Syntax Analysis

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Outline

- Grammars, derivations, and parse trees
- Introduction to bottom-up parsing
- Shift reduce parsing
- SLR(1) parsing
- Conceptual issues in LR parsing
- CLR(1) parsing
- LALR(1) parsing

Grammars, derivations, and parse trees

Introduction to Parsing

parsing-slides-sanyal-part1.pdf

Shift Reduce Parsing

Shift Reduce Parsing

parsing-slides-sanyal-part2.pdf

Shift Reduce Parsing Example

$E \rightarrow E + T$
$E \rightarrow T$
$T \rightarrow T * F$
$T \rightarrow F$
$F \rightarrow (E)$
$F \rightarrow \text{id}$

Step	Stack $\rightarrow$	Remaining Input	Action

Shift Reduce Parsing Example

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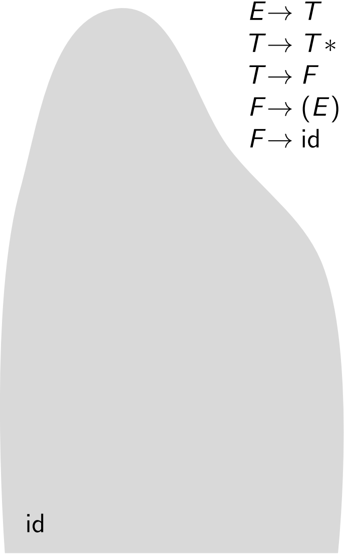
Step	Stack $\rightarrow$	Remaining Input	Action
1	\$	id $*$ (id $+$ id)\$	shift id

Shift Reduce Parsing Example



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Step	Stack $\rightarrow$	Remaining Input	Action
1	\$	$\text{id} * (\text{id} + \text{id})\$$	shift id
2	\$id	$* (\text{id} + \text{id})\$$	reduce by $F \rightarrow \text{id}$

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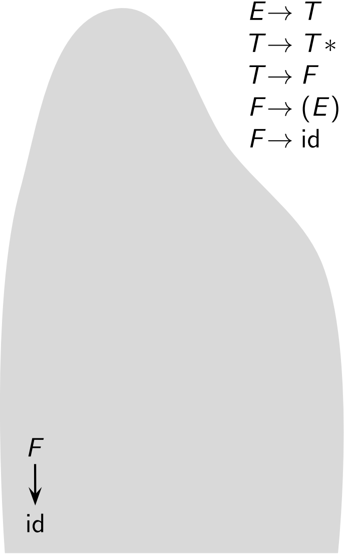
Step	Stack →	Remaining Input	Action
1	\$	id $*$ (id + id)\$	shift id
2	\$id	$*$ (id + id)\$	reduce by $F \rightarrow \text{id}$
3	\$$F$	$*$ (id + id)\$	reduce by $T \rightarrow F$



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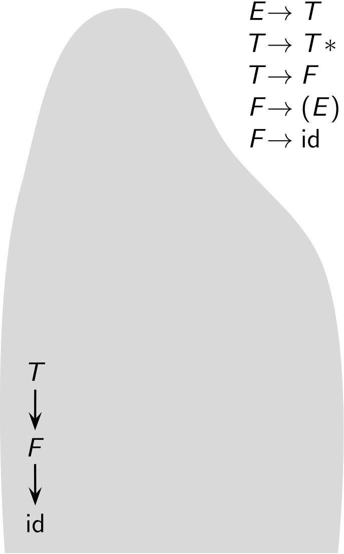

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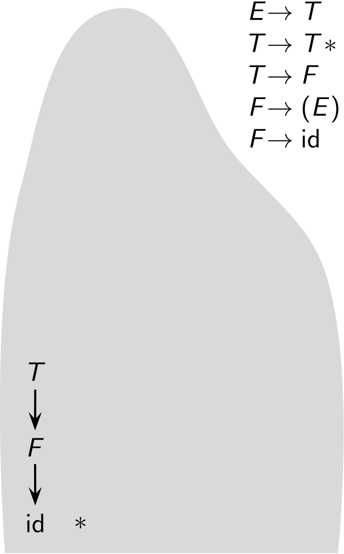

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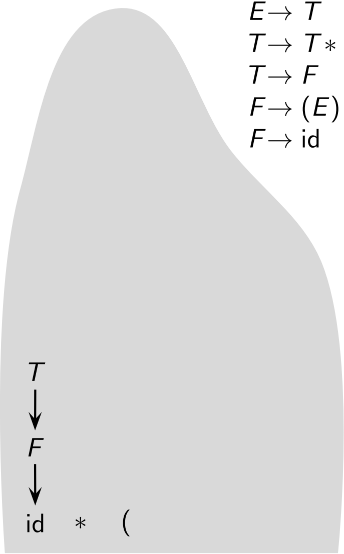
6	$\$T * ($	$\text{id} + \text{id})\$$	shift id



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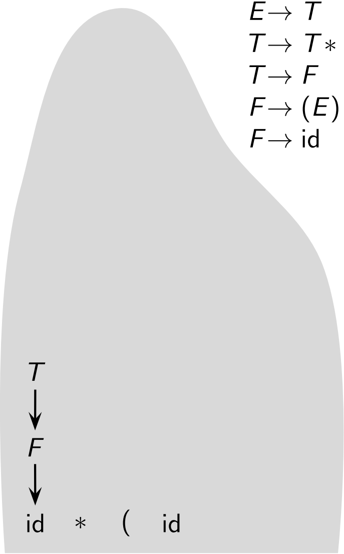

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7	\$$T *$ (id	+ id)\$	reduce by $F \rightarrow$ id

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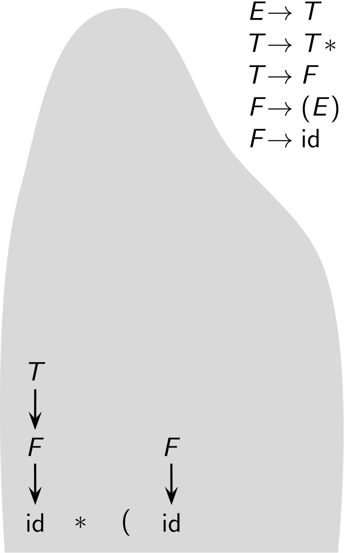

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5	\$$T$ $*$	(id $+$ id)\$	shift (
6	\$$T$ $*$ (	id $+$ id)\$	shift id
7	\$$T$ $*$ (id	$+$ id)\$	reduce by $F \rightarrow$ id
8	\$$T$ $*$ (F	$+$ id)\$	reduce by $T \rightarrow F$

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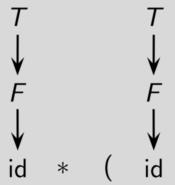

Observations

- A shift corresponds to creating a leaf node in the parse tree whereas a reduce corresponds to creating an internal node
- In every step i, concatenation of the stack and the remaining input gives a right sentential form (rsf_i)
- For every step i, $\text{rsf}_{i+1} \overset{rm}{\Rightarrow} \text{rsf}_i$
- In every step, the partial parse tree constructed until then consists of a forest of trees
- In every step, the stack holds the root nodes of the trees contained in the forest

  A reduce action may amount to joining some of these trees

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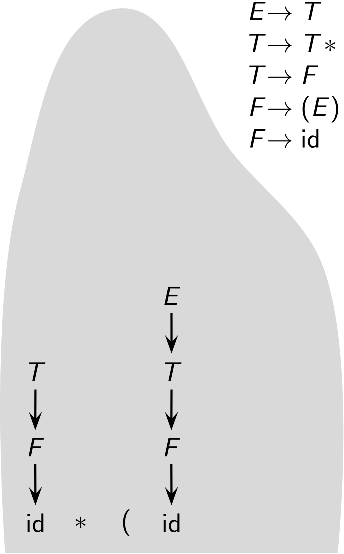

Step	Stack $\rightarrow$	Remaining Input	Action
1	\$	$\text{id} * (\text{id} + \text{id})\$$	shift id
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4	$\$T$	$* (\text{id} + \text{id})\$$	shift $*$
5	$\$T\ *$	$(\text{id} + \text{id})\$$	shift (
6	$\$T * ($	$\text{id} + \text{id})\$$	shift id
7	$\$T * (\text{id}$	$+ \text{id})\$$	reduce by $F \rightarrow \text{id}$
8	$\$T * (F$	$+ \text{id})\$$	reduce by $T \rightarrow F$
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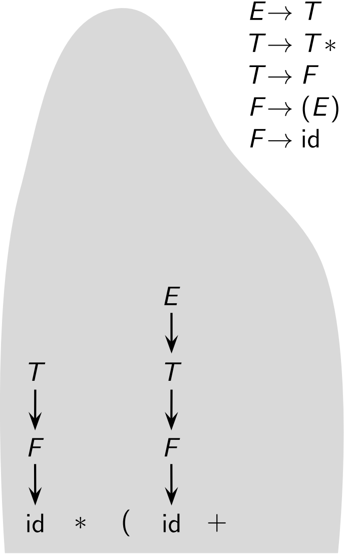

Step	Stack →	Remaining Input	Action
1	\$	id ∗ (id + id)\$	shift id
2	\$id	∗ (id + id)\$	reduce by $F \rightarrow \text{id}$
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4	\$$T$	∗ (id + id)\$	shift ∗
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6	\$$T$ ∗ (	id + id)\$	shift id
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9	\$$T$ ∗ (T	+ id)\$	reduce by $E \rightarrow T$
10	\$$T$ ∗ (E	+ id)\$	shift +

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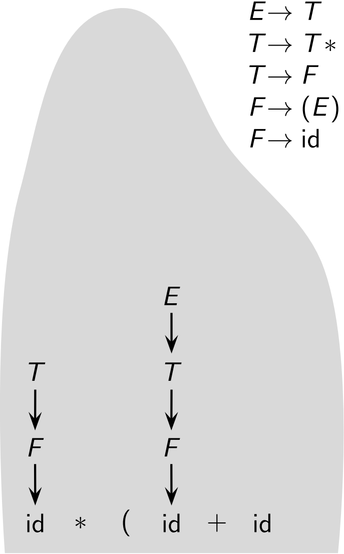

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6	$\$T * ($	$\text{id} + \text{id})\$$	shift id
7	$\$T * (\text{id}$	$+ \text{id})\$$	reduce by $F \rightarrow \text{id}$
8	$\$T * (F$	$+ \text{id})\$$	reduce by $T \rightarrow F$
9	$\$T * (T$	$+ \text{id})\$$	reduce by $E \rightarrow T$
10	$\$T * (E$	$+ \text{id})\$$	shift $+$
11	$\$T * (E +$	$\text{id})\$$	shift id

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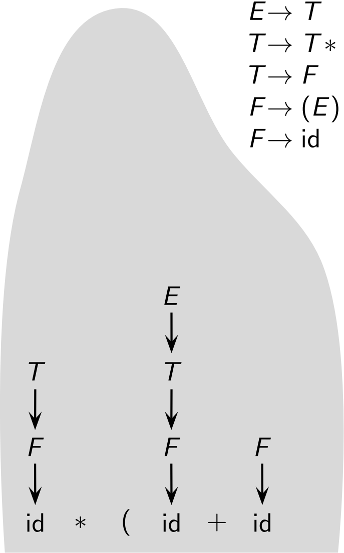

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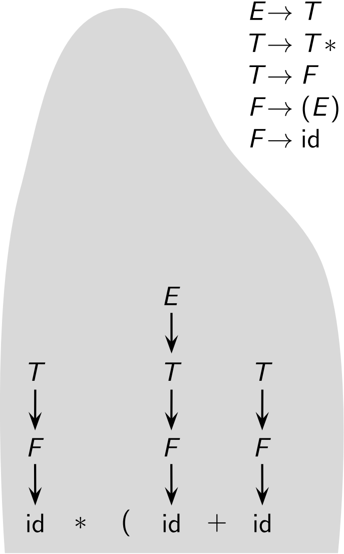

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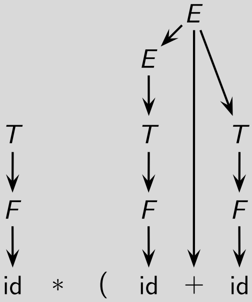

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6	$\$T * ($	$\text{id} + \text{id})\$$	shift id
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11	$\$T * (E +$	$\text{id})\$$	shift id
12	$\$T * (E + \text{id}$	$)\$$	reduce by $F \rightarrow \text{id}$
13	$\$T * (E + F$	$)\$$	reduce by $T \rightarrow F$
14	$\$T * (E + T$	$)\$$	reduce by $E \rightarrow E + T$

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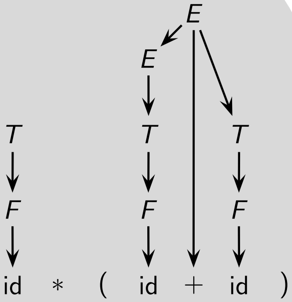

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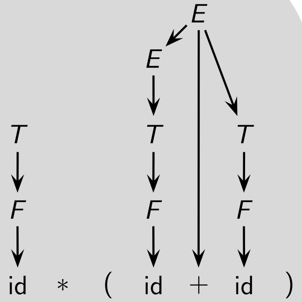

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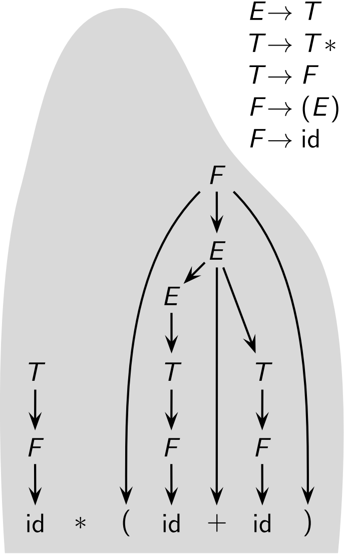

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16	$\$T * (E)$	$\$$	reduce by $F \rightarrow (E)$
17	$\$T * F$	$\$$	reduce by $T \rightarrow T * F$

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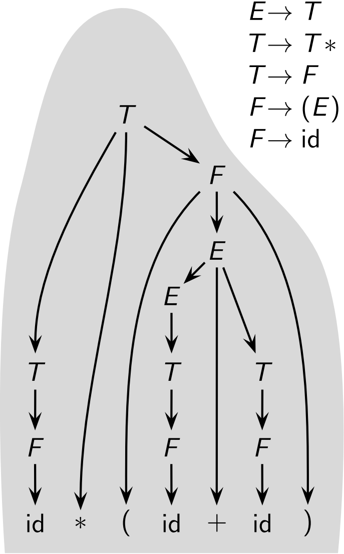

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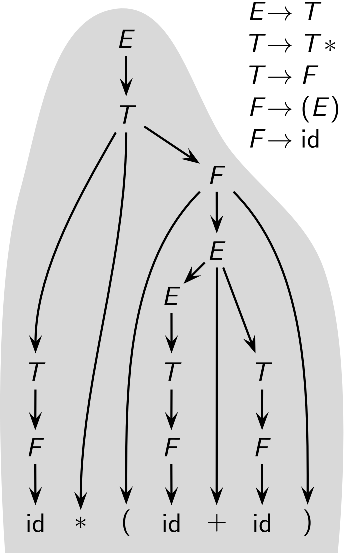
10	$\$T * (E$	$+ \mathrm{id})\$$	shift $+$
11	$\$T * (E +$	$\mathrm{id})\$$	shift id
12	$\$T * (E + \mathrm{id}$	$)\$$	reduce by $F \rightarrow \mathrm{id}$
13	$\$T * (E + F$	$)\$$	reduce by $T \rightarrow F$
14	$\$T * (E + T$	$)\$$	reduce by $E \rightarrow E + T$
15	$\$T * (E$	$)\$$	shift)
16	$\$T * (E)$	$\$$	reduce by $F \rightarrow (E)$
17	$\$T * F$	$\$$	reduce by $T \rightarrow T * F$
18	$\$T$	$\$$	reduce by $E \rightarrow T$

Shift Reduce Parsing Example

$E \rightarrow E + T$
$E \rightarrow T$
$T \rightarrow T * F$
$T \rightarrow F$
$F \rightarrow (E)$
$F \rightarrow \text{id}$



Step	Stack $\rightarrow$	Remaining Input	Action
1	\$	$\text{id} * (\text{id} + \text{id})\$$	shift id
2	\$id	$* (\text{id} + \text{id})\$$	reduce by $F \rightarrow \text{id}$
3	$\$F$	$* (\text{id} + \text{id})\$$	reduce by $T \rightarrow F$
4	$\$T$	$* (\text{id} + \text{id})\$$	shift $*$
5	$\$T *$	$(\text{id} + \text{id})\$$	shift (
6	$\$T * ($	$\text{id} + \text{id})\$$	shift id
7	$\$T * (\text{id}$	$+ \text{id})\$$	reduce by $F \rightarrow \text{id}$
8	$\$T * (F$	$+ \text{id})\$$	reduce by $T \rightarrow F$
9	$\$T * (T$	$+ \text{id})\$$	reduce by $E \rightarrow T$
10	$\$T * (E$	$+ \text{id})\$$	shift $+$
11	$\$T * (E +$	$\text{id})\$$	shift id
12	$\$T * (E + \text{id}$	$)\$$	reduce by $F \rightarrow \text{id}$
13	$\$T * (E + F$	$)\$$	reduce by $T \rightarrow F$
14	$\$T * (E + T$	$)\$$	reduce by $E \rightarrow E + T$
15	$\$T * (E$	$)\$$	shift)
16	$\$T * (E)$	\$	reduce by $F \rightarrow (E)$
17	$\$T * F$	\$	reduce by $T \rightarrow T * F$
18	$\$T$	\$	reduce by $E \rightarrow T$
19	$\$E$	\$	accept

SLR(1) Parsing Example

1. $E \rightarrow E + T$
2. $E \rightarrow T$
3. $T \rightarrow T * F$
4. $T \rightarrow F$
5. $F \rightarrow (E)$
6. $F \rightarrow \texttt{id}$

