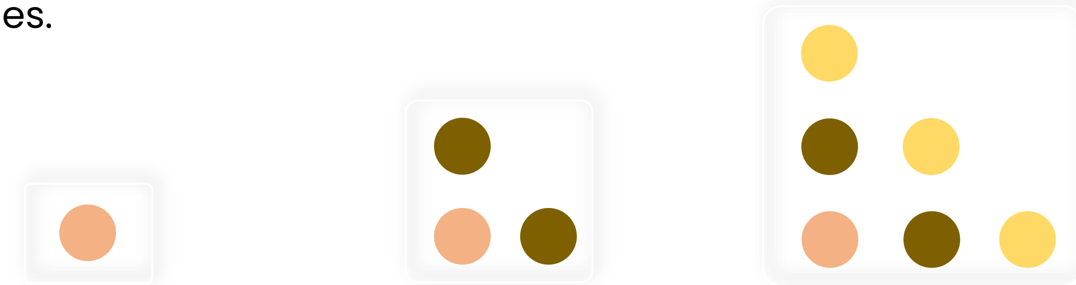
Squares and Square Roots

Triangular Numbers:

A triangular number is the one whose dot pattern can be arranged as triangles.



1 $\qquad$ $1 + 2 = 3$ $\qquad$ $1 + 2 + 3 = 6$

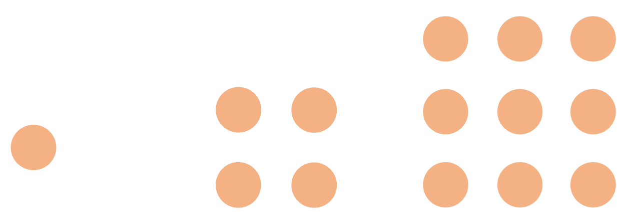
Here, $1, 3$ and 6 are triangular numbers.

Square Numbers:

Square numbers are obtained when a number is multiplied to itself.

$$a \times a = a^2$$
$$3 \times 3 = 3^2$$

1^2 $\qquad$ 2^2 $\qquad$ 3^2



The square of an odd number is always an odd number, and the square of an even number is always an even number.

The numbers $1, 4, 9, 16, \ldots$ are square numbers, also called as perfect squares.

Properties of Square Numbers:

Unit digit of a number	Unit digit of its square
0	0
3	9
5	5
2 or 8	4
4 or 6	6
1 or 9	1

Numbers between Square Numbers:

Between n^2 and $(n+1)^2$, there are $2n$ non square numbers.

Example:

Between 9 and 16 there lies 6 numbers.
$9 = 3^2$ and $16 = 4^2$.
So, between 3^2 and 4^2 there lies 2 × 3 = 6 numbers.

$$\underset{3^2}{9} \quad \boxed{10 \quad 11 \quad 12 \quad 13 \quad 14 \quad 15} \quad \underset{4^2}{16}$$

Adding Consecutive Odd Numbers:

The sum of first $\boldsymbol{n}$ odd natural numbers is $\boldsymbol{n^2}$.

$$\begin{aligned}
1 &= 1 = 1^2 \\
1+3 &= 4 = 2^2 \\
1+3+5 &= 9 = 3^2 \\
1+3+5+7 &= 16 = 4^2 \\
1+3+5+7+9 &= 25 = 5^2 \\
1+3+5+7+9+11 &= 36 = 6^2
\end{aligned}$$

If a natural number cannot be expressed as a sum of successive odd natural numbers starting with 1, then it is not a perfect square.

Some More Patterns:

Some more interesting patterns related to squares of numbers are as follows:

$$\begin{aligned}
1^2 &= 1 \\
11^2 &= 1\;2\;1 \\
111^2 &= 1\;2\;3\;2\;1 \\
1111^2 &= 1\;2\;3\;4\;3\;2\;1 \\
11111^2 &= 1\;2\;3\;4\;5\;4\;3\;2\;1
\end{aligned}$$

$$\begin{aligned}
7^2 &= 49 \\
67^2 &= 4489 \\
667^2 &= 444889 \\
6667^2 &= 44448889 \\
66667^2 &= 4444488889
\end{aligned}$$

Finding Square Roots:

1. Using inverse operation of squaring
2. Using repeated subtraction
3. Using prime factorisation
4. Using long division method

1. Using Inverse Operation of Squaring:

- Finding a square root is the inverse operation of squaring.
- The positive square root of a number is denoted by the symbol $\sqrt{\ }$.

$$2^2 = 2 \times 2 = 4 \quad \Rightarrow \quad \sqrt{4} = 2$$

$$3^2 = 3 \times 3 = 9 \quad \Rightarrow \quad \sqrt{9} = 3$$

$$4^2 = 4 \times 4 = 16 \quad \Rightarrow \quad \sqrt{16} = 4$$

$$5^2 = 5 \times 5 = 25 \quad \Rightarrow \quad \sqrt{25} = 5$$

If a perfect square is of n-digits then, its square root will have:

- $\frac{n}{2}$ digits if n is even.
- $\left(\frac{n+1}{2}\right)$ digits if n is odd.

