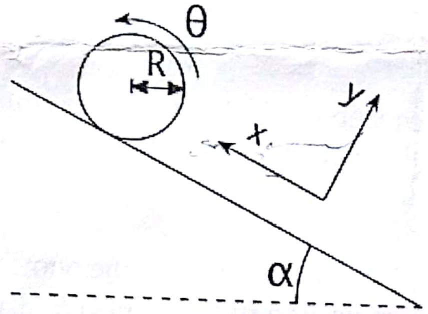
End Semester Examination – Monsoon 2017
IIIT-Hyderabad
Subject: Science I (ISC201)

Total: 50 marks

Time: 3 hrs

1 A wheel of mass m, radius R, and radius of gyration R_G is released at the top of a hill. Assume that the wheel does not slip as it rolls down the hill (refer Figure). Using Lagrange's equations, derive the equations of motion of this system. (8M)



2 Consider the random walk problem in one dimension and suppose that the probability of a single displacement between s and s+ds is given by

$$w(s)\,ds = \frac{1}{\pi}\frac{b}{s^2+b^2}\,ds$$

Calculate the probability $P(x)\,dx$ that the total displacement after N steps lies between x and $x+dx$. Does $P(x)$ become Gaussian when N becomes large? (4M)

3 a) For a quantum particle of mass m moving on the surface of sphere, express the kinetic energy operator in terms of the spherical polar coordinates.
b) Determine the energy and angular momentum of a quantum particle of mass m travelling on a circular ring. (7M)

4 a) Find the probability that the electron in the ground-state H atom is less than a distance a_0 from the nucleus. The wavefunction of 1s electron is $\psi = \frac{e^{-r/a_o}}{\sqrt{\pi}\,a_o^{3/2}}$.
b) Find the expectation value of $1/r$ for 1s electron. (3M)

5 What are Euler angles? How do you use them to describe the rotational dynamics of a rigid body? (4M)

6 Given the Lagrangian of an isolated system, derive the conservation laws resulting from the (a) homogeneity of time, (b) homogeneity of space, (c) isotropy of space. (6M)

7. Demonstrate that the uncertainity principle (relating Δx and Δp) is satisfied in the ground- state of a particle in a one-dimensional box. (5M)

8. The dynamics of a quantum particle of mass m moving one-dimensionally in a potential $V(x)$ is governed by the Hamiltonian $H_0 = \frac{p^2}{2m} + V(x)$, where $p = -i\hbar \frac{d}{dx}$ is the momentum operator. Let $E_n^{(0)}$, $n = 1, 2, 3, \ldots$, be the eigenvalues (i.e., energy of the n^{th} state) of H_0. Now consider a new Hamiltonian $H = H_0 + \lambda p/m$, where λ is given parameter. Given m and $E_n^{(0)}$, find the eigenvalues of H. (8M)

$\lambda = \frac{h}{mv}$ $\lambda = \frac{h}{p}$

9. Using Langevin's equation, calculate the mean square displacement of a solute in a solvent. Discuss the short-time and long-time behavior of the mean square displacement. (3M)

10. Using Bohr theory, find the frequency of the photon emitted by a hydrogen atom due to the transtion of electron from the level $n+1$ to the level n and frequency of revolution of the electron in n^{th} level. Show that at larger values of n, both the frequency of photon and frequency of revolution are approximately same. (2M)