Assignment 0

1. a $S_n = \frac{n(a_1 + a_n)}{2}$

$$\frac{512 \times (2 + 1024)}{2} = 262{,}656$$

b. $S_n = \frac{a_0(1 - q^n)}{1 - q}$

$$= \frac{3(1 - 3^{20})}{1 - 3}$$

c. $n + 3 - 5 + 1 = n - 1$

d. $5 + 6 + 7 \cdots\cdots n + 3$

$$S_n = \frac{(n+3-5+1)(5+n+3)}{2}$$

$$= \frac{(n-1)(n+8)}{2}$$

e. $2^5 + 2^6 + \cdots 2^{n+2}$

$$S_n = \frac{2^5(1-2^{n+2-5+1})}{1-2}$$

$$= \frac{2^5(1-2^{n-2})}{1-2} = 2^5(2^{n-2}-1)$$

$$= 2^{n+3} - 2^5$$

f. $\sum_{i=2}^{n}(i+1)\sum_{j=3}^{n}(j+2)$

$$\sum_{i=2}^{n} \sum_{j=3}^{n} (ij + 2i + j + 2)$$

$$\sum_{i=2}^{n} \sum_{j=3}^{n} ij$$

$$= \sum_{i=2}^{n} i \sum_{j=3}^{n} j$$

$$= \frac{(n-2+1)(n+2)}{2} \cdot \frac{(n-3+1)(n+3)}{2} \quad ①$$

$$\sum_{i=2}^{n} \sum_{j=3}^{n} 2i$$

$$(n-2+1) \sum_{i=2}^{n} 2i = (n-1) 2 \cdot \sum_{i=2}^{n} i = (n-1) \cdot 2 \cdot \frac{(n-2+1)(2+n)}{2}$$

$$(n-1)^2 (n+2) \quad ②$$

$$\sum_{i=2}^{n} \sum_{j=3}^{n} j = (n-1) \frac{(n-3+1)(3+n)}{2} \quad ③$$

$$= \frac{(n-2)(n+3)(n-1)}{2}$$

final:

$$\sum_{i=2}^{n} \sum_{j=3}^{n} 2 = (n-1) \cdot 2 \cdot (n-2) \quad ④$$

$$= 2(n-1)(n-2)$$

① + ② + ③ + ④

2. a

$$(i^3+3)^3 = (i^3+3)(i^6+6i^3+9)$$

$$= i^9+6i^6+9i^3+3i^6+18i^3+27$$

$$= i^9+9i^6+27i^3+27$$

$$\sum_3^{n-3}(i^9+9i^6+27i^3+27)$$

$$\in \Theta(n \cdot n^9) \quad \text{i.e.} \quad \Theta(n^{10})$$

b. $$\sum_{2023}^{n} \lg i^{1024}$$

$$= 1024 \sum_{2023}^{n} \lg i \quad \in \Theta(n \lg n)$$