$G^{+}_{3\times 4}$

$$W = G^{+} d$$
$$= (3\times 4)(4\times 1) = (3\times 1)$$

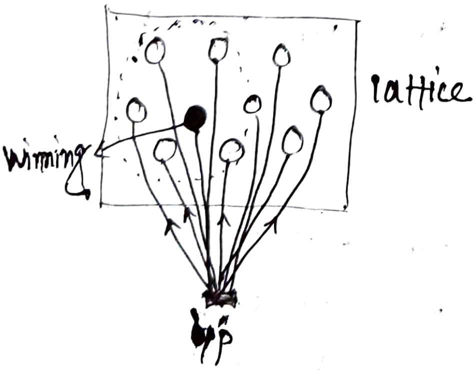
$$W = \begin{bmatrix} -2.5018 \\ -2.5018 \\ +2.8404 \end{bmatrix}_{3\times 1}$$

$\left.\begin{matrix}-2.5018\\-2.5018\end{matrix}\right\}$ weights, $+2.8404 \rightarrow$ bias

Comparision between MLP & RBF:

RBF	MLP
(i) single hidden layer	(i) one or more hidden layer
(ii) Radial basis fns.	(ii) Activation fns.
(iii) hidden layer → non-linear, o/p " → linear	(iii) Non-linear activations
(iv) compute Euclidean norm of i/p & center	(iv) inner product of i/p & synaptic weights
(v) Local approximation to non-linear i/p-o/p mapping.	(v) Global approximation to non-linear i/p-o/p mapping.

Self-Organizing Map (SOM):



competitive learning
↓
winner takes all

(i) Competition
(ii) Co-operation
(iii) Synaptic Adaption

(i) competition:

$$\vec{x} = [x_1, x_2, \ldots, x_m]$$

$$\vec{w}_j = [w_{j1}, w_{j2}, \ldots, w_{jm}], \quad j = 1, 2, \ldots, l$$

l: total # neurons in networks

$\vec{w}^T \cdot \vec{x} \rightarrow$ inner product

↳ best matching criterion

↳ maximize the inner product

equivalent to minimize the Euclidean distance

$$\min_j \| x - w_j \|$$

23/10/25 ADSP

Affine projection Algorithm:

Time n

$n+1$	$n+2$	$n+P$
$y(n+1)$	$y(n+2)$	$y(n+P)$
$\vec{x}(n+1)$	$\vec{x}(n+2)$	$\vec{x}(n+P)$

} using this data, we need to find $\vec{w}(n)$

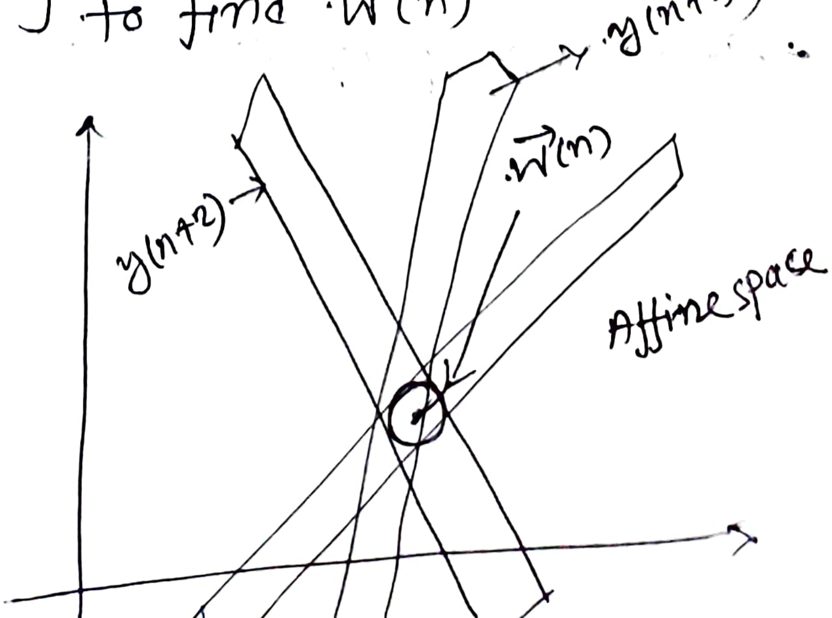
total data = N

#data in one batch = P

#batch = $\frac{N}{P}$

$y(n+1) = x^T(n+1)\, \vec{w}(n+1)$

$y(n+2) = x^T(n+2)\, \vec{w}(n+1)$



24/10/25 soft computing

$$i(x) = \arg\min_j \| x - w_j \| , \quad j = 1, 2, \ldots l$$

→ winning neuron can be detected

② **Co-operation process:**

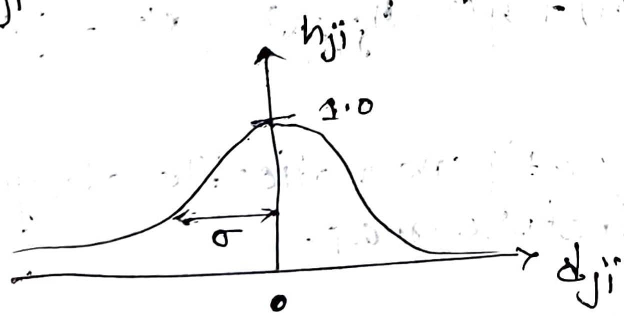
$$h_{ji}(x) = \exp\left(-\frac{d_{ji}}{2\sigma^2}\right)$$

$h_{ji}(x)$ → topological neighbourhood centered on winning neuron i.

d_{ji} → latteral distance between winning neuron & excited neuron j

$$d_{ji}^2 = \| r_j - r_i \|^2$$; r_j → position of the excited neuron j

r_i → position of the winning neuron i



③ **Adaptive process:**

$$\Delta w_j = \eta h_{ji} (x - w_j)$$

$$w_j(n+1) = w_j(n) + \eta(n) h_{ji(x)}(n) (\vec{x} - \vec{w_j}(n))$$

Summary of the algorithm:

(i) **Initialization**: weight vectors are to be initialized $w_j(0)$, where $j = 1, 2, \dots l$

$\{x_i\}_{i=1}^{N}$, N: # datapoints
l: # neurons in lattice

(ii) **Sampling**: Draw a sample $\vec{x}$ from the input space with certain probability.

