A. (2) $(w^R)^i = (w^i)^R$ prove for $i \geq 0$ using induction

Basic $(i=0)$:

$(w^R)^0 = \lambda$

$(w^0)^R = \lambda^R = \lambda$

Induction hypothesis: Assume for some $k \in N$ & $k < i$

$(w^R)^k = (w^k)^R$

Induction step:

prove for some $k+1 \in N$

$(w^R)^{k+1} = (w^{k+1})^R$

$$(w^R)^{k+1} = (w^R)^k \cdot (w^R)^1$$

$$= (w^k)^R \cdot w^R$$

$$= (w \cdot w^k)^R = (w^{k+1})^R$$

$\Downarrow$

using $(uv)^R = v^R u^R$

$\therefore$ $(w^R)^i = (w^i)^R$ holds for anystring w & all $i \geq 0$

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a

$L_0 = \{b\}$

$L_1 = \{bb, bab, bba\}$

$L_2 = \{bbb, bbab, bbba, babb, babab, babba, bbaba, bbab, bbaab, bbaba, bbbaa\}$

b the string bbaaba doesn't belong's to L because as given in recursive step the no of a's & b's in string which are equal cannot be generated by recursive step from base $\{b\}$

c

~~Same~~ the string bbaaaa bb is not in L because as defined in (a) It can't generate strings having ~~equal~~ even no of a's & b's (or) equal no of a's & equal no of b's

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$L_0 = \{\lambda\}$

$L_1 = \{aab\}$

$L_2 = \{aaaabb\}$

$L_3 = \{aaaaaabbb\}$

b

(b) the set of strings in language L generated by recursive definition have twice no. of a's than no of b's in string

$w, w \in L$

$w \in L$

$\boxed{n_a(w) = 2 \cdot n_b(w)}$ $w \in L$

(c) Basic: if let $w \in L$ and $w = \lambda$

then

$n_b(w) = 0$

$n_a(w) = 2 * 0 = 0$

Induction hypothesis: Assue that $n_a(k) = 2 * n_b(k)$ for some $k \in L$

Induction step: prove the for string $aakb$ that $n_a(aakb) = 2 \cdot n_b(aak_b)$ for some $aakb \in L$

$$n_a(aakb) = |aa| + n_a(k)$$

$$= 2 + n_a(k) \quad \text{— (1)}$$

$$= 2 + 2 \cdot n_b(k)$$

$$= 2(1 + n_b(k)) = 2 n_b(aakb)$$

from (2)

$$n_b(aak_b) = [n_b(k) + 1]$$

$$= n_b(k) + 1$$

$$= n_b(k) + 1 \quad \text{— (2)}$$

∴ It is clear from (1) & (2) the no of a's got added twice & no of b's added is one.

A: (18) $\Sigma = \{a, b, c\}$

regular expression of length greater than three

$\Rightarrow \{a \cup b \cup c\}^{+} \{a \cup b \cup c\}^{+} \{a \cup b \cup c\}^{+}$

$\{a \cup b \cup c\}^{+}$

$\Rightarrow \{a \cup b \cup c\}^{+} = \{a \cup b \cup c\}^{+} \cdot \{a \cup b \cup c\}^{+} \{a \cup b \cup c\}^{+}$

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$(a \cup b)^{?} b (a \cup b)^*$

$= \{a \cup b\}^{?} \{b\} \{a \cup b\}^*$

39 d

$$(a \cup b)^* = (a^* \cup ba^*)^*$$

$$(a \cup b)^* \xRightarrow{12.3} (a \cup ba^*)^*$$

$$\xRightarrow{12.1} (a^* \cup ba^*)^*$$

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$(([0-9]+[\ ][A-Z][a-z]+[\ ][A-Z][a-z]+[.])$