Step	Stack →	Remaining Input	Action

SLR(1) Parsing Example

1. $E \rightarrow E + T$
2. $E \rightarrow T$
3. $T \rightarrow T * F$
4. $T \rightarrow F$
5. $F \rightarrow (E)$
6. $F \rightarrow$ `id`

State	Action						Goto		
	`id`	+	*	(	)	$	E	T	F
0	$s5$			$s4$			$c1$	$c2$	$c3$
1		$s6$				acc			
2		$r2$	$s7$		$r2$	$r2$			
3		$r4$	$r4$		$r4$	$r4$			
4	$s5$			$s4$			$c8$	$c2$	$c3$
5		$r6$	$r6$		$r6$	$r6$			
6	$s5$			$s4$				$c9$	$c3$
7	$s5$			$s4$					$c10$
8		$s6$			$s11$				
9		$r1$	$s7$		$r1$	$r1$			
10		$r3$	$r3$		$r3$	$r3$			
11		$r5$	$r5$		$r5$	$r5$			

Step	Stack →	Remaining Input	Action
1	$0	id $*$ (id $+$ id)$	shift 5

SLR(1) Parsing Example

1. $E \rightarrow E + T$
2. $E \rightarrow T$
3. $T \rightarrow T * F$
4. $T \rightarrow F$
5. $F \rightarrow (E)$
6. $F \rightarrow \texttt{id}$

State	Action						Goto		
	`id`	+	*	(	)	$	E	T	F
0	$s5$			$s4$			$c1$	$c2$	$c3$
1		$s6$				acc			
2		$r2$	$s7$		$r2$	$r2$			
3		$r4$	$r4$		$r4$	$r4$			
4	$s5$			$s4$			$c8$	$c2$	$c3$
5		$r6$	$r6$		$r6$	$r6$			
6	$s5$			$s4$				$c9$	$c3$
7	$s5$			$s4$					$c10$
8		$s6$			$s11$				
9		$r1$	$s7$		$r1$	$r1$			
10		$r3$	$r3$		$r3$	$r3$			
11		$r5$	$r5$		$r5$	$r5$			

Step	Stack →	Remaining Input	Action
1	$0	id $*$ (id + id)$	shift 5
2	$0 id 5	$*$ (id + id)$	reduce by 6

SLR(1) Parsing Example

1. $E \rightarrow E + T$ | 2. $E \rightarrow T$
3. $T \rightarrow T * F$ | 4. $T \rightarrow F$
5. $F \rightarrow (E)$ | 6. $F \rightarrow \texttt{id}$

State	Action						Goto		
	id	+	*	(	)	$	E	T	F
0	$s5$			$s4$			$c1$	$c2$	$c3$
1		$s6$				acc			
2		$r2$	$s7$		$r2$	$r2$			
3		$r4$	$r4$		$r4$	$r4$			
4	$s5$			$s4$			$c8$	$c2$	$c3$
5		$r6$	$r6$		$r6$	$r6$			
6	$s5$			$s4$				$c9$	$c3$
7	$s5$			$s4$					$c10$
8		$s6$			$s11$				
9		$r1$	$s7$		$r1$	$r1$			
10		$r3$	$r3$		$r3$	$r3$			
11		$r5$	$r5$		$r5$	$r5$			

Step	Stack →	Remaining Input	Action
1	\$0	id $*$ (id + id)\$	shift 5
2	\$0 id 5	$*$ (id + id)\$	reduce by 6
3	\$0 F	$*$ (id + id)\$	goto 3

SLR(1) Parsing Example

1.	$E \rightarrow E + T$	2.	$E \rightarrow T$
3.	$T \rightarrow T * F$	4.	$T \rightarrow F$
5.	$F \rightarrow (E)$	6.	$F \rightarrow \texttt{id}$

State	Action						Goto		
	id	+	*	(	)	$	*E*	*T*	*F*
0	*s*5			*s*4			*c*1	*c*2	*c*3
1		*s*6				*acc*			
2		*r*2	*s*7		*r*2	*r*2			
3		*r*4	*r*4		*r*4	*r*4			
4	*s*5			*s*4			*c*8	*c*2	*c*3
5		*r*6	*r*6		*r*6	*r*6			
6	*s*5			*s*4				*c*9	*c*3
7	*s*5			*s*4					*c*10
8		*s*6			*s*11				
9		*r*1	*s*7		*r*1	*r*1			
10		*r*3	*r*3		*r*3	*r*3			
11		*r*5	*r*5		*r*5	*r*5			

Step	Stack →	Remaining Input	Action
1	$0	id ∗ (id + id)$	shift 5
2	$0 id 5	∗ (id + id)$	reduce by 6
3	$0 *F*	∗ (id + id)$	goto 3
4	$0 *F* 3	∗ (id + id)$	reduce 4

SLR(1) Parsing Example

1. $E \rightarrow E + T$
2. $E \rightarrow T$
3. $T \rightarrow T * F$
4. $T \rightarrow F$
5. $F \rightarrow (E)$
6. $F \rightarrow \texttt{id}$

State	Action						Goto		
	id	+	*	(	)	$	E	T	F
0	s5			s4			c1	c2	c3
1		s6				acc			
2		r2	s7		r2	r2			
3		r4	r4		r4	r4			
4	s5			s4			c8	c2	c3
5		r6	r6		r6	r6			
6	s5			s4				c9	c3
7	s5			s4					c10
8		s6			s11				
9		r1	s7		r1	r1			
10		r3	r3		r3	r3			
11		r5	r5		r5	r5			

Step	Stack →	Remaining Input	Action
1	$0	id ∗ (id + id)$	shift 5
2	$0 id 5	∗ (id + id)$	reduce by 6
3	$0 F	∗ (id + id)$	goto 3
4	$0 F 3	∗ (id + id)$	reduce 4
5	$0 T	∗ (id + id)$	goto 2

SLR(1) Parsing Example

1.	$E \rightarrow E + T$	2.	$E \rightarrow T$
3.	$T \rightarrow T * F$	4.	$T \rightarrow F$
5.	$F \rightarrow (E)$	6.	$F \rightarrow \mathtt{id}$

State	Action						Goto		
	id	+	*	(	)	$	*E*	*T*	*F*
0	*s*5			*s*4			*c*1	*c*2	*c*3
1		*s*6				*acc*			
2		*r*2	*s*7		*r*2	*r*2			
3		*r*4	*r*4		*r*4	*r*4			
4	*s*5			*s*4			*c*8	*c*2	*c*3
5		*r*6	*r*6		*r*6	*r*6			
6	*s*5			*s*4				*c*9	*c*3
7	*s*5			*s*4					*c*10
8		*s*6			*s*11				
9		*r*1	*s*7		*r*1	*r*1			
10		*r*3	*r*3		*r*3	*r*3			
11		*r*5	*r*5		*r*5	*r*5			

Step	Stack →	Remaining Input	Action
1	$\$0$	$\mathrm{id} * (\mathrm{id} + \mathrm{id})\$$	shift 5
2	$\$0\ \mathrm{id}\ 5$	$* (\mathrm{id} + \mathrm{id})\$$	reduce by 6
3	$\$0\ F$	$* (\mathrm{id} + \mathrm{id})\$$	goto 3
4	$\$0\ F\ 3$	$* (\mathrm{id} + \mathrm{id})\$$	reduce 4
5	$\$0\ T$	$* (\mathrm{id} + \mathrm{id})\$$	goto 2
6	$\$0\ T\ 2$	$* (\mathrm{id} + \mathrm{id})\$$	shift 7

SLR(1) Parsing Example

1. $E \rightarrow E + T$
2. $E \rightarrow T$
3. $T \rightarrow T * F$
4. $T \rightarrow F$
5. $F \rightarrow (E)$
6. $F \rightarrow \texttt{id}$

State	Action						Goto		
	id	+	*	(	)	$	E	T	F
0	s5			s4			c1	c2	c3
1		s6				acc			
2		r2	s7		r2	r2			
3		r4	r4		r4	r4			
4	s5			s4			c8	c2	c3
5		r6	r6		r6	r6			
6	s5			s4				c9	c3
7	s5			s4					c10
8		s6			s11				
9		r1	s7		r1	r1			
10		r3	r3		r3	r3			
11		r5	r5		r5	r5			

Step	Stack →	Remaining Input	Action
1	$\$0$	$\text{id} * (\text{id} + \text{id})\$$	shift 5
2	$\$0\ \text{id}\ 5$	$* (\text{id} + \text{id})\$$	reduce by 6
3	$\$0\ F$	$* (\text{id} + \text{id})\$$	goto 3
4	$\$0\ F\ 3$	$* (\text{id} + \text{id})\$$	reduce 4
5	$\$0\ T$	$* (\text{id} + \text{id})\$$	goto 2
6	$\$0\ T\ 2$	$* (\text{id} + \text{id})\$$	shift 7
7	$\$0\ T\ 2 * 7$	$(\text{id} + \text{id})\$$	shift 4

SLR(1) Parsing Example

1. $E \rightarrow E + T$
2. $E \rightarrow T$
3. $T \rightarrow T * F$
4. $T \rightarrow F$
5. $F \rightarrow (E)$
6. $F \rightarrow \texttt{id}$

State	Action						Goto		
	id	+	∗	(	)	$	E	T	F
0	$s5$			$s4$			$c1$	$c2$	$c3$
1		$s6$				acc			
2		$r2$	$s7$		$r2$	$r2$			
3		$r4$	$r4$		$r4$	$r4$			
4	$s5$			$s4$			$c8$	$c2$	$c3$
5		$r6$	$r6$		$r6$	$r6$			
6	$s5$			$s4$				$c9$	$c3$
7	$s5$			$s4$					$c10$
8		$s6$			$s11$				
9		$r1$	$s7$		$r1$	$r1$			
10		$r3$	$r3$		$r3$	$r3$			
11		$r5$	$r5$		$r5$	$r5$			

Step	Stack →	Remaining Input	Action
1	$0	id ∗ (id + id)$	shift 5
2	$0 id 5	∗ (id + id)$	reduce by 6
3	$0 F	∗ (id + id)$	goto 3
4	$0 F 3	∗ (id + id)$	reduce 4
5	$0 T	∗ (id + id)$	goto 2
6	$0 T 2	∗ (id + id)$	shift 7
7	$0 T 2 ∗ 7	(id + id)$	shift 4
8	$0 T 2 ∗ 7 (4	id + id)$	shift 5

SLR(1) Parsing Example

1. $E \rightarrow E + T$
3. $T \rightarrow T * F$
5. $F \rightarrow (E)$
2. $E \rightarrow T$
4. $T \rightarrow F$
6. $F \rightarrow$ `id`

State	Action						Goto		
	`id`	+	*	(	)	$	E	T	F
0	$s5$			$s4$			$c1$	$c2$	$c3$
1		$s6$				acc			
2		$r2$	$s7$		$r2$	$r2$			
3		$r4$	$r4$		$r4$	$r4$			
4	$s5$			$s4$			$c8$	$c2$	$c3$
5		$r6$	$r6$		$r6$	$r6$			
6	$s5$			$s4$				$c9$	$c3$
7	$s5$			$s4$					$c10$
8		$s6$			$s11$				
9		$r1$	$s7$		$r1$	$r1$			
10		$r3$	$r3$		$r3$	$r3$			
11		$r5$	$r5$		$r5$	$r5$			

Step	Stack →	Remaining Input	Action
1	$0	id ∗ (id + id)$	shift 5
2	$0 id 5	∗ (id + id)$	reduce by 6
3	$0 F	∗ (id + id)$	goto 3
4	$0 F 3	∗ (id + id)$	reduce 4
5	$0 T	∗ (id + id)$	goto 2
6	$0 T 2	∗ (id + id)$	shift 7
7	$0 T 2 ∗ 7	(id + id)$	shift 4
8	$0 T 2 ∗ 7 (4	id + id)$	shift 5
9	$0 T 2 ∗ 7 (4 id 5	+ id)$	reduce 6

SLR(1) Parsing Example

1. $E \rightarrow E + T$
2. $E \rightarrow T$
3. $T \rightarrow T * F$
4. $T \rightarrow F$
5. $F \rightarrow (E)$
6. $F \rightarrow$ `id`

State	Action						Goto		
	`id`	+	*	(	)	$	E	T	F
0	$s5$			$s4$			$c1$	$c2$	$c3$
1		$s6$				acc			
2		$r2$	$s7$		$r2$	$r2$			
3		$r4$	$r4$		$r4$	$r4$			
4	$s5$			$s4$			$c8$	$c2$	$c3$
5		$r6$	$r6$		$r6$	$r6$			
6	$s5$			$s4$				$c9$	$c3$
7	$s5$			$s4$					$c10$
8		$s6$			$s11$				
9		$r1$	$s7$		$r1$	$r1$			
10		$r3$	$r3$		$r3$	$r3$			
11		$r5$	$r5$		$r5$	$r5$			

Step	Stack →	Remaining Input	Action
1	\$0	id $*$ (id $+$ id)\$	shift 5
2	\$0 id 5	$*$ (id $+$ id)\$	reduce by 6
3	\$0 F	$*$ (id $+$ id)\$	goto 3
4	\$0 F 3	$*$ (id $+$ id)\$	reduce 4
5	\$0 T	$*$ (id $+$ id)\$	goto 2
6	\$0 T 2	$*$ (id $+$ id)\$	shift 7
7	\$0 T 2 $*$ 7	(id $+$ id)\$	shift 4
8	\$0 T 2 $*$ 7 (4	id $+$ id)\$	shift 5
9	\$0 T 2 $*$ 7 (4 id 5	$+$ id)\$	reduce 6
10	\$0 T 2 $*$ 7 (4 F	$+$ id)\$	goto 3

SLR(1) Parsing Example

1. $E \rightarrow E + T$
2. $E \rightarrow T$
3. $T \rightarrow T * F$
4. $T \rightarrow F$
5. $F \rightarrow (E)$
6. $F \rightarrow$ `id`

State	Action						Goto		
	id	+	*	(	)	$	E	T	F
0	$s5$			$s4$			$c1$	$c2$	$c3$
1		$s6$				acc			
2		$r2$	$s7$		$r2$	$r2$			
3		$r4$	$r4$		$r4$	$r4$			
4	$s5$			$s4$			$c8$	$c2$	$c3$
5		$r6$	$r6$		$r6$	$r6$			
6	$s5$			$s4$				$c9$	$c3$
7	$s5$			$s4$					$c10$
8		$s6$			$s11$				
9		$r1$	$s7$		$r1$	$r1$			
10		$r3$	$r3$		$r3$	$r3$			
11		$r5$	$r5$		$r5$	$r5$			

Step	Stack →	Remaining Input	Action
1	$\$0$	$\text{id} * (\text{id} + \text{id})\$$	shift 5
2	$\$0\ \text{id}\ 5$	$* (\text{id} + \text{id})\$$	reduce by 6
3	$\$0\ F$	$* (\text{id} + \text{id})\$$	goto 3
4	$\$0\ F\ 3$	$* (\text{id} + \text{id})\$$	reduce 4
5	$\$0\ T$	$* (\text{id} + \text{id})\$$	goto 2
6	$\$0\ T\ 2$	$* (\text{id} + \text{id})\$$	shift 7
7	$\$0\ T\ 2 * 7$	$(\text{id} + \text{id})\$$	shift 4
8	$\$0\ T\ 2 * 7\ (4$	$\text{id} + \text{id})\$$	shift 5
9	$\$0\ T\ 2 * 7\ (4\ \text{id}\ 5$	$+ \text{id})\$$	reduce 6
10	$\$0\ T\ 2 * 7\ (4\ F$	$+ \text{id})\$$	goto 3
11	$\$0\ T\ 2 * 7\ (4\ F\ 3$	$+ \text{id})\$$	reduce 4

SLR(1) Parsing Example

1.	$E \rightarrow E + T$	2.	$E \rightarrow T$
3.	$T \rightarrow T * F$	4.	$T \rightarrow F$
5.	$F \rightarrow (E)$	6.	$F \rightarrow \mathtt{id}$

State	Action						Goto		
	id	+	*	(	)	$	E	T	F
0	s5			s4			c1	c2	c3
1		s6				acc			
2		r2	s7		r2	r2			
3		r4	r4		r4	r4			
4	s5			s4			c8	c2	c3
5		r6	r6		r6	r6			
6	s5			s4				c9	c3
7	s5			s4					c10
8		s6			s11				
9		r1	s7		r1	r1			
10		r3	r3		r3	r3			
11		r5	r5		r5	r5			

Step	Stack →	Remaining Input	Action
1	$\$0$	$\mathrm{id} * (\mathrm{id} + \mathrm{id})\$$	shift 5
2	$\$0\ \mathrm{id}\ 5$	$* (\mathrm{id} + \mathrm{id})\$$	reduce by 6
3	$\$0\ F$	$* (\mathrm{id} + \mathrm{id})\$$	goto 3
4	$\$0\ F\ 3$	$* (\mathrm{id} + \mathrm{id})\$$	reduce 4
5	$\$0\ T$	$* (\mathrm{id} + \mathrm{id})\$$	goto 2
6	$\$0\ T\ 2$	$* (\mathrm{id} + \mathrm{id})\$$	shift 7
7	$\$0\ T\ 2 * 7$	$(\mathrm{id} + \mathrm{id})\$$	shift 4
8	$\$0\ T\ 2 * 7\ (4$	$\mathrm{id} + \mathrm{id})\$$	shift 5
9	$\$0\ T\ 2 * 7\ (4\ \mathrm{id}\ 5$	$+ \mathrm{id})\$$	reduce 6
10	$\$0\ T\ 2 * 7\ (4\ F$	$+ \mathrm{id})\$$	goto 3
11	$\$0\ T\ 2 * 7\ (4\ F\ 3$	$+ \mathrm{id})\$$	reduce 4
12	$\$0\ T\ 2 * 7\ (4\ T$	$+ \mathrm{id})\$$	goto 2