(iii) **Similarity Matching**:

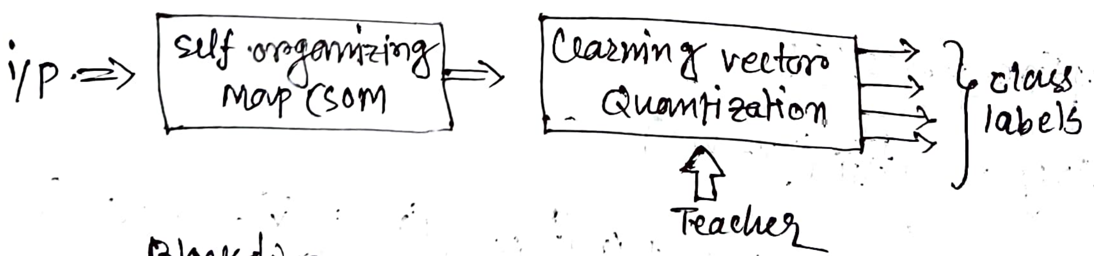
$$i(x) = \arg\min_j \|x(n) - w_j\|, \; j = 1, 2, \dots, l$$

(iv) **Updating**:

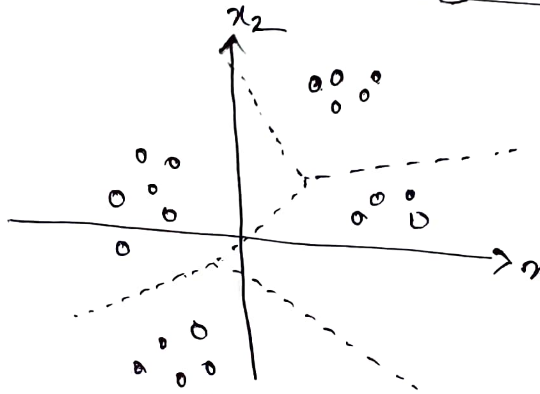
$$w_j(n+1) = w_j(n) + \eta(n)\, h_{j, i_x}(n)\, (x(n) - w_j(n))$$

(v) **Continuation**: continue until no noticeable changes in the feature map.

*Learning vector Quantization (LVQ):



Block diagram of improved pattern classification using SOM & LVQ



$X = [x_1, x_2]^T$

Voronoi diagram involving four cells

Code book - NN rule

LVQ → supervised learning technique

26/10/25 Soft computing

LVQ algorithm:

① Assume, voronoi vector – w_c → closest to the i/p vector x_i

C_{w_i} → class which associated voronoi vector w_c

C_{x_i} → class label of i/p vector x_i

→ If $C_{w_i} = C_{x_i}$, then

$$w_c(n+1) = w_c(n) + \alpha_n [x_i - w_c(n)]$$

where, $0 < \alpha_n < 1$

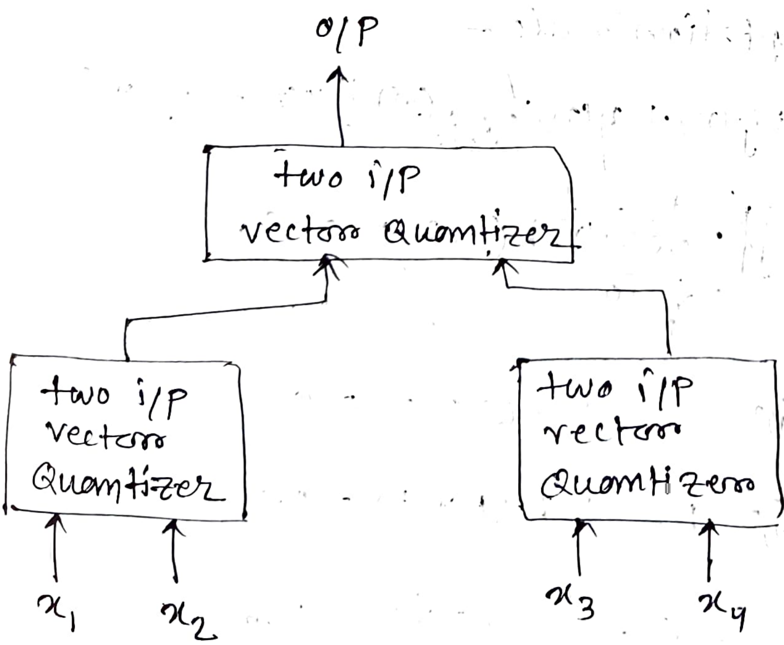
→ If $C_{w_i} \neq C_{x_i}$ then

$$w_c(n+1) = w_c(n) - \alpha_n [x_i - w_c(n)]$$

② The other voronoi vectors are not modified.

(Hierarchical vector Quantization)

Two-stage Hierarchical LVQ:



Example

Two dimensional NN : 9 neurons $\rightarrow$ defined by coordinate values

$i_1 = [0,0]$; $i_2 = [1,0]$; $i_3 = [2,0]$; $i_4 = [0,1]$; $i_5 = [1,1]$

$i_6 = [2,1]$; $i_7 = [0,2]$; $i_8 = [1,2]$; $i_9 = [2,2]$

At a certain step t, the weight vectors w_j are,

$\vec{w_1} = [4,5]^T$; $\vec{w_2} = [5,3]^T$; $\vec{w_3} = [4,6]$; $\vec{w_4} = [1,1]^T$;

$\vec{w_5} = [2,2]^T$; $\vec{w_6} = [0,0]^T$; $\vec{w_7} = [6,4]^T$; $\vec{w_8} = [3,5]^T$;

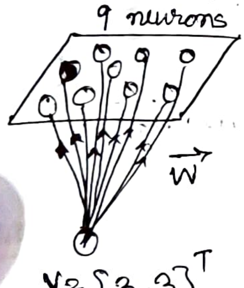
$\vec{w_9} = [5,4]^T$

At time t, we're i/p vector $v = [3,3]^T$

calculate weights at time $(t+1)$, using sum adaptation rule. Assume, $\eta(t) = 1$; $h(r,s,t) = 1$ if $|i_r - i_s| = 0$

