THEORY FOUNDATIONS, CS-5800 Exam 1

Select 4 questions.

Your name:

1. (10 points) Define (formally):
   (i) the multiplication of natural numbers, using s (successor function) and addition (recursive definition)
   (ii) for sets X and Y, when is $card(X) = card(Y)$ (also denoted $|X| = |Y|$)?
   (iii) a *regular expression* (recursive definition)
   (iv) a *context-free grammar*
   (v) the *Chomsky normal form* of a context-free grammar G

2. (10 points) Prove that $(w^R)^R = w$ for all strings $w \in \Sigma^*$, using induction on the length of the string.

3. (10 points) Give a regular expression for
   (i) the set of the strings of a's and b's that have two or more a's
   (ii) the set of the strings of a's and b's that have an even number of a's
   (iii) the set of the strings of a's and b's that have exactly two b's and start with a

4. (10 points) Answer the following T/F questions:
   (i) $(\mathbf{a} \cup \mathbf{b})^* = \mathbf{a}^* \cup \mathbf{b}^*$ (T/F)
   (ii) $\mathbf{a}^* \cap \mathbf{b}^* = \emptyset$ (T/F)
   (iii) $\mathbf{a}^*(\mathbf{a}^* \cup \mathbf{b}^*) = \{a, b\}^*\mathbf{a}^*$ (T/F)
   (iv) $\emptyset^* = \emptyset$ (T/F)

5. (10 points) For the grammar G with production rules
$$S \to aSa \mid bSb \mid a \mid b \mid \lambda$$
   (i) What type of grammar is G in the Chomsky hierarchy of languages and grammars?
   (ii) Give a leftmost derivation of the string $babbab$. Show both your derivation and the corresponding syntax tree.
   (iii) Give a characterization of the strings in words (English).

6. (10 points) For the NFA M with the following transition table:

δ	a	b
$\to * q_0$	$\{q_2\}$	$\{q_0, q_1\}$
q_1	$\{q_2\}$	$\{q_1\}$
$* \ q_2$	$\{q_1\}$	$\{q_0\}$

   (i) (3 points) Give a computation on the string $w = abbaaba$ in M. Is w accepted?
   (ii) (7 points) Construct an equivalent DFA M.