INTERNATIONAL INSTITUTE OF INFORMATION TECHNOLOGY

Instructor: Lakshmi Burra	Maths III
Time 3 Hours (9:00-12:00) End -Semester Examination	Monsoon 2017-Math Core
	Date: November 22, 2017
	Total Marks :100

Instructions:

- Answer all questions
- Calculators are allowed
- Normal Distribution Table for required problems

Section A

1) a) Determine the principal value of the argument of: $(9+9i)^3$

b) Find and graph all the roots in the complex plane: $\sqrt[3]{3+4i}$ (2+2=4 marks)

2) a) Find out whether $f(z)$ is continuous at $z=0$ if $\begin{cases} f(z) = \frac{[Re(z^2)]}{|z|^2} & z \neq 0 \\ f(z) = 0 & z = 0 \end{cases}$

b) Verify whether $f(z)$ is analytic and if so find $f'(z)$

i) $f(z) = \sqrt{r}\, e^{\frac{i\theta}{2}}$ $(r>0, \quad \alpha < \theta < \alpha + 2\pi)$

ii) $f(z) = e^{-\theta}\cos(\ln r) + i\, e^{-\theta}\sin(\ln r)$ $(r>0, \ 0<\theta<2\pi)$ (4+4+4=12 marks)

3) Show that $u(x,y)$ is harmonic in some domain and find a harmonic conjugate $v(x,y)$ when

i) $u(x,y) = 2x(1-y)$ ii) $u(x,y) = 2x - x^3 + 3xy^2$ (5+5= 10 marks)

4) a) Let C denote the line segment from $z=i$ to $z=1$. By observing that of all the points on that line segment, the midpoint is the closest to the origin, show that $\left|\int_C \frac{dz}{z^4}\right| \leq 4\sqrt{2}$, without evaluating the integral.

b) Find the Taylor or Laurent Series with centre $z=z_0$ of $f(z) = \frac{1}{z^2}$, $z_0 = i$ (4+5=9 marks)

5) Find the value of $\int_C g(z)$ around the circle C: $|z-i|=2$ in the positive sense when

a) $g(z) = \frac{1}{(z^2+4)^2}$. (5 marks)

b) Evaluate the following Real Integrals **using Contour Integration** :

a) $\int_0^{2\pi} \frac{1+4\cos\theta}{17-8\cos\theta}\, d\theta$ b) $\int_{-\infty}^{\infty} \frac{x^2+1}{x^4+1}\, dx$ (5+5=10 marks)

Section B

6) An elementary school is offering 3 language classes: one in Spanish, one in French, and one in German. The classes are open to any of the 100 students in the school. There are 28 students in the Spanish class, 26 in the French class, and 16 in the German class. There are 12 students that are in both Spanish and French, 4 that are in both Spanish and German, and 6 that are in both French and German. In addition, there are 2 students taking all 3 classes.

(a) If a student is chosen randomly, what is the probability that he or she is not in any of the language classes?
(b) If a student is chosen randomly, what is the probability that he or she is taking exactly one language class?
(c) If 2 students are chosen randomly, what is the probability that at least 1 is taking a language class? **(6 marks)**

7) An instructor gives her class a set of 10 problems with the information that the final exam will consist of a random selection of 5 of them. If a student has figured out how to do 7 of the problems, what is the probability that he or she will answer correctly

(a) all 5 problems?
(b) at least 4 of the problems?

(4 marks)

8) At a certain stage of a criminal investigation, the inspector in charge is 60 percent convinced of the guilt of a certain suspect. Suppose, however, that a *new* piece of evidence which shows that the criminal has a certain characteristic (such as left-handedness, baldness, or brown hair) is uncovered. If 20 percent of the population possesses this characteristic, how certain of the guilt of the suspect should the inspector now be if it turns out that the suspect has the characteristic?

(8 marks)

9) Five men and 5 women are ranked according to their scores on an examination. Assume that no two scores are alike and all 10! possible rankings are equally likely. Let X denote the highest ranking achieved by a woman. (For instance, $X = 1$ if the top-ranked person is female.) Find $P\{X = i\}$, $i = 1, 2, 3, \ldots, 8, 9, 10$.

(8 marks)

10) A student is getting ready to take an important oral examination and is concerned about the possibility of having an "on" day or an "off" day. He figures that if he has an "on" day, then each of his examiners will pass him, independently of each other, with **probability 0.8,** whereas if he has an "off" day, this probability will be reduced **to 0.4**. Suppose that the student will pass the examination if a majority of the examiners pass him. If the student feels that he is twice as likely to have an "off" day as he is to have an "on" day, should he request an examination with 3 examiners or with 5 examiners?

(8 marks)

11) A certain typing agency employs 2 typists. The average number of errors per article is 3 when typed by the first typist and 4.2 when typed by the second. If your article is equally likely to be typed by either typist, approximate the probability that it will have no errors.

(8 marks)

12) The life of a certain type of automobile tire is normally distributed with mean 34,000 miles and standard deviation 4000 miles.

(a) What is the probability that such a tire lasts over 40,000 miles?
(b) What is the probability that it lasts between 30,000 and 35,000 miles?

(8 marks)

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