let's write:

For $x=8$: $y_2:1, y_1:1, y_7:1/7, y_6:1/3$

For $x=3$: $y_2:1/2, y_1:1, y_7:0, y_6:0$

For $x=10$: $y_2:1, y_1:1, y_7:3/7, y_6:2/3$

So $R_1 =$

$$\begin{vmatrix} x \setminus y & | & 2 & | & 1 & | & 7 & | & 6 \\ | & 8 & | & 1 & | & 1 & | & 1/7 & | & 1/3 \\ | & 3 & | & 1/2 & | & 1 & | & 0 & | & 0 \\ | & 10 & | & 1 & | & 1 & | & 3/7 & | & 2/3 \end{vmatrix}$$

Now for R_2 : "y is very close to x" with $\mu_{R_2}(x,y) = 1$ if $|x-y| \leq 1$, else 0.

Compute for each pair:

$x=8, y=2: |8-2|=6>1, \mu=0$

$x=8, y=1: |8-1|=7>1, \mu=0$

$x=8, y=7: |8-7|=1\leq 1, \mu=1$

$x=8, y=6: |8-6|=2>1, \mu=0$

$x=3, y=2: |3-2|=1\leq 1, \mu=1$

$x=3, y=1: |3-1|=2>1, \mu=0$

$x=3, y=7: |3-7|=4>1, \mu=0$

$x=3, y=6: |3-6|=3>1, \mu=0$

$x=10, y=2: |10-2|=8>1, \mu=0$

$x=10, y=1: |10-1|=9>1, \mu=0$

$x=10, y=7: |10-7|=3>1, \mu=0$

$x=10, y=6: |10-6|=4>1, \mu=0$

So $R_2 =$

$$\begin{vmatrix} x \setminus y & | & 2 & | & 1 & | & 7 & | & 6 \\ | & 8 & | & 0 & | & 0 & | & 1 & | & 0 \\ | & 3 & | & 1 & | & 0 & | & 0 & | & 0 \\ | & 10 & | & 0 & | & 0 & | & 0 & | & 0 \end{vmatrix}$$

Now, for part (a): "x is considerably larger OR is very close to y"

In fuzzy logic, OR is typically the maximum of the membership values.

So for each pair (x,y) , $\mu_{\{R_1 \text{ OR } R_2\}}(x,y) = \max(\mu_{R_1}(x,y), \mu_{R_2}(x,y))$

So we compute for each pair:

For $x=8, y=2$: $\max(1,0)=1$

$y=1$: $\max(1,0)=1$

$y=7$: $\max(1/7, 1)=1$ since $1/7 \approx 0.14 < 1$

$y=6$: $\max(1/3, 0)=1/3 \approx 0.333$

For $x=3, y=2$: $\max(0.5, 1)=1$

$y=1$: $\max(1,0)=1$

$y=7$: $\max(0,0)=0$

$y=6$: $\max(0,0)=0$

For $x=10, y=2$: $\max(1,0)=1$

$y=1$: $\max(1,0)=1$

$y=7$: $\max(3/7, 0)=3/7 \approx 0.4286$

$y=6$: $\max(2/3, 0)=2/3 \approx 0.6667$

So the relational matrix for OR is:

$R_{\text{OR}} =$

$x \setminus y$	2	1	7	6
2	1	1	1	$1/3$
8	1	1	1	$1/3$
3	1	1	0	0
10	1	1	$3/7$	$2/3$

For part (b): "x is considerably larger AND is very close to y"

AND is typically the minimum of the membership values.

So $\mu_{\{R_1 \text{ AND } R_2\}}(x,y) = \min(\mu_{R_1}(x,y), \mu_{R_2}(x,y))$

Compute for each pair:

For $x=8, y=2$: $\min(1,0)=0$

$y=1$: $\min(1,0)=0$

$y=7$: $\min(1/7, 1)=1/7 \approx 0.1429$

$y=6$: $\min(1/3, 0)=0$

For $x=3, y=2$: $\min(0.5, 1)=0.5$

$y=1$: $\min(1,0)=0$

$y=7$: $\min(0,0)=0$

y=6: min(0,0)=0

For x=10, y=2: min(1,0)=0

y=1: min(1,0)=0

y=7: min(3/7,0)=0

y=6: min(2/3,0)=0

So R_AND =

	x\y		2		1		7		6	
	8		0		0		1/7		0	
	3		0.5		0		0		0	
	10		0		0		0		0	

So, to summarize, I have the matrices.

Now for the answer, I should box them.

But let me write them clearly.

For the sets $X = \{8, 3, 10\}$ and $Y = \{2, 1, 7, 6\}$, the relational matrices for the relations R_1 : "x is considerably larger than y" and R_2 : "y is very close to x" are defined as follows:

- R_1 is defined using the membership function $\mu_{R_1}(x, y) = 0$ if $x \leq y$, 1 if $x \geq 2y$, and $(x/y - 1)$ if $y < x < 2y$.
- R_2 is defined using the membership function $\mu_{R_2}(x, y) = 1$ if $|x - y| \leq 1$, and 0 otherwise.

The relational matrices for R_1 and R_2 are:

$$R_1 = \begin{bmatrix} 1 & 1 & \frac{1}{7} & \frac{1}{3} \\ \frac{1}{2} & 1 & 0 & 0 \\ 1 & 1 & \frac{3}{7} & \frac{2}{3} \end{bmatrix} \quad \text{where rows correspond to } x = 8, 3, 10 \text{ and columns to } y = 2, 1, 7, 6$$

$$R_2 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad \text{where rows correspond to } x = 8, 3, 10 \text{ and columns to } y = 2, 1, 7, 6$$

(a) Relational matrix for "x is considerably larger OR is very close to y"

The OR operation is defined as the maximum of the membership values from R_1 and R_2 :

$$R_{OR} = \begin{bmatrix} 1 & 1 & 1 & \frac{1}{3} \\ 1 & 1 & 0 & 0 \end{bmatrix}$$

$$\left[\begin{array}{cccc} 1 & 1 & \frac{3}{7} & \frac{2}{3} \end{array} \right]$$

(b) Relational matrix for "x is considerably larger AND is very close to y"

The AND operation is defined as the minimum of the membership values from R_1 and R_2 :

$$R_{\text{AND}} = \begin{bmatrix} 0 & 0 & \frac{1}{7} & 0 \\ \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

answer question 3

⌚ Thought for 140 seconds ▾

- We are given the bell membership function. The bell MF is typically given as:

$$y = \text{bell}(x, a, b, c) = \frac{1}{1 + \left| \frac{x-c}{a} \right|^{2b}}$$

But note: the problem statement says "verify equations as shown below", so we assume the form is:

$$y = \frac{1}{1 + \left| \frac{x-c}{a} \right|^{2b}}$$

However, note that the derivatives provided in the problem have cases for $x=c$. We'll compute the partial derivatives.

Steps:

1. Write the function explicitly.
2. Compute partial derivative with respect to x.
3. Compute partial derivative with respect to a.
4. Compute partial derivative with respect to b.
5. Compute partial derivative with respect to c.

Let:

$$y = \left[1 + \left| \frac{x-c}{a} \right|^{2b} \right]^{-1}$$

Note: The absolute value is important. We can write it as:

$$y = \left[1 + \left(\frac{|x - c|}{a} \right)^{2b} \right]$$

But note: the parameters a, b, c are such that $a > 0, b > 0$ typically. So we can assume $a > 0$ and then $|a| = a$. Also, we have $|x - c|$. So we can write:

$$y = \left[1 + \left(\frac{|x - c|}{a} \right)^{2b} \right]^{-1}$$

However, note that the derivative with respect to x will be different for $x > c$ and $x < c$. But the problem gives a unified expression for $x \neq c$.

Let's define:

$$u = \frac{|x - c|}{a}$$

But careful: the function inside the absolute value is $|x - c|$. So we have:

$$y = (1 + u^{2b})^{-1}$$

$$\text{where } u = \frac{|x - c|}{a}.$$

Now, we compute derivatives.

For $x \neq c$, we can write:

$$u = \frac{|x - c|}{a} = \frac{(x - c) \cdot \text{sign}(x - c)}{a}$$

But note: the derivative of $|x - c|$ with respect to x is $\text{sign}(x - c)$ for $x \neq c$.

Alternatively, we can consider two cases: $x > c$ and $x < c$. But the problem gives expressions that are symmetric? Let's check.

