

Assignment 5

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Outline

1 Problem

2 Solution

Problem Statement

(Papoulis chap-12 - 12.1) Find the mean and variance of the random variable

$$n_T = \frac{1}{2T} \int_{-T}^T x(t) dt$$

$$\text{where } x(t) = 10 + v(t)$$

For $T=5$ and for $T=100$, assume

$$E[v(t)] = 0$$

$$R_v(\tau) = 2\delta(\tau)$$

Solution

$$x(t) = 10 + v(t) \quad (1)$$

$$R_v(\tau) = 2\delta(\tau) \quad (2)$$

From question

$$E[v(t)] = 0 \quad (3)$$

$$E(x(t)) = E(10 + v(t)) \quad (4)$$

$$E[n_T] = E[x_T] = 10 \quad (5)$$

$$C_x(\tau) = 2\delta(\tau) \quad (6)$$

$$\sigma_{n_T}^2 = \frac{1}{2T} \int_{-T}^T C_x(\tau) \left(1 - \frac{\tau}{2T}\right) d\tau$$

From equation 6

$$\frac{1}{2T} \int_{-T}^T C_x(\tau) \left(1 - \frac{\tau}{2T}\right) d\tau = \frac{1}{2T} \int_{-T}^T 2\delta(\tau) \left(1 - \frac{\tau}{2T}\right) d\tau$$

$$\frac{1}{2T} \int_{-T}^T 2\delta(\tau) \left(1 - \frac{\tau}{2T}\right) d\tau = \frac{1}{T}$$

So variance σ^2 is $\frac{1}{T}$