

Your Quiz will have 3 questions, one from each of the three sections given here.

Please come with sufficient papers to write your answers.

No books or smart phones are allowed.

Each question carries 10 marks, for a total of 30 marks.

### Section A

Solve the linear program using the simplex method.

1.

$$\begin{aligned} \max & (3x + 4y) \\ \text{s.t. } & 5x + 7y \leq 15 \\ & 3x - y \leq 2 \end{aligned}$$

2.

$$\begin{aligned} \max & (2x + 5y) \\ \text{s.t. } & x + 7y \leq 15 \\ & 3x - 4y \leq 6 \end{aligned}$$

3.

$$\begin{aligned} \max & (4x - 7y) \\ \text{s.t. } & 4x + y \leq 10 \\ & -2x + 3y \leq 8 \end{aligned}$$

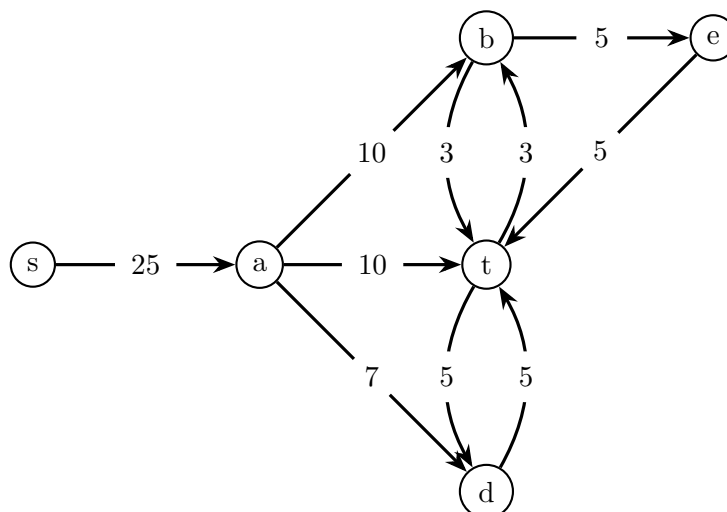
4.

$$\begin{aligned} \max & (-3x + 5y) \\ \text{s.t. } & 3x + 7y \leq 15 \\ & -x + 2y \leq 2 \end{aligned}$$

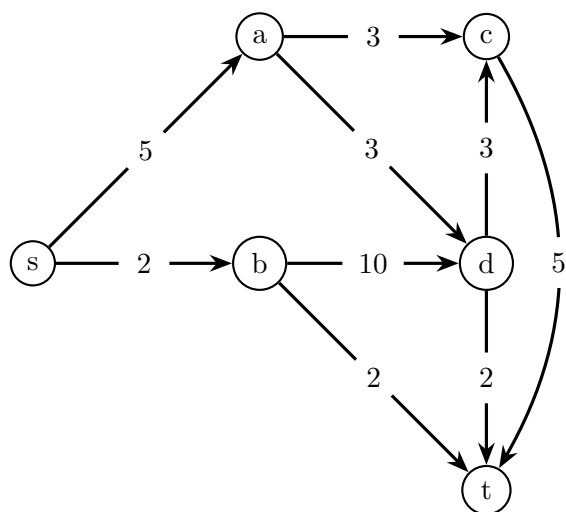
### Section B

Using the Ford-Fulkerson algorithm, compute the maximum flow from  $s$  to  $t$  in the graph.

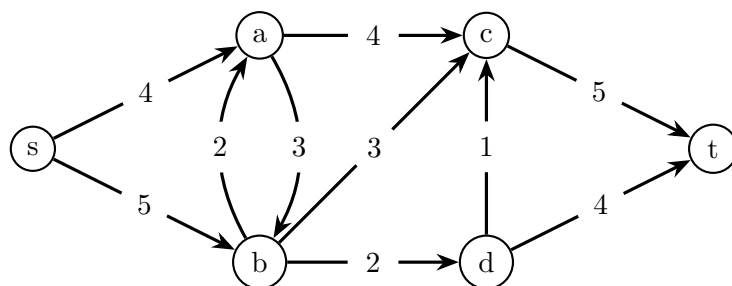
1.



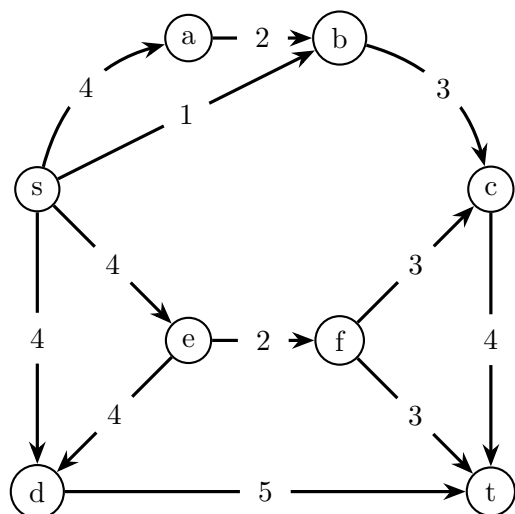
2.



3.



4.



### Section C

1. (a) (3 marks) If we measure the qubit  $\alpha|0\rangle + \beta|1\rangle$  in the basis of  $|0\rangle$  and  $|1\rangle$ , what is the probability of the result being  $|0\rangle$ ? What is it for  $|1\rangle$ ?  
 (b) (7 marks) Describe the three polarizers experiment.
2. (a) (3 marks) What is the significance of unitary operators in quantum computing?

- (b) (7 marks) Write down the matrix that performs the discrete Fourier transform. Show that it is unitary. (Assume that the matrix is appropriately normalized, i.e. multiply a scalar factor to the matrix to make it unitary.)
- 3. (a) (3 marks) Mention two quantum gates (other than the identity gate) and write their matrix representation.  
(b) (7 marks) Find the best approximation of  $\sqrt{2}$  as a rational fraction  $p/q$ , where  $p$  and  $q$  are positive integers and  $q \leq 30$ . (Hint: use continued fractions.)
- 4. (10 marks) Suppose we have an efficient randomized algorithm to find the order of an element  $a$  in a group  $G$ . Show that, using this, we can come up with an efficient randomized algorithm that, given a composite number  $N$ , finds a non-trivial factor of  $N$ .