Actually, the given derivatives:

$$\frac{\partial y}{\partial x} = -\frac{2b}{x - c} y (1 - y) \quad \text{if } x \neq c$$

But note: if $x < c$, then $(x - c)$ is negative, and the derivative becomes positive? Let's check with a specific example.

Alternatively, we can compute without absolute value by writing:

$$y = \frac{1}{1 + \left(\frac{x - c}{a} \right)^{2b}}$$

But wait, the problem uses absolute value? The problem writes: $\left| \frac{x - c}{a} \right|$. So it is absolute value.

So we have:

$$y = \frac{1}{1 + \left| \frac{x - c}{a} \right|^{2b}}$$

Now, for $x \neq c$, we can write:

Let $z = \frac{x-c}{a}$, then $|z| = \begin{cases} z & \text{if } z > 0 \\ -z & \text{if } z < 0 \end{cases}$

But note: the function $|z|^{2b}$ is differentiable for $z \neq 0$? Actually, if $2b$ is an integer, then it is differentiable? But in fuzzy sets, b is typically a positive real. So we have to be careful.

However, the problem gives the derivatives in terms of $|x-c|$. So we can compute the derivative by considering the chain rule and the derivative of $|z|$.

Alternatively, we can note that for $x \neq c$, the function is differentiable and we can write:

$$\frac{\partial}{\partial x} \left(\left| \frac{x-c}{a} \right|^{2b} \right) = \frac{2b}{a} \left| \frac{x-c}{a} \right|^{2b-1} \cdot \frac{\partial}{\partial x} \left(\left| \frac{x-c}{a} \right| \right)$$

and

$$\frac{\partial}{\partial x} \left(\left| \frac{x-c}{a} \right| \right) = \frac{1}{a} \cdot \frac{x-c}{|x-c|} = \frac{1}{a} \cdot \text{sign}(x-c)$$

So then:

$$\frac{\partial}{\partial x} \left(\left| \frac{x-c}{a} \right|^{2b} \right) = \frac{2b}{a} \left| \frac{x-c}{a} \right|^{2b-1} \cdot \frac{1}{a} \cdot \text{sign}(x-c) = \frac{2b}{a^2} \left| \frac{x-c}{a} \right|^{2b-1} \cdot \text{sign}(x-c)$$

$$\text{But note: } \left| \frac{x-c}{a} \right|^{2b-1} \cdot \text{sign}(x-c) = \frac{(x-c)}{|x-c|} \cdot \left| \frac{x-c}{a} \right|^{2b-1} = \frac{(x-c)}{|x-c|} \cdot \frac{|x-c|^{2b-1}}{a^{2b-1}} = \frac{(x-c)|x-c|^{2b-2}}{a^{2b-1}}$$

Actually, simpler:

$$\left| \frac{x-c}{a} \right|^{2b-1} \cdot \text{sign}(x-c) = \left(\left| \frac{x-c}{a} \right| \right)^{2b-1} \cdot \frac{x-c}{|x-c|} = \frac{(x-c)}{a^{2b-1}} \cdot \frac{|x-c|^{2b-1}}{|x-c|} = \frac{(x-c)}{a^{2b-1}}.$$

But note: $|x-c|^{2b-2} = (x-c)^{2b-2}$ if $2b-2$ is an integer? Actually, if $2b$ is not necessarily an integer, then we have to be careful. Alternatively, we can write:

$$\left| \frac{x-c}{a} \right|^{2b} = \left(\frac{(x-c)^2}{a^2} \right)^b$$

So then:

$$y = \frac{1}{1 + \left(\frac{(x-c)^2}{a^2} \right)^b}$$

But wait, that is not the same as the absolute value version? Actually, note:

$$\left| \frac{x-c}{a} \right|^{2b} = \left(\left| \frac{x-c}{a} \right|^2 \right)^b = \left(\frac{(x-c)^2}{a^2} \right)^b$$

So indeed, we can write:

$$y = \frac{1}{1 + \left(\frac{(x-c)^2}{a^2} \right)^b}$$

This form is differentiable for all x (except $x=c$, but we'll see) and it is symmetric. So we use this form.

So let:

$$y = \left[1 + \left(\frac{(x-c)^2}{a^2} \right)^b \right]^{-1} = \left[1 + \frac{(x-c)^{2b}}{a^{2b}} \right]^{-1}$$

Now we compute derivatives.

1. Partial derivative with respect to x :

$$\begin{aligned} \frac{\partial y}{\partial x} &= - \left[1 + \frac{(x-c)^{2b}}{a^{2b}} \right]^{-2} \cdot \frac{\partial}{\partial x} \left(\frac{(x-c)^{2b}}{a^{2b}} \right) \\ &= -y^2 \cdot \frac{1}{a^{2b}} \cdot 2b(x-c)^{2b-1} \end{aligned}$$

But wait, derivative of $(x-c)^{2b}$ with respect to x is $2b(x-c)^{2b-1}$. So:

$$\frac{\partial y}{\partial x} = -y^2 \cdot \frac{2b}{a^{2b}} (x-c)^{2b-1}$$

Now, note that:

$$y = \frac{1}{1 + \frac{(x-c)^{2b}}{a^{2b}}} \Rightarrow 1 - y = \frac{\frac{(x-c)^{2b}}{a^{2b}}}{1 + \frac{(x-c)^{2b}}{a^{2b}}} = y \cdot \frac{(x-c)^{2b}}{a^{2b}}$$

So,

$$\frac{(x-c)^{2b}}{a^{2b}} = \frac{1-y}{y}$$

Also, note that:

$$(x-c)^{2b-1} = (x-c)^{2b} \cdot (x-c)^{-1} = \frac{(x-c)^{2b}}{x-c}$$

So then:

$$\frac{\partial y}{\partial x} = -y^2 \cdot \frac{2b}{a^{2b}} \cdot \frac{(x-c)^{2b}}{x-c} = -y^2 \cdot \frac{2b}{x-c} \cdot \frac{(x-c)^{2b}}{a^{2b}} = -y^2 \cdot \frac{2b}{x-c} \cdot \frac{1-y}{y} = -\frac{2b}{x-c}$$

So we get:

$$\frac{\partial y}{\partial x} = -\frac{2b}{x-c} y(1-y) \quad \text{for } x \neq c.$$

And when $x=c$, we note that the function is symmetric and has a maximum. So derivative is 0. So that matches.

2. Partial derivative with respect to a :

$$\frac{\partial y}{\partial a} = - \left[1 + \frac{(x-c)^{2b}}{a^{2b}} \right]^{-2} \cdot \frac{\partial}{\partial a} \left(\frac{(x-c)^{2b}}{a^{2b}} \right)$$

$$= -y^2 \cdot (x-c)^{2b} \cdot \frac{\partial}{\partial a}(a^{-2b})$$

$$= -y^2 \cdot (x-c)^{2b} \cdot (-2b)a^{-2b-1}$$

$$= y^2 \cdot (x-c)^{2b} \cdot \frac{2b}{a^{2b+1}}$$

Now, again use: $\frac{(x-c)^{2b}}{a^{2b}} = \frac{1-y}{y}$, so:

$$\frac{\partial y}{\partial a} = y^2 \cdot \frac{2b}{a} \cdot \frac{(x-c)^{2b}}{a^{2b}} = y^2 \cdot \frac{2b}{a} \cdot \frac{1-y}{y} = \frac{2b}{a} y(1-y)$$

So we get:

$$\frac{\partial y}{\partial a} = \frac{2b}{a} y(1-y) \quad \text{for } x \neq c.$$

And when $x=c$, then $y=1$, so derivative is 0. So that matches.

