

**The estimation for**

the channels is done using the linear minimum mean square  
error.

what is linear minimum mean square error in A. Training and Channel Estimation of the paper

Linear Minimum Mean Square Error (LMMSE) is a widely used channel estimation technique in wireless communication systems, including cell-free Massive MIMO setups. In the context of the paper, LMMSE is employed during the **training and channel estimation phase** to estimate the channels between Access Points (APs) and User Equipments (UEs). Here's a detailed explanation of LMMSE and its role in the paper:

**What is LMMSE?**

LMMSE is a statistical estimation method that minimizes the mean square error (MSE) between the actual channel and its estimated value, while considering prior information about the channel. It provides a more accurate estimate compared to simpler methods like Least Squares (LS), especially when there is knowledge of channel statistics such as average channel gain and noise variance.

**How LMMSE Works in Channel Estimation**

1. **Uplink Pilot Transmission**:
   * Each UE transmits a known pilot sequence to the APs during the training phase.
   * These pilot signals are used by APs to estimate the channel coefficients.
2. **Channel Model**:
   * The channel between AP and UE is modeled as:where:
     + represents large-scale fading (path loss and shadowing).
     + represents small-scale fading (Rayleigh fading).
3. **Received Signal Model**:
   * The received signal at AP during pilot transmission is:where:
     + : Pilot signal power.
     + : Pilot duration.
     + : Noise term.
4. **LMMSE Estimation Formula**:
   * The LMMSE estimator uses prior knowledge of channel statistics (mean, variance, etc.) to estimate the channel coefficient:
   * This formula accounts for both signal strength and noise variance, providing a more accurate estimate than LS.

**Advantages of LMMSE in the Paper**

* **Improved Accuracy**:
  + By incorporating statistical knowledge of large-scale fading () and noise variance, LMMSE achieves better accuracy compared to LS estimators.  
    - **Pilot Contamination Mitigation**:  
    - LMMSE helps reduce interference caused by overlapping pilot sequences from multiple UEs, which is a common issue in Massive MIMO systems.  
    - **Efficient Resource Utilization**:  
    - Accurate channel estimation ensures that APs can effectively serve UEs with minimal interference, improving spectral efficiency.

**Role of LMMSE in Training and Channel Estimation**

In the paper's system model:

* During training, UEs transmit pilot sequences to APs.
* APs use LMMSE to estimate the channels based on received pilot signals and known statistics of large-scale fading ().  
  - These estimated channels are then used for AP selection and spectral efficiency computation in subsequent steps of the CAPS algorithm.

**Summary**

LMMSE plays a critical role in accurately estimating channels during the training phase in cell-free Massive MIMO systems. By minimizing mean square error using statistical knowledge of the channel, it enhances system performance by reducing pilot contamination and ensuring efficient AP-user pairing. This method underpins the CAPS algorithm's ability to select access points based on reliable channel estimates.

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