CS 5800 – theory foundations
WIN ID: 004543153

8 $f: R^+ \to R^+$ defined by $f(x) = 1/x$

prove f is onetoone & onto

onetoone: (injection)

let $m, n \in R^+$

prove $f(m) = f(n)$ i.e $m = n$

$\cancel{\frac{1}{m} = \frac{1}{n}}$

$\boxed{n = m}$

$\therefore$ f is oneto one

onto (surjective):

let $y \in R^+$ (codomain/range) then exsists $x \in R^+$ (domain of f) such that

$f(x) = y$

$\frac{1}{x} = y$

$y = \frac{1}{x}$

$\therefore$ ~~for every to $x \in R^+$ (domain)~~ ~~There exists~~

for every $y \in R^+$ (range) there exsist $x \in D_f$ (in domain of f)

$\therefore$ f is oneto one & onto

(29) Given a recursive definition of the relation is equal to on $N \times N$ using the operator's.

Basic: $(0,0) \in EQ$, where i,j are equal $(i,j) \in EQ$

recursive step: if $(i,j) \in EQ$, then $(s(i), s(j)) \in EQ$

Observe: $(i,j) \in EQ$ if and only if it can be obtained in finite no of appliation of recursive step from $(0,0) \in EQ$

---

(17) prove that set of even integers is denumerable (Countable)

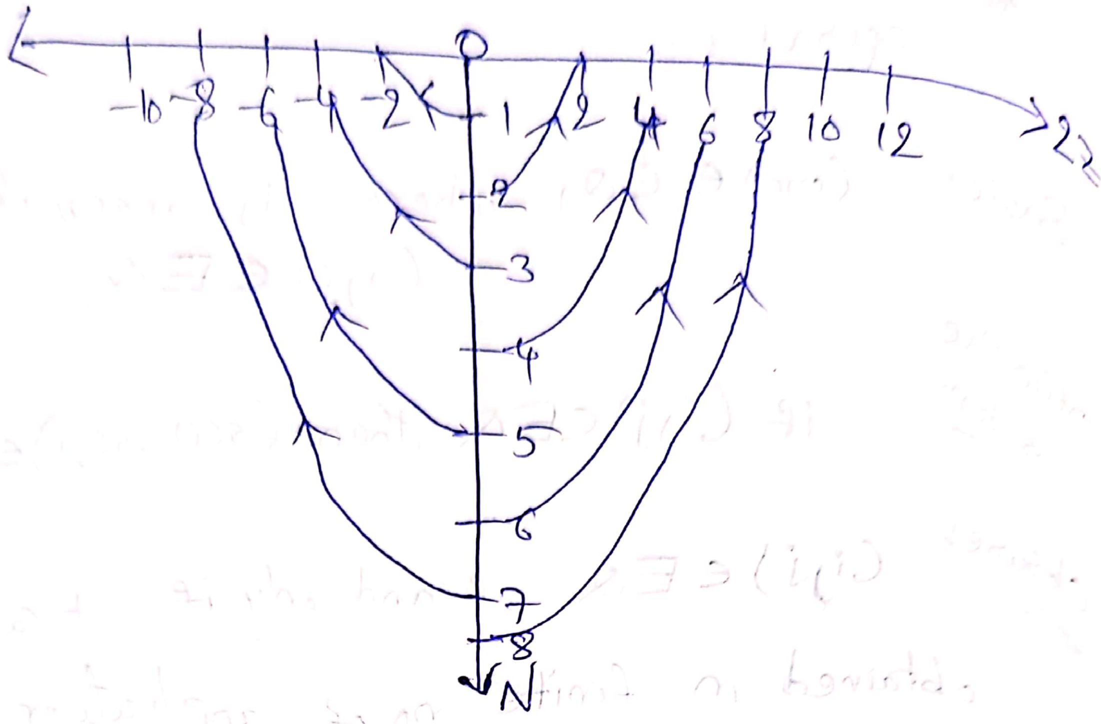
solution: let $Z = \{ \ldots, -6, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4 \ldots \}$

$2Z = \{ \ldots, -12, -10, -8, -6, -4, -2, 0, 2, 4, 6, 8, 10, \ldots \}$

$N = \{ 0, 1, 2, 3, 4, \ldots \}$

$$f(n) = \begin{cases} n, & \text{if } n \text{ is even} \\ -(n+1), & \text{if } n \text{ is odd} \end{cases}$$