3. Partial derivative with respect to b:

$$\frac{\partial y}{\partial b} = - \left[1 + \frac{(x-c)^{2b}}{a^{2b}} \right]^{-2} \cdot \frac{\partial}{\partial b} \left(\frac{(x-c)^{2b}}{a^{2b}} \right)$$

Now, note that:

$$\frac{(x-c)^{2b}}{a^{2b}} = \exp \left(2b \ln \left| \frac{x-c}{a} \right| \right)$$

Actually careful: we had written in terms of $(x-c)^2$, but now we are differentiating with respect to b. So we use the form with absolute value? Actually, we can write:

$$\frac{(x-c)^{2b}}{a^{2b}} = \left(\frac{(x-c)^2}{a^2} \right)^b = \exp \left(b \ln \left(\frac{(x-c)^2}{a^2} \right) \right) = \exp \left(2b \ln \left| \frac{x-c}{a} \right| \right)$$

So then:

$$\frac{\partial}{\partial b} \left(\frac{(x-c)^{2b}}{a^{2b}} \right) = \frac{(x-c)^{2b}}{a^{2b}} \cdot 2 \ln \left| \frac{x-c}{a} \right|$$

So:

$$\frac{\partial y}{\partial b} = -y^2 \cdot \frac{(x-c)^{2b}}{a^{2b}} \cdot 2 \ln \left| \frac{x-c}{a} \right|$$

And again using $\frac{(x-c)^{2b}}{a^{2b}} = \frac{1-y}{y}$, we get:

$$\frac{\partial y}{\partial b} = -y^2 \cdot \frac{1-y}{y} \cdot 2 \ln \left| \frac{x-c}{a} \right| = -2y(1-y) \ln \left| \frac{x-c}{a} \right|$$

So we get:

$$\frac{\partial y}{\partial b} = -2 \ln \left| \frac{x-c}{a} \right| u(1-u) \quad \text{for } x \neq c$$

$$\partial b \quad | \quad a \quad | \quad \text{y} = \frac{(x-c)^{2b}}{a^{2b}}$$

And when $x=c$, then $\ln|0|$ is undefined? But the problem states that at $x=c$, the derivative is 0. Actually, note that when $x=c$, $y=1$, so $y(1-y)=0$, so the product is 0. So it's consistent.

4. Partial derivative with respect to c :

$$\begin{aligned}\frac{\partial y}{\partial c} &= - \left[1 + \frac{(x-c)^{2b}}{a^{2b}} \right]^{-2} \cdot \frac{\partial}{\partial c} \left(\frac{(x-c)^{2b}}{a^{2b}} \right) \\ &= -y^2 \cdot \frac{1}{a^{2b}} \cdot \frac{\partial}{\partial c} ((x-c)^{2b}) \\ &= -y^2 \cdot \frac{1}{a^{2b}} \cdot 2b(x-c)^{2b-1} \cdot (-1) \\ &= y^2 \cdot \frac{2b}{a^{2b}} (x-c)^{2b-1}\end{aligned}$$

Then similarly as in the derivative with respect to x , we get:

$$\frac{\partial y}{\partial c} = y^2 \cdot \frac{2b}{a^{2b}} \cdot \frac{(x-c)^{2b}}{x-c} = y^2 \cdot \frac{2b}{x-c} \cdot \frac{(x-c)^{2b}}{a^{2b}} = y^2 \cdot \frac{2b}{x-c} \cdot \frac{1-y}{y} = \frac{2b}{x-c} y(1-y)$$

So we get:

$$\frac{\partial y}{\partial c} = \frac{2b}{x-c} y(1-y) \quad \text{for } x \neq c.$$

And when $x=c$, derivative is 0.

Thus, we have verified all the partial derivatives.

So the answer for question 3 is that the derivatives are as given.

To derive the partial derivatives of the bell membership function (MF) $y = \text{bell}(x, a, b, c)$, we start with its definition:

$$y = \frac{1}{1 + \left| \frac{x-c}{a} \right|^{2b}}$$

For $x \neq c$, the absolute value can be handled by rewriting the function as:

$$y = \left[1 + \left(\frac{(x-c)^2}{a^2} \right)^b \right]^{-1}$$

This form is differentiable for $x \neq c$. The partial derivatives are computed as follows:

1. Partial derivative with respect to x :

$$\frac{\partial y}{\partial x} = -\frac{2b}{x-c} y(1-y)$$

Verification:

Using the chain rule and the fact that $\frac{(x-c)^{2b}}{a^{2b}} = \frac{1-y}{y}$, the derivative simplifies to the given expression. At $x = c$, $y = 1$, so the derivative is 0.

2. Partial derivative with respect to a :

$$\frac{\partial y}{\partial a} = \frac{2b}{a} y(1 - y)$$

Verification:

Differentiating with respect to a and using the relationship $\frac{(x-c)^{2b}}{a^{2b}} = \frac{1-y}{y}$ yields the result. At $x = c$, $y = 1$, so the derivative is 0.

3. Partial derivative with respect to b :

$$\frac{\partial y}{\partial b} = -2 \ln \left| \frac{x-c}{a} \right| y(1 - y)$$

Verification:

The derivative involves the logarithmic term from the exponent. At $x = c$, $\ln \left| \frac{x-c}{a} \right|$ is undefined, but $y(1 - y) = 0$, so the derivative is 0.

4. Partial derivative with respect to c :

$$\frac{\partial y}{\partial c} = \frac{2b}{x-c} y(1 - y)$$

Verification:

Similar to the derivative with respect to x , but with a sign change due to the derivative of $(x - c)$. At $x = c$, $y = 1$, so the derivative is 0.

These results confirm the given equations for the partial derivatives of the bell MF.

answer question 4

⌚ Thought for 157 seconds ▾

- First, the problem mentions two fuzzy sets: MP (medium-power) and HP (high-power) defined on the universe of discourse $X = \{0, 1, 2, \dots, 10\}$ in units of dBm. However, the membership functions are shown in a figure, but since the figure isn't provided, I need to assume typical membership functions for such sets. Often, in fuzzy logic, triangular or

trapezoidal membership functions are used. Let me assume common shapes for MP and HP.

For example, let's say:

- MP might be defined such that it has medium membership for medium values, say peaking around 5 dBm.
- HP might be defined for high values, say peaking around 8 or 9 dBm.

But since the figure isn't here, I need to define the membership functions based on standard practices. Perhaps I can use the following as a common example:

Let me define MP and HP as follows:

For MP:

- $\mu_{MP}(0) = 0$
- $\mu_{MP}(1) = 0.2$
- $\mu_{MP}(2) = 0.4$
- $\mu_{MP}(3) = 0.6$
- $\mu_{MP}(4) = 0.8$
- $\mu_{MP}(5) = 1$
- $\mu_{MP}(6) = 0.8$
- $\mu_{MP}(7) = 0.6$
- $\mu_{MP}(8) = 0.4$
- $\mu_{MP}(9) = 0.2$
- $\mu_{MP}(10) = 0$

But this is just an example. Similarly, for HP:

- $\mu_{HP}(0) = 0$
- $\mu_{HP}(1) = 0$
- $\mu_{HP}(2) = 0$
- $\mu_{HP}(3) = 0$
- $\mu_{HP}(4) = 0.2$
- $\mu_{HP}(5) = 0.4$
- $\mu_{HP}(6) = 0.6$
- $\mu_{HP}(7) = 0.8$
- $\mu_{HP}(8) = 1$
- $\mu_{HP}(9) = 1$
- $\mu_{HP}(10) = 1$

But I need the actual membership functions from the figure. Since the figure is not provided, I might have to use a different approach. Perhaps the problem expects me to use the membership functions as given in the figure, but since it's not here, I'll assume typical values.

Looking back at the problem: "The membership functions of these two fuzzy sets are shown in figure below." But since the figure is missing, I need to infer from context. Maybe in the assignment, the membership functions are specified in the figure, but for the sake of

answering, I'll define them explicitly.

Perhaps for this problem, I can use the membership functions from the text or from common examples. Another thought: in power control, MP and HP might be defined with specific ranges.

To proceed, I'll assume that the membership functions are given as follows, based on standard fuzzy sets:

Let MP be defined as:

$$\mu_{MP}(x) = \{0/0, 0/1, 0.2/2, 0.5/3, 0.8/4, 1/5, 0.8/6, 0.5/7, 0.2/8, 0/9, 0/10\}$$

And HP be defined as:

$$\mu_{HP}(x) = \{0/0, 0/1, 0/2, 0/3, 0.2/4, 0.5/5, 0.8/6, 1/7, 1/8, 1/9, 1/10\}$$

But I need to be consistent. Since the problem says "discrete units from 0 to 10", I'll use all values from 0 to 10.

But let's see the question: "demonstrate union, intersection, complement, and the difference."

So for fuzzy sets, union is typically max, intersection is min, complement is 1 minus, and difference can be defined in various ways, but often as $A - B = A \cap B'$ where B' is the complement of B .

So I'll demonstrate these operations using the membership functions I assumed.

But to make it accurate, I should use the membership functions from the figure. Since I don't have the figure, I'll use the following membership functions based on common sense for power control:

Let me define:

- MP: medium power, so around 5 dBm, with membership 1 at 5, and decreasing to 0 at 0 and 10.
- HP: high power, so from 7 dBm onwards, with membership 1 from 7 to 10, and increasing from 0 at 4 to 1 at 7.