$\cup ([0-9]+[\ ][A-Z][a-z]+[\ ][\text{Street/Avenue/Rd}]$

$[.]))$

(B) ~~7~~, 6 ~~(d)~~ (11), ~~14 (d)~~, ~~17~~, 29, 33 ~~(a)~~, ~~38~~

~~1-8 (a-14-0)~~

(2) $S \to ASB \mid \lambda$

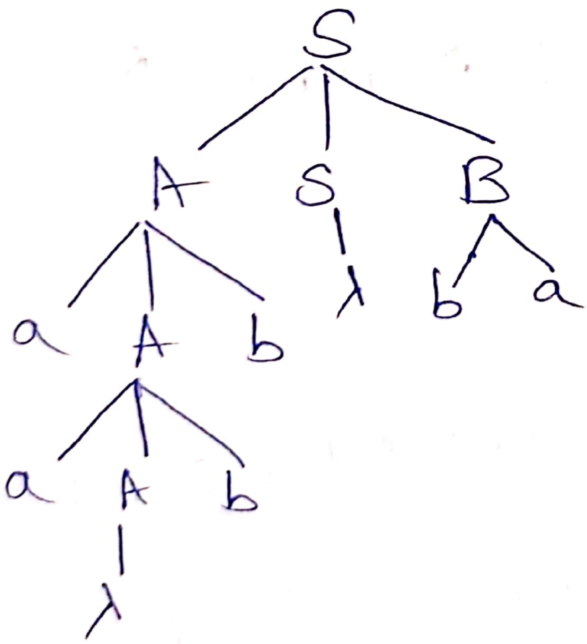
$A \to aAb \mid \lambda$

$B \to bBa \mid ba$

(a)

$S \Rightarrow ASB$	$S \to ASB$
$\Rightarrow \underline{aAb}SB$	$A \to aAb$
$\Rightarrow a\underline{aAb}bSB$	$A \to aAb$
$\Rightarrow aabbSB$	$A \to \lambda$
$\Rightarrow aabbB$	$S \to \lambda$
$\Rightarrow aabb\underline{ba}$	$B \to ba$

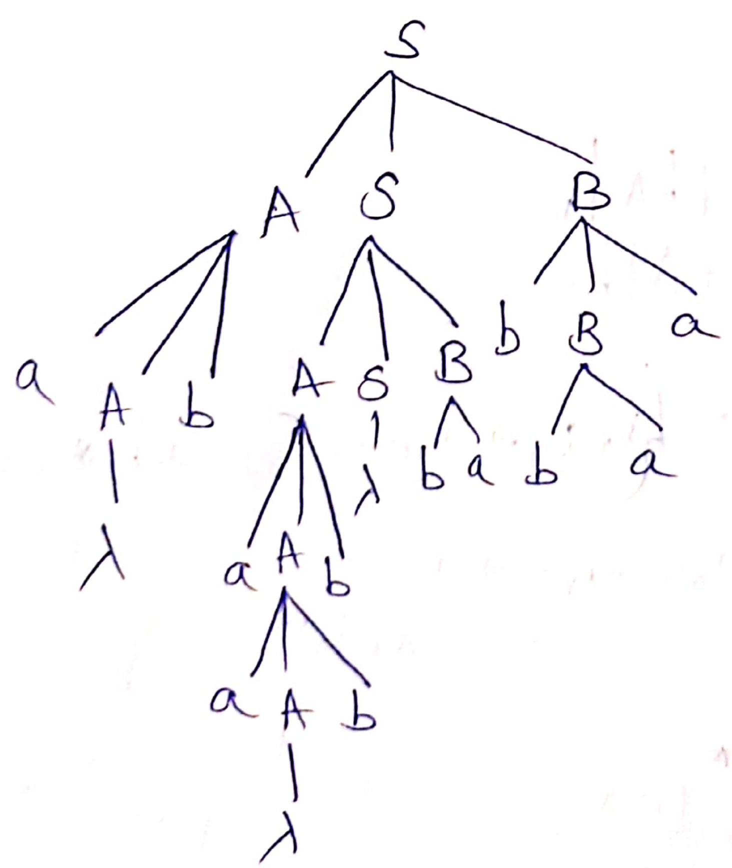
(c) (a) derivation tree for (a)



(b)

