

AI110:Assignment 4

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Outline

1 Problem

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Problem Statement

(Papoulis chap-12 - 12.4) Show that if a process is normal and distribution-ergodic as in (12-35), then it is also mean-ergodic.

Defination from 12-35

$$\frac{1}{T} \int_0^T F(x, x : \tau) d\tau \longrightarrow F^2(x)$$

for $T \rightarrow \infty$

Solution

The process $x(t)$ is normal and such that

$$F(x, x : \tau) \longrightarrow F^2(x) \quad (1)$$

as $\tau \rightarrow \infty$

We shall show that it is mean - ergodic. It suffices to show that

$$C(\tau) \rightarrow 0$$

for $\tau \rightarrow \infty$ We are assuming that $\eta = 0$

$$C(0)=1$$

$$f(x_1, x_2; \tau) = \frac{1}{2\pi\sqrt{1-r^2}} \exp \left| -\frac{1}{2(1-r^2)}(x_1^2 - 2rx_1x_2 + x_2^2) \right| \quad (2)$$

$$= \frac{1}{2\pi\sqrt{1-r^2}} \exp \left| -\frac{1}{2(1-r^2)}(x_1 - rx_2)^2 \right| e^{-\frac{x_2^2}{2}} \quad (3)$$

Clearly, $f(x,y) = f(y,x)$, hence

$$\begin{aligned} F(x+dx, x+dx; \tau) - F(x, x, \tau) &= 2 \int_{-\infty}^x f(\epsilon, x) d\epsilon dx \\ &= \frac{1}{\pi\sqrt{1-r^2}} \int_{-\infty}^x \exp \left| -\frac{1}{2(1-r^2)}(\epsilon - rx)^2 \right| d\epsilon e^{-x^2/2} dx \end{aligned} \quad (4)$$

further

$$F^2(x + dx) - F^2(x) = 2F(x)f(x)dx$$

From above 1 it follows that $G\left\{\frac{x-rx}{\sqrt{1-r^2}}\right\} \longrightarrow G(x)$

for $\tau \rightarrow \infty$

Hence, $r(\tau) \rightarrow 0$

as $\tau \rightarrow \infty$