SLR(1) Parsing Example

1. $E \rightarrow E + T$ | 2. $E \rightarrow T$
3. $T \rightarrow T * F$ | 4. $T \rightarrow F$
5. $F \rightarrow (E)$ | 6. $F \rightarrow \texttt{id}$

State	Action						Goto		
	id	+	*	(	)	$	E	T	F
0	s5			s4			c1	c2	c3
1		s6				acc			
2		r2	s7		r2	r2			
3		r4	r4		r4	r4			
4	s5			s4			c8	c2	c3
5		r6	r6		r6	r6			
6	s5			s4				c9	c3
7	s5			s4					c10
8		s6			s11				
9		r1	s7		r1	r1			
10		r3	r3		r3	r3			
11		r5	r5		r5	r5			

Step	Stack →	Remaining Input	Action
1	$\$0$	$\mathrm{id} * (\mathrm{id} + \mathrm{id})\$$	shift 5
2	$\$0\ \mathrm{id}\ 5$	$* (\mathrm{id} + \mathrm{id})\$$	reduce by 6
3	$\$0\ F$	$* (\mathrm{id} + \mathrm{id})\$$	goto 3
4	$\$0\ F\ 3$	$* (\mathrm{id} + \mathrm{id})\$$	reduce 4
5	$\$0\ T$	$* (\mathrm{id} + \mathrm{id})\$$	goto 2
6	$\$0\ T\ 2$	$* (\mathrm{id} + \mathrm{id})\$$	shift 7
7	$\$0\ T\ 2 * 7$	$(\mathrm{id} + \mathrm{id})\$$	shift 4
8	$\$0\ T\ 2 * 7\ (4$	$\mathrm{id} + \mathrm{id})\$$	shift 5
9	$\$0\ T\ 2 * 7\ (4\ \mathrm{id}\ 5$	$+ \mathrm{id})\$$	reduce 6
10	$\$0\ T\ 2 * 7\ (4\ F$	$+ \mathrm{id})\$$	goto 3
11	$\$0\ T\ 2 * 7\ (4\ F\ 3$	$+ \mathrm{id})\$$	reduce 4
12	$\$0\ T\ 2 * 7\ (4\ T$	$+ \mathrm{id})\$$	goto 2
13	$\$0\ T\ 2 * 7\ (4\ T\ 2$	$+ \mathrm{id})\$$	reduce 2

SLR(1) Parsing Example

1. $E \rightarrow E + T$ | 2. $E \rightarrow T$
3. $T \rightarrow T * F$ | 4. $T \rightarrow F$
5. $F \rightarrow (E)$ | 6. $F \rightarrow \texttt{id}$

State	Action						Goto		
	id	+	*	(	)	$	E	T	F
0	s5			s4			c1	c2	c3
1		s6				acc			
2		r2	s7		r2	r2			
3		r4	r4		r4	r4			
4	s5			s4			c8	c2	c3
5		r6	r6		r6	r6			
6	s5			s4				c9	c3
7	s5			s4					c10
8		s6			s11				
9		r1	s7		r1	r1			
10		r3	r3		r3	r3			
11		r5	r5		r5	r5			

Step	Stack →	Remaining Input	Action
1	$0	id ∗ (id + id)$	shift 5
2	$0 id 5	∗ (id + id)$	reduce by 6
3	$0 F	∗ (id + id)$	goto 3
4	$0 F 3	∗ (id + id)$	reduce 4
5	$0 T	∗ (id + id)$	goto 2
6	$0 T 2	∗ (id + id)$	shift 7
7	$0 T 2 ∗ 7	(id + id)$	shift 4
8	$0 T 2 ∗ 7 (4	id + id)$	shift 5
9	$0 T 2 ∗ 7 (4 id 5	+ id)$	reduce 6
10	$0 T 2 ∗ 7 (4 F	+ id)$	goto 3
11	$0 T 2 ∗ 7 (4 F 3	+ id)$	reduce 4
12	$0 T 2 ∗ 7 (4 T	+ id)$	goto 2
13	$0 T 2 ∗ 7 (4 T 2	+ id)$	reduce 2
14	$0 T 2 ∗ 7 (4 E	+ id)$	goto 8

SLR(1) Parsing Example

1. $E \rightarrow E + T$
2. $E \rightarrow T$
3. $T \rightarrow T * F$
4. $T \rightarrow F$
5. $F \rightarrow (E)$
6. $F \rightarrow \texttt{id}$

State	Action						Goto		
	id	+	*	(	)	$	E	T	F
0	s5			s4			c1	c2	c3
1		s6				acc			
2		r2	s7		r2	r2			
3		r4	r4		r4	r4			
4	s5			s4			c8	c2	c3
5		r6	r6		r6	r6			
6	s5			s4				c9	c3
7	s5			s4					c10
8		s6			s11				
9		r1	s7		r1	r1			
10		r3	r3		r3	r3			
11		r5	r5		r5	r5			

Step	Stack →	Remaining Input	Action
1	$0	id ∗ (id + id)$	shift 5
2	$0 id 5	∗ (id + id)$	reduce by 6
3	$0 F	∗ (id + id)$	goto 3
4	$0 F 3	∗ (id + id)$	reduce 4
5	$0 T	∗ (id + id)$	goto 2
6	$0 T 2	∗ (id + id)$	shift 7
7	$0 T 2 ∗ 7	(id + id)$	shift 4
8	$0 T 2 ∗ 7 (4	id + id)$	shift 5
9	$0 T 2 ∗ 7 (4 id 5	+ id)$	reduce 6
10	$0 T 2 ∗ 7 (4 F	+ id)$	goto 3
11	$0 T 2 ∗ 7 (4 F 3	+ id)$	reduce 4
12	$0 T 2 ∗ 7 (4 T	+ id)$	goto 2
13	$0 T 2 ∗ 7 (4 T 2	+ id)$	reduce 2
14	$0 T 2 ∗ 7 (4 E	+ id)$	goto 8
15	$0 T 2 ∗ 7 (4 E 8	+ id)$	shift 6

IIT Bombay
cs302: Implementation of Programming Languages

Topic:

Syntax Analysis

Section:

Grammars, Derivations, and Parse Trees

Shift Reduce Parsing

SLR(1) Parsing

Conceptual Issues in Parsing

CLR(1) Parsing

LALR(1) Parsing

SLR(1) Parsing Example (Continued)

1. $E \rightarrow E + T$
3. $T \rightarrow T * F$
5. $F \rightarrow (E)$

2. $E \rightarrow T$
4. $T \rightarrow F$
6. $F \rightarrow$ `id`

State	Action						Goto		
	id	+	*	(	)	$	E	T	F
0	$s5$			$s4$			$c1$	$c2$	$c3$
1		$s6$				acc			
2		$r2$	$s7$		$r2$	$r2$			
3		$r4$	$r4$		$r4$	$r4$			
4	$s5$			$s4$			$c8$	$c2$	$c3$
5		$r6$	$r6$		$r6$	$r6$			
6	$s5$			$s4$				$c9$	$c3$
7	$s5$			$s4$					$c10$
8		$s6$			$s11$				
9		$r1$	$s7$		$r1$	$r1$			
10		$r3$	$r3$		$r3$	$r3$			
11		$r5$	$r5$		$r5$	$r5$			

Step	Stack →	Input	Action
16	$0 T 2 $*$ 7 (4 E 8 + 6	id)$	shift 5

SLR(1) Parsing Example (Continued)

1. $E \rightarrow E + T$
2. $E \rightarrow T$
3. $T \rightarrow T * F$
4. $T \rightarrow F$
5. $F \rightarrow (E)$
6. $F \rightarrow$ `id`

State	Action						Goto		
	id	+	*	(	)	$	E	T	F
0	$s5$			$s4$			$c1$	$c2$	$c3$
1		$s6$				acc			
2		$r2$	$s7$		$r2$	$r2$			
3		$r4$	$r4$		$r4$	$r4$			
4	$s5$			$s4$			$c8$	$c2$	$c3$
5		$r6$	$r6$		$r6$	$r6$			
6	$s5$			$s4$				$c9$	$c3$
7	$s5$			$s4$					$c10$
8		$s6$			$s11$				
9		$r1$	$s7$		$r1$	$r1$			
10		$r3$	$r3$		$r3$	$r3$			
11		$r5$	$r5$		$r5$	$r5$			

Step	Stack →	Input	Action
16	$0 T 2 $*$ 7 (4 E 8 + 6	id)$	shift 5
17	$0 T 2 $*$ 7 (4 E 8 + 6 id 5	)$	reduce 6

SLR(1) Parsing Example (Continued)

1. $E \rightarrow E + T$
2. $E \rightarrow T$
3. $T \rightarrow T * F$
4. $T \rightarrow F$
5. $F \rightarrow (E)$
6. $F \rightarrow$ `id`

State	Action						Goto		
	id	+	*	(	)	\$	E	T	F
0	$s5$			$s4$			$c1$	$c2$	$c3$
1		$s6$				acc			
2		$r2$	$s7$		$r2$	$r2$			
3		$r4$	$r4$		$r4$	$r4$			
4	$s5$			$s4$			$c8$	$c2$	$c3$
5		$r6$	$r6$		$r6$	$r6$			
6	$s5$			$s4$				$c9$	$c3$
7	$s5$			$s4$					$c10$
8		$s6$			$s11$				
9		$r1$	$s7$		$r1$	$r1$			
10		$r3$	$r3$		$r3$	$r3$			
11		$r5$	$r5$		$r5$	$r5$			

Step	Stack →	Input	Action
16	$\$0\ T\ 2 * 7\ (4\ E\ 8 + 6$	id)\$	shift 5
17	$\$0\ T\ 2 * 7\ (4\ E\ 8 + 6\ \mathrm{id}\ 5$	)\$	reduce 6
18	$\$0\ T\ 2 * 7\ (4\ E\ 8 + 6\ F$	)\$	goto 3

SLR(1) Parsing Example (Continued)

1.	$E \rightarrow E + T$	2.	$E \rightarrow T$
3.	$T \rightarrow T * F$	4.	$T \rightarrow F$
5.	$F \rightarrow (E)$	6.	$F \rightarrow$ `id`

State	Action						Goto		
	id	+	*	(	)	$	E	T	F
0	s5			s4			c1	c2	c3
1		s6				acc			
2		r2	s7		r2	r2			
3		r4	r4		r4	r4			
4	s5			s4			c8	c2	c3
5		r6	r6		r6	r6			
6	s5			s4				c9	c3
7	s5			s4					c10
8		s6			s11				
9		r1	s7		r1	r1			
10		r3	r3		r3	r3			
11		r5	r5		r5	r5			

Step	Stack $\rightarrow$	Input	Action
16	$\$0\ T\,2 * 7\,(4\,E\,8 + 6$	id)$	shift 5
17	$\$0\ T\,2 * 7\,(4\,E\,8 + 6\,\mathrm{id}\,5$	)$	reduce 6
18	$\$0\ T\,2 * 7\,(4\,E\,8 + 6\,F$	)$	goto 3
19	$\$0\ T\,2 * 7\,(4\,E\,8 + 6\,F\,3$	)$	reduce 4

SLR(1) Parsing Example (Continued)

1. $E \rightarrow E + T$
3. $T \rightarrow T * F$
5. $F \rightarrow (E)$

2. $E \rightarrow T$
4. $T \rightarrow F$
6. $F \rightarrow \texttt{id}$

State	Action						Goto		
	id	+	*	(	)	$	*E*	*T*	*F*
0	*s*5			*s*4			*c*1	*c*2	*c*3
1		*s*6				*acc*			
2		*r*2	*s*7		*r*2	*r*2			
3		*r*4	*r*4		*r*4	*r*4			
4	*s*5			*s*4			*c*8	*c*2	*c*3
5		*r*6	*r*6		*r*6	*r*6			
6	*s*5			*s*4				*c*9	*c*3
7	*s*5			*s*4					*c*10
8		*s*6			*s*11				
9		*r*1	*s*7		*r*1	*r*1			
10		*r*3	*r*3		*r*3	*r*3			
11		*r*5	*r*5		*r*5	*r*5			

Step	Stack →	Input	Action
16	$\$0\ T\ 2 * 7\ (4\ E\ 8 + 6$	id)$	shift 5
17	$\$0\ T\ 2 * 7\ (4\ E\ 8 + 6\ \mathrm{id}\ 5$	)$	reduce 6
18	$\$0\ T\ 2 * 7\ (4\ E\ 8 + 6\ F$	)$	goto 3
19	$\$0\ T\ 2 * 7\ (4\ E\ 8 + 6\ F\ 3$	)$	reduce 4
20	$\$0\ T\ 2 * 7\ (4\ E\ 8 + 6\ T$	)$	goto 9

SLR(1) Parsing Example (Continued)

1. $E \to E + T$
2. $E \to T$
3. $T \to T * F$
4. $T \to F$
5. $F \to (E)$
6. $F \to \texttt{id}$

State	Action						Goto		
	id	+	*	(	)	$	E	T	F
0	$s5$			$s4$			$c1$	$c2$	$c3$
1		$s6$				acc			
2		$r2$	$s7$		$r2$	$r2$			
3		$r4$	$r4$		$r4$	$r4$			
4	$s5$			$s4$			$c8$	$c2$	$c3$
5		$r6$	$r6$		$r6$	$r6$			
6	$s5$			$s4$				$c9$	$c3$
7	$s5$			$s4$					$c10$
8		$s6$			$s11$				
9		$r1$	$s7$		$r1$	$r1$			
10		$r3$	$r3$		$r3$	$r3$			
11		$r5$	$r5$		$r5$	$r5$			

Step	Stack →	Input	Action
16	$\$0\ T\ 2 * 7\ (4\ E\ 8 + 6$	id)$	shift 5
17	$\$0\ T\ 2 * 7\ (4\ E\ 8 + 6\ \mathrm{id}\ 5$	)$	reduce 6
18	$\$0\ T\ 2 * 7\ (4\ E\ 8 + 6\ F$	)$	goto 3
19	$\$0\ T\ 2 * 7\ (4\ E\ 8 + 6\ F\ 3$	)$	reduce 4
20	$\$0\ T\ 2 * 7\ (4\ E\ 8 + 6\ T$	)$	goto 9
20	$\$0\ T\ 2 * 7\ (4\ E\ 8 + 6\ T\ 9$	)$	reduce 1

SLR(1) Parsing Example (Continued)

1. $E \rightarrow E + T$
2. $E \rightarrow T$
3. $T \rightarrow T * F$
4. $T \rightarrow F$
5. $F \rightarrow (E)$
6. $F \rightarrow \texttt{id}$

State	Action						Goto		
	id	+	*	(	)	$	E	T	F
0	s5			s4			c1	c2	c3
1		s6				acc			
2		r2	s7		r2	r2			
3		r4	r4		r4	r4			
4	s5			s4			c8	c2	c3
5		r6	r6		r6	r6			
6	s5			s4				c9	c3
7	s5			s4					c10
8		s6			s11				
9		r1	s7		r1	r1			
10		r3	r3		r3	r3			
11		r5	r5		r5	r5			

Step	Stack →	Input	Action
16	$\$0\ T\ 2 * 7\ (4\ E\ 8 + 6$	id)$	shift 5
17	$\$0\ T\ 2 * 7\ (4\ E\ 8 + 6\ \text{id}\ 5$	)$	reduce 6
18	$\$0\ T\ 2 * 7\ (4\ E\ 8 + 6\ F$	)$	goto 3
19	$\$0\ T\ 2 * 7\ (4\ E\ 8 + 6\ F\ 3$	)$	reduce 4
20	$\$0\ T\ 2 * 7\ (4\ E\ 8 + 6\ T$	)$	goto 9
20	$\$0\ T\ 2 * 7\ (4\ E\ 8 + 6\ T\ 9$	)$	reduce 1
21	$\$0\ T\ 2 * 7\ (4\ E$	)$	goto 8

SLR(1) Parsing Example (Continued)

1. $E \rightarrow E + T$
2. $E \rightarrow T$
3. $T \rightarrow T * F$
4. $T \rightarrow F$
5. $F \rightarrow (E)$
6. $F \rightarrow$ `id`

State	Action						Goto		
	id	+	*	(	)	$	E	T	F
0	$s5$			$s4$			$c1$	$c2$	$c3$
1		$s6$				acc			
2		$r2$	$s7$		$r2$	$r2$			
3		$r4$	$r4$		$r4$	$r4$			
4	$s5$			$s4$			$c8$	$c2$	$c3$
5		$r6$	$r6$		$r6$	$r6$			
6	$s5$			$s4$				$c9$	$c3$
7	$s5$			$s4$					$c10$
8		$s6$			$s11$				
9		$r1$	$s7$		$r1$	$r1$			
10		$r3$	$r3$		$r3$	$r3$			
11		$r5$	$r5$		$r5$	$r5$			

Step	Stack →	Input	Action
16	$\$0\ T\ 2 * 7\ (\ 4\ E\ 8 + 6$	id)$	shift 5
17	$\$0\ T\ 2 * 7\ (\ 4\ E\ 8 + 6\ \mathrm{id}\ 5$	)$	reduce 6
18	$\$0\ T\ 2 * 7\ (\ 4\ E\ 8 + 6\ F$	)$	goto 3
19	$\$0\ T\ 2 * 7\ (\ 4\ E\ 8 + 6\ F\ 3$	)$	reduce 4
20	$\$0\ T\ 2 * 7\ (\ 4\ E\ 8 + 6\ T$	)$	goto 9
20	$\$0\ T\ 2 * 7\ (\ 4\ E\ 8 + 6\ T\ 9$	)$	reduce 1
21	$\$0\ T\ 2 * 7\ (\ 4\ E$	)$	goto 8
22	$\$0\ T\ 2 * 7\ (\ 4\ E\ 8$	)$	shift 11

SLR(1) Parsing Example (Continued)

1. $E \rightarrow E + T$
2. $E \rightarrow T$
3. $T \rightarrow T * F$
4. $T \rightarrow F$
5. $F \rightarrow (E)$
6. $F \rightarrow$ `id`