$S \Rightarrow ASB$	
$\Rightarrow ASbBa$	$B \to bBa$
$\Rightarrow ASbbaa$	$B \to ba$
$\Rightarrow AASBbbaa$	$S \to ASB$
$\Rightarrow AASbabbaa$	$B \to ba$
$\Rightarrow AAbabbaa$	$S \to \lambda$
$\Rightarrow AaAbbabbaa$	$A \to aAb$
$\Rightarrow AaaAbbbabbaa$	$A \to aAb$
$\Rightarrow Aaabbbabbaa$	$A \to \lambda$
$\Rightarrow aAbaabbbabbaa$	$A \to aAb$
$\Rightarrow abaabbbabbaa$	$A \to \lambda$

(C).(b) derivation tree for above problem (b)



(d) $L(G) = \{a^m b^m b^n a^n \mid m,n \geq 0\}$

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(11)

$S \to ABA$

$A \to aA \mid Aa \mid \lambda$

$B \to bB \mid b \mid \lambda$

B 6 d

$S \rightarrow aSb \mid A$

$A \rightarrow cAd \mid cBd$

$B \rightarrow aBb \mid ab$

$L = \{ a^i c^j a^k b^k d^j b^i \mid k, j \geq 1, i \geq 0 \}$

$$\boxed{L = \{ a^i c^j a^k b^k d^j b^i \mid k, j \geq 1, i \geq 0 \}}$$

B 14 d

$S \rightarrow aS \mid bA \mid \lambda$

$A \rightarrow aA \mid bS$

$L(G) = \{ \lambda, a, aa, aaa, \ldots, ab(a \cup b)^* \}$

regular expression

$= (a \cup b)^*$

B 17

$(\{a \cup b\}^* \{aa\} \{a \cup b\}^* \{aa\} \{a \cup b\}^*)$

(29)

$S \Rightarrow aSaa \mid B$

$B \rightarrow bbBdd \mid C$

$C \rightarrow bd$

$L = \{ a^n b^m d^m a^{2n} \mid n \geq 0, m \geq 1 \}$

Basic :-

(B)

(33) (a) $S \to aaS \mid aaaaaS \mid \lambda$

regular expression

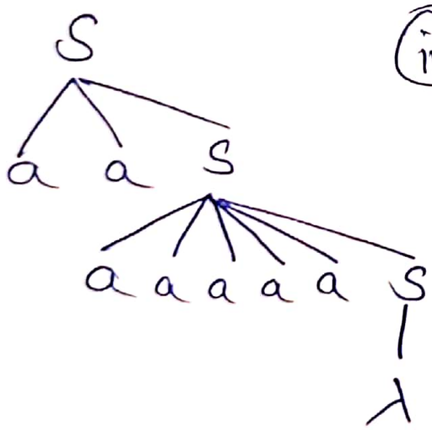
$S = \{\lambda, aa, aaaa, aaaaa, aaaaaa, aaaaaaa$

$S = \{a^i \mid i \geq 2 \ \& \ i \neq 3\}$

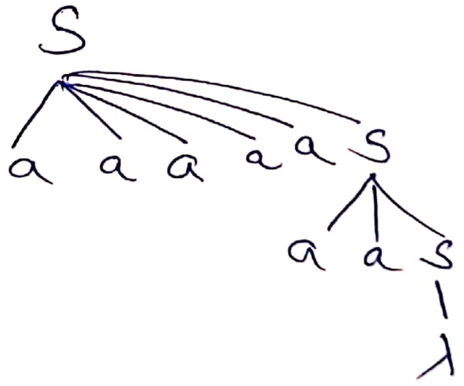
$S = \{a^i - \{aaa\} \mid i \geq 2 \ \& \ i \neq 3\}$

Consider string aaaaaaa

(i)

