CS 600 Advanced Algorithms

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Homework 10

1 R-19.8.3 Use a Chernoff bound to bound the probability that more than 4% of the 1 million children born in a given large city have this birth defect.

For $i = 1, ..., 10^6$, Compute μ
$\mu = E(X) = \Sigma_{i=1}^{n} p_i = 0.02 * 10^6 = 2 * 10^4$
$Pr(X > (1+\delta))\mu \leq [\frac{e^\delta}{(1+\delta)^{(1+\delta)}}]^\mu$, where $\delta > 1$. Here, $\delta = 1$.
$Pr(X > 2 * 2 * 10^4) \leq [\frac{e^\sigma}{(1+\sigma)^{(1+\sigma)}}]^\mu = [\frac{e^1}{(2)^2}]^{2*10^4} = [\frac{e}{4}]^{20000}$

Using a Chernoff bound, the probability that more than 4% of the 1 million children born in a given large city have this birth defect is bound by 0.6796^{20000}.

2 C-19.8.18 Consider a modification of the Fisher-Yates random shuffling algorithm where we replace the call to $random(k+1)$ with random(n). Show that this algorithm does not generate every permutation with equal probability.

Assume n = 3. We'll begin with 123. After one step, we have 123, 132, and 321 with a chance of $\frac{1}{3}$. Step 2 yields 123, 213, and 132, each with a chance of $\frac{1}{9}$, and so on. Finally, each permutation of 123 occurs with a chance of $\frac{i}{27}$, where i is the number of times that permutation might occur if we follow the depth-3 tree of possibilities, which has degree-3 and 27 external nodes. But each permutation must occur with a chance of $\frac{1}{3!} = \frac{1}{6}$, and there is no way to make a fraction of the form $\frac{i}{27}$ equal to $\frac{1}{6}$, where i is an integer.

3 A-19.8.35 Suppose that an attacker is sending a large number of packets in a denial-of-service attack to some recipient, and every one of the d routers in the path from the sender to the recipient is performing probabilistic packet marking.

a) The likelihood that a router will execute probabilistic packet marking is given as $p \leq \frac{1}{2}$. Now, for the packet to survive with a mark created by the furthest router, other routers must not execute probabilistic packet marketing on that packet, and the chance of this is given by $(1-p)$. As a result, the overall likelihood that the router farthest away from the destination will mark a packet and that this mark will survive all the way to the recipient is $p(1-p)^{d-1}$, where d is the total number of routers.

b) This is the same as the coupon collector problem. Let $X = X_1 + X_2 + ... + X_d$ be a random variable that we must gather in order to identify all d routers, where X_i is the number of packets required to transition from having i-1 distinct router addresses to having i distinct addresses. After receiving i-1 different addresses, the probability of receiving a new one is $p_i = \frac{d-(i-1)}{d}$, with the expected value of $E[X_i] = \frac{1}{p_i}$. $E[X] = nH_n$ via linearity of expectation, where H_n is the nth harmonic number, which may be approximated as $lnd \leq H_n \leq lnd + 1$. Now, tail estimates that the recipient will get all of the addresses after collecting $(cdlnd)$ packets for $c \geq 2$. Thus, the upper bound on the expected number of packets that the recipient needs to collect to get all d routers addresses is $O(d \log d)$.