State	Action						Goto		
	id	+	*	(	)	$	E	T	F
0	s5			s4			c1	c2	c3
1		s6				acc			
2		r2	s7		r2	r2			
3		r4	r4		r4	r4			
4	s5			s4			c8	c2	c3
5		r6	r6		r6	r6			
6	s5			s4				c9	c3
7	s5			s4					c10
8		s6			s11				
9		r1	s7		r1	r1			
10		r3	r3		r3	r3			
11		r5	r5		r5	r5			

Step	Stack →	Input	Action
16	$\$0\ T\ 2 * 7\ (4\ E\ 8 + 6$	id)$	shift 5
17	$\$0\ T\ 2 * 7\ (4\ E\ 8 + 6\ \mathrm{id}\ 5$	)$	reduce 6
18	$\$0\ T\ 2 * 7\ (4\ E\ 8 + 6\ F$	)$	goto 3
19	$\$0\ T\ 2 * 7\ (4\ E\ 8 + 6\ F\ 3$	)$	reduce 4
20	$\$0\ T\ 2 * 7\ (4\ E\ 8 + 6\ T$	)$	goto 9
20	$\$0\ T\ 2 * 7\ (4\ E\ 8 + 6\ T\ 9$	)$	reduce 1
21	$\$0\ T\ 2 * 7\ (4\ E$	)$	goto 8
22	$\$0\ T\ 2 * 7\ (4\ E\ 8$	)$	shift 11
23	$\$0\ T\ 2 * 7\ (4\ E\ 8)\ 11$	$	reduce 5

SLR(1) Parsing Example (Continued)

1. $E \rightarrow E + T$
3. $T \rightarrow T * F$
5. $F \rightarrow (E)$
2. $E \rightarrow T$
4. $T \rightarrow F$
6. $F \rightarrow \mathtt{id}$

State	Action						Goto		
	id	+	∗	(	)	$	E	T	F
0	$s5$			$s4$			$c1$	$c2$	$c3$
1		$s6$				acc			
2		$r2$	$s7$		$r2$	$r2$			
3		$r4$	$r4$		$r4$	$r4$			
4	$s5$			$s4$			$c8$	$c2$	$c3$
5		$r6$	$r6$		$r6$	$r6$			
6	$s5$			$s4$				$c9$	$c3$
7	$s5$			$s4$					$c10$
8		$s6$			$s11$				
9		$r1$	$s7$		$r1$	$r1$			
10		$r3$	$r3$		$r3$	$r3$			
11		$r5$	$r5$		$r5$	$r5$			

Step	Stack →	Input	Action
16	$\$0\ T\ 2 * 7\ (4\ E\ 8 + 6$	id)$	shift 5
17	$\$0\ T\ 2 * 7\ (4\ E\ 8 + 6\ \mathrm{id}\ 5$	)$	reduce 6
18	$\$0\ T\ 2 * 7\ (4\ E\ 8 + 6\ F$	)$	goto 3
19	$\$0\ T\ 2 * 7\ (4\ E\ 8 + 6\ F\ 3$	)$	reduce 4
20	$\$0\ T\ 2 * 7\ (4\ E\ 8 + 6\ T$	)$	goto 9
20	$\$0\ T\ 2 * 7\ (4\ E\ 8 + 6\ T\ 9$	)$	reduce 1
21	$\$0\ T\ 2 * 7\ (4\ E$	)$	goto 8
22	$\$0\ T\ 2 * 7\ (4\ E\ 8$	)$	shift 11
23	$\$0\ T\ 2 * 7\ (4\ E\ 8)\ 11$	$	reduce 5
24	$\$0\ T\ 2 * 7\ F$	$	goto 10

SLR(1) Parsing Example (Continued)

1. $E \rightarrow\ E + T$ | 2. $E \rightarrow\ T$
3. $T \rightarrow\ T * F$ | 4. $T \rightarrow\ F$
5. $F \rightarrow\ (E)$ | 6. $F \rightarrow\ \mathtt{id}$

State	Action						Goto		
	id	+	*	(	)	$	E	T	F
0	s5			s4			c1	c2	c3
1		s6				acc			
2		r2	s7		r2	r2			
3		r4	r4		r4	r4			
4	s5			s4			c8	c2	c3
5		r6	r6		r6	r6			
6	s5			s4				c9	c3
7	s5			s4					c10
8		s6			s11				
9		r1	s7		r1	r1			
10		r3	r3		r3	r3			
11		r5	r5		r5	r5			

Step	Stack $\rightarrow$	Input	Action
16	$\$0\ T\ 2 * 7\ (4\ E\ 8 + 6$	id)\$	shift 5
17	$\$0\ T\ 2 * 7\ (4\ E\ 8 + 6\ \mathrm{id}\ 5$	)\$	reduce 6
18	$\$0\ T\ 2 * 7\ (4\ E\ 8 + 6\ F$	)\$	goto 3
19	$\$0\ T\ 2 * 7\ (4\ E\ 8 + 6\ F\ 3$	)\$	reduce 4
20	$\$0\ T\ 2 * 7\ (4\ E\ 8 + 6\ T$	)\$	goto 9
20	$\$0\ T\ 2 * 7\ (4\ E\ 8 + 6\ T\ 9$	)\$	reduce 1
21	$\$0\ T\ 2 * 7\ (4\ E$	)\$	goto 8
22	$\$0\ T\ 2 * 7\ (4\ E\ 8$	)\$	shift 11
23	$\$0\ T\ 2 * 7\ (4\ E\ 8)\ 11$	\$	reduce 5
24	$\$0\ T\ 2 * 7\ F$	\$	goto 10
25	$\$0\ T\ 2 * 7\ F\ 10$	\$	reduce 3

SLR(1) Parsing Example (Continued)

1. $E \rightarrow E + T$
2. $E \rightarrow T$
3. $T \rightarrow T * F$
4. $T \rightarrow F$
5. $F \rightarrow (E)$
6. $F \rightarrow$ `id`

State	Action						Goto		
	id	+	*	(	)	$	E	T	F
0	s5			s4			c1	c2	c3
1		s6				acc			
2		r2	s7		r2	r2			
3		r4	r4		r4	r4			
4	s5			s4			c8	c2	c3
5		r6	r6		r6	r6			
6	s5			s4				c9	c3
7	s5			s4					c10
8		s6			s11				
9		r1	s7		r1	r1			
10		r3	r3		r3	r3			
11		r5	r5		r5	r5			

Step	Stack →	Input	Action
16	$0 T 2 * 7 (4 E 8 + 6	id)$	shift 5
17	$0 T 2 * 7 (4 E 8 + 6 id 5	)$	reduce 6
18	$0 T 2 * 7 (4 E 8 + 6 F	)$	goto 3
19	$0 T 2 * 7 (4 E 8 + 6 F 3	)$	reduce 4
20	$0 T 2 * 7 (4 E 8 + 6 T	)$	goto 9
20	$0 T 2 * 7 (4 E 8 + 6 T 9	)$	reduce 1
21	$0 T 2 * 7 (4 E	)$	goto 8
22	$0 T 2 * 7 (4 E 8	)$	shift 11
23	$0 T 2 * 7 (4 E 8) 11	$	reduce 5
24	$0 T 2 * 7 F	$	goto 10
25	$0 T 2 * 7 F 10	$	reduce 3
26	$0 T	$	goto 2

SLR(1) Parsing Example (Continued)

1. $E \rightarrow E + T$
2. $E \rightarrow T$
3. $T \rightarrow T * F$
4. $T \rightarrow F$
5. $F \rightarrow (E)$
6. $F \rightarrow$ `id`

State	Action						Goto		
	id	+	*	(	)	\$	E	T	F
0	s5			s4			c1	c2	c3
1		s6				acc			
2		r2	s7		r2	r2			
3		r4	r4		r4	r4			
4	s5			s4			c8	c2	c3
5		r6	r6		r6	r6			
6	s5			s4				c9	c3
7	s5			s4					c10
8		s6			s11				
9		r1	s7		r1	r1			
10		r3	r3		r3	r3			
11		r5	r5		r5	r5			

Step	Stack →	Input	Action
16	$\$0\ T\ 2 * 7\ (4\ E\ 8 + 6$	id)\$	shift 5
17	$\$0\ T\ 2 * 7\ (4\ E\ 8 + 6\ \mathrm{id}\ 5$	)\$	reduce 6
18	$\$0\ T\ 2 * 7\ (4\ E\ 8 + 6\ F$	)\$	goto 3
19	$\$0\ T\ 2 * 7\ (4\ E\ 8 + 6\ F\ 3$	)\$	reduce 4
20	$\$0\ T\ 2 * 7\ (4\ E\ 8 + 6\ T$	)\$	goto 9
20	$\$0\ T\ 2 * 7\ (4\ E\ 8 + 6\ T\ 9$	)\$	reduce 1
21	$\$0\ T\ 2 * 7\ (4\ E$	)\$	goto 8
22	$\$0\ T\ 2 * 7\ (4\ E\ 8$	)\$	shift 11
23	$\$0\ T\ 2 * 7\ (4\ E\ 8)\ 11$	\$	reduce 5
24	$\$0\ T\ 2 * 7\ F$	\$	goto 10
25	$\$0\ T\ 2 * 7\ F\ 10$	\$	reduce 3
26	$\$0\ T$	\$	goto 2
27	$\$0\ T\ 2$	\$	reduce 2

SLR(1) Parsing Example (Continued)

1. $E \rightarrow E + T$ | 2. $E \rightarrow T$
3. $T \rightarrow T * F$ | 4. $T \rightarrow F$
5. $F \rightarrow (E)$ | 6. $F \rightarrow$ `id`

State	Action						Goto		
	id	+	*	(	)	$	E	T	F
0	$s5$			$s4$			$c1$	$c2$	$c3$
1		$s6$				acc			
2		$r2$	$s7$		$r2$	$r2$			
3		$r4$	$r4$		$r4$	$r4$			
4	$s5$			$s4$			$c8$	$c2$	$c3$
5		$r6$	$r6$		$r6$	$r6$			
6	$s5$			$s4$				$c9$	$c3$
7	$s5$			$s4$					$c10$
8		$s6$			$s11$				
9		$r1$	$s7$		$r1$	$r1$			
10		$r3$	$r3$		$r3$	$r3$			
11		$r5$	$r5$		$r5$	$r5$			

Step	Stack →	Input	Action
16	$\$0\ T\ 2 * 7\ (4\ E\ 8 + 6$	id)$	shift 5
17	$\$0\ T\ 2 * 7\ (4\ E\ 8 + 6\ \mathrm{id}\ 5$	)$	reduce 6
18	$\$0\ T\ 2 * 7\ (4\ E\ 8 + 6\ F$	)$	goto 3
19	$\$0\ T\ 2 * 7\ (4\ E\ 8 + 6\ F\ 3$	)$	reduce 4
20	$\$0\ T\ 2 * 7\ (4\ E\ 8 + 6\ T$	)$	goto 9
20	$\$0\ T\ 2 * 7\ (4\ E\ 8 + 6\ T\ 9$	)$	reduce 1
21	$\$0\ T\ 2 * 7\ (4\ E$	)$	goto 8
22	$\$0\ T\ 2 * 7\ (4\ E\ 8$	)$	shift 11
23	$\$0\ T\ 2 * 7\ (4\ E\ 8)\ 11$	$	reduce 5
24	$\$0\ T\ 2 * 7\ F$	$	goto 10
25	$\$0\ T\ 2 * 7\ F\ 10$	$	reduce 3
26	$\$0\ T$	$	goto 2
27	$\$0\ T\ 2$	$	reduce 2
28	$\$0\ E$	$	goto 1

SLR(1) Parsing Example (Continued)

1. $E \rightarrow E + T$ | 2. $E \rightarrow T$
3. $T \rightarrow T * F$ | 4. $T \rightarrow F$
5. $F \rightarrow (E)$ | 6. $F \rightarrow$ `id`

State	Action						Goto		
	id	+	*	(	)	$	E	T	F
0	$s5$			$s4$			$c1$	$c2$	$c3$
1		$s6$				acc			
2		$r2$	$s7$		$r2$	$r2$			
3		$r4$	$r4$		$r4$	$r4$			
4	$s5$			$s4$			$c8$	$c2$	$c3$
5		$r6$	$r6$		$r6$	$r6$			
6	$s5$			$s4$				$c9$	$c3$
7	$s5$			$s4$					$c10$
8		$s6$			$s11$				
9		$r1$	$s7$		$r1$	$r1$			
10		$r3$	$r3$		$r3$	$r3$			
11		$r5$	$r5$		$r5$	$r5$			

Step	Stack $\rightarrow$	Input	Action
16	$\$0\;T\;2 * 7\;(4\;E\;8 + 6$	id)$	shift 5
17	$\$0\;T\;2 * 7\;(4\;E\;8 + 6\;\text{id}\;5$	)$	reduce 6
18	$\$0\;T\;2 * 7\;(4\;E\;8 + 6\;F$	)$	goto 3
19	$\$0\;T\;2 * 7\;(4\;E\;8 + 6\;F\;3$	)$	reduce 4
20	$\$0\;T\;2 * 7\;(4\;E\;8 + 6\;T$	)$	goto 9
20	$\$0\;T\;2 * 7\;(4\;E\;8 + 6\;T\;9$	)$	reduce 1
21	$\$0\;T\;2 * 7\;(4\;E$	)$	goto 8
22	$\$0\;T\;2 * 7\;(4\;E\;8$	)$	shift 11
23	$\$0\;T\;2 * 7\;(4\;E\;8)\;11$	$	reduce 5
24	$\$0\;T\;2 * 7\;F$	$	goto 10
25	$\$0\;T\;2 * 7\;F\;10$	$	reduce 3
26	$\$0\;T$	$	goto 2
27	$\$0\;T\;2$	$	reduce 2
28	$\$0\;E$	$	goto 1
29	$\$0\;E\;1$	$	accept

SLR(1) Parsing

SLR(1) Parsing

parsing-slides-sanyal-part3.pdf

Shift Reduce Parsing: From Intuitions to Formal Algorithms

We undertake this journey in six steps using the ambiguous grammar of expressions. It illustrates how yacc allows disambiguating a grammar without rewriting it

$$\begin{array}{lcl} E & \rightarrow & E + E \\ E & \rightarrow & E * E \\ E & \rightarrow & \text{id} \end{array}$$

1. We assume that both $+$ and $*$ are left associative and $*$ takes precedence over $+$

   We see the influence of these choices on derivations by considering four inputs

   $\text{id} + \text{id} + \text{id}$, $\text{id} * \text{id} * \text{id}$, $\text{id} + \text{id} * \text{id}$, and $\text{id} * \text{id} + \text{id}$.
2. We see the meaning of a shift reduce parser tracing the rightmost derivation in reverse

   We see the meaning of handle pruning in tracing the rightmost derivation
3. We define the notions of viable prefixes for discovering handles
4. We define valid items to recognize viable prefixes
5. We define FOLLOW sets to define a criterion of handle pruning
6. We see the algorithm that constructs valid items

Shift Reduce Parsing: From Intuitions to Formal Algorithms

1 $E \rightarrow E + E$
2 $E \rightarrow E * E$
3 $E \rightarrow \text{id}$

Shift Reduce Parsing: From Intuitions to Formal Algorithms

1 $E \rightarrow E + E$
2 $E \rightarrow E * E$
3 $E \rightarrow \text{id}$

Step	Stack $\rightarrow$	Input	Action
1	\$	id + id ∗ id\$	shift
2	\$id	+ id ∗ id\$	reduce by 3
3	\$$E$	+ id ∗ id\$	shift
4	\$$E$ +	id ∗ id\$	shift
5	\$$E$ + id	∗ id\$	reduce by 3
6	\$$E + E$	∗ id\$	shift
7	\$$E + E$ ∗	id\$	shift
8	\$$E + E$ ∗ id	\$	reduce by 3
9	\$$E + E * E$	\$	reduce by 2
10	\$$E + E$	\$	reduce by 1
11	\$$E$	\$	accept

IIT Bombay
cs302: Implementation of Programming Languages

Topic:
Syntax Analysis

Section:
Grammars, Derivations, and Parse Trees
Shift Reduce Parsing
SLR(1) Parsing
Conceptual Issues in Parsing
CLR(1) Parsing
LALR(1) Parsing

Shift Reduce Parsing: From Intuitions to Formal Algorithms

1 $E \rightarrow E + E$
2 $E \rightarrow E * E$
3 $E \rightarrow \text{id}$

Shift reduce conflicts resolved using precedence and associativity

	id	$+$	$*$	\$	E
0	$s2$				$c1$
1		$s3$	$s4$	acc	
2		$r3$	$r3$	$r3$	
3	$s2$				$c5$
4	$s2$				$c6$
5		$\cancel{s3}/r1$	$s4/\cancel{r1}$	$r1$	
6		$\cancel{s3}/r2$	$\cancel{s4}/r2$	$r2$	

Step	Stack $\rightarrow$	Input	Action
1	\$	id $+$ id $*$ id\$	shift
2	\$id	$+$ id $*$ id\$	reduce by 3
3	\$$E$	$+$ id $*$ id\$	shift
4	\$$E$ $+$	id $*$ id\$	shift
5	\$$E$ $+$ id	$*$ id\$	reduce by 3
6	\$$E + E$	$*$ id\$	shift
7	\$$E + E *$	id\$	shift
8	\$$E + E *$ id	\$	reduce by 3
9	\$$E + E * E$	\$	reduce by 2
10	\$$E + E$	\$	reduce by 1
11	\$$E$	\$	accept

Step	Stack $\rightarrow$	Input	Action
1	\$0	id $+$ id $*$ id\$	$s2$
2	\$0 id 2	$+$ id $*$ id\$	$r3$ and $c1$
3	\$0 E 1	$+$ id $*$ id\$	$s3$
4	\$0 E 1 $+$ 3	id $*$ id\$	$s2$
5	\$0 E 1 $+$ 3 id 2	$*$ id\$	$r3$ and $c5$
6	\$0 E 1 $+$ 3 E 5	$*$ id\$	$s4$
7	\$0 E 1 $+$ 3 E 5 $*$ 4	id\$	$s2$
8	\$0 E 1 $+$ 3 E 5 $*$ 4 id 2	\$	$r3$ and $c6$
9	\$0 E 1 $+$ 3 E 5 $*$ 4 E6	\$	$r2$ and $c5$
10	\$0 E 1 $+$ 3 E 5	\$	$r1$ and $c1$
11	\$0 E 1	\$	$accept$