2. Using Repeated Subtraction:

Square root of a perfect square can be found by repeatedly subtracting consecutive odd numbers till we get 0. The number of odd numbers subtracted is the square root of the given number.

Example: $\sqrt{25} = 5$

$25 - 1 = 24$ —— 1
$24 - 3 = 21$ —— 2
$21 - 5 = 16$ —— 3
$16 - 7 = 9$ —— 4
$9 - 9 = 0$ —— 5

3. Using Prime Factorisation:

- Express the number as the product of its prime factors.
  Example: $144 = 2 \times 2 \times 2 \times 2 \times 3 \times 3$

- Pair these prime factors such that both the numbers in a pair are the same.
  $144 = (2 \times 2) \times (2 \times 2) \times (3 \times 3)$

- Take one number from each pair and multiply them to get the square root.
  $\sqrt{144} = 2 \times 2 \times 3 = 12$

$$\begin{array}{r|l} 2 & 144 \\ \hline 2 & 72 \\ \hline 2 & 36 \\ \hline 2 & 18 \\ \hline 3 & 9 \\ \hline 3 & 3 \\ \hline & 1 \end{array}$$

4. Using Long Division Method:

Square root of 529 using long division.

i. Divide the number into groups of 2, starting from the ones place.

$$529 \Rightarrow \overline{5}\ \overline{29}$$

ii. Find the largest number whose square is less than or equal to the number under the extreme left bar. Take this number as the divisor and the quotient with the number under the extreme left bar as the dividend. Divide and get the remainder.

Quotient

Divisor

$$\begin{array}{r|l} & 2 \\ \hline 2 & \overline{5}\ \ \overline{29} \\ & -4 \end{array}$$

4. Using Long Division Method:

iii. Bring down the number under the next bar to the right of the remainder. So, the new dividend is 129.

$$
\begin{array}{r|l}
 & \quad 2 \\ \hline
2 & \overline{5}\ \ \overline{29} \\
 & -4 \\ \hline
 & \ \ 1\ \ 29
\end{array}
$$

iv. Double the quotient and enter it with a blank on its right.

$$
\begin{array}{r|l}
 & \quad 2 \\ \hline
2 & \overline{5}\ \ \overline{29} \\
 & -4 \\ \hline
4\,\_\_ & \ \ 1\ \ 29
\end{array}
$$

v. Guess a largest possible digit to fill the blank which will also become the new digit in the quotient, such that when the new divisor is multiplied to the new quotient the product is less than or equal to the dividend.

$$
\begin{array}{r|l}
 & \quad 2\ 3 \\ \hline
2 & \overline{5}\ \ \overline{29} \\
 & -4 \\ \hline
4\ 3 & 129 \\
 & 129 \\ \hline
 & \ \ 0
\end{array}
$$

vi. Since the remainder is 0 and no digits are left in the given number, therefore,

$$\sqrt{529} = 23$$

Q. What will be the unit digit of square of 669?

A. The unit digit of the square of a number ending with 9 is always 1.

Hence, the unit digit of square of 669 is 1.

Q. Is 1473 a perfect square? Give reasons.

A. The square of any natural number does not end with $2, 3, 7$ or 8.

Hence, 1473 is not a perfect square.

Q. Without adding, find the sum of $1 + 3 + 5 + 7 + 9$.

A. We know that the sum of first n odd natural number is n^2.
$1 + 3 + 5 + 7 + 9$ is the sum of first 5 odd natural numbers.
Hence, $1 + 3 + 5 + 7 + 9 = 5^2 = 25$

Q. Find the number of digits in the square root of 65025.

A. If a perfect square is of n-digits then, its square root will have:

$\frac{n}{2}$ digits if n is even.

$\left(\frac{n+1}{2}\right)$ if n is odd.

Here, 65025 has 5 digits. Hence, the number of digits in the square root of 65025 is 3.

Q. The square root of 169 can be confirmed after repeated subtraction of the starting ________ odd numbers.

A.

The $\sqrt{169}$ can be obtained by:

$169 - 1 = 168$
$168 - 3 = 165$
$165 - 5 = 160$
$160 - 7 = 153$
$153 - 9 = 144$
$144 - 11 = 133$
$133 - 13 = 120$
$120 - 15 = 105$
$105 - 17 = 88$
$88 - 19 = 69$
$69 - 21 = 48$
$48 - 23 = 25$
$25 - 25 = 0$

By subtracting the first 13 consecutive odd numbers from 169, zero is obtained.
Hence, $\sqrt{169} = 13$

Q. The smallest prime number to be multiplied with 32 to obtain a perfect square is ________.

A.

Here, $32 = 2 \times 2 \times 2 \times 2 \times 2$
$32 = 2^2 \times 2^2 \times 2$

Here, multiplying 32 by 2 will complete the pair for the last 2 and resulting number will be the perfect square.
$2^2 \times 2^2 \times 2 \times 2 = 64$
64 is a perfect square.

The smallest prime number to be multiplied with 32 to obtain a perfect square is 2.

2	32
2	16
2	8
2	4
2	2
	1