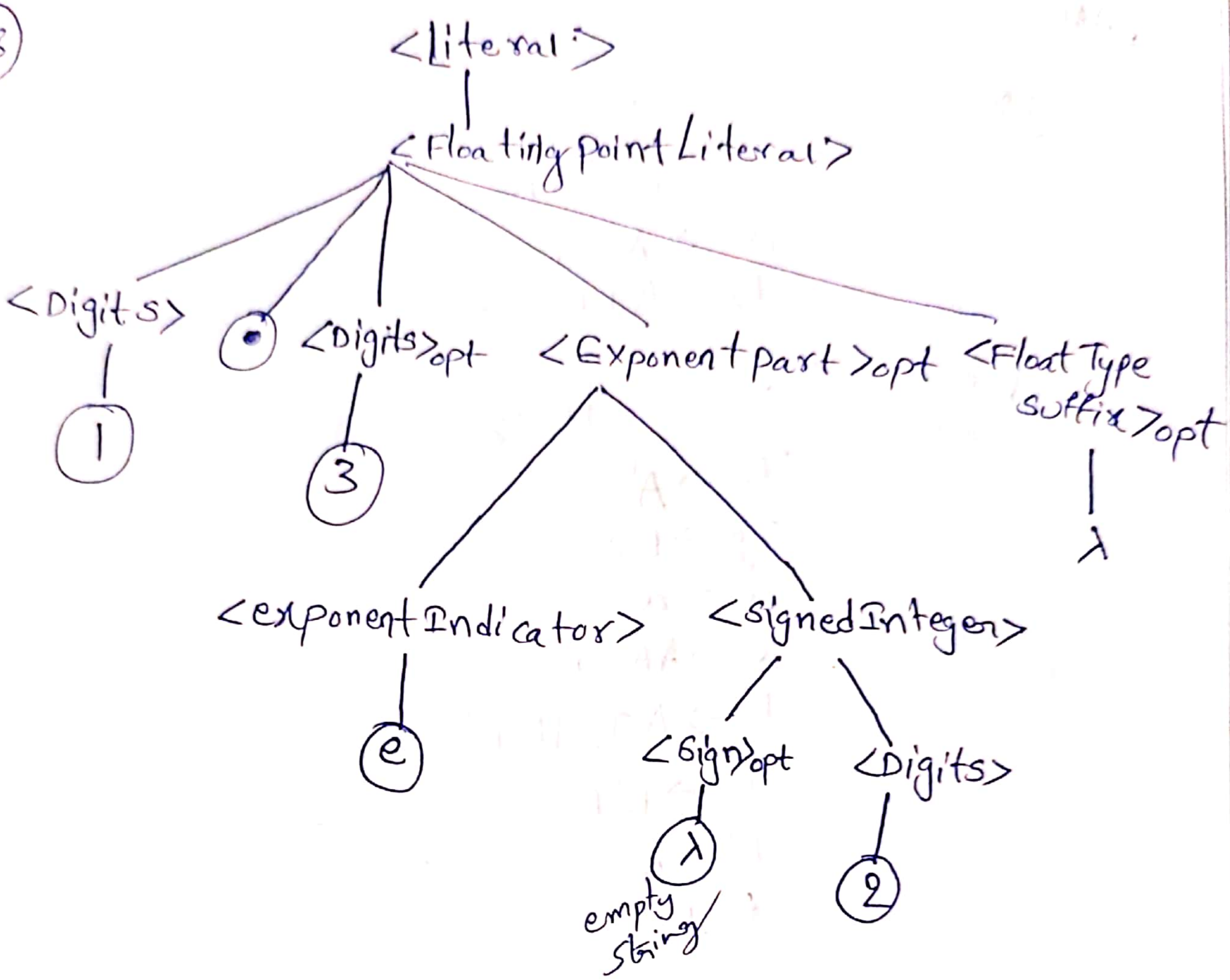


(ii)



$\therefore$ from (i) & (ii) there are two left most derivation's tree's for same string so the grammar is ambiguous

B. 38



∴ Concatenate all leaf nodes except λ

$\boxed{1.3e2}$ is the required string

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18 chomsky Normal form

$G: S \to aA \mid ABa$

$A \to AA \mid a$

$B \to AbB \mid bb$

chomsky Normal form

$S \to A'A \mid AT_2$

$T_2 \to BA'$

$A' \to a$

$A \to AA \mid a$

$B \to AT_3 \mid B'B'$

$T_3 \to B'B$

$B' \to b$