Shift Reduce Parsing: From Intuitions to Formal Algorithms

1 $E \rightarrow E + E$
2 $E \rightarrow E * E$
3 $E \rightarrow \text{id}$

Combining the reduce and the following cover operation into a single step for convenience

	id	$+$	$*$	\$	E
	s2				$c1$
		$s3$	$s4$	acc	
		$r3$	$r3$	$r3$	
					$c5$
	2				$c6$
5		$\cancel{s3}/r1$	$s4/\cancel{r1}$	$r1$	
6		$\cancel{s3}/r2$	$\cancel{s4}/r2$	$r2$	

Step	Stack →	Input	Action
1	\$	id + id ∗ id\$	shift
2	\$id	+ id ∗ id\$	reduce by 3
3	\$$E$	+ id ∗ id\$	shift
4	\$$E$ +	id ∗ id\$	shift
5	\$$E$ + id	∗ id\$	reduce by 3
6	\$$E + E$	∗ id\$	shift
7	\$$E + E$ ∗	id\$	shift
8	\$$E + E$ ∗ id	\$	reduce by 3
9	\$$E + E * E$	\$	reduce by 2
10	\$$E + E$	\$	reduce by 1
11	\$$E$	\$	accept

Step	Stack →	Input	Action
1	\$0	id + id ∗ id\$	$s2$
2	\$0 id 2	+ id ∗ id\$	$r3$ and $c1$
3	\$0 E 1	+ id ∗ id\$	$s3$
4	\$0 E 1 + 3	id ∗ id\$	$s2$
5	\$0 E 1 + 3 id 2	∗ id\$	$r3$ and $c5$
6	\$0 E 1 + 3 E 5	∗ id\$	$s4$
7	\$0 E 1 + 3 E 5 ∗ 4	id\$	$s2$
8	\$0 E 1 + 3 E 5 ∗ 4 id 2	\$	$r3$ and $c6$
9	\$0 E 1 + 3 E 5 ∗ 4 E 6	\$	$r2$ and $c5$
10	\$0 E 1 + 3 E 5	\$	$r1$ and $c1$
11	\$0 E 1	\$	$accept$

Shift Reduce Parsing: From Intuitions to Formal Algorithms

1 $E \rightarrow E + E$
2 $E \rightarrow E * E$
3 $E \rightarrow \text{id}$

How do we make this journey?

	id	$+$	$*$	\$	E
0	$s2$				$c1$
1		$s3$	$s4$	acc	
2		$r3$	$r3$	$r3$	
3	$s2$				$c5$
4	$s2$				$c6$
5		$\xcancel{s3}/r1$	$s4/\xcancel{r1}$	$r1$	
6		$\xcancel{s3}/r2$	$\xcancel{s4}/r2$	$r2$	

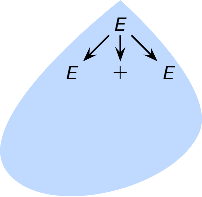
Step	Stack $\rightarrow$	Input	Action
1	\$	$\text{id} + \text{id} * \text{id}\$$	shift
2	\$id	$+ \text{id} * \text{id}\$$	reduce by 3
3	$\$E$	$+ \text{id} * \text{id}\$$	shift
4	$\$E +$	$\text{id} * \text{id}\$$	shift
5	$\$E + \text{id}$	$* \text{id}\$$	reduce by 3
6	$\$E + E$	$* \text{id}\$$	shift
7	$\$E + E *$	$\text{id}\$$	shift
8	$\$E + E * \text{id}$	\$	reduce by 3
9	$\$E + E * E$	\$	reduce by 2
10	$\$E + E$	\$	reduce by 1
11	$\$E$	\$	accept

Step	Stack $\rightarrow$	Input	Action
1	$\$0$	$\text{id} + \text{id} * \text{id}\$$	$s2$
2	$\$0\ \text{id}\ 2$	$+ \text{id} * \text{id}\$$	$r3$ and $c1$
3	$\$0\ E\ 1$	$+ \text{id} * \text{id}\$$	$s3$
4	$\$0\ E\ 1 + 3$	$\text{id} * \text{id}\$$	$s2$
5	$\$0\ E\ 1 + 3\ \text{id}\ 2$	$* \text{id}\$$	$r3$ and $c5$
6	$\$0\ E\ 1 + 3\ E\ 5$	$* \text{id}\$$	$s4$
7	$\$0\ E\ 1 + 3\ E\ 5 * 4$	$\text{id}\$$	$s2$
8	$\$0\ E\ 1 + 3\ E\ 5 * 4\ \text{id}\ 2$	\$	$r3$ and $c6$
9	$\$0\ E\ 1 + 3\ E\ 5 * 4\ E6$	\$	$r2$ and $c5$
10	$\$0\ E\ 1 + 3\ E\ 5$	\$	$r1$ and $c1$
11	$\$0\ E\ 1$	\$	$accept$

Step 1: Precedence and Associativity Rule Out Undesirable Derivations

Input id + id + id

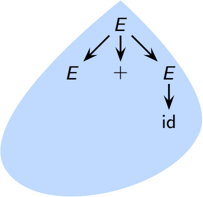
$$E \overset{rm}{\Rightarrow} E + E$$



Step 1: Precedence and Associativity Rule Out Undesirable Derivations

Input id + id + id

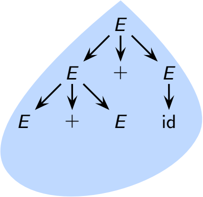
$$\begin{aligned} E &\overset{rm}{\Rightarrow} E + E \\ &\overset{rm}{\Rightarrow} E + \text{id} \end{aligned}$$



Step 1: Precedence and Associativity Rule Out Undesirable Derivations

Input id + id + id

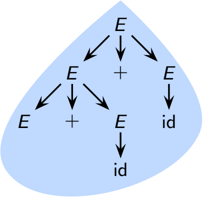
$$\begin{aligned} E &\overset{rm}{\Rightarrow} E + E \\ &\overset{rm}{\Rightarrow} E + \text{id} \\ &\overset{rm}{\Rightarrow} E + E + \text{id} \end{aligned}$$



Step 1: Precedence and Associativity Rule Out Undesirable Derivations

Input id + id + id

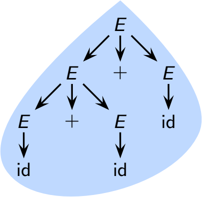
$$
\begin{aligned}
E &\overset{rm}{\Rightarrow} E + E\\
&\overset{rm}{\Rightarrow} E + \text{id}\\
&\overset{rm}{\Rightarrow} E + E + \text{id}\\
&\overset{rm}{\Rightarrow} E + \text{id} + \text{id}
\end{aligned}
$$



Step 1: Precedence and Associativity Rule Out Undesirable Derivations

Input id + id + id

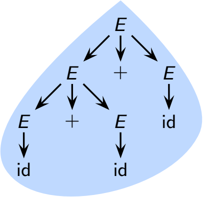
$$
\begin{aligned}
E &\overset{rm}{\Rightarrow} E + E \\
&\overset{rm}{\Rightarrow} E + \text{id} \\
&\overset{rm}{\Rightarrow} E + E + \text{id} \\
&\overset{rm}{\Rightarrow} E + \text{id} + \text{id} \\
&\overset{rm}{\Rightarrow} \text{id} + \text{id} + \text{id}
\end{aligned}
$$



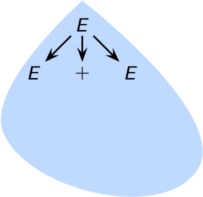
Step 1: Precedence and Associativity Rule Out Undesirable Derivations

Input id + id + id

$$
\begin{aligned}
E &\overset{rm}{\Rightarrow} E + E \\
&\overset{rm}{\Rightarrow} E + \text{id} \\
&\overset{rm}{\Rightarrow} E + E + \text{id} \\
&\overset{rm}{\Rightarrow} E + \text{id} + \text{id} \\
&\overset{rm}{\Rightarrow} \text{id} + \text{id} + \text{id}
\end{aligned}
$$



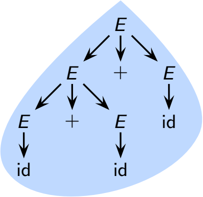
$$E \overset{rm}{\Rightarrow} E + E$$



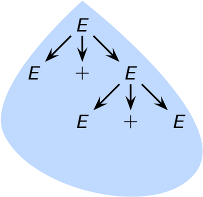
Step 1: Precedence and Associativity Rule Out Undesirable Derivations

Input id + id + id

$$\begin{aligned} E &\overset{rm}{\Rightarrow} E + E \\ &\overset{rm}{\Rightarrow} E + \text{id} \\ &\overset{rm}{\Rightarrow} E + E + \text{id} \\ &\overset{rm}{\Rightarrow} E + \text{id} + \text{id} \\ &\overset{rm}{\Rightarrow} \text{id} + \text{id} + \text{id} \end{aligned}$$



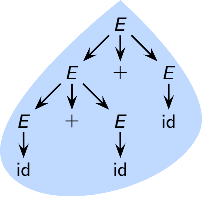
$$\begin{aligned} E &\overset{rm}{\Rightarrow} E + E \\ &\overset{rm}{\Rightarrow} E + E + E \end{aligned}$$



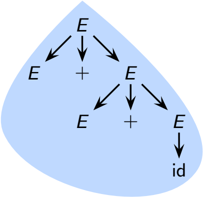
Step 1: Precedence and Associativity Rule Out Undesirable Derivations

Input id + id + id

$$\begin{aligned} E &\overset{rm}{\Rightarrow} E + E \\ &\overset{rm}{\Rightarrow} E + \text{id} \\ &\overset{rm}{\Rightarrow} E + E + \text{id} \\ &\overset{rm}{\Rightarrow} E + \text{id} + \text{id} \\ &\overset{rm}{\Rightarrow} \text{id} + \text{id} + \text{id} \end{aligned}$$



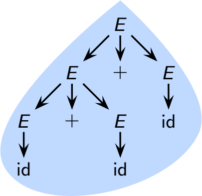
$$\begin{aligned} E &\overset{rm}{\Rightarrow} E + E \\ &\overset{rm}{\Rightarrow} E + E + E \\ &\overset{rm}{\Rightarrow} E + E + \text{id} \end{aligned}$$



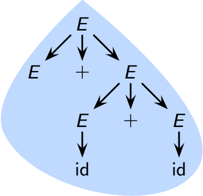
Step 1: Precedence and Associativity Rule Out Undesirable Derivations

$\boxed{\text{Input id} + \text{id} + \text{id}}$

$$
\begin{aligned}
E &\overset{rm}{\Rightarrow} E + E \\
&\overset{rm}{\Rightarrow} E + \text{id} \\
&\overset{rm}{\Rightarrow} E + E + \text{id} \\
&\overset{rm}{\Rightarrow} E + \text{id} + \text{id} \\
&\overset{rm}{\Rightarrow} \text{id} + \text{id} + \text{id}
\end{aligned}
$$



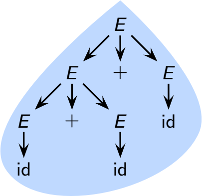
$$
\begin{aligned}
E &\overset{rm}{\Rightarrow} E + E \\
&\overset{rm}{\Rightarrow} E + E + E \\
&\overset{rm}{\Rightarrow} E + E + \text{id} \\
&\overset{rm}{\Rightarrow} E + \text{id} + \text{id}
\end{aligned}
$$



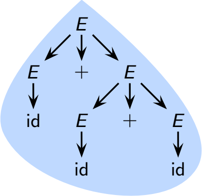
Step 1: Precedence and Associativity Rule Out Undesirable Derivations

Input id + id + id

$$
\begin{aligned}
E &\overset{rm}{\Rightarrow} E + E\\
&\overset{rm}{\Rightarrow} E + \text{id}\\
&\overset{rm}{\Rightarrow} E + E + \text{id}\\
&\overset{rm}{\Rightarrow} E + \text{id} + \text{id}\\
&\overset{rm}{\Rightarrow} \text{id} + \text{id} + \text{id}
\end{aligned}
$$



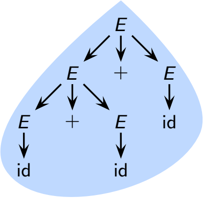
$$
\begin{aligned}
E &\overset{rm}{\Rightarrow} E + E\\
&\overset{rm}{\Rightarrow} E + E + E\\
&\overset{rm}{\Rightarrow} E + E + \text{id}\\
&\overset{rm}{\Rightarrow} E + \text{id} + \text{id}\\
&\overset{rm}{\Rightarrow} \text{id} + \text{id} + \text{id}
\end{aligned}
$$



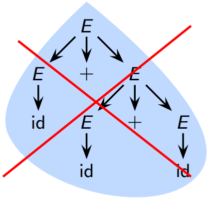
Step 1: Precedence and Associativity Rule Out Undesirable Derivations

Input id + id + id

$$\begin{aligned} E &\overset{rm}{\Rightarrow} E + E \\ &\overset{rm}{\Rightarrow} E + \text{id} \\ &\overset{rm}{\Rightarrow} E + E + \text{id} \\ &\overset{rm}{\Rightarrow} E + \text{id} + \text{id} \\ &\overset{rm}{\Rightarrow} \text{id} + \text{id} + \text{id} \end{aligned}$$



~~$E \overset{rm}{\Rightarrow} E + E$~~
~~$\overset{rm}{\Rightarrow} E + E + E$~~
~~$\overset{rm}{\Rightarrow} E + E + \text{id}$~~
~~$\overset{rm}{\Rightarrow} E + \text{id} + \text{id}$~~
~~$\overset{rm}{\Rightarrow} \text{id} + \text{id} + \text{id}$~~

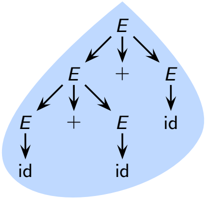


+ is left associative

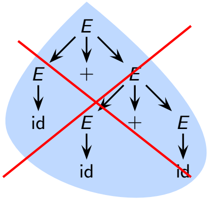
Step 1: Precedence and Associativity Rule Out Undesirable Derivations

Input id + id + id

$$\begin{aligned} E &\overset{rm}{\Rightarrow} E + E \\ &\overset{rm}{\Rightarrow} E + \text{id} \\ &\overset{rm}{\Rightarrow} E + E + \text{id} \\ &\overset{rm}{\Rightarrow} E + \text{id} + \text{id} \\ &\overset{rm}{\Rightarrow} \text{id} + \text{id} + \text{id} \end{aligned}$$



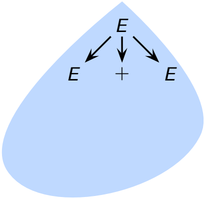
$$\xcancel{\begin{aligned} E &\overset{rm}{\Rightarrow} E + E \\ &\overset{rm}{\Rightarrow} E + E + E \\ &\overset{rm}{\Rightarrow} E + E + \text{id} \\ &\overset{rm}{\Rightarrow} E + \text{id} + \text{id} \\ &\overset{rm}{\Rightarrow} \text{id} + \text{id} + \text{id} \end{aligned}}$$



+ is left associative

Input id ∗ id + id

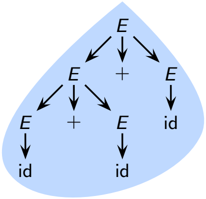
$$E \overset{rm}{\Rightarrow} E + E$$



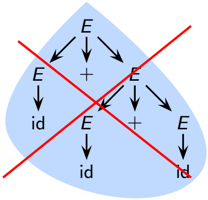
Step 1: Precedence and Associativity Rule Out Undesirable Derivations

Input id + id + id

$$
\begin{aligned}
E &\overset{rm}{\Rightarrow} E + E \\
&\overset{rm}{\Rightarrow} E + \text{id} \\
&\overset{rm}{\Rightarrow} E + E + \text{id} \\
&\overset{rm}{\Rightarrow} E + \text{id} + \text{id} \\
&\overset{rm}{\Rightarrow} \text{id} + \text{id} + \text{id}
\end{aligned}
$$



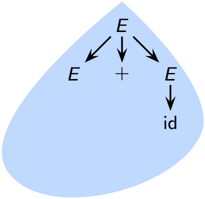
$$
\begin{aligned}
E &\overset{rm}{\Rightarrow} E + E \\
&\overset{rm}{\Rightarrow} E + E + E \\
&\overset{rm}{\Rightarrow} E + E + \text{id} \\
&\overset{rm}{\Rightarrow} E + \text{id} + \text{id} \\
&\overset{rm}{\Rightarrow} \text{id} + \text{id} + \text{id}
\end{aligned}
$$



+ is left associative

Input id ∗ id + id

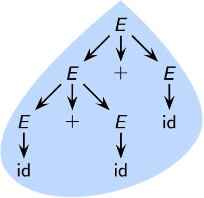
$$
\begin{aligned}
E &\overset{rm}{\Rightarrow} E + E \\
&\overset{rm}{\Rightarrow} E + \text{id}
\end{aligned}
$$



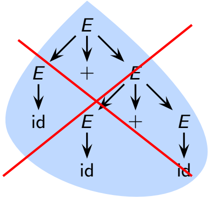
Step 1: Precedence and Associativity Rule Out Undesirable Derivations

Input id $+$ id $+$ id

$$\begin{array}{ll} E & \overset{rm}{\Rightarrow} E + E \\ & \overset{rm}{\Rightarrow} E + \text{id} \\ & \overset{rm}{\Rightarrow} E + E + \text{id} \\ & \overset{rm}{\Rightarrow} E + \text{id} + \text{id} \\ & \overset{rm}{\Rightarrow} \text{id} + \text{id} + \text{id} \end{array}$$



