Q1 $w(s)\,ds = \frac{1}{\sqrt{2\pi\sigma^2}}\, e^{-(s-l)^2/2\sigma^2} ds$

a) $\bar{x} = \int_{-\infty}^{\infty} \frac{s}{\sqrt{2\pi\sigma^2}}\, e^{-(s-l)^2/2\sigma^2} ds$

$$\left.\begin{aligned} \frac{s-l}{\sqrt{2\sigma^2}} &= y \\ ds &= \sqrt{2\sigma^2}\, y \end{aligned}\right\} \quad s = \sqrt{2\sigma^2}\, y + l$$

$$\sqrt{2\sigma^2} \int_{-\infty}^{\infty} \frac{\sqrt{2\sigma^2}\, y + l}{\sqrt{2\pi\sigma^2}}\, e^{-y^2} dy$$

$$\frac{\sqrt{2\sigma^2}}{\sqrt{2\pi\sigma^2}} \left[\int_{-\infty}^{\infty} 2\sigma^2 y e^{-y^2} dy + \int_{-\infty}^{\infty} l e^{-y^2} dy \right]$$

$$\frac{\sqrt{2\sigma^2}}{\sqrt{2\pi\sigma^2}} \left[\left[-\sigma^2 e^{-y^2} \right]_{-\infty}^{\infty} + l\sqrt{\pi} \right]$$

$$\frac{1}{\sqrt{\pi}} \left[0 + l\sqrt{\pi} \right]$$

$$= l$$

$$\boxed{\bar{x} = l}$$

Mean displacement after N steps = Nl

b) $\overline{(x-\bar{x})^2} = \int \frac{(s-\bar{x})^2}{\sqrt{2\pi\sigma^2}} e^{-(s-\ell)^2/2\sigma^2} ds$

$\frac{1}{\sqrt{2\pi\sigma^2}} \int (s-\ell)^2 e^{-(s-\ell)^2/2\sigma^2} ds$

$\frac{s-\ell}{\sqrt{2\sigma^2}} = y$

$ds = \sqrt{2\sigma} y$

$\frac{2\sigma^2}{\sqrt{\pi}} \int y^2 e^{-y^2} \sqrt{2\sigma^2}\, y$

$\frac{2\sigma^2}{\sqrt{\pi}} \left[y \frac{e^{-y^2}}{2} \Big|_{-\infty}^{\infty} + \int_{-\infty}^{\infty} \frac{e^{-y^2}}{2} dy \right]$

$\frac{2\sigma^2}{\sqrt{\pi}} \times \frac{\sqrt{\pi}}{2}$

$= \sigma^2$

$\boxed{\overline{(x-\bar{x})^2} = \sigma^2}$

Diskersion after N steps $= N\sigma^2$

~~PDF P(x) $\frac{1}{2b}$ $\ell-b$ $\ell+b$~~

σ^2 2 a)

$$f(x) = \begin{cases} \frac{1}{2b} & \ell-b \le x \le \ell+b \\ 0 & \text{otherwise} \end{cases}$$

$\bar{x} = \int x f(x) dx$

$= \frac{1}{2b} \int_{\ell-b}^{\ell+b} x\, dx$

$$= \frac{1}{2b}\left[\frac{x^2}{2}\right]_{l-b}^{l+b}$$

$$= \frac{4l\,b}{4b}$$

$$\boxed{\bar{x} = l}$$

Mean displacement after N steps $= Nl$

b) $$(x-\bar{x})^2 = \int_{-\infty}^{\infty} (s-l)^2 f(s)\, ds$$

$$= \int_{l-b}^{l+b} (s-l)^2 \frac{1}{2b}\, ds$$

$$= \frac{(s-l)^3}{2b3}\Bigg|_{l-b}^{l+b}$$

$$= \frac{b^3}{6b} - \frac{(-b)^3}{6b}$$

$$= \frac{b^2}{6} + \frac{b^2}{6}$$

$$= \frac{b^2}{3}$$

$$\boxed{(x-\bar{x})^2 = \frac{b^2}{3}}$$

Dispersion after N steps $= \frac{Nb^2}{3}$