Assignment 6

Sri Charvi (BT21BTECH11008)

17 June 2022

Outline

Question

Show that

$$\lim_{N\to+\infty} \log \frac{\Delta_{N+1}}{\Delta_N} = \lim_{N\to+\infty} \frac{\log \Delta_N}{N} = \frac{1}{2\pi}\int_{-\pi}^{\pi} \log S_s(\omega)\, d\omega$$

Solution

The Nth order MS estimation error P_N equals $\dfrac{\Delta_{N+1}}{\Delta_N}$

$$\lim_{N \to +\infty} \log P_N = \frac{1}{2\sigma} \int_{-\pi}^{\pi} \log S_s(\omega)\, d\omega \tag{1}$$

$$= \lim_{N \to +\infty} \log \frac{\Delta_{N+1}}{\Delta_N} \tag{2}$$

And we know

$$\lim_{N \to +\infty} \frac{1}{N} \sum_{n=1}^{N} \log \frac{\Delta_{N+1}}{\Delta_N} = \lim_{N \to +\infty} \log \frac{\Delta_{N+1}}{\Delta_N} \tag{3}$$

and the result follows because

$$\frac{1}{N}\sum_{n=1}^{N}(\log\Delta_{N+1}-\log\Delta_{N})=\frac{\log\Delta_{N+1}}{N}-\frac{\log\Delta_{1}}{N} \tag{4}$$

and the last term tends to zero as N $\rightarrow \infty$
Hence Proved .