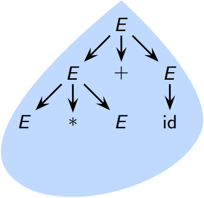
$$\xcancel{\begin{array}{ll} E & \overset{rm}{\Rightarrow} E + E \\ & \overset{rm}{\Rightarrow} E + E + E \\ & \overset{rm}{\Rightarrow} E + E + \text{id} \\ & \overset{rm}{\Rightarrow} E + \text{id} + \text{id} \\ & \overset{rm}{\Rightarrow} \text{id} + \text{id} + \text{id} \end{array}}$$



$+$ is left associative

Input id $*$ id $+$ id

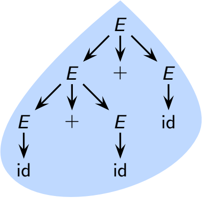
$$\begin{array}{ll} E & \overset{rm}{\Rightarrow} E + E \\ & \overset{rm}{\Rightarrow} E + \text{id} \\ & \overset{rm}{\Rightarrow} E * E + \text{id} \end{array}$$



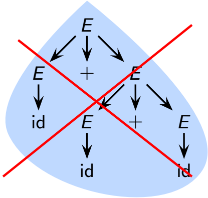
Step 1: Precedence and Associativity Rule Out Undesirable Derivations

Input id + id + id

$$\begin{aligned} E &\overset{rm}{\Rightarrow} E + E \\ &\overset{rm}{\Rightarrow} E + \mathrm{id} \\ &\overset{rm}{\Rightarrow} E + E + \mathrm{id} \\ &\overset{rm}{\Rightarrow} E + \mathrm{id} + \mathrm{id} \\ &\overset{rm}{\Rightarrow} \mathrm{id} + \mathrm{id} + \mathrm{id} \end{aligned}$$



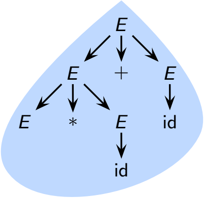
$$\xcancel{\begin{aligned} E &\overset{rm}{\Rightarrow} E + E \\ &\overset{rm}{\Rightarrow} E + E + E \\ &\overset{rm}{\Rightarrow} E + E + \mathrm{id} \\ &\overset{rm}{\Rightarrow} E + \mathrm{id} + \mathrm{id} \\ &\overset{rm}{\Rightarrow} \mathrm{id} + \mathrm{id} + \mathrm{id} \end{aligned}}$$



+ is left associative

Input id ∗ id + id

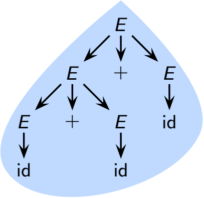
$$\begin{aligned} E &\overset{rm}{\Rightarrow} E + E \\ &\overset{rm}{\Rightarrow} E + \mathrm{id} \\ &\overset{rm}{\Rightarrow} E * E + \mathrm{id} \\ &\overset{rm}{\Rightarrow} E * \mathrm{id} + \mathrm{id} \end{aligned}$$



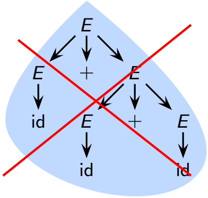
Step 1: Precedence and Associativity Rule Out Undesirable Derivations

$\boxed{\text{Input id} + \text{id} + \text{id}}$

$$\begin{aligned} E &\overset{rm}{\Rightarrow} E + E \\ &\overset{rm}{\Rightarrow} E + \text{id} \\ &\overset{rm}{\Rightarrow} E + E + \text{id} \\ &\overset{rm}{\Rightarrow} E + \text{id} + \text{id} \\ &\overset{rm}{\Rightarrow} \text{id} + \text{id} + \text{id} \end{aligned}$$



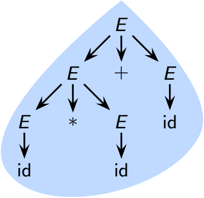
$$\xcancel{\begin{aligned} E &\overset{rm}{\Rightarrow} E + E \\ &\overset{rm}{\Rightarrow} E + E + E \\ &\overset{rm}{\Rightarrow} E + E + \text{id} \\ &\overset{rm}{\Rightarrow} E + \text{id} + \text{id} \\ &\overset{rm}{\Rightarrow} \text{id} + \text{id} + \text{id} \end{aligned}}$$



$\boxed{+ \text{ is left associative}}$

$\boxed{\text{Input id} * \text{id} + \text{id}}$

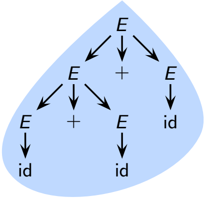
$$\begin{aligned} E &\overset{rm}{\Rightarrow} E + E \\ &\overset{rm}{\Rightarrow} E + \text{id} \\ &\overset{rm}{\Rightarrow} E * E + \text{id} \\ &\overset{rm}{\Rightarrow} E * \text{id} + \text{id} \\ &\overset{rm}{\Rightarrow} \text{id} * \text{id} + \text{id} \end{aligned}$$



Step 1: Precedence and Associativity Rule Out Undesirable Derivations

Input id + id + id

$$\begin{aligned} E &\overset{rm}{\Rightarrow} E + E \\ &\overset{rm}{\Rightarrow} E + \text{id} \\ &\overset{rm}{\Rightarrow} E + E + \text{id} \\ &\overset{rm}{\Rightarrow} E + \text{id} + \text{id} \\ &\overset{rm}{\Rightarrow} \text{id} + \text{id} + \text{id} \end{aligned}$$



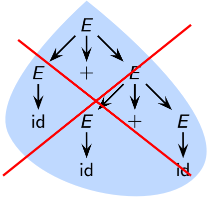
~~$E \overset{rm}{\Rightarrow} E + E$~~

~~$\overset{rm}{\Rightarrow} E + E + E$~~

~~$\overset{rm}{\Rightarrow} E + E + \text{id}$~~

~~$\overset{rm}{\Rightarrow} E + \text{id} + \text{id}$~~

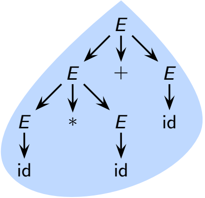
~~$\overset{rm}{\Rightarrow} \text{id} + \text{id} + \text{id}$~~



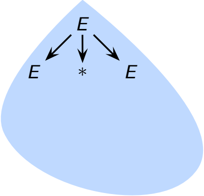
+ is left associative

Input id ∗ id + id

$$\begin{aligned} E &\overset{rm}{\Rightarrow} E + E \\ &\overset{rm}{\Rightarrow} E + \text{id} \\ &\overset{rm}{\Rightarrow} E * E + \text{id} \\ &\overset{rm}{\Rightarrow} E * \text{id} + \text{id} \\ &\overset{rm}{\Rightarrow} \text{id} * \text{id} + \text{id} \end{aligned}$$



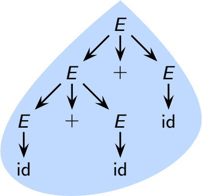
$$E \overset{rm}{\Rightarrow} E * E$$



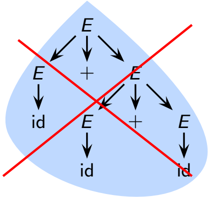
Step 1: Precedence and Associativity Rule Out Undesirable Derivations

Input id + id + id

$$\begin{aligned} E &\overset{rm}{\Rightarrow} E + E \\ &\overset{rm}{\Rightarrow} E + \text{id} \\ &\overset{rm}{\Rightarrow} E + E + \text{id} \\ &\overset{rm}{\Rightarrow} E + \text{id} + \text{id} \\ &\overset{rm}{\Rightarrow} \text{id} + \text{id} + \text{id} \end{aligned}$$



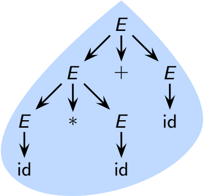
$$\begin{aligned} E &\overset{rm}{\Rightarrow} E + E \\ &\overset{rm}{\Rightarrow} E + E + E \\ &\overset{rm}{\Rightarrow} E + E + \text{id} \\ &\overset{rm}{\Rightarrow} E + \text{id} + \text{id} \\ &\overset{rm}{\Rightarrow} \text{id} + \text{id} + \text{id} \end{aligned}$$



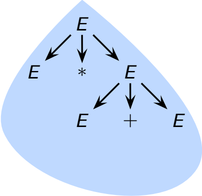
+ is left associative

Input id ∗ id + id

$$\begin{aligned} E &\overset{rm}{\Rightarrow} E + E \\ &\overset{rm}{\Rightarrow} E + \text{id} \\ &\overset{rm}{\Rightarrow} E * E + \text{id} \\ &\overset{rm}{\Rightarrow} E * \text{id} + \text{id} \\ &\overset{rm}{\Rightarrow} \text{id} * \text{id} + \text{id} \end{aligned}$$



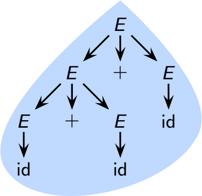
$$\begin{aligned} E &\overset{rm}{\Rightarrow} E * E \\ &\overset{rm}{\Rightarrow} E * E + E \end{aligned}$$



Step 1: Precedence and Associativity Rule Out Undesirable Derivations

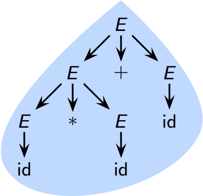
$\boxed{\text{Input id} + \text{id} + \text{id}}$

$$\begin{aligned} E &\overset{rm}{\Rightarrow} E + E \\ &\overset{rm}{\Rightarrow} E + \text{id} \\ &\overset{rm}{\Rightarrow} E + E + \text{id} \\ &\overset{rm}{\Rightarrow} E + \text{id} + \text{id} \\ &\overset{rm}{\Rightarrow} \text{id} + \text{id} + \text{id} \end{aligned}$$

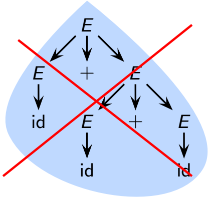


$\boxed{\text{Input id} * \text{id} + \text{id}}$

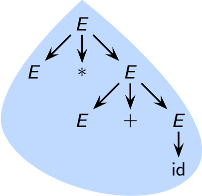
$$\begin{aligned} E &\overset{rm}{\Rightarrow} E + E \\ &\overset{rm}{\Rightarrow} E + \text{id} \\ &\overset{rm}{\Rightarrow} E * E + \text{id} \\ &\overset{rm}{\Rightarrow} E * \text{id} + \text{id} \\ &\overset{rm}{\Rightarrow} \text{id} * \text{id} + \text{id} \end{aligned}$$



$$\cancel{\begin{aligned} E &\overset{rm}{\Rightarrow} E + E \\ &\overset{rm}{\Rightarrow} E + E + E \\ &\overset{rm}{\Rightarrow} E + E + \text{id} \\ &\overset{rm}{\Rightarrow} E + \text{id} + \text{id} \\ &\overset{rm}{\Rightarrow} \text{id} + \text{id} + \text{id} \end{aligned}}$$



$$\begin{aligned} E &\overset{rm}{\Rightarrow} E * E \\ &\overset{rm}{\Rightarrow} E * E + E \\ &\overset{rm}{\Rightarrow} E * E + \text{id} \end{aligned}$$

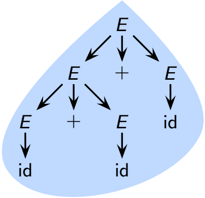


$\boxed{+ \text{ is left associative}}$

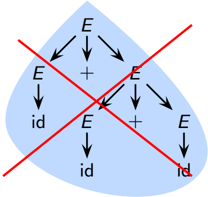
Step 1: Precedence and Associativity Rule Out Undesirable Derivations

Input id + id + id

$$\begin{aligned} E &\overset{rm}{\Rightarrow} E + E \\ &\overset{rm}{\Rightarrow} E + \text{id} \\ &\overset{rm}{\Rightarrow} E + E + \text{id} \\ &\overset{rm}{\Rightarrow} E + \text{id} + \text{id} \\ &\overset{rm}{\Rightarrow} \text{id} + \text{id} + \text{id} \end{aligned}$$



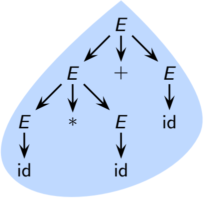
~~$E \overset{rm}{\Rightarrow} E + E$~~
~~$\overset{rm}{\Rightarrow} E + E + E$~~
~~$\overset{rm}{\Rightarrow} E + E + \text{id}$~~
~~$\overset{rm}{\Rightarrow} E + \text{id} + \text{id}$~~
~~$\overset{rm}{\Rightarrow} \text{id} + \text{id} + \text{id}$~~



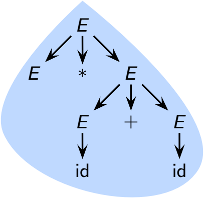
+ is left associative

Input id ∗ id + id

$$\begin{aligned} E &\overset{rm}{\Rightarrow} E + E \\ &\overset{rm}{\Rightarrow} E + \text{id} \\ &\overset{rm}{\Rightarrow} E * E + \text{id} \\ &\overset{rm}{\Rightarrow} E * \text{id} + \text{id} \\ &\overset{rm}{\Rightarrow} \text{id} * \text{id} + \text{id} \end{aligned}$$



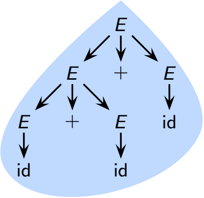
$$\begin{aligned} E &\overset{rm}{\Rightarrow} E * E \\ &\overset{rm}{\Rightarrow} E * E + E \\ &\overset{rm}{\Rightarrow} E * E + \text{id} \\ &\overset{rm}{\Rightarrow} E * \text{id} + \text{id} \end{aligned}$$



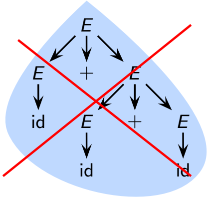
Step 1: Precedence and Associativity Rule Out Undesirable Derivations

$\boxed{\text{Input id} + \text{id} + \text{id}}$

$$\begin{aligned} E &\overset{rm}{\Rightarrow} E + E \\ &\overset{rm}{\Rightarrow} E + \text{id} \\ &\overset{rm}{\Rightarrow} E + E + \text{id} \\ &\overset{rm}{\Rightarrow} E + \text{id} + \text{id} \\ &\overset{rm}{\Rightarrow} \text{id} + \text{id} + \text{id} \end{aligned}$$



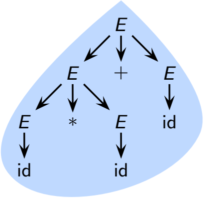
$$\xcancel{\begin{aligned} E &\overset{rm}{\Rightarrow} E + E \\ &\overset{rm}{\Rightarrow} E + E + E \\ &\overset{rm}{\Rightarrow} E + E + \text{id} \\ &\overset{rm}{\Rightarrow} E + \text{id} + \text{id} \\ &\overset{rm}{\Rightarrow} \text{id} + \text{id} + \text{id} \end{aligned}}$$



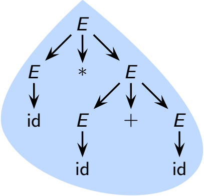
$\boxed{+ \text{ is left associative}}$

$\boxed{\text{Input id} * \text{id} + \text{id}}$

$$\begin{aligned} E &\overset{rm}{\Rightarrow} E + E \\ &\overset{rm}{\Rightarrow} E + \text{id} \\ &\overset{rm}{\Rightarrow} E * E + \text{id} \\ &\overset{rm}{\Rightarrow} E * \text{id} + \text{id} \\ &\overset{rm}{\Rightarrow} \text{id} * \text{id} + \text{id} \end{aligned}$$



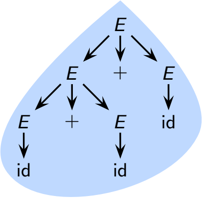
$$\begin{aligned} E &\overset{rm}{\Rightarrow} E * E \\ &\overset{rm}{\Rightarrow} E * E + E \\ &\overset{rm}{\Rightarrow} E * E + \text{id} \\ &\overset{rm}{\Rightarrow} E * \text{id} + \text{id} \\ &\overset{rm}{\Rightarrow} \text{id} * \text{id} + \text{id} \end{aligned}$$



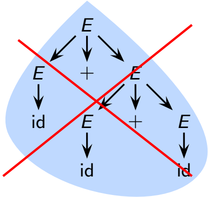
Step 1: Precedence and Associativity Rule Out Undesirable Derivations

Input id + id + id

$$\begin{aligned} E &\overset{rm}{\Rightarrow} E + E \\ &\overset{rm}{\Rightarrow} E + \text{id} \\ &\overset{rm}{\Rightarrow} E + E + \text{id} \\ &\overset{rm}{\Rightarrow} E + \text{id} + \text{id} \\ &\overset{rm}{\Rightarrow} \text{id} + \text{id} + \text{id} \end{aligned}$$



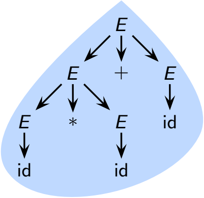
$$\xcancel{\begin{aligned} E &\overset{rm}{\Rightarrow} E + E \\ &\overset{rm}{\Rightarrow} E + E + E \\ &\overset{rm}{\Rightarrow} E + E + \text{id} \\ &\overset{rm}{\Rightarrow} E + \text{id} + \text{id} \\ &\overset{rm}{\Rightarrow} \text{id} + \text{id} + \text{id} \end{aligned}}$$