$h(r,s,t) = 0.5$ if $|i_r - i_s| = 1$

$h(r,s,t) = 0$ otherwise



Solution: sum adaptation rule -

$$w_j(n+1) = w_j(n) + \eta(n)\, h_{j,i_x}(n)\, [x(n) - w_j(n)]$$

$$\left\| \overset{i_1}{\begin{bmatrix}0\\0\end{bmatrix}} - \overset{i_2}{\begin{bmatrix}1\\0\end{bmatrix}} \right\| = \left\| \begin{bmatrix}-1\\0\end{bmatrix} \right\| = \sqrt{(-1)^2+(0)^2} \quad h(1,2,t) = 0.5$$
$$= \sqrt{1+0}$$
$$= 1$$

$$\left\| \overset{i_1}{\begin{bmatrix}0\\0\end{bmatrix}} - \overset{i_3}{\begin{bmatrix}2\\0\end{bmatrix}} \right\| = \sqrt{(-2)^2+(0)^2} = 2 \quad ; \; h(1,3,t) = 0$$

$$\left\| \overset{i_1}{\begin{bmatrix}0\\0\end{bmatrix}} - \overset{i_4}{\begin{bmatrix}0\\1\end{bmatrix}} \right\| = \sqrt{0^2+(-1)^2} = 1 \quad ; \; h(1,4,t) = 0.5$$

$$\left\| \begin{bmatrix}3\\3\end{bmatrix} - \overset{w_1}{\begin{bmatrix}4\\5\end{bmatrix}} \right\| = \sqrt{(-1)^2+(-2)^2} = \sqrt{1+4} = \sqrt{5} = 2.236$$

$$\left\| \begin{bmatrix}3\\3\end{bmatrix} - \overset{w_2}{\begin{bmatrix}5\\3\end{bmatrix}} \right\| = \sqrt{(-2)^2+(0)^2} = \sqrt{4+0} = 2$$

$$\left\| \begin{bmatrix}3\\3\end{bmatrix} - \overset{w_3}{\begin{bmatrix}4\\6\end{bmatrix}} \right\| = \sqrt{(-1)^2+(-3)^2} = \sqrt{1+9} = \sqrt{10} = 3.162$$

$\left\|\begin{bmatrix}3\\3\end{bmatrix}-\overset{w_4}{\begin{bmatrix}1\\1\end{bmatrix}}\right\| = \sqrt{2^2+2^2} = \sqrt{8} = 2.828$

$\left\|\begin{bmatrix}3\\3\end{bmatrix}-\overset{w_5}{\begin{bmatrix}2\\2\end{bmatrix}}\right\| = \sqrt{1^2+1^2} = \sqrt{2} = 1.414$ ✓

$\left\|\begin{bmatrix}3\\3\end{bmatrix}-\overset{w_6}{\begin{bmatrix}0\\0\end{bmatrix}}\right\| = \sqrt{3^2+3^2} = \sqrt{18} = 4.242$

$\left\|\begin{bmatrix}3\\3\end{bmatrix}-\overset{w_7}{\begin{bmatrix}6\\4\end{bmatrix}}\right\| = \sqrt{(-3)^2+(-1)^2} = \sqrt{10} = 3.162$

$\left\|\begin{bmatrix}3\\3\end{bmatrix}-\overset{w_8}{\begin{bmatrix}3\\5\end{bmatrix}}\right\| = \sqrt{0^2+(-2)^2} = 2$

$\left\|\begin{bmatrix}3\\3\end{bmatrix}-\overset{w_9}{\begin{bmatrix}5\\4\end{bmatrix}}\right\| = \sqrt{(-2)^2+(-1)^2} = \sqrt{5} = 2.236$

$\left\|\begin{bmatrix}0\\0\end{bmatrix}-\begin{bmatrix}1\\1\end{bmatrix}\right\|^2 = \sqrt{(-1)^2+(-1)^2} = 2 \Rightarrow 0$

$\left\|\begin{bmatrix}1\\0\end{bmatrix}-\begin{bmatrix}1\\1\end{bmatrix}\right\|^2 = (0)^2+(-1)^2 = 1 \Rightarrow 0.5$

$\left\|\begin{bmatrix}2\\0\end{bmatrix}-\begin{bmatrix}1\\1\end{bmatrix}\right\|^2 = 1^2+(-1)^2 = 2 \Rightarrow 0$

$\left\|\begin{bmatrix}0\\1\end{bmatrix}-\begin{bmatrix}1\\1\end{bmatrix}\right\|^2 = (-1)^2+0^2 = 1 \Rightarrow 0.5$

$\left\|\begin{bmatrix}2\\1\end{bmatrix}-\begin{bmatrix}1\\1\end{bmatrix}\right\|^2 = (1)^2+(0)^2 = 1 \Rightarrow 0.5$

$\left\|\begin{bmatrix}0\\2\end{bmatrix}-\begin{bmatrix}1\\1\end{bmatrix}\right\|^2 = (-1)^2+(1)^2 = 2 \Rightarrow 0$

$\left\|\begin{bmatrix}1\\2\end{bmatrix}-\begin{bmatrix}1\\1\end{bmatrix}\right\|^2 = 0^2+1^2 = 1 \Rightarrow 0.5$

$\left\|\underset{19}{\begin{bmatrix}2\\2\end{bmatrix}}-\underset{\text{winning}}{\begin{bmatrix}1\\1\end{bmatrix}}\right\|^2 = 1^2+1^2 = 2 \Rightarrow 0$

x_{min}, 8, 2

$\vec{w}_1' = \begin{bmatrix}4\\5\end{bmatrix} + 1*0 = \begin{bmatrix}4\\5\end{bmatrix}$

$\vec{w}_2' = \begin{bmatrix}5\\3\end{bmatrix} + 1*0.5\left(\begin{bmatrix}3\\3\end{bmatrix}-\begin{bmatrix}5\\3\end{bmatrix}\right) = \begin{bmatrix}5\\3\end{bmatrix} + 0.5\begin{bmatrix}-2\\0\end{bmatrix} = \begin{bmatrix}5\\3\end{bmatrix} + \begin{bmatrix}-1\\0\end{bmatrix} = \begin{bmatrix}4\\3\end{bmatrix}$

$\vec{w}_3' = \begin{bmatrix}4\\6\end{bmatrix} + 1*0 = \begin{bmatrix}4\\6\end{bmatrix}$