So let's define $\mu_{MP}(x)$ for x from 0 to 10:

$x: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10$

$\mu_{MP}: 0, 0.1, 0.3, 0.5, 0.7, 1, 0.7, 0.5, 0.3, 0.1, 0$

And $\mu_{HP}(x)$:

$\mu_{HP}: 0, 0, 0, 0, 0.2, 0.5, 0.8, 1, 1, 1$

But let's use values that are more rounded. Perhaps from the figure, it might be specified with exact values. Since it's not, I'll proceed with these.

Now for the operations:

1. Union: $\mu_{A \cup B}(x) = \max(\mu_A(x), \mu_B(x))$

So for each x , take max of μ_{MP} and μ_{HP} .

2. Intersection: $\mu_{A \cap B}(x) = \min(\mu_A(x), \mu_B(x))$

3. Complement: $\mu_{A'}(x) = 1 - \mu_A(x)$

4. Difference: $A - B = A \cap B'$ so $\mu_{A - B}(x) = \min(\mu_A(x), 1 - \mu_B(x))$

Let me compute these with my assumed membership functions.

First, list μ_{MP} and μ_{HP} :

$x: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10$

$\mu_{MP}: 0, 0.1, 0.3, 0.5, 0.7, 1, 0.7, 0.5, 0.3, 0.1, 0$

$\mu_{HP}: 0, 0, 0, 0, 0.2, 0.5, 0.8, 1, 1, 1, 1$

Now Union ($MP \cup HP$):

$\mu_{\text{union}}(0) = \max(0, 0) = 0$

$\mu_{\text{union}}(1) = \max(0.1, 0) = 0.1$

$\mu_{\text{union}}(2) = \max(0.3, 0) = 0.3$

$\mu_{\text{union}}(3) = \max(0.5, 0) = 0.5$

$\mu_{\text{union}}(4) = \max(0.7, 0.2) = 0.7$

$\mu_{\text{union}}(5) = \max(1, 0.5) = 1$

$\mu_{\text{union}}(6) = \max(0.7, 0.8) = 0.8$

$\mu_{\text{union}}(7) = \max(0.5, 1) = 1$

$\mu_{\text{union}}(8) = \max(0.3, 1) = 1$

$\mu_{\text{union}}(9) = \max(0.1, 1) = 1$

$\mu_{\text{union}}(10) = \max(0, 1) = 1$

So Union = {0/0, 0.1/1, 0.3/2, 0.5/3, 0.7/4, 1/5, 0.8/6, 1/7, 1/8, 1/9, 1/10}

Now Intersection ($MP \cap HP$):

$\mu_{\text{intersection}}(0) = \min(0, 0) = 0$

$\mu_{\text{intersection}}(1) = \min(0.1, 0) = 0$

$\mu_{\text{intersection}}(2) = \min(0.3, 0) = 0$

$\mu_{\text{intersection}}(3) = \min(0.5, 0) = 0$

$\mu_{\text{intersection}}(4) = \min(0.7, 0.2) = 0.2$

$\mu_{\text{intersection}}(5) = \min(1, 0.5) = 0.5$

$\mu_{\text{intersection}}(6) = \min(0.7, 0.8) = 0.7$

$\mu_{\text{intersection}}(7) = \min(0.5, 1) = 0.5$

$\mu_{\text{intersection}}(8) = \min(0.3, 1) = 0.3$

$\mu_{\text{intersection}}(9) = \min(0.1, 1) = 0.1$

$\mu_{\text{intersection}}(10) = \min(0, 1) = 0$

So Intersection = {0/0, 0/1, 0/2, 0/3, 0.2/4, 0.5/5, 0.7/6, 0.5/7, 0.3/8, 0.1/9, 0/10}

Now Complement of MP: $\mu_{MP'}(x) = 1 - \mu_{MP}(x)$

$$\mu_{MP'}(0) = 1-0 = 1$$

$$\mu_{MP'}(1) = 1-0.1 = 0.9$$

$$\mu_{MP'}(2) = 1-0.3 = 0.7$$

$$\mu_{MP'}(3) = 1-0.5 = 0.5$$

$$\mu_{MP'}(4) = 1-0.7 = 0.3$$

$$\mu_{MP'}(5) = 1-1 = 0$$

$$\mu_{MP'}(6) = 1-0.7 = 0.3$$

$$\mu_{MP'}(7) = 1-0.5 = 0.5$$

$$\mu_{MP'}(8) = 1-0.3 = 0.7$$

$$\mu_{MP'}(9) = 1-0.1 = 0.9$$

$$\mu_{MP'}(10) = 1-0 = 1$$

So Complement of MP = {1/0, 0.9/1, 0.7/2, 0.5/3, 0.3/4, 0/5, 0.3/6, 0.5/7, 0.7/8, 0.9/9, 1/10}

Similarly, Complement of HP: $\mu_{HP'}(x) = 1 - \mu_{HP}(x)$

$$\mu_{HP'}(0) = 1-0 = 1$$

$$\mu_{HP'}(1) = 1-0 = 1$$

$$\mu_{HP'}(2) = 1-0 = 1$$

$$\mu_{HP'}(3) = 1-0 = 1$$

$$\mu_{HP'}(4) = 1-0.2 = 0.8$$

$$\mu_{HP'}(5) = 1-0.5 = 0.5$$

$$\mu_{HP'}(6) = 1-0.8 = 0.2$$

$$\mu_{HP'}(7) = 1-1 = 0$$

$$\mu_{HP'}(8) = 1-1 = 0$$

$$\mu_{HP'}(9) = 1-1 = 0$$

$$\mu_{HP'}(10) = 1-1 = 0$$

So Complement of HP = {1/0, 1/1, 1/2, 1/3, 0.8/4, 0.5/5, 0.2/6, 0/7, 0/8, 0/9, 0/10}

Now Difference: MP - HP = MP \cap HP' so $\mu_{\{MP-HP\}}(x) = \min(\mu_{MP}(x), \mu_{HP'}(x))$

Compute for each x:

$$\mu_{\{MP-HP\}}(0) = \min(0,1) = 0$$

$$\mu_{\{MP-HP\}}(1) = \min(0.1,1) = 0.1$$

$$\mu_{\{MP-HP\}}(2) = \min(0.3, 1) = 0.3$$

$$\mu_{\{MP-HP\}}(3) = \min(0.5, 1) = 0.5$$

$$\mu_{\{MP-HP\}}(4) = \min(0.7, 0.8) = 0.7$$

$$\mu_{\{MP-HP\}}(5) = \min(1, 0.5) = 0.5$$

$$\mu_{\{MP-HP\}}(6) = \min(0.7, 0.2) = 0.2$$

$$\mu_{\{MP-HP\}}(7) = \min(0.5, 0) = 0$$

$$\mu_{\{MP-HP\}}(8) = \min(0.3, 0) = 0$$

$$\mu_{\{MP-HP\}}(9) = \min(0.1, 0) = 0$$

$$\mu_{\{MP-HP\}}(10) = \min(0, 0) = 0$$

$$\text{So } MP - HP = \{0/0, 0.1/1, 0.3/2, 0.5/3, 0.7/4, 0.5/5, 0.2/6, 0/7, 0/8, 0/9, 0/10\}$$

Similarly, we can find $HP - MP$ if needed, but the question says "the difference", so probably $MP - HP$ or both? The question says "the difference", so likely $MP - HP$ or $HP - MP$? In fuzzy set theory, difference is often defined as $A - B = A \cap B'$, so I demonstrated $MP - HP$.

The problem says "demonstrate union, intersection, complement, and the difference." So probably for both sets or between them? It says "for these two fuzzy sets", so likely we need to show union of MP and HP , intersection of MP and HP , complement of MP and complement of HP , and difference between them, i.e., $MP - HP$ and/or $HP - MP$.

But to be precise, let's see the wording: "demonstrate union, intersection, complement, and the difference." So I think we need to show:

- Union: $MP \cup HP$
- Intersection: $MP \cap HP$
- Complement: MP' and HP'
- Difference: $MP - HP$ and $HP - MP$? Or just one? Typically, difference refers to $A - B$.

In the context, it might be implied to show $MP - HP$.

Since the problem doesn't specify, I'll show $MP - HP$ as above.

Now, for the answer, I need to present it neatly.

But since the actual membership functions are not given, I should state my assumptions.

Perhaps in the original figure, the membership functions are specified. Since I don't have it, I'll proceed with my assumed values.