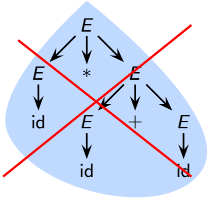
+ is left associative

Input id $*$ id + id

$$\begin{aligned} E &\overset{rm}{\Rightarrow} E + E \\ &\overset{rm}{\Rightarrow} E + \text{id} \\ &\overset{rm}{\Rightarrow} E * E + \text{id} \\ &\overset{rm}{\Rightarrow} E * \text{id} + \text{id} \\ &\overset{rm}{\Rightarrow} \text{id} * \text{id} + \text{id} \end{aligned}$$



$$\xcancel{\begin{aligned} E &\overset{rm}{\Rightarrow} E * E \\ &\overset{rm}{\Rightarrow} E * E + E \\ &\overset{rm}{\Rightarrow} E * E + \text{id} \\ &\overset{rm}{\Rightarrow} E * \text{id} + \text{id} \\ &\overset{rm}{\Rightarrow} \text{id} * \text{id} + \text{id} \end{aligned}}$$



$*$ has a higher precedence than +

Step 1: Precedence and Associativity Rule Out Undesirable Derivations

Input id + id + id

Input id $*$ id + id

$E \xrightarrow{rm} E + E$

$E \xrightarrow{rm} E + E$

The moral of the story

- Right sentential forms containing the strings $E + E + E$, $E * E * E$, and $E * E + E$ are ruled out by our choice of precedence and associativity
- The grouping that we want is $(E + E) + E$, $(E * E) * E$, and $(E * E) + E$ so the non-terminals in the parenthesis should be derived first
- However, the parenthesized term does not occur in the rightmost position and hence it cannot be derived first in a rightmost derivation
- The string $E + E * E$ can appear in a rightmost derivation because the grouping is $E + (E * E)$ and the parenthesized term occurs in the rightmost position

+ is left associative

$*$ has a higher precedence than +

Step 2: Shift Reduce Actions, Rightmost Derivations, and Handles

$$E \overset{rm}{\Rightarrow} E + E \overset{rm}{\Rightarrow} E + E * E \overset{rm}{\Rightarrow} E + E * \text{id} \overset{rm}{\Rightarrow} E + \text{id} * \text{id} \overset{rm}{\Rightarrow} \text{id} + \text{id} * \text{id}$$

1 $E \rightarrow E + E$
2 $E \rightarrow E * E$
3 $E \rightarrow \text{id}$

Step	Stack $\rightarrow$	Input	Action
1	\$	$\text{id} + \text{id} * \text{id}\$$	shift
2	\$id	$+ \text{id} * \text{id}\$$	reduce by 3
3	$\$E$	$+ \text{id} * \text{id}\$$	shift
4	$\$E +$	$\text{id} * \text{id}\$$	shift
5	$\$E + \text{id}$	$* \text{id}\$$	reduce by 3
6	$\$E + E$	$* \text{id}\$$	shift
7	$\$E + E *$	id\$	shift
8	$\$E + E * \text{id}$	\$	reduce by 3
9	$\$E + E * E$	\$	reduce by 2
10	$\$E + E$	\$	reduce by 1
11	$\$E$	\$	accept

Step 2: Shift Reduce Actions, Rightmost Derivations, and Handles

$$E \overset{rm}{\Rightarrow} E+E \overset{rm}{\Rightarrow} E+E*E \overset{rm}{\Rightarrow} E+E*\text{id} \overset{rm}{\Rightarrow} E+\text{id}*\text{id} \overset{rm}{\Rightarrow} \boxed{\text{id}+\text{id}*\text{id}}$$

1 $E \rightarrow E+E$
2 $E \rightarrow E*E$
3 $E \rightarrow \text{id}$

Step	Stack $\rightarrow$	Input	Action
1	\$	$\text{id}+\text{id}*\text{id}\$$	shift
2	\$id	$+\text{id}*\text{id}\$$	reduce by 3
3	$\$E$	$+\text{id}*\text{id}\$$	shift
4	$\$E+$	$\text{id}*\text{id}\$$	shift
5	$\$E+\text{id}$	$*\text{id}\$$	reduce by 3
6	$\$E+E$	$*\text{id}\$$	shift
7	$\$E+E*$	id\$	shift
8	$\$E+E*\text{id}$	\$	reduce by 3
9	$\$E+E*E$	\$	reduce by 2
10	$\$E+E$	\$	reduce by 1
11	$\$E$	\$	accept

IIT Bombay
cs302: Implementation of Programming Languages

Topic:

Syntax Analysis

Section:

Grammars, Derivations, and Parse Trees

Shift Reduce Parsing

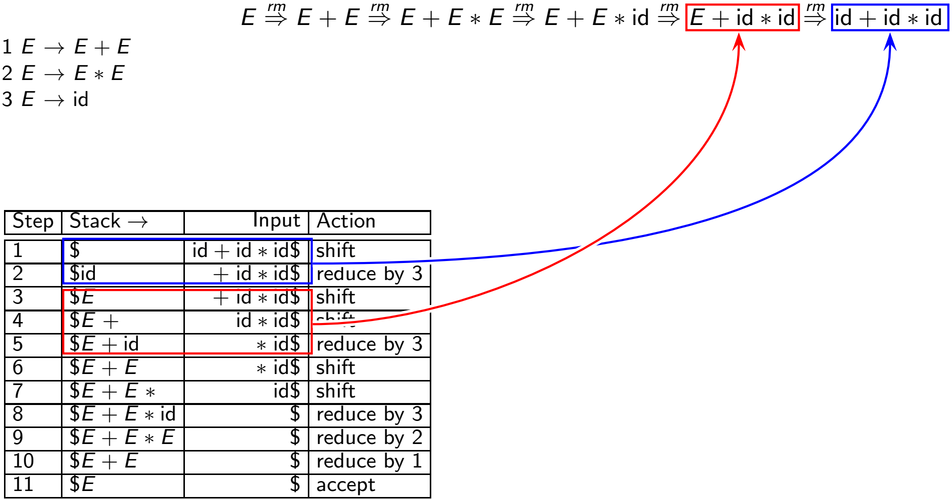
SLR(1) Parsing

Conceptual Issues in Parsing

CLR(1) Parsing

LALR(1) Parsing

Step 2: Shift Reduce Actions, Rightmost Derivations, and Handles

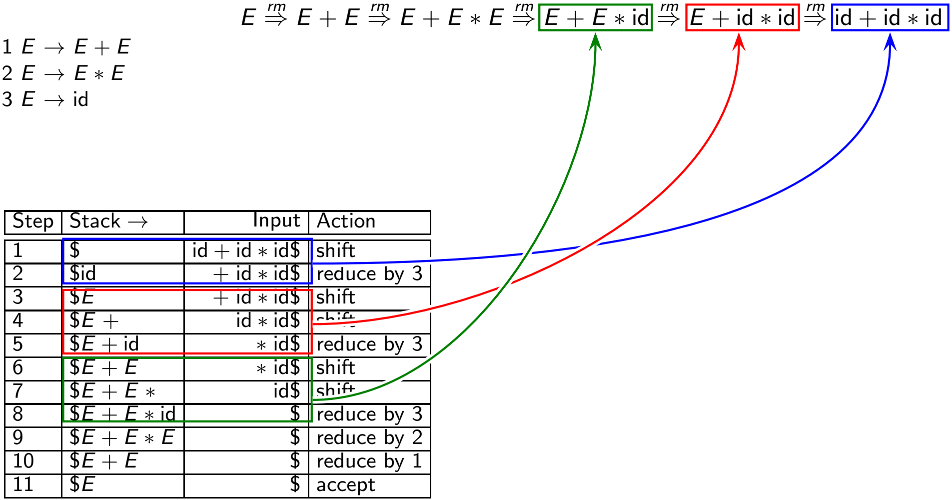


$$E \overset{rm}{\Rightarrow} E + E \overset{rm}{\Rightarrow} E + E * E \overset{rm}{\Rightarrow} E + E * \text{id} \overset{rm}{\Rightarrow} \boxed{E + \text{id} * \text{id}} \overset{rm}{\Rightarrow} \boxed{\text{id} + \text{id} * \text{id}}$$

1 $E \rightarrow E + E$
2 $E \rightarrow E * E$
3 $E \rightarrow \text{id}$

Step	Stack →	Input	Action
1	\$	id + id ∗ id\$	shift
2	\$id	+ id ∗ id\$	reduce by 3
3	$\$E$	+ id ∗ id\$	shift
4	$\$E +$	id ∗ id\$	shift
5	$\$E +$ id	∗ id\$	reduce by 3
6	$\$E + E$	∗ id\$	shift
7	$\$E + E *$	id\$	shift
8	$\$E + E *$ id	\$	reduce by 3
9	$\$E + E * E$	\$	reduce by 2
10	$\$E + E$	\$	reduce by 1
11	$\$E$	\$	accept

Step 2: Shift Reduce Actions, Rightmost Derivations, and Handles



$$E \overset{rm}{\Rightarrow} E + E \overset{rm}{\Rightarrow} E + E * E \overset{rm}{\Rightarrow} \boxed{E + E * \text{id}} \overset{rm}{\Rightarrow} \boxed{E + \text{id} * \text{id}} \overset{rm}{\Rightarrow} \boxed{\text{id} + \text{id} * \text{id}}$$

1 $E \rightarrow E + E$
2 $E \rightarrow E * E$
3 $E \rightarrow \text{id}$

Step	Stack →	Input	Action
1	\$	id + id ∗ id\$	shift
2	\$id	+ id ∗ id\$	reduce by 3
3	\$$E$	+ id ∗ id\$	shift
4	\$$E$ +	id ∗ id\$	shift
5	\$$E$ + id	∗ id\$	reduce by 3
6	\$$E$ + E	∗ id\$	shift
7	\$$E$ + E ∗	id\$	shift
8	\$$E$ + E ∗ id	\$	reduce by 3
9	\$$E$ + E ∗ E	\$	reduce by 2
10	\$$E$ + E	\$	reduce by 1
11	\$$E$	\$	accept

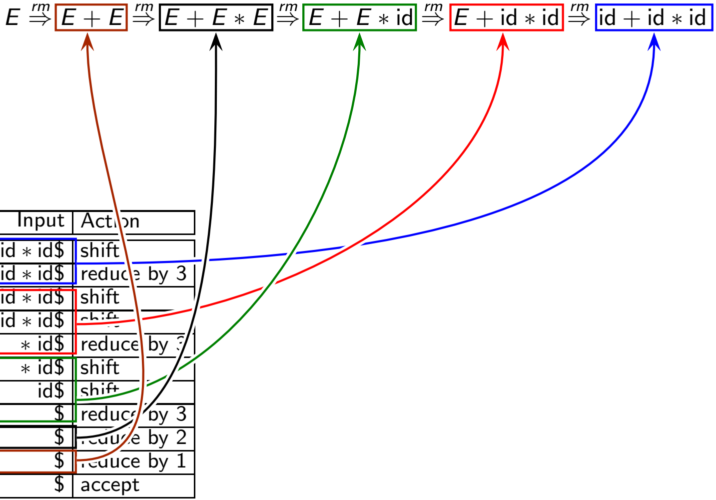
Step 2: Shift Reduce Actions, Rightmost Derivations, and Handles

$$E \overset{rm}{\Rightarrow} E+E \overset{rm}{\Rightarrow} \boxed{E+E*E} \overset{rm}{\Rightarrow} \boxed{E+E*\text{id}} \overset{rm}{\Rightarrow} \boxed{E+\text{id}*\text{id}} \overset{rm}{\Rightarrow} \boxed{\text{id}+\text{id}*\text{id}}$$

1 $E \rightarrow E+E$
2 $E \rightarrow E*E$
3 $E \rightarrow \text{id}$

Step	Stack $\rightarrow$	Input	Action
1	\$	id + id ∗ id\$	shift
2	\$id	+ id ∗ id\$	reduce by 3
3	\$$E$	+ id ∗ id\$	shift
4	\$$E$ +	id ∗ id\$	shift
5	\$$E$ + id	∗ id\$	reduce by 3
6	\$$E$ + E	∗ id\$	shift
7	\$$E$ + E ∗	id\$	shift
8	\$$E$ + E ∗ id	\$	reduce by 3
9	\$$E$ + E ∗ E	\$	reduce by 2
10	\$$E$ + E	\$	reduce by 1
11	\$$E$	\$	accept

Step 2: Shift Reduce Actions, Rightmost Derivations, and Handles



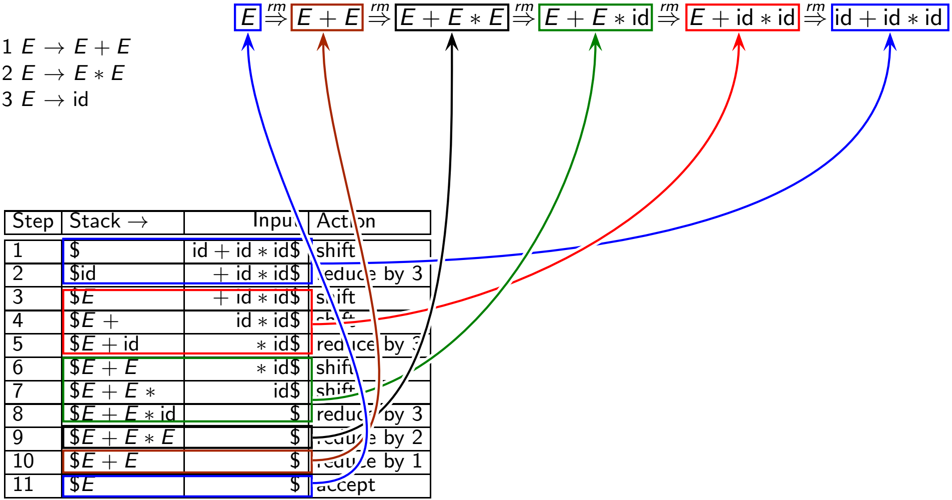
$$E \overset{rm}{\Rightarrow} \boxed{E+E} \overset{rm}{\Rightarrow} \boxed{E+E*E} \overset{rm}{\Rightarrow} \boxed{E+E*\text{id}} \overset{rm}{\Rightarrow} \boxed{E+\text{id}*\text{id}} \overset{rm}{\Rightarrow} \boxed{\text{id}+\text{id}*\text{id}}$$

1 $E \rightarrow E + E$
2 $E \rightarrow E * E$
3 $E \rightarrow \text{id}$

Step	Stack →	Input	Action
1	\$	id + id ∗ id\$	shift
2	\$id	+ id ∗ id\$	reduce by 3
3	\$$E$	+ id ∗ id\$	shift
4	\$$E$ +	id ∗ id\$	shift
5	\$$E$ + id	∗ id\$	reduce by 3
6	\$$E$ + E	∗ id\$	shift
7	\$$E$ + E ∗	id\$	shift
8	\$$E$ + E ∗ id	\$	reduce by 3
9	\$$E$ + E ∗ E	\$	reduce by 2
10	\$$E$ + E	\$	reduce by 1
11	\$$E$	\$	accept

Step 2: Shift Reduce Actions, Rightmost Derivations, and Handles

1 $E \rightarrow E + E$
2 $E \rightarrow E * E$
3 $E \rightarrow \text{id}$

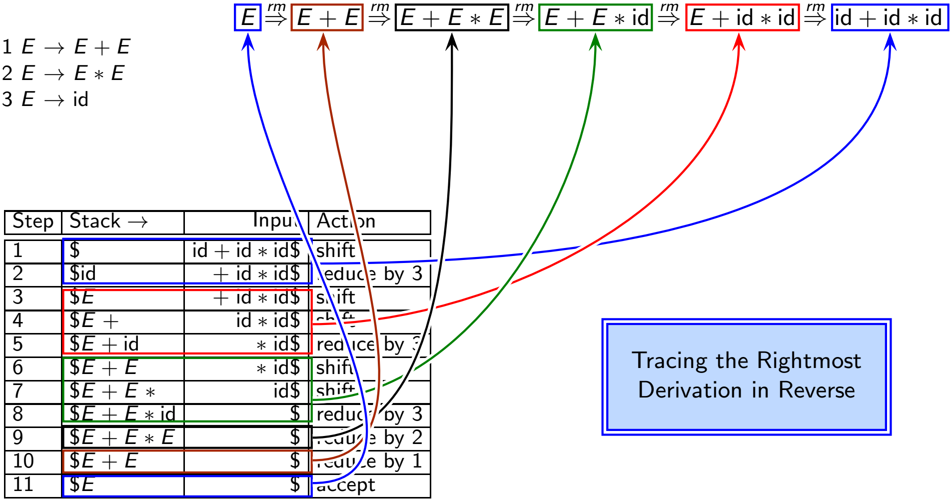


$$\boxed{E} \overset{rm}{\Rightarrow} \boxed{E + E} \overset{rm}{\Rightarrow} \boxed{E + E * E} \overset{rm}{\Rightarrow} \boxed{E + E * \text{id}} \overset{rm}{\Rightarrow} \boxed{E + \text{id} * \text{id}} \overset{rm}{\Rightarrow} \boxed{\text{id} + \text{id} * \text{id}}$$

Step	Stack →	Input	Action
1	\$	id + id ∗ id\$	shift
2	\$id	+ id ∗ id\$	reduce by 3
3	\$$E$	+ id ∗ id\$	shift
4	\$$E$ +	id ∗ id\$	shift
5	\$$E$ + id	∗ id\$	reduce by 3
6	\$$E$ + E	∗ id\$	shift
7	\$$E$ + E ∗	id\$	shift
8	\$$E$ + E ∗ id	\$	reduce by 3
9	\$$E$ + E ∗ E	\$	reduce by 2
10	\$$E$ + E	\$	reduce by 1
11	\$$E$	\$	accept

Step 2: Shift Reduce Actions, Rightmost Derivations, and Handles

1 $E \rightarrow E + E$
2 $E \rightarrow E * E$
3 $E \rightarrow \text{id}$



$$\boxed{E} \overset{rm}{\Rightarrow} \boxed{E+E} \overset{rm}{\Rightarrow} \boxed{E+E*E} \overset{rm}{\Rightarrow} \boxed{E+E*\text{id}} \overset{rm}{\Rightarrow} \boxed{E+\text{id}*\text{id}} \overset{rm}{\Rightarrow} \boxed{\text{id}+\text{id}*\text{id}}$$