$\vec{w}_4' = \begin{bmatrix}1\\1\end{bmatrix} + 1*0.5\left(\begin{bmatrix}3\\3\end{bmatrix}-\begin{bmatrix}1\\1\end{bmatrix}\right) = \begin{bmatrix}1\\1\end{bmatrix} + 0.5\begin{bmatrix}2\\2\end{bmatrix} = \begin{bmatrix}1\\1\end{bmatrix} + \begin{bmatrix}1\\1\end{bmatrix} = \begin{bmatrix}2\\2\end{bmatrix}$

$\vec{w}_5' = \begin{bmatrix}2\\2\end{bmatrix} + 1*0.5\left(\begin{bmatrix}3\\3\end{bmatrix}-\begin{bmatrix}2\\2\end{bmatrix}\right) = \begin{bmatrix}2\\2\end{bmatrix} + 0.5\begin{bmatrix}1\\1\end{bmatrix} = \begin{bmatrix}2.5\\2.5\end{bmatrix}$

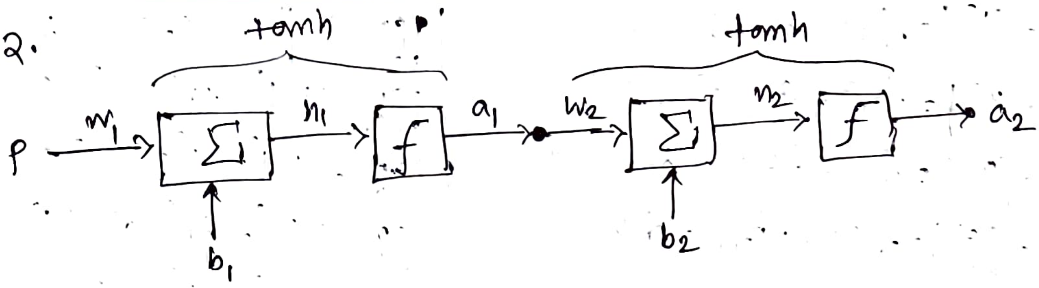
$\vec{w}_6' = \begin{bmatrix}0\\0\end{bmatrix} + 1*0.5\left(\begin{bmatrix}3\\3\end{bmatrix}-\begin{bmatrix}0\\0\end{bmatrix}\right) = \begin{bmatrix}0\\0\end{bmatrix} + 0.5\begin{bmatrix}3\\3\end{bmatrix} = \begin{bmatrix}1.5\\1.5\end{bmatrix}$

$\vec{w}_7' = \begin{bmatrix}6\\4\end{bmatrix} + \cancel{1*0.5\left(\begin{bmatrix}3\\3\end{bmatrix}-\begin{bmatrix}6\\4\end{bmatrix}\right) = \begin{bmatrix}6\\4\end{bmatrix} + 0.5\begin{bmatrix}3\\1\end{bmatrix} = \begin{bmatrix}6\\4\end{bmatrix} + \begin{bmatrix}1.5\\0.5\end{bmatrix} = \begin{bmatrix}7.5\\4.5\end{bmatrix}}$

$\vec{w}_8' = \begin{bmatrix}3\\5\end{bmatrix} + 1*0.5\left(\begin{bmatrix}3\\3\end{bmatrix}-\begin{bmatrix}3\\5\end{bmatrix}\right) = \begin{bmatrix}3\\5\end{bmatrix} + 0.5\begin{bmatrix}0\\-2\end{bmatrix} = \begin{bmatrix}3\\5\end{bmatrix} + \begin{bmatrix}0\\-1\end{bmatrix} = \begin{bmatrix}3\\4\end{bmatrix}$

$\vec{w}_9 = \begin{bmatrix}5\\4\end{bmatrix} + 1*0 = \begin{bmatrix}5\\4\end{bmatrix}$

Q.2.



Initial weights: $w_1(0) = -1$, $b_1(0) = 1$

$w_2(0) = -2$, $b_2(0) = 1$

An i/p pair is given: $(p = -1, t = 1)$

perform one iteration of BP algorithm with $\eta = 1$

$$\tanh(z) = \frac{e^{z} - e^{-z}}{e^{z} + e^{-z}} \quad ; \quad \nabla \tanh(z) = 1 - \tanh^2(z)$$

Forward pass:

$z_1 = p_1 w_1 + b_1 = (-1) * (-1) + 1 = 2$; $\tanh(z_1) = \dfrac{e^{2} - e^{-2}}{e^{2} + e^{-2}}$

$a_1 = 0.964$

$z_2 = a_1 w_2 + b_2 = (0.964) * (-2) + 1 = -0.928$; $\tanh(z_2) = \dfrac{e^{-0.928} - e^{0.928}}{e^{-0.928} + e^{0.928}}$

$a_2 = 0.729$

Error $= 1 - 0.729 = 0.271$; $\mathcal{E} = \frac{1}{2}\, \text{error}^2$

$= \frac{1}{2}(t - a_2)^2$

Backward pass:

$w_2(1) = w_2(0) - \eta * \dfrac{\partial \mathcal{E}}{\partial w_2(0)}$; $\dfrac{\partial \mathcal{E}}{\partial w_2} = \dfrac{\partial \mathcal{E}}{\partial a_2} \dfrac{\partial a_2}{\partial z_2} \dfrac{\partial z_2}{\partial w_2}$

$= -2 - 1 * (-0.452)$

$= -2 + 0.452 = 1.548$

$= -1 * (1 - 0.531) * 0.964$

$a_2 = \tanh(\cancel{\ } z_2)$; $\dfrac{\partial a_2}{\partial z_2} = 1 - \tanh^2(z_2)$

$z_2 = a_1 w_2 + b_2$; $\dfrac{\partial z_2}{\partial w_2} = a_1$

$= -0.452$

$b_2(1) = b_2(0) - \eta * \dfrac{\partial \mathcal{E}}{\partial b_2(0)}$; $\dfrac{\partial \mathcal{E}}{\partial b_2} = \dfrac{\partial \mathcal{E}}{\partial a_2} * \dfrac{\partial a_2}{\partial z_2} * \dfrac{\partial z_2}{\partial b_2}$