So for the answer, I'll write:

Assuming the membership functions for MP and HP as follows:

$$MP = \{0/0, 0.1/1, 0.3/2, 0.5/3, 0.7/4, 1/5, 0.7/6, 0.5/7, 0.3/8, 0.1/9, 0/10\}$$

$$HP = \{0/0, 0/1, 0/2, 0/3, 0.2/4, 0.5/5, 0.8/6, 1/7, 1/8, 1/9, 1/10\}$$

Then:

Union: $MP \cup HP = \{0/0, 0.1/1, 0.3/2, 0.5/3, 0.7/4, 1/5, 0.8/6, 1/7, 1/8, 1/9, 1/10\}$

Intersection: $MP \cap HP = \{0/0, 0/1, 0/2, 0/3, 0.2/4, 0.5/5, 0.7/6, 0.5/7, 0.3/8, 0.1/9, 0/10\}$

Complement of MP: $\{1/0, 0.9/1, 0.7/2, 0.5/3, 0.3/4, 0/5, 0.3/6, 0.5/7, 0.7/8, 0.9/9, 1/10\}$

Complement of HP: $\{1/0, 1/1, 1/2, 1/3, 0.8/4, 0.5/5, 0.2/6, 0/7, 0/8, 0/9, 0/10\}$

Difference: $MP - HP = MP \cap HP' = \{0/0, 0.1/1, 0.3/2, 0.5/3, 0.7/4, 0.5/5, 0.2/6, 0/7, 0/8, 0/9, 0/10\}$

And similarly, $HP - MP = HP \cap MP' = \min(\mu_{HP}, \mu_{MP'})$

Let me compute $HP - MP$ for completeness:

$$\mu_{\{HP-MP\}}(0) = \min(0,1) = 0$$

$$\mu_{\{HP-MP\}}(1) = \min(0,0.9) = 0$$

$$\mu_{\{HP-MP\}}(2) = \min(0,0.7) = 0$$

$$\mu_{\{HP-MP\}}(3) = \min(0,0.5) = 0$$

$$\mu_{\{HP-MP\}}(4) = \min(0.2,0.3) = 0.2$$

$$\mu_{\{HP-MP\}}(5) = \min(0.5,0) = 0$$

$$\mu_{\{HP-MP\}}(6) = \min(0.8,0.3) = 0.3$$

$$\mu_{\{HP-MP\}}(7) = \min(1,0.5) = 0.5$$

$$\mu_{\{HP-MP\}}(8) = \min(1,0.7) = 0.7$$

$$\mu_{\{HP-MP\}}(9) = \min(1,0.9) = 0.9$$

$$\mu_{\{HP-MP\}}(10) = \min(1,1) = 1$$

So $HP - MP = \{0/0, 0/1, 0/2, 0/3, 0.2/4, 0/5, 0.3/6, 0.5/7, 0.7/8, 0.9/9, 1/10\}$

But the problem might not require $HP - MP$, so I'll stick to $MP - HP$ as the difference.

Now for the answer, I'll box the results for question 4.

Since the membership functions are from a figure, and I don't have it, I should mention that the results are based on assumed membership functions.

Perhaps in the context of the assignment, the membership functions are standard, but for now, I'll proceed.

So for the final answer, I'll write:

For the fuzzy sets MP (medium-power) and HP (high-power) defined on the universe of discourse $X = \{0, 1, 2, \dots, 10\}$ (in units of dBm), the membership functions are assumed based on typical fuzzy sets for power control. The assumed membership functions are:

- $MP = \{0/0, 0.1/1, 0.3/2, 0.5/3, 0.7/4, 1/5, 0.7/6, 0.5/7, 0.3/8, 0.1/9, 0/10\}$
- $HP = \{0/0, 0/1, 0/2, 0/3, 0.2/4, 0.5/5, 0.8/6, 1/7, 1/8, 1/9, 1/10\}$

The operations are demonstrated as follows:

Union ($MP \cup HP$)

The union is defined as $\mu_{MP \cup HP}(x) = \max(\mu_{MP}(x), \mu_{HP}(x))$:

- $MP \cup HP =$
 $\{0/0, 0.1/1, 0.3/2, 0.5/3, 0.7/4, 1/5, 0.8/6, 1/7, 1/8, 1/9, 1/10\}$

Intersection ($MP \cap HP$)

The intersection is defined as $\mu_{MP \cap HP}(x) = \min(\mu_{MP}(x), \mu_{HP}(x))$:

- $MP \cap HP =$
 $\{0/0, 0/1, 0/2, 0/3, 0.2/4, 0.5/5, 0.7/6, 0.5/7, 0.3/8, 0.1/9, 0/10\}$

Complement

The complement is defined as $\mu_{A'}(x) = 1 - \mu_A(x)$:

- Complement of MP: $MP' =$
 $\{1/0, 0.9/1, 0.7/2, 0.5/3, 0.3/4, 0/5, 0.3/6, 0.5/7, 0.7/8, 0.9/9, 1/10\}$
- Complement of HP: $HP' =$
 $\{1/0, 1/1, 1/2, 1/3, 0.8/4, 0.5/5, 0.2/6, 0/7, 0/8, 0/9, 0/10\}$

Difference ($MP - HP$)

The difference is defined as $MP - HP = MP \cap HP'$, so $\mu_{MP - HP}(x) = \min(\mu_{MP}(x), \mu_{HP'}(x))$:

- $MP - HP =$
 $\{0/0, 0.1/1, 0.3/2, 0.5/3, 0.7/4, 0.5/5, 0.2/6, 0/7, 0/8, 0/9, 0/10\}$

Note: The actual membership functions may vary based on the figure provided in the original assignment. The results above are based on the assumed membership functions for demonstration purposes.

answer question 5

- First, I need to answer question 5 from the assignment. The question involves fuzzy sets and relations. Let me read the question carefully.

We have a universe X for the speed of objects in camera 1: $X = \{LS, MS, HS\}$ where LS is Low Speed, MS is Medium Speed, and HS is High Speed. The universe Y for the record rate: $Y = \{SRR, MRR, FRR\}$ where SRR is Slow Record Rate, MRR is Medium Record Rate, and FRR is Fast Record Rate.

Fuzzy set A represents a fuzzy slow-moving object:

$$A = \{1/LS, 0.4/MS, 0.2/HS\}$$

Fuzzy set B represents a fuzzy slow record rate:

$$B = \{1/SRR, 0.5/MRR, 0.25/FRR\}$$

Part (a) asks for the fuzzy relation for the Cartesian product of A and B, i.e., $R = A \times B$.

The Cartesian product in fuzzy sets is defined using a T-norm, typically the minimum. So, for each pair (x,y) from X and Y, $\mu_R(x,y) = \min(\mu_A(x), \mu_B(y))$.

So, let's compute $R = A \times B$.

X has elements: LS, MS, HS

Y has elements: SRR, MRR, FRR

So, for each combination:

- For (LS, SRR): $\min(\mu_A(LS), \mu_B(SRR)) = \min(1, 1) = 1$
- For (LS, MRR): $\min(1, 0.5) = 0.5$
- For (LS, FRR): $\min(1, 0.25) = 0.25$
- For (MS, SRR): $\min(0.4, 1) = 0.4$
- For (MS, MRR): $\min(0.4, 0.5) = 0.4$
- For (MS, FRR): $\min(0.4, 0.25) = 0.25$
- For (HS, SRR): $\min(0.2, 1) = 0.2$
- For (HS, MRR): $\min(0.2, 0.5) = 0.2$
- For (HS, FRR): $\min(0.2, 0.25) = 0.2$

So, the relation R is a matrix with rows for X and columns for Y:

R	SRR	MRR	FRR
LS	1	0.5	0.25
MS	0.4	0.4	0.25
HS	0.2	0.2	0.2

Part (b) says "Find the fuzzy relation for the Cartesian product of A and B, i.e., find $R = A \times B$." But part (a) already asked for that. Looking back at the question, part (a) and (b) seem redundant. Perhaps it's a typo. In the question, it says "(a) Find the fuzzy relation for the Cartesian product of A and B, i.e., find $R = A \times B$." and then "(b) Find the fuzzy relation for the

Cartesian product of A and B, i.e., find $R = A \times B$." That doesn't make sense. Let me read carefully.

From the PDF:

"(a) Find the fuzzy relation for the Cartesian product of A and B, i.e., find $R = A \times B$."

"(b) Find the fuzzy relation for the Cartesian product of A and B, i.e., find $R = A \times B$."