$\therefore$ A set is contably infinite if its elements can be put in a one to one correspondence with natural numbers



Prove f is one to one [redacted]

onetoone

$f(m) = f(n)$, let $m, n \in N$

for even integers $f(n) = n$

$$\boxed{m = n}$$

for odd integers $f(n) = -(n+1)$

$$-(m+1) = -(n+1)$$

$$m+1 = n+1$$

$$\boxed{m = n}$$

$\therefore$ f is one to one

40) prove $1+2^n<3^n$, for all $n>2$

Basic: $n=3$

$L.H.S = 1+2^3$
$= 9$

$R.H.S = 3^3$
$= 27$

$L.H.S < R.H.S$ that is $1+2^n<3^n$ for $n=3$

Induction Hypothesis: let $k \in N$ Assume That for some k, $1+2^k<3^n$ where $n=k$

Induction Step: prove for some $k+1 \in N$, $1+2^{k+1}<3^{k+1}$ where $n=k+1$

$L.H.S = 1+2^{k+1}$

add & subtract 1

$L.H.S = 2+2^{k+1}-1$

$= 2(1+2^k)-1$ ~~divide by 2~~ ~~for both L.H.S & R.H.S~~

$R.H.S = 3.3^k$

$\therefore$ from induction hypothesis

$1+2^k<3^k$

multiply 2 on bothsides

$2(1+2^k)<2.3^k$

$\therefore$ from the abovestep

$2(1+2^k)-1<3.3^k$ from Induction hypothesis

(4) let $X = \{n^3 + 3n^2 + 3n \mid n \geq 0\}$
$Y = \{n^3 - 1 \mid n > 0\}$
prove that $X = Y$

Answer:

prove $X \subseteq Y$ & $Y \subseteq X$ (i.e $X = Y$)

let $k \in N$

$X = n^3 + 3n^2 + 3n$

add and subtract 1

$X = n^3 + 3n^2 + 3n + 1 - 1$

$X = (n+1)^3 - 1$

$\therefore n = k$

$\boxed{X = (k+1)^3 - 1}$ — (1)

let $k+1 \in N$

$\cancel{Y = (k+1)^3 - 1}$, $Y = n^3 - 1$

$\boxed{Y = (k+1)^3 - 1}$ where $n = k+1$

— (2)

$\therefore$ from (1) & (2)

$X \subseteq Y$ & $Y \subseteq X$ it implies $X = Y$ for ~~all n~~ some k

(15) a binary relation $\equiv$ is defined on ordered pairs of natural numbers as follows $[m,n] \equiv [j,k]$ if, and only $m+k=n+j$. prove that $\equiv$ is an equivalence relation in $N \times N$

Ans: To prove it we have to prove the it satisfies Reflexivity, Symmetry, Transitivity.

(i) Reflexivity: let $(m,n) \in N \times N$

$\therefore (m,n)$ is an ordered pair in $N \times N$

$\therefore$ prove that $[m,n] \equiv [m,n]$

$$\underset{m}{m} + \underset{n}{n}, \quad \underset{j}{m}, \underset{k}{n}$$

$$m+n = n+m$$

$\therefore$ which is true for $m+n$ (or) $n+m$

(ii) Symmetry: let (m,n) & $(j,k) \in N \times N$

& (m,n), (j,k) be ordered pair's in $N \times N$

to prove it is symmetric we have to show

$[m,n] \equiv [j,k]$ &

$[j,k] \equiv [m,n]$

That is $m+k = n+j$ & $j+n = k+m$

$\therefore$ symmetric property holds

(iii) Transitivity:

let $(m,n) \in N \times N$
$(j,k) \in N \times N$
$(p,q) \in N \times N$ and are ordered pairs

To prove Transitivity holds we have to show that $[m,n] \equiv [j,k]$, $[j,k] \equiv [p,q]$
~~and~~ i.e $[m,n] \equiv [p,q]$

$[m,n] \equiv [j,k]$

$m+k = n+j$

from symmetricity

$[j,k] \equiv [p,q]$

$j+q = k+p$

from symmetricity.

add above both properties

$$m+k+j+q = n+j+k+p$$
$$m+\cancel{k}+\cancel{j}-\cancel{j}-\cancel{k}+q = n+p$$
$$\boxed{m+q = n+p}$$

$\therefore$ transitivity holds ~~for both~~

$\therefore$ the relation satifies for ordered pairs ~~on~~ reflexivity, symmetry, transitivity is an equivalence relation.