Discrete Assignment

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1. $8a_n - 6a_{n-1} - 11a_{n-2} + 3a_{n-3} = 0$ — ①

Degree = 3

Let $a_n = r^n$ be the solⁿ type of — ①

Subs $a_n = r^n$ in ①

$8r^n - 6r^{n-1} - 11r^{n-2} + 3r^{n-3} = 0$

divide by r^{n-3}

$8r^3 - 6r^2 - 11r + 3 = 0$

$r = -1, \frac{3}{2}, \frac{1}{4}$

real and distinct roots

General solⁿ

$$a_n = \alpha_1(-1)^n + \alpha_2\left(\frac{3}{2}\right)^n + \alpha_3\left(\frac{1}{4}\right)^n$$

2. $a_n = n2^{n-2}$ — ①

$a_n = 2a_{n-1} + 2^{n-2}$

subs ①

$$a_n = 2\left[(n-1)2^{n-3}\right] + 2^{n-2}$$

$$\Rightarrow 2n\frac{2^n}{2^3} - \frac{2 \times 2^n}{2^3} + \frac{2^n}{2^2}$$

$$\Rightarrow n\frac{2^n}{2^2} - \frac{2^n}{2^2} + \frac{2^n}{2^2}$$

$$\frac{n2^n}{2^2} = n2^{n-2}$$

$\therefore a_n = n2^{n-2}$ is a solution

3. $B(0) = 200$ (initial number of bacteria)

$B(1) = 3B(0) = 3(200) = 600$

$B(2) = 3B(1) = 3(600) = 1800$

$B(3) = 3B(2) = 3(1800) = 5400$

$B(4) = 3B(3) = 3(5400) = 16200$

$B(5) = 3B(4) = 3(16200) = 48600$

$B(6) = 3B(5) = 3(48600) = 145800$

$B(7) = 3B(6) = 3(145800) = 437400$

$B(8) = 3B(7) = 3(437400) = 1312200$

$B(9) = 3B(8) = 3(1312200) = 3936600$

$B(10) = 3B(9) = 3(3936600) = 11809800$

so, the reccurence relation is $B_n = 3B_{n-1}$

$\therefore$ After 10 hours, there will be 11809800 bacteria in the colony.

6 $a_n + a_{n-1} - 2a_{n-2} = n+2$, $a_0 = 1$, $a_1 = 2$ —(II)

$a_n = a_n^{(h)} + a_n^{(P)}$

$a_n + a_{n-1} - 2a_{n-2} = 0$

degree $= 2$

Let $a_n = r^n$ be the sol^n

subs $a_n = r^n$

$r^n + r^{n-1} - 2r^{n-2} = 0$ —(1)

divide by r^{n-2}

$r^2 + r - 2 = 0$

$r^2 - r + 2r - 2 = 0$

$r(r-1) + 2(r-1) = 0$

$r = 1, -2$

General sol^n

$a_n^{(h)} = \alpha_1 (1)^n + \alpha_2 (-2)^n$

$a_n^{(P)} = n+2$

$a_n^{(P)} = [P_1 n + P_0](1)^n \times n$

subs in (II)

$a_n^{(P)} + a_{n-1}^{(P)} - 2a_{n-2}^{(P)} = n+2$

$[P_1 n + P_0](1)^n \times n + [P_1(n-1) + P_0](1)^{n-1} \times (n-1)$

$-2[P_1(n-2) + P_0](1)^{n-2} \times (n-2)$

$= n+2$

Put $n=0$

$0 - [-P_1 + P_0] + 4[-2P_1 + P_0] = 2$

$P_1 - P_0 - 8P_1 + 4P_0 = 2$

$-7P_1 + 3P_0 = 2$ — (III)

Put $n=1$

$[P_1 + P_0] + 0 + 2[-P_1 + P_0] = 1+2$

$-P_1 + 3P_0 = 3$ — (IV)

(IV) − (III)

$6P_1 = 3-2$

$6P_1 = 1$

$P_1 = \frac{1}{6}$

Putting value of P_1 in (iv)

$3P_0 = 3 + \frac{1}{6}$

$3P_0 = \frac{19}{6}$

$P_0 = \frac{19}{18}$

$\therefore \quad a_n^{(P)} = \left[\frac{1}{6}n + \frac{19}{18}\right](1)^n n$

$$\left[\frac{1}{6}n^2 + \frac{19}{18}n\right](1)^n$$

$a_n^{(h)} = \alpha_1(1)^n + \alpha_2(-2)^n + \left[\frac{1}{6}n^2 + \frac{19}{18}n\right](1)^n$

Putting
$a_0 = 1$ & $n = 0$

$$1 = \alpha_1 + \alpha_2 \quad \text{—} \text{(v)}$$

Putting
$a_1 = 2$ & $n = 1$

$$2 = \alpha_1 - 2\alpha_2 + \frac{1}{6} + \frac{19}{18} \quad \text{—} \text{(vi)}$$

(vi) - (v)

$$1 = -3\alpha_2 + \frac{22}{18}$$

$$3\alpha_2 = \frac{22-18}{18}$$

$$3\alpha_2 = \frac{4}{18} \Rightarrow \alpha_2 = \frac{2}{27}$$

$\alpha_1 = 1 - \alpha_2$

$= 1 - \frac{2}{27}$

$= \frac{25}{27}$

$a_n = \frac{25}{27}(1)^n + \frac{2}{27}(-2)^n + \left[\frac{1}{6}n^2 + \frac{19}{18}n\right](1)^n$

7.

$a_n - 2a_{n-1} = 0$, $a_0 = 2$

multiply both side by x^n

$a_n x^n - 2a_{n-1}x^n - 1x^n = 0$

apply Σ on both side from $n=1$ to ∞

$$\sum_{n=1}^{\infty} a_n x^n - 2\sum_{n=1}^{\infty} a_{n-1}x^n - \sum_{n=1}^{\infty}(1)x^n = 0$$

$$\sum_{n=1}^{\infty} a_n x^n - 2x\sum_{n=1}^{\infty} a_{n-1}x^{n-1} - \sum_{n=1}^{\infty} x^n + 1 = 0$$

$$G(x) - a_0 - 2x[G(x)] - \frac{1}{1-x} + 1 = 0$$

$$G(x)[1-2x] = \frac{1}{1-x} + 1$$

$$G(x) = \frac{1}{(1-x)(1-2x)} + \frac{1}{(1-2x)}$$

by partial fraction

$$\frac{1}{(1-x)(1-2x)} = \frac{A}{(1-x)} + \frac{B}{(1-2x)}$$

$$1 = A(1-2x) + B(1-x)$$

$$A+B = 1 \quad \text{— (i)}$$

$$-2A - B = 0 \quad \text{— (ii)}$$

$$A = -1$$

$$B = 2$$

$$\frac{1}{(1-x)(1-2x)} = \frac{-1}{(1-x)} + \frac{2}{1-2x}$$

$$G(x) = \frac{-1}{(1-x)} + \frac{2}{(1-2x)} + \frac{1}{(1-2x)}$$

$$\frac{-1}{1-x} + \frac{3}{1-2x}$$

$$\sum_{n=0}^{\infty} a_n x^n = -1\sum_{n=0}^{\infty} x^n + 3\sum_{n=0}^{\infty} 2^n x^n$$

$$\therefore \quad a_n = -1 + 3(2)^n$$