$= 1 - 1 * (-0.469)$

$= 1.469$

$= -1 * (1 - 0.531) * 1$

$= -0.469$

$$w_1(1) = w_1(0) - \eta \frac{\partial E_0}{\partial w_1(0)} \quad ; \quad \frac{\partial E_0}{\partial w_1} = \frac{\partial w_1}{\partial a_2} \frac{\partial a_2}{\partial z_2} \frac{\partial z_2}{\partial a_1} \frac{\partial a_1}{\partial z_1} \frac{\partial z_1}{\partial w_1}$$

Product

Tap(a, b) =

29/10/25 Soft computing

Fuzzy set :-

$P = \{x \mid x > 6 \text{ ft.}\}$ where x = height

$X \rightarrow$ collection of objects $\rightarrow$ denoted by x

then, fuzzy set

$$A = \{(x, \mu_A(x)) \mid x \in X\}$$

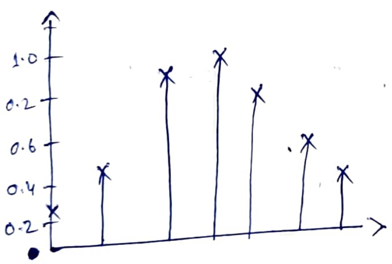
where,

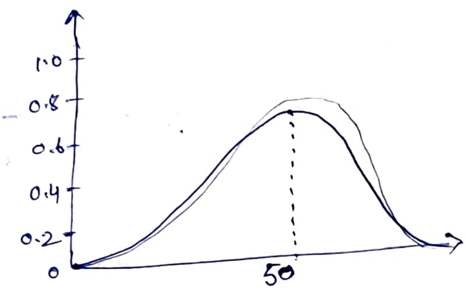
$\mu_A(x) \rightarrow$ membership function for the fuzzy set A

$$0 \leq \mu_A(x) \leq 1$$

$X \rightarrow$ Universe of discourse / universe

MF on a discrete universe





X = #children

A = sensible number of children in a family

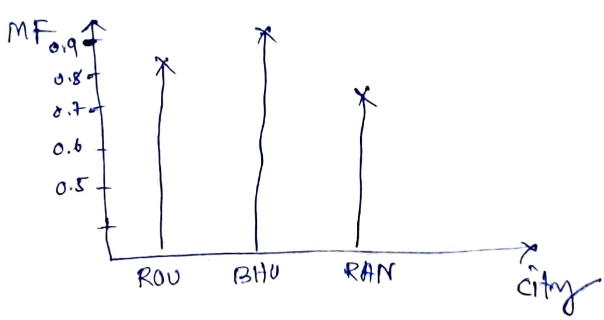
X = Age

B = about 50y old

Fuzzy set with discrete Universe :

Best city to live given by set C

$C = \{$(Rourkela, 0.8), (Bhubaneswar, 0.9), (Ranchi, 0.7)$\}$



Sensible # students

$B = \{(2, 0.1), (5, 0.2), (10, 0.3), (15, 0.6), (18, 0.7), (20, 0.8),$
$(22, 0.9), (50, 0.4), (60, 0.3)\}$

Alternative Expression:

$C = 0.8/\text{Rourkela} + 0.9/\text{Bhubaneswar} + 0.7/\text{Ranchi}$

$B = 0.1/2 + 0.2/5 + 0.3/10 + 0.6/15 + 0.7/18 + 0.8/20 +$
$0.9/22 + 0.4/50 + 0.3/60$

Fuzzy set with continuous Universe:

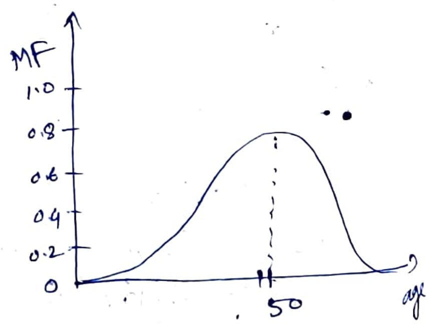
$X = R^+$ be the set of possible ages of human in a village

$B \rightarrow$ Fuzzy set "about 50y old".

$B = \{(x, \mu_B(x)) \mid x \in X\}$

where,

$$\mu_B(x) = \frac{1}{1 + \left(\frac{x-50}{10}\right)^4} \leftarrow \text{Bell MF}$$



Linguistic variable and Linguistic values:

$X \rightarrow$ age

fuzzy set can be defined as –

"young" "Middle age"

$\uparrow$ $\uparrow$

$\mu_{young}(x)$ $\mu_{middle\ age}(x)$

where,

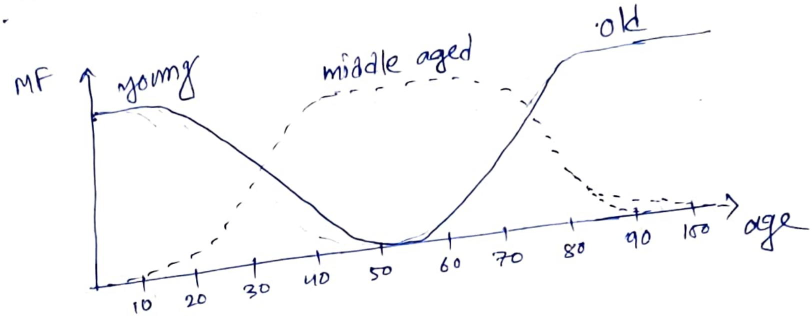
"Age" $\rightarrow$ Linguistic variable

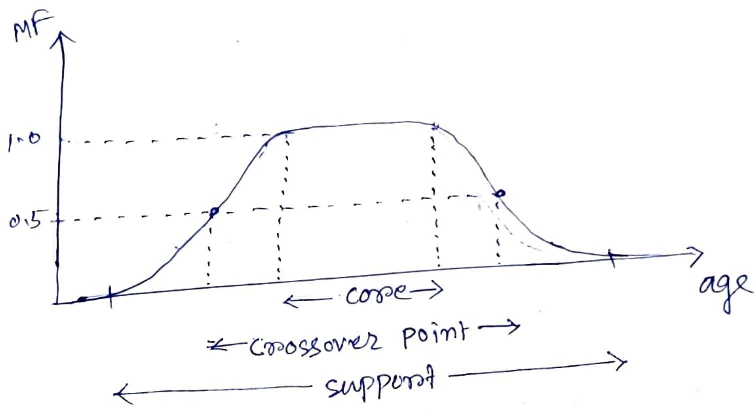
"young", "middle age" $\rightarrow$ Linguistic values

1/4, 1/5, 0/6

loric Product

ap la..