Step	Stack →	Input	Action
1	\$	id + id ∗ id\$	shift
2	\$id	+ id ∗ id\$	reduce by 3
3	\$$E$	+ id ∗ id\$	shift
4	\$$E$ +	id ∗ id\$	shift
5	\$$E$ + id	∗ id\$	reduce by 3
6	\$$E$ + E	∗ id\$	shift
7	\$$E$ + E ∗	id\$	shift
8	\$$E$ + E ∗ id	\$	reduce by 3
9	\$$E$ + E ∗ E	\$	reduce by 2
10	\$$E$ + E	\$	reduce by 1
11	\$$E$	\$	accept

Tracing the Rightmost Derivation in Reverse

Step 2: Shift Reduce Actions, Rightmost Derivations, and Handles

$$E \overset{rm}{\Rightarrow} E + E \overset{rm}{\Rightarrow} E + E * E \overset{rm}{\Rightarrow} E + E * \text{id} \overset{rm}{\Rightarrow} E + \text{id} * \text{id} \overset{rm}{\Rightarrow} \text{id} + \text{id} * \text{id}$$

1 $E \rightarrow E + E$
2 $E \rightarrow E * E$
3 $E \rightarrow \text{id}$

Step	Stack →	Input	Action
1	\$	id + id ∗ id\$	shift
2	\$id	+ id ∗ id\$	reduce by 3
3	\$$E$	+ id ∗ id\$	shift
4	\$$E$ +	id ∗ id\$	shift
5	\$$E$ + id	∗ id\$	reduce by 3
6	\$$E + E$	∗ id\$	shift
7	\$$E + E$ ∗	id\$	shift
8	\$$E + E$ ∗ id	\$	reduce by 3
9	\$$E + E * E$	\$	reduce by 2
10	\$$E + E$	\$	reduce by 1
11	\$$E$	\$	accept

Rightmost derivations are traced in reverse by identifying handles in right sentential forms (beginning with the sentence) and pruning them for constructing the previous right sentential form

Step 2: Shift Reduce Actions, Rightmost Derivations, and Handles

$$E \overset{rm}{\Rightarrow} E + E \overset{rm}{\Rightarrow} E + E * E \overset{rm}{\Rightarrow} E + E * \text{id} \overset{rm}{\Rightarrow} \boxed{E} + \text{id} * \text{id} \overset{rm}{\Rightarrow} \boxed{\boxed{\text{id}} + \text{id} * \text{id}}$$

1 $E \rightarrow E + E$
2 $E \rightarrow E * E$
3 $E \rightarrow \text{id}$

Handle

Right Sentential Form

Step	Stack →	Input	Action
1	\$	id + id ∗ id\$	shift
2	\$id	+ id ∗ id\$	reduce by 3
3	\$$E$	+ id ∗ id\$	shift
4	\$$E$ +	id ∗ id\$	shift
5	\$$E$ + id	∗ id\$	reduce by 3
6	\$$E$ + E	∗ id\$	shift
7	\$$E$ + E ∗	id\$	shift
8	\$$E$ + E ∗ id	\$	reduce by 3
9	\$$E$ + E ∗ E	\$	reduce by 2
10	\$$E$ + E	\$	reduce by 1
11	\$$E$	\$	accept

Step 2: Shift Reduce Actions, Rightmost Derivations, and Handles

$$E \overset{rm}{\Rightarrow} E + E \overset{rm}{\Rightarrow} E + E * E \overset{rm}{\Rightarrow} E + \boxed{E} * \text{id} \overset{rm}{\Rightarrow} \boxed{E + \boxed{\text{id}} * \text{id}} \overset{rm}{\Rightarrow} \text{id} + \text{id} * \text{id}$$

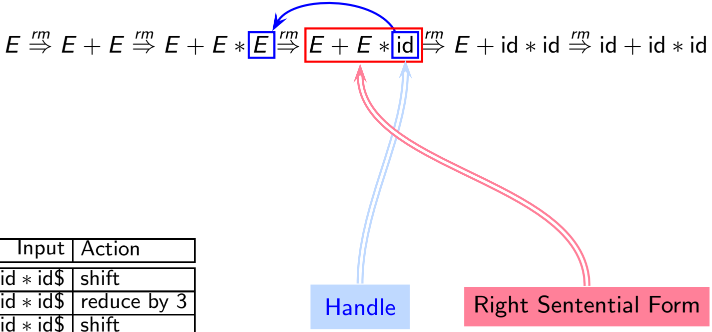
1 $E \rightarrow E + E$
2 $E \rightarrow E * E$
$\boxed{3\ E \rightarrow \text{id}}$

Handle

Right Sentential Form

Step	Stack →	Input	Action
1	\$	id + id ∗ id\$	shift
2	\$id	+ id ∗ id\$	reduce by 3
3	\$$E$	+ id ∗ id\$	shift
4	\$$E$ +	id ∗ id\$	shift
5	\$$E$ + id	∗ id\$	reduce by 3
6	\$$E + E$	∗ id\$	shift
7	\$$E + E$ ∗	id\$	shift
8	\$$E + E$ ∗ id	\$	reduce by 3
9	\$$E + E * E$	\$	reduce by 2
10	\$$E + E$	\$	reduce by 1
11	\$$E$	\$	accept

Step 2: Shift Reduce Actions, Rightmost Derivations, and Handles

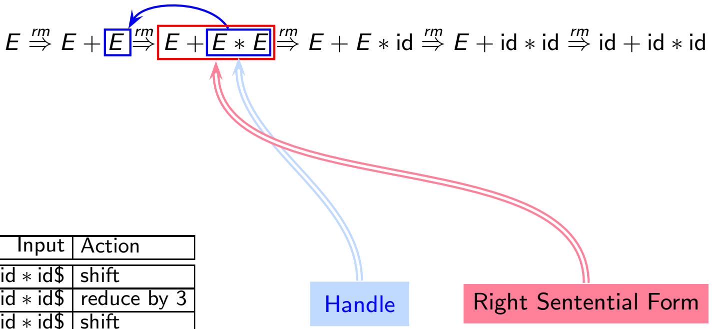


$$E \overset{rm}{\Rightarrow} E + E \overset{rm}{\Rightarrow} E + E * \boxed{E} \overset{rm}{\Rightarrow} \boxed{E + E * \boxed{\text{id}}} \overset{rm}{\Rightarrow} E + \text{id} * \text{id} \overset{rm}{\Rightarrow} \text{id} + \text{id} * \text{id}$$

1 $E \to E + E$
2 $E \to E * E$
$\boxed{3\ E \to \text{id}}$

Step	Stack →	Input	Action
1	\$	id + id ∗ id\$	shift
2	\$id	+ id ∗ id\$	reduce by 3
3	\$*E*	+ id ∗ id\$	shift
4	\$*E* +	id ∗ id\$	shift
5	\$*E* + id	∗ id\$	reduce by 3
6	\$*E* + *E*	∗ id\$	shift
7	\$*E* + *E* ∗	id\$	shift
8	\$*E* + *E* ∗ id	\$	reduce by 3
9	\$*E* + *E* ∗ *E*	\$	reduce by 2
10	\$*E* + *E*	\$	reduce by 1
11	\$*E*	\$	accept

Step 2: Shift Reduce Actions, Rightmost Derivations, and Handles



$$E \overset{rm}{\Rightarrow} E + \boxed{E} \overset{rm}{\Rightarrow} \boxed{E + \boxed{E * E}} \overset{rm}{\Rightarrow} E + E * \text{id} \overset{rm}{\Rightarrow} E + \text{id} * \text{id} \overset{rm}{\Rightarrow} \text{id} + \text{id} * \text{id}$$

Handle

Right Sentential Form

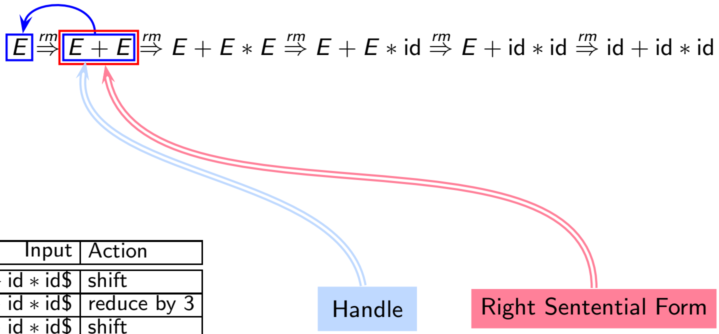
1 $E \rightarrow E + E$

$\boxed{2\ E \rightarrow E * E}$

3 $E \rightarrow \text{id}$

Step	Stack →	Input	Action
1	\$	id + id ∗ id\$	shift
2	\$id	+ id ∗ id\$	reduce by 3
3	\$$E$	+ id ∗ id\$	shift
4	\$$E$ +	id ∗ id\$	shift
5	\$$E$ + id	∗ id\$	reduce by 3
6	\$$E + E$	∗ id\$	shift
7	\$$E + E$ ∗	id\$	shift
8	\$$E + E$ ∗ id	\$	reduce by 3
9	\$$E + E * E$	\$	reduce by 2
10	\$$E + E$	\$	reduce by 1
11	\$$E$	\$	accept

Step 2: Shift Reduce Actions, Rightmost Derivations, and Handles



$$\boxed{E} \overset{rm}{\Rightarrow} \boxed{E+E} \overset{rm}{\Rightarrow} E+E*E \overset{rm}{\Rightarrow} E+E*\text{id} \overset{rm}{\Rightarrow} E+\text{id}*\text{id} \overset{rm}{\Rightarrow} \text{id}+\text{id}*\text{id}$$

Handle

Right Sentential Form

$\boxed{1\ E \rightarrow E+E}$
$2\ E \rightarrow E*E$
$3\ E \rightarrow \text{id}$

Step	Stack →	Input	Action
1	\$	id + id ∗ id\$	shift
2	\$id	+ id ∗ id\$	reduce by 3
3	\$$E$	+ id ∗ id\$	shift
4	\$$E$ +	id ∗ id\$	shift
5	\$$E$ + id	∗ id\$	reduce by 3
6	\$$E + E$	∗ id\$	shift
7	\$$E + E$ ∗	id\$	shift
8	\$$E + E$ ∗ id	\$	reduce by 3
9	\$$E + E * E$	\$	reduce by 2
10	\$$E + E$	\$	reduce by 1
11	\$$E$	\$	accept

Step 3: Identifying Handles in Right Sentential Forms

- Our goal is to discover a prefix of right sentential form that ends with a handle
- Viable Prefix. A prefix of a right sentential form that does not extend beyond the handle
    - It is either a string with no handle, or
    - a string that ends with the handle
- By suffixing appropriate symbols to a viable prefix of the first kind, we can create a viable prefix of the second kind
- By suffixing terminal symbols to the viable prefix of the second kind, we can create a right sentential form
- The set of viable prefixes forms a regular language, thus they can be recognized by a DFA
- The handles in a viable prefix can be identified using a stack
- We keep pushing viable prefixes on the stack until the handle appears on the top of the stack

Step 3: Viable Prefixes for Our Grammar (After Incorporating Precedences and Associativities)

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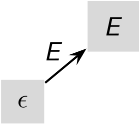
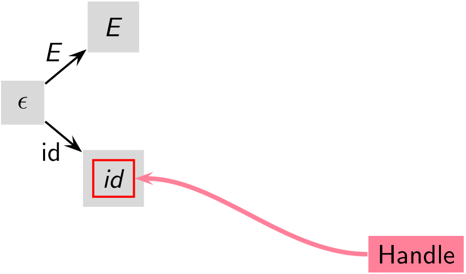
Step 3: Viable Prefixes for Our Grammar (After Incorporating Precedences and Associativities)

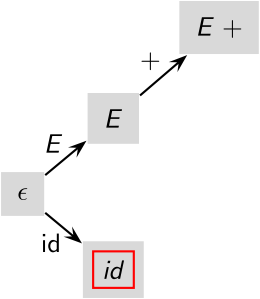


Step 3: Viable Prefixes for Our Grammar (After Incorporating Precedences and Associativities)

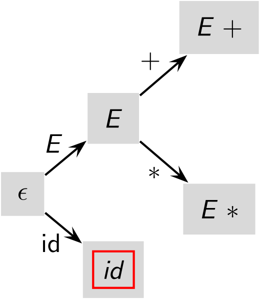


Viable prefix id must be reduced to E and no grammar symbol can be suffixed to it (because there is no rule with a symbol after id)

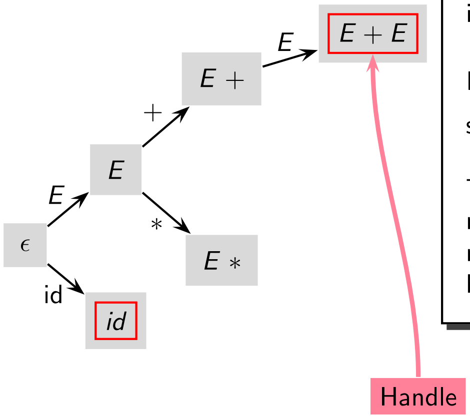
Step 3: Viable Prefixes for Our Grammar (After Incorporating Precedences and Associativities)



Step 3: Viable Prefixes for Our Grammar (After Incorporating Precedences and Associativities)



Step 3: Viable Prefixes for Our Grammar (After Incorporating Precedences and Associativities)

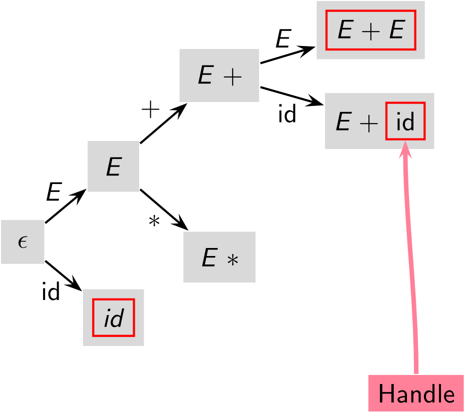


Viable prefix $E+E$ must be reduced to E if it is not followed by a "*"

If $E+E$ is followed by a "*", "*" should be shifted and $E+E$ should not be reduced

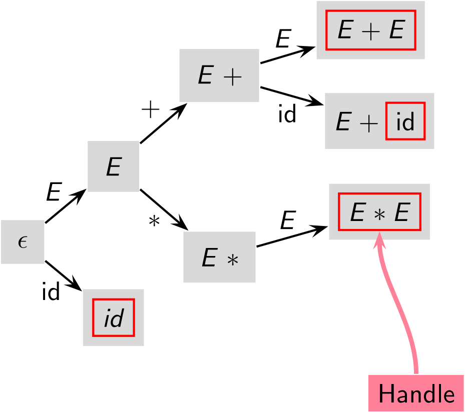
The occurrence of a potential handle does not mean it should be reduced, the next terminal symbol decides whether it is an actual handle (and if so, it should be reduced)

Step 3: Viable Prefixes for Our Grammar (After Incorporating Precedences and Associativities)



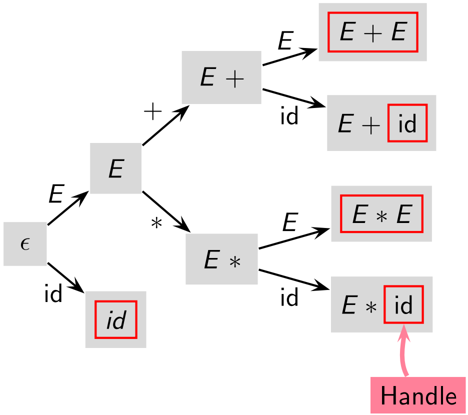
Viable prefix $E+\text{id}$ must be reduced to $E+E$ and no grammar symbol can be suffixed to it

Step 3: Viable Prefixes for Our Grammar (After Incorporating Precedences and Associativities)



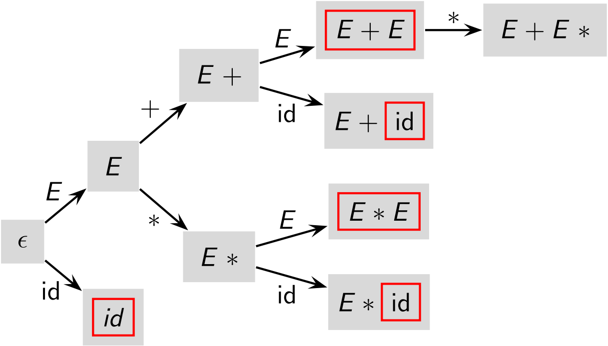
Viable prefix $E * E$ must be reduced to E and no grammar symbol can be suffixed to it

Step 3: Viable Prefixes for Our Grammar (After Incorporating Precedences and Associativities)

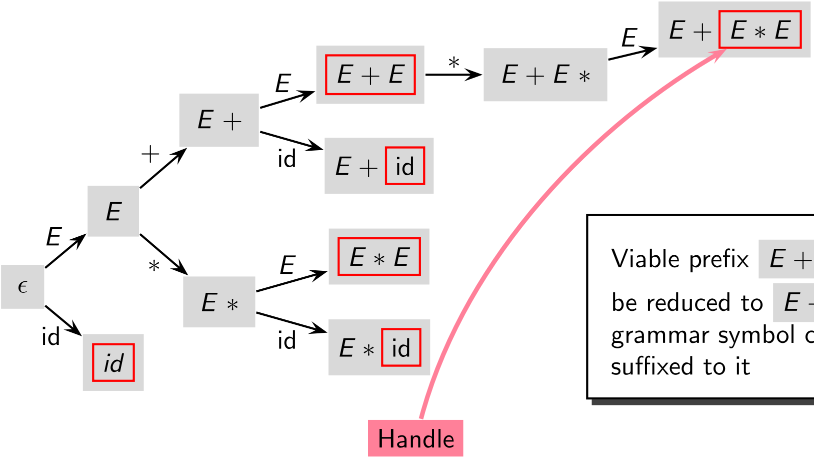


Viable prefix $E * \text{id}$ must be reduced to $E * E$ and no grammar symbol can be suffixed to it

Step 3: Viable Prefixes for Our Grammar (After Incorporating Precedences and Associativities)