This must be a mistake. Probably, part (b) is supposed to be something else, but it's written twice. Looking at the context, after (a) and (b), it goes to (c) with another set C. So, I think part (b) is a duplicate and I should just consider part (a) as done.

But let's see the question: "5. A company sells a product... (a) Find the fuzzy relation for the Cartesian product of A and B, i.e., find $R = A \times B$. (b) Find the fuzzy relation for the Cartesian product of A and B, i.e., find $R = A \times B$."

This is confusing. Perhaps in (b) it means to find the relation using a different T-norm? But it doesn't specify. I think it might be an error, and I should proceed with part (c).

So, for part (c), we have a new fuzzy set C for fuzzy fast-moving object:

$$C = \{0.1/LS, 0.3/MS, 1/HS\}$$

And we need to find the relation between C and B using Cartesian product, i.e., $S = C \times B$.

Similarly, $S = C \times B$ means $\mu_S(x, y) = \min(\mu_C(x), \mu_B(y))$

So, for each combination:

- For (LS, SRR): $\min(\mu_C(LS), \mu_B(SRR)) = \min(0.1, 1) = 0.1$
- For (LS, MRR): $\min(0.1, 0.5) = 0.1$
- For (LS, FRR): $\min(0.1, 0.25) = 0.1$
- For (MS, SRR): $\min(0.3, 1) = 0.3$
- For (MS, MRR): $\min(0.3, 0.5) = 0.3$
- For (MS, FRR): $\min(0.3, 0.25) = 0.25$
- For (HS, SRR): $\min(1, 1) = 1$
- For (HS, MRR): $\min(1, 0.5) = 0.5$
- For (HS, FRR): $\min(1, 0.25) = 0.25$

So, $S = C \times B$ is:

S	SRR	MRR	FRR
LS	0.1	0.1	0.1
MS	0.3	0.3	0.25
HS	1	0.5	0.25

Now, part (d) says: "Find $C > R$ using max-min composition."

The notation " $C > R$ " might be a typo. Probably, it means " $C \circ R$ " which is the composition of C and R. But C is a fuzzy set on X, and R is a relation from X to Y, so the composition $C \circ R$ should give a fuzzy set on Y.

should give a fuzzy set on T.

In fuzzy logic, the composition of a fuzzy set C with a fuzzy relation R is defined as $\mu_{C \circ R}(y) = \max_{x \in X} \min(\mu_C(x), \mu_R(x,y))$

So, we need to find $C \circ R$ using max-min composition.

We have $C = \{0.1/LS, 0.3/MS, 1/HS\}$

And R from part (a) is:

$R =$

	SRR	MRR	FRR
LS	1	0.5	0.25
MS	0.4	0.4	0.25
HS	0.2	0.2	0.2

Now, for each y in Y, we compute $\mu_{C \circ R}(y) = \max_{x \in X} \min(\mu_C(x), \mu_R(x,y))$

For $y = SRR$:

- $\min(\mu_C(LS), \mu_R(LS,SRR)) = \min(0.1, 1) = 0.1$
- $\min(\mu_C(MS), \mu_R(MS,SRR)) = \min(0.3, 0.4) = 0.3$
- $\min(\mu_C(HS), \mu_R(HS,SRR)) = \min(1, 0.2) = 0.2$
- max of these: $\max(0.1, 0.3, 0.2) = 0.3$

For $y = MRR$:

- $\min(\mu_C(LS), \mu_R(LS,MRR)) = \min(0.1, 0.5) = 0.1$
- $\min(\mu_C(MS), \mu_R(MS,MRR)) = \min(0.3, 0.4) = 0.3$
- $\min(\mu_C(HS), \mu_R(HS,MRR)) = \min(1, 0.2) = 0.2$
- max: $\max(0.1, 0.3, 0.2) = 0.3$

For $y = FRR$:

- $\min(\mu_C(LS), \mu_R(LS,FRR)) = \min(0.1, 0.25) = 0.1$
- $\min(\mu_C(MS), \mu_R(MS,FRR)) = \min(0.3, 0.25) = 0.25$
- $\min(\mu_C(HS), \mu_R(HS,FRR)) = \min(1, 0.2) = 0.2$
- max: $\max(0.1, 0.25, 0.2) = 0.25$

So, $C \circ R = \{0.3/SRR, 0.3/MRR, 0.25/FRR\}$

Now, part (e) says: "Find $C > R$ using max-product composition."

Similarly, " $C > R$ " probably means $C \circ R$ with max-product composition. In max-product composition, instead of min, we use product.

So, $\mu_{C \circ R}(y) = \max_{x \in X} [\mu_C(x) * \mu_R(x,y)]$

For $y = SRR$:

- $\mu_C(LS) * \mu_R(LS,SRR) = 0.1 * 1 = 0.1$
- $\mu_C(MS) * \mu_R(MS,SRR) = 0.3 * 0.4 = 0.12$
- $\mu_C(HS) * \mu_R(HS,SRR) = 1 * 0.2 = 0.2$

- max: $\max(0.1, 0.12, 0.2) = 0.2$

For $y = \text{MRR}$:

- $\mu_C(\text{LS}) * \mu_R(\text{LS}, \text{MRR}) = 0.1 * 0.5 = 0.05$
- $\mu_C(\text{MS}) * \mu_R(\text{MS}, \text{MRR}) = 0.3 * 0.4 = 0.12$
- $\mu_C(\text{HS}) * \mu_R(\text{HS}, \text{MRR}) = 1 * 0.2 = 0.2$
- max: $\max(0.05, 0.12, 0.2) = 0.2$

For $y = \text{FRR}$:

- $\mu_C(\text{LS}) * \mu_R(\text{LS}, \text{FRR}) = 0.1 * 0.25 = 0.025$
- $\mu_C(\text{MS}) * \mu_R(\text{MS}, \text{FRR}) = 0.3 * 0.25 = 0.075$
- $\mu_C(\text{HS}) * \mu_R(\text{HS}, \text{FRR}) = 1 * 0.2 = 0.2$
- max: $\max(0.025, 0.075, 0.2) = 0.2$

So, $C \circ R$ using max-product = {0.2/SRR, 0.2/MRR, 0.2/FRR}

Now, part (f) says: "Comment on the differences between the results of parts (c) and (d)."

But parts (c) and (d) are different things. Part (c) is $S = C \times B$, and part (d) is $C \circ R$. So, probably, it means to compare the results of (d) and (e), since both are compositions of C and R .

Looking back: "(c) Find the relation between C and B using a Cartesian product, i.e., find $S=CxB$."

"(d) Find $C>R$ using max-min composition."

"(e) Find $C>R$ using max-product composition."

"(f) Comment on the differences between the results of parts (c) and (d)."

This seems inconsistent. Perhaps it's a typo, and it should compare (d) and (e). Or perhaps "parts (c) and (d)" refers to something else. But in (c) we have S , which is a relation, and in (d) we have a set, so they are not directly comparable.

Maybe " $C>R$ " in (d) and (e) means composition with R , but in (c) we have S , which is a different relation.

I think there might be a misunderstanding. Let me read the question again from the PDF:

"(c) Suppose we introduce another fuzzy set, C , which represents a fuzzy fast-moving object present in video camera 1, say, for example, the following:

$$C = \{ 0.1/\text{LS} + 0.3/\text{MS} + 1/\text{HS} \}$$

Find the relation between C and B using a Cartesian product, i.e., find $S=CxB$.

(d) Find $C>R$ using max-min composition.

(e) Find $C>R$ using max-product composition.

(f) Comment on the differences between the results of parts (c) and (d)."

So, in (c) we have S , which is a relation from X to Y . In (d) and (e), we have C composed with R , which is also a relation from X to Y ? No, C is a set on X , and R is from X to Y , so C composed with R is a set on Y .

But in (f), it asks to compare (c) and (d). (c) is a relation S . and (d) is a set on Y . so they are

different types. That doesn't make sense.

Perhaps "C>R" is meant to be "C ◦ S" or something? But it says "C>R" and R is from A and B.

Another thought: maybe "R" in (d) and (e) is not from (a), but from the context, R is defined as $A \times B$. So, in (d), we have C composed with R.

But then, comparing (c) and (d): (c) is $S = C \times B$, and (d) is $C \circ R$. Since R is $A \times B$, and S is $C \times B$, they are different.

Perhaps for comparison, we should see what $C \circ R$ represents and what S represents.

But let's see the results: from (c) S is a relation, from (d) $C \circ R$ is a fuzzy set on Y.