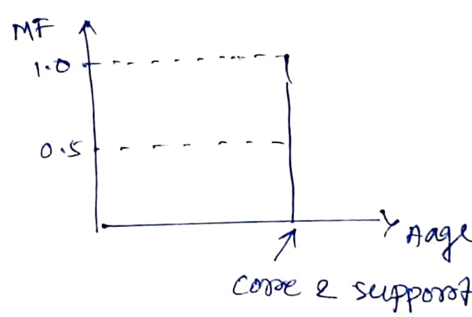
support: $\text{support}(A) = \{x \mid \mu_A(x) > 0\}$

core: $\text{core}(A) = \{x \mid \mu_A(x) = 1\}$

Normality: Normal fuzzy set $\longrightarrow$ a point $x \in X$ s.t $\mu_A(x) = 1$

cross-over points : $\text{crossover}(A) = \{x \mid \mu_A(x) = 0.5\}$

Fuzzy singletone: A fuzzy set whose support is a single point in X with $\mu_A(x) = 1$



α-cut & strong α-cut :

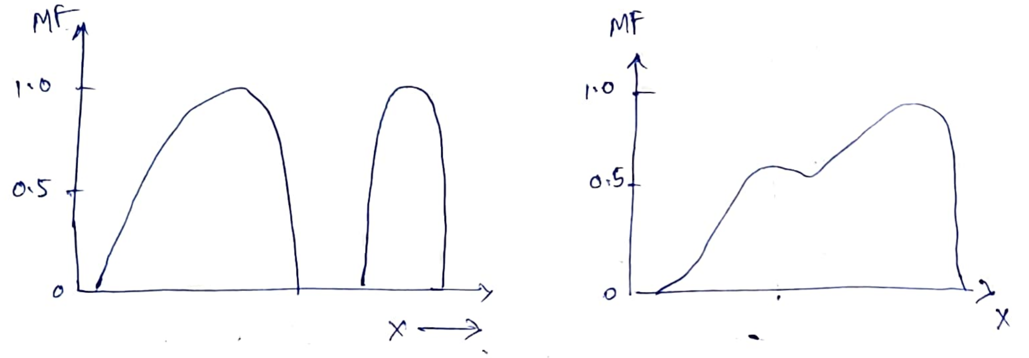
(α-level cut / α-cut) $\longrightarrow$ $A_\alpha = \{x \mid \mu_A(x) \geq \alpha\}$, $\text{support}(A) = A'_0$

(strong α-cut) $\longrightarrow$ $A'_\alpha = \{x \mid \mu_A(x) > \alpha\}$ $\text{core}(A) = A_1$

Convexity:

Fuzzy set A is convex iff for any $x_1, x_2 \in X$ and $\lambda \in [0,1]$

$$\mu_A(\lambda x_1 + (1-\lambda)x_2) \geq \min\{\mu_A(x_1), \mu_A(x_2)\}$$



Convex

Bandwidth of normal and convex fuzzy set

$$\text{width}(A) = |x_2 - x_1| \quad \text{where, } \mu_A(x_1) = \mu_A(x_2) = 0.5$$

Symmetry:

fuzzy set will be symmetric fuzzy set if its MF is symmetric around a certain point $x = c$

$$\mu_A(c+x) = \mu_A(c-x) \quad \forall x \in X$$

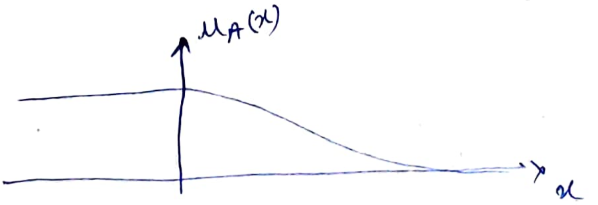
Open Left, Open Right, Closed Membership Function:

fuzzy set A is open left if $\lim_{x \to +\infty} \mu_A(x) = 0$ and

$$\lim_{x \to -\infty} \mu_A(x) = 1$$

open right: if $\lim_{x \to +\infty} \mu_A(x) = 1$ and $\lim_{x \to -\infty} \mu_A(x) = 0$

closed MF: if $\lim_{x \to +\infty} \mu_A(x) = 0$ & $\lim_{x \to -\infty} \mu_A(x) = 0$

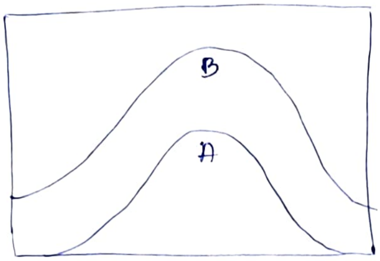


30/10/25
soft computing

Set theoretic operation:

● Subset/Containment: Fuzzy set A is contained in fuzzy set B (or ~~for~~ fuzzy set A is subset of fuzzy set B) iff $\mu_A(x) \leq \mu_B(x)$ $\forall x$,

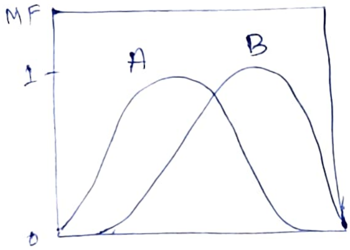
$$A \subseteq B \Rightarrow \mu_A(x) \leq \mu_B(x)$$

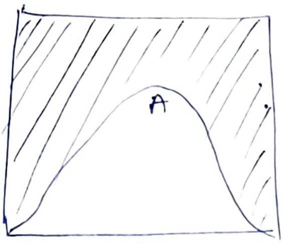


subset

● Union: The union of two fuzzy set A and B will be fuzzy set C or $C = A \cup B$ or $C = A$ OR B

$$\mu_C(x) := \max(\mu_A(x), \mu_B(x))$$
$$= \mu_A(x) \vee \mu_B(x)$$





¬A

● Intersection: $C = A \cap B \Rightarrow C = A$ AND B

$$\mu_C(x) = \min(\mu_A(x), \mu_B(x)) = \mu_A(x) \wedge \mu_B(x)$$

● Complement: complement of fuzzy set $A \rightarrow \bar{A}$ or NOT $A \Rightarrow \neg A$

$$\mu_{\bar{A}}(x) = 1 - \mu_A(x)$$

● Cartesian product and co-product:

Assume, A and B two fuzzy set in X and Y respectively.

Cartesian product of A and B $\rightarrow A \times B$ in the product space $X \times Y$

$$\mu_{A \times B}(x,y) = \min(\mu_A(x), \mu_B(y))$$

Cartesian co-product of A & B $\rightarrow A + B$

$$\mu_{A+B}(x,y) = \max(\mu_A(x), \mu_B(y))$$

MF formulation and parameterization:

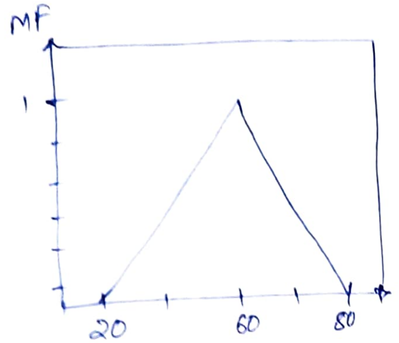
MF's in one dimension:

① Triangular MF's:

This MF is specified by 3 parameters (a, b, c)

~~triangle (a, b, c)~~

$$\text{triangle}(x; a, b, c) = \begin{cases} 0 & ; \quad x \le a \\ \dfrac{x-a}{b-a} & ; \quad a \le x \le b \\ \dfrac{c-x}{c-b} & ; \quad b \le x \le c \\ 0 & ; \quad c \le x \end{cases}$$

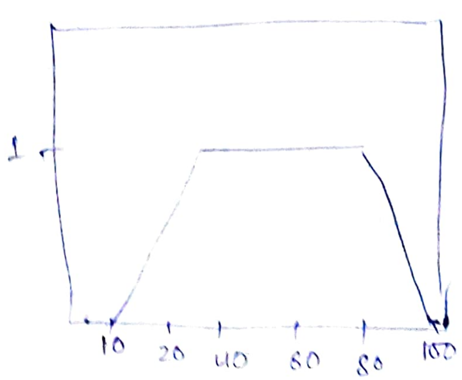


triangle $(x; 20, 60, 80)$

② Trapezoidal MF:

specified by 4 parameters $\{a, b, c, d\}$

$$\text{trapezoid}(x; a, b, c, d) = \begin{cases} 0 & ; \quad x \le a \\ \dfrac{x-a}{b-a} & ; \quad a \le x \le b \\ 1 & ; \quad b \le x \le c \\ \dfrac{d-x}{d-c} & ; \quad c \le x \le d \\ 0 & ; \quad x \ge d \end{cases}$$

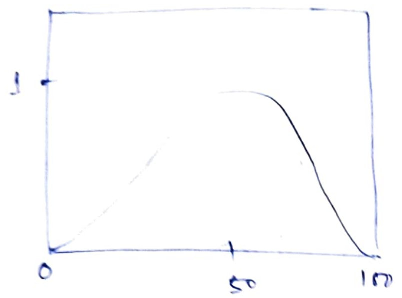


trapezoid $(x; 10, 20, 60, 95)$

③ Gaussian MF:

specified by two parameters (c, σ)

$$\text{Gaussian}(x; c, \sigma) = e^{-\frac{1}{2}\left(\frac{x-c}{\sigma}\right)^2}$$

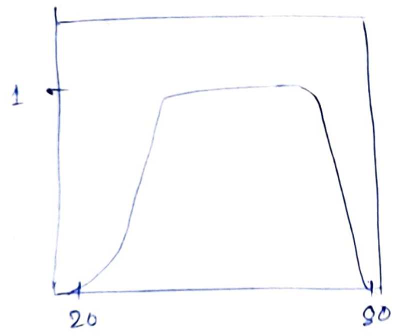


Gaussian $(x; 50, 20)$

④ Generalized Bell MF:

specified by 3 parameters $\{a, b, c\}$

$$\text{bell}(x; a, b, c) = \frac{1}{1 + \left|\frac{x-c}{a}\right|^{2b}}$$



⑤ Sigmoidal ~~Mob~~ MF:

$$\delta(x; a, c) = \frac{1}{1 + \exp(-a(x-c))}$$

a : controls the slope @ crossover point $x = c$

c : cross over point

⑥ Closed and asymmetric MF based on Sigmoidal MF:

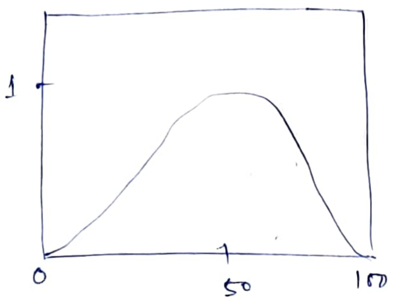
$y_1 = \delta(x; 1, -5)$

$y_2 = \delta(x; 2, 5)$

③ Gaussian MF :

specified by two parameters (c, σ)

$$\text{Gaussian}(x; c, \sigma) = e^{-\frac{1}{2}\left(\frac{x-c}{\sigma}\right)^2}$$

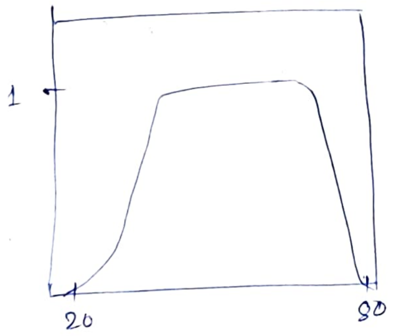


Gaussian $(x; 50, 20)$

④ Generalized Bell MF :

specified by 3 parameters $\{a, b, c\}$

$$\text{bell}(x; a, b, c) = \frac{1}{1 + \left|\frac{x-c}{a}\right|^{2b}}$$



⑤ Sigmoidal ~~Meb~~ MF :

$$s(x; a, c) = \frac{1}{1 + \exp(-a(x-c))}$$

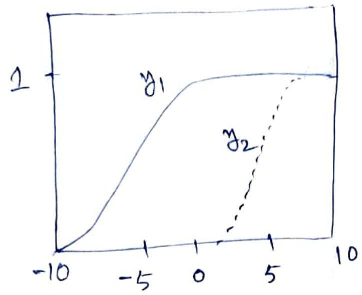
a : controls the slope @ crossover point $x = c$

c : cross over point

⑥ Closed and asymmetric MF based on Sigmoidal MF :

$y_1 = s(x; 1, -5)$

$y_2 = s(x; 2, 5)$



⑦ Left-Right MF: (3 parameters $\{\alpha, \beta, c\}$)

$$LR(x; c, \alpha, \beta) = \begin{cases} F_L\left(\frac{c-x}{\alpha}\right) & ; x \le c \\ F_R\left(\frac{x-c}{\beta}\right) & ; x \geq c \end{cases}$$

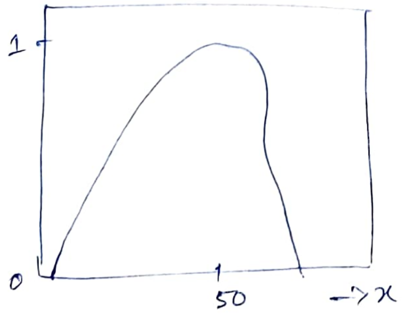
where, $F_L(x)$ & $F_R(x)$ monotonically decreasing function defined on $[0, \alpha)$ with $F_L(0) = F_R(0) = 1$

and $\lim_{x \to \infty} F_L(x) = \lim_{x \to \infty} F_R(x) = 0$

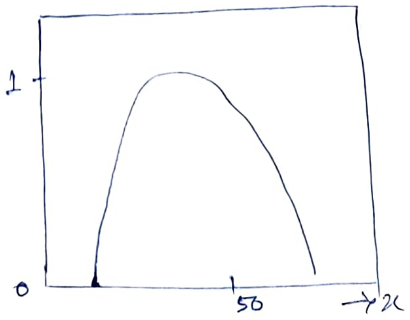
e.g.: L-R MF

$$F_L(x) = \sqrt{\max(0, 1-x^2)}$$

$$F_R(x) = e^{-x^3}$$

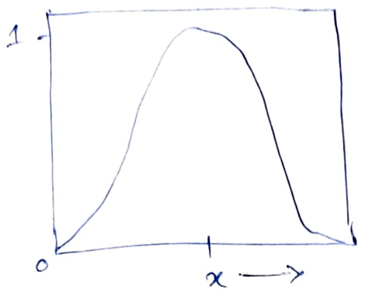


$LR(x; 65, 60, 10)$

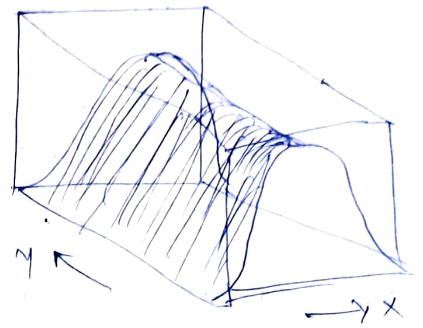


$LR(x; 25, 10, 40)$

2D - MF:



Base Fuzzy set A



Cylindical Extension of A

Cylindrical Extension of 1D Fuzzy set:

A is a fuzzy set in X

Then its cylindrical extension in $X \times Y$ in a fuzzy set $C(A)$

$$C(A) = \int_{X \times Y} \mu_A(x) / (x, y)$$

Projection of Fuzzy sets:

$R \longrightarrow$ 2D fuzzy set on $X \times Y$

then projections of R onto X and Y are defined as,

$$R_X = \int_X \left[\max_y \mu_R(x, y) \right] / x$$

$$R_Y = \int_Y \left[\max_X \mu_R(x, y) \right] / y \quad \text{respectively.}$$

Composite and Non-composite MFs:

e.g. fuzzy set $A \rightarrow$ "(x, y) is near $(3, 4)$"

$$\mu_A(x, y) = \exp\left[-\left(\frac{x-3}{2}\right)^2 - (y-4)^2 \right]$$

$$\mu_A(x, y) = \exp\left[-\left(\frac{x-3}{2}\right)^2 \right] \exp\left[-\frac{(y-4)}{1} \right]^2$$

$$= \text{Gaussian}(x; 3, 2) \ \text{Gaussian}(y; 4, 1)$$

Fuzzy set A → composite MF

If, fuzzy set A is define by following MF

$$\mu_A(x, y) = \frac{1}{1 + |x-3|\,|y-4|^{2.5}} \quad \Big\} \text{ non composite MF}$$

Derivative of parameterized MF's :

Assume, $y = \text{Gaussian}(x; \sigma, c) = e^{-\frac{1}{2}\left(\frac{x-c}{\sigma}\right)^2}$

$$\frac{\partial y}{\partial x} = -\frac{(x-c)}{\sigma^2}\, y$$

$$\frac{\partial y}{\partial \sigma} = \frac{(x-c)^2}{\sigma^3}\, y$$

$$\frac{\partial y}{\partial c} = \frac{(x-c)}{\sigma^2}\, y$$

1/10/25
Soft computing

Fuzzy Rules and Fuzzy Reasoning :

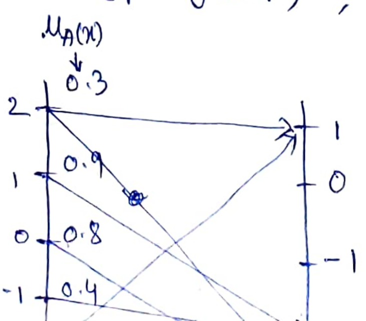
Extension principle :

$$A = \mu_A(x_1)/x_1 + \mu_A(x_2)/x_2 + \cdots + \mu_A(x_n)/x_n$$

$$f(A) = B = \mu_A(x_1)/y_1 + \mu_A(x_2)/y_2 + \cdots + \mu_A(x_n)/y_n$$

where,

$$y_i = f(x_i) \; ; \; i = 1, 2, \ldots, n$$



$$A = \underset{\mu_A(x)}{0.1}\,|\,\underset{x}{-2} + 0.4|-1 + 0.8|0 +$$
$$0.9|1 + 0.3|2$$

$$f(x) = x^2 - 3$$