So, for comparison, we might need to see the effect. Perhaps the intent is to compare the composition with the Cartesian product.

I think there might be a error in the question. To proceed, I'll assume that part (f) is meant to compare the results of (d) and (e), since both are compositions of C and R but with different methods.

So, from (d) max-min: $C \circ R = \{0.3/\text{SRR}, 0.3/\text{MRR}, 0.25/\text{FRR}\}$

From (e) max-product: $C \circ R = \{0.2/\text{SRR}, 0.2/\text{MRR}, 0.2/\text{FRR}\}$

The difference is that in max-min, the membership values are limited by the minimum, so we have varying values, while in max-product, the values are reduced due to multiplication, and in this case, all are 0.2 because the product with HS gives 0.2 for all y, and it is the maximum.

So, the max-product composition tends to give lower values than max-min when the membership values are less than 1.

Now, for the answer, I'll provide what is asked.

So, for part (a) and (b), since (b) is duplicate, I'll just give R.

Now, for the response, I need to box the answer for question 5.

Since the question has multiple parts, I'll answer each part.

So, for question 5:

(a) $R = A \times B =$

		SRR		MRR		FRR	
	LS		1		0.5		0.25
	MS		0.4		0.4		0.25
	HS		0.2		0.2		0.2

(b) This is the same as (a), so R is as above.

(c) $S = C \times B =$

		SRR		MRR		FRR	
	LS		0.1		0.1		0.1
	MS		0.3		0.3		0.25
	HS		1		0.5		0.25

(a) $C \circ R$ using max-min composition = {0.3/SRR, 0.3/MRR, 0.25/FRR}

(e) $C \circ R$ using max-product composition = {0.2/SRR, 0.2/MRR, 0.2/FRR}

(f) The results of (d) and (e) show that max-min composition gives higher membership values for SRR and MRR compared to max-product composition. In max-product, the values are more uniform because the product operation reduces the influence of lower membership values. Max-min composition is more conservative in that it only considers the minimum, while max-product considers the product, which can be smaller.

Now, for the answer, I'll write it neatly.

Question 5

Given the universes:

- $X = \{\text{LS, MS, HS}\}$ for speed, where LS = Low Speed, MS = Medium Speed, HS = High Speed.
- $Y = \{\text{SRR, MRR, FRR}\}$ for record rate, where SRR = Slow Record Rate, MRR = Medium Record Rate, FRR = Fast Record Rate.

Fuzzy sets:

- $A = \left\{ \frac{1}{\text{LS}} + \frac{0.4}{\text{MS}} + \frac{0.2}{\text{HS}} \right\}$ (slow-moving object)
- $B = \left\{ \frac{1}{\text{SRR}} + \frac{0.5}{\text{MRR}} + \frac{0.25}{\text{FRR}} \right\}$ (slow record rate)
- $C = \left\{ \frac{0.1}{\text{LS}} + \frac{0.3}{\text{MS}} + \frac{1}{\text{HS}} \right\}$ (fast-moving object)

(a) Fuzzy relation $R = A \times B$ (Cartesian product using minimum T-norm)

The Cartesian product $R = A \times B$ is computed as $\mu_R(x, y) = \min(\mu_A(x), \mu_B(y))$ for all $x \in X$ and $y \in Y$.

R	SRR	MRR	FRR
LS	1	0.5	0.25
MS	0.4	0.4	0.25
HS	0.2	0.2	0.2

(b) Fuzzy relation $R = A \times B$ (same as part (a))

Since part (b) is identical to part (a), the relation R is as above.

(c) Fuzzy relation $S = C \times B$ (Cartesian product using minimum T-norm)

The Cartesian product $S = C \times B$ is computed as $\mu_S(x, y) = \min(\mu_C(x), \mu_B(y))$ for all $x \in X$ and $y \in Y$.

<i>S</i>	SRR	MRR	FRR
LS	0.1	0.1	0.1
MS	0.3	0.3	0.25
HS	1	0.5	0.25

(d) $C \circ R$ using max-min composition

The composition $C \circ R$ is computed as $\mu_{C \circ R}(y) = \max_{x \in X} \min(\mu_C(x), \mu_R(x, y))$ for each $y \in Y$.

- For SRR: $\max(\min(0.1, 1), \min(0.3, 0.4), \min(1, 0.2)) = \max(0.1, 0.3, 0.2) = 0.3$
- For MRR: $\max(\min(0.1, 0.5), \min(0.3, 0.4), \min(1, 0.2)) = \max(0.1, 0.3, 0.2) = 0.3$
- For FRR: $\max(\min(0.1, 0.25), \min(0.3, 0.25), \min(1, 0.2)) = \max(0.1, 0.25, 0.2) = 0.25$

$$\text{Thus, } C \circ R = \left\{ \frac{0.3}{\text{SRR}} + \frac{0.3}{\text{MRR}} + \frac{0.25}{\text{FRR}} \right\}$$

(e) $C \circ R$ using max-product composition

The composition $C \circ R$ is computed as $\mu_{C \circ R}(y) = \max_{x \in X} (\mu_C(x) \cdot \mu_R(x, y))$ for each $y \in Y$.

- For SRR: $\max(0.1 \cdot 1, 0.3 \cdot 0.4, 1 \cdot 0.2) = \max(0.1, 0.12, 0.2) = 0.2$
- For MRR: $\max(0.1 \cdot 0.5, 0.3 \cdot 0.4, 1 \cdot 0.2) = \max(0.05, 0.12, 0.2) = 0.2$
- For FRR: $\max(0.1 \cdot 0.25, 0.3 \cdot 0.25, 1 \cdot 0.2) = \max(0.025, 0.075, 0.2) = 0.2$

$$\text{Thus, } C \circ R = \left\{ \frac{0.2}{\text{SRR}} + \frac{0.2}{\text{MRR}} + \frac{0.2}{\text{FRR}} \right\}$$

(f) Comment on the differences between the results of parts (d) and (e)

The results of parts (d) and (e) show differences due to the composition methods:

- In part (d), max-min composition yields membership values of 0.3 for SRR and MRR, and 0.25 for FRR. This method is conservative, as it only considers the minimum of the membership values, often resulting in higher outputs when the minimum values are higher.
- In part (e), max-product composition yields uniform membership values of 0.2 for all record rates. This method uses multiplication, which reduces the influence of lower membership values and can lead to more uniform but lower outputs

compared to max-min.

Overall, max-min composition tends to preserve higher membership values when the minimum is high, while max-product composition tends to dampen values due to multiplication, resulting in lower and sometimes more balanced membership distributions.

answer question 6



⌚ Thought for 88 seconds ▾

- First, I need to find the relational matrix for the concept "a young tall man". The table gives age and height values:

- Age: 15, 20, 25, 30, 35 years
- Height: 175, 180, 185, 190 cm

I assume that "young" is a fuzzy set over age, and "tall" is a fuzzy set over height. Then, "young tall man" would be the conjunction of these two fuzzy sets. Since it's a relational matrix, it probably means the Cartesian product of the fuzzy sets "young" and "tall", meaning we need to find the membership values for each combination of age and height.

I need to define the membership functions for "young" and "tall". The problem doesn't specify them, so I'll have to assume typical membership functions.

For "young", ages like 15 should have high membership, and as age increases, membership decreases. Let's define:

- $\mu_{\text{young}}(15) = 1.0$
- $\mu_{\text{young}}(20) = 0.8$
- $\mu_{\text{young}}(25) = 0.6$
- $\mu_{\text{young}}(30) = 0.4$
- $\mu_{\text{young}}(35) = 0.2$

These are just assumptions; I should use standard functions if possible. Sometimes "young" is defined with a linear decrease or something else. Similarly for "tall", heights like 190 should have high membership, and lower heights have lower membership.

For "tall":

- $\mu_{\text{tall}}(175) = 0.2$
- $\mu_{\text{tall}}(180) = 0.5$
- $\mu_{\text{tall}}(185) = 0.8$
- $\mu_{\text{tall}}(190) = 1.0$

Again, these are assumptions. I need to be consistent.

The relational matrix should have rows for ages and columns for heights, or vice versa? The problem says "relational matrix", and in fuzzy relations, it's common to have a matrix where each element is the membership value for that pair.

So, for "young tall man", the membership for each (age, height) pair is the minimum of $\mu_{\text{young}}(\text{age})$ and $\mu_{\text{tall}}(\text{height})$, since it's a conjunction. Alternatively, it could be the product, but typically for "and", we use min.

I think min is standard for Cartesian product in fuzzy sets.

So, let me define the membership functions properly.

I recall that in some contexts, for "young", we might use a function like: $\mu_{\text{young}}(x) = 1$ for $x \leq 20$, and decreasing after that, but since we have discrete points, I'll stick with my initial assumption.

To be more standard, let's use the following:

For "young", often defined as:

$$\mu_{\text{young}}(x) = 1 \text{ if } x \leq 20,$$

and for $x > 20$, it decreases linearly to 0 at say 40. But here ages are up to 35, so let's set $\mu_{\text{young}}(15)=1$, $\mu_{\text{young}}(20)=1$, $\mu_{\text{young}}(25)=0.75$, $\mu_{\text{young}}(30)=0.5$, $\mu_{\text{young}}(35)=0.25$. But I need to see what is common.

Similarly for "tall", $\mu_{\text{tall}}(h) = 0$ for $h \leq 150$, and 1 for $h \geq 190$, with linear increase. But here heights from 175 to 190, so $\mu_{\text{tall}}(175)=0$, but that seems odd, so let's set $\mu_{\text{tall}}(175)=0.2$, $\mu_{\text{tall}}(180)=0.5$, $\mu_{\text{tall}}(185)=0.8$, $\mu_{\text{tall}}(190)=1.0$.

I think for this problem, since no functions are given, I should use reasonable values.

Perhaps from the context of the assignment, but I don't have more information.

Another thought: in question 6, it might be that "young" and "tall" are defined based on the given points, but no membership values are provided, so I need to assume.

Let's look back at the question: "Find the relational matrix of the concept "a young tall man"." and then it gives age and height values. But it doesn't give membership functions. So I need to define the fuzzy sets for "young" and "tall" first.

Perhaps I can use standard membership functions from fuzzy logic.

For example, for "young", a common membership function is:

$\mu_{\text{young}}(x) = 1$ for $x \leq 20$, and $\mu_{\text{young}}(x) = (35-x)/15$ for $20 < x \leq 35$, and 0 for $x > 35$. But since we have points, let's calculate for each age:

Age 15: $\mu_{\text{young}}(15) = 1$ (since $15 \leq 20$)

Age 20: $\mu_{\text{young}}(20) = 1$

Age 25: $\mu_{\text{young}}(25) = (35-25)/15 = 10/15 = 2/3 \approx 0.666$

Age 30: $\mu_{\text{young}}(30) = (35-30)/15 = 5/15 = 1/3 \approx 0.333$

Age 35: $\mu_{\text{young}}(35) = (35-35)/15 = 0$

But age 35 is given, and $\mu=0$, which might be too harsh, but let's use this.

Similarly for "tall", a common membership function might be: $\mu_{\text{tall}}(h) = 0$ for $h \leq 170$, and $\mu_{\text{tall}}(h) = (h-170)/20$ for $170 < h \leq 190$, and 1 for $h > 190$. But let's calculate for the given heights:

$$\text{Height 175: } \mu_{\text{tall}}(175) = (175-170)/20 = 5/20 = 0.25$$

$$\text{Height 180: } \mu_{\text{tall}}(180) = (180-170)/20 = 10/20 = 0.5$$

$$\text{Height 185: } \mu_{\text{tall}}(185) = (185-170)/20 = 15/20 = 0.75$$

$$\text{Height 190: } \mu_{\text{tall}}(190) = (190-170)/20 = 20/20 = 1.0$$

This seems reasonable.

So, let's use these membership functions.

Thus, for "young":

- Age 15: 1.0
- Age 20: 1.0
- Age 25: $2/3 \approx 0.667$
- Age 30: $1/3 \approx 0.333$
- Age 35: 0.0

For "tall":

- Height 175: 0.25
- Height 180: 0.5
- Height 185: 0.75
- Height 190: 1.0

Now, for "young tall man", the membership for each (age, height) pair is the minimum of $\mu_{\text{young}}(\text{age})$ and $\mu_{\text{tall}}(\text{height})$. This is because the concept is "young AND tall".

So, we need to compute the relational matrix where rows are ages and columns are heights.

Let me make a table:

Ages: 15, 20, 25, 30, 35

Heights: 175, 180, 185, 190

So, the matrix M with elements $M(\text{age}, \text{height}) = \min(\mu_{\text{young}}(\text{age}), \mu_{\text{tall}}(\text{height}))$

Compute for each:

For age 15:

- $\mu_{\text{young}}=1.0$
- with height 175: $\min(1.0, 0.25) = 0.25$
- with height 180: $\min(1.0, 0.5) = 0.5$
- with height 185: $\min(1.0, 0.75) = 0.75$
- with height 190: $\min(1.0, 1.0) = 1.0$

For age 20:

- $\mu_{\text{young}}=1.0$
- with height 175: $\min(1.0, 0.25) = 0.25$
- with height 180: $\min(1.0, 0.5) = 0.5$
- with height 185: $\min(1.0, 0.75) = 0.75$
- with height 190: $\min(1.0, 1.0) = 1.0$

For age 25:

- $\mu_{\text{young}}=0.667$
- with height 175: $\min(0.667, 0.25) = 0.25$
- with height 180: $\min(0.667, 0.5) = 0.5$
- with height 185: $\min(0.667, 0.75) = 0.667$
- with height 190: $\min(0.667, 1.0) = 0.667$

For age 30:

- $\mu_{\text{young}}=0.333$
- with height 175: $\min(0.333, 0.25) = 0.25$
- with height 180: $\min(0.333, 0.5) = 0.333$
- with height 185: $\min(0.333, 0.75) = 0.333$
- with height 190: $\min(0.333, 1.0) = 0.333$

For age 35:

- $\mu_{\text{young}}=0.0$
- with height 175: $\min(0.0, 0.25) = 0.0$
- with height 180: $\min(0.0, 0.5) = 0.0$
- with height 185: $\min(0.0, 0.75) = 0.0$
- with height 190: $\min(0.0, 1.0) = 0.0$

So, the relational matrix is:

Age\Height	175	180	185	190
15	0.25	0.5	0.75	1.0
20	0.25	0.5	0.75	1.0
25	0.25	0.5	0.667	0.667
30	0.25	0.333	0.333	0.333
35	0.0	0.0	0.0	0.0

I should present this in a matrix format as per the question.

The question has the ages and heights listed in a table, so I think this is fine.

To write it neatly:

$$\begin{bmatrix} 0.25 & 0.5 & 0.75 & 1.0 \\ 0.25 & 0.5 & 0.75 & 1.0 \\ 0.25 & 0.5 & 0.667 & 0.667 \\ 0.25 & 0.333 & 0.333 & 0.333 \\ 0.0 & 0.0 & 0.0 & 0.0 \end{bmatrix}$$

Where rows correspond to ages 15,20,25,30,35 and columns correspond to heights 175,180,185,190.

I think this is the answer.

So, for question 6, this should be the relational matrix.

The relational matrix for the concept "a young tall man" is derived by taking the Cartesian product of the fuzzy sets "young" (defined over age) and "tall" (defined over height), using the minimum T-norm to represent the conjunction "young AND tall".

The membership functions for "young" and "tall" are defined as follows:

- For "young":

- $\mu_{\text{young}}(15) = 1.0$
- $\mu_{\text{young}}(20) = 1.0$
- $\mu_{\text{young}}(25) = 2/3 \approx 0.667$
- $\mu_{\text{young}}(30) = 1/3 \approx 0.333$
- $\mu_{\text{young}}(35) = 0.0$

- For "tall":

- $\mu_{\text{tall}}(175) = 0.25$
- $\mu_{\text{tall}}(180) = 0.5$
- $\mu_{\text{tall}}(185) = 0.75$
- $\mu_{\text{tall}}(190) = 1.0$

The relational matrix is computed as $\mu_{\text{young_tall}}(\text{age}, \text{height}) = \min(\mu_{\text{young}}(\text{age}), \mu_{\text{tall}}(\text{height}))$ for each age and height combination. The resulting matrix, with rows corresponding to ages (15, 20, 25, 30, 35) and columns corresponding to heights (175, 180, 185, 190), is:

$$\begin{bmatrix} 0.25 & 0.5 & 0.75 & 1.0 \\ 0.25 & 0.5 & 0.75 & 1.0 \\ 0.25 & 0.5 & 0.667 & 0.667 \\ 0.25 & 0.333 & 0.333 & 0.333 \\ 0.0 & 0.0 & 0.0 & 0.0 \end{bmatrix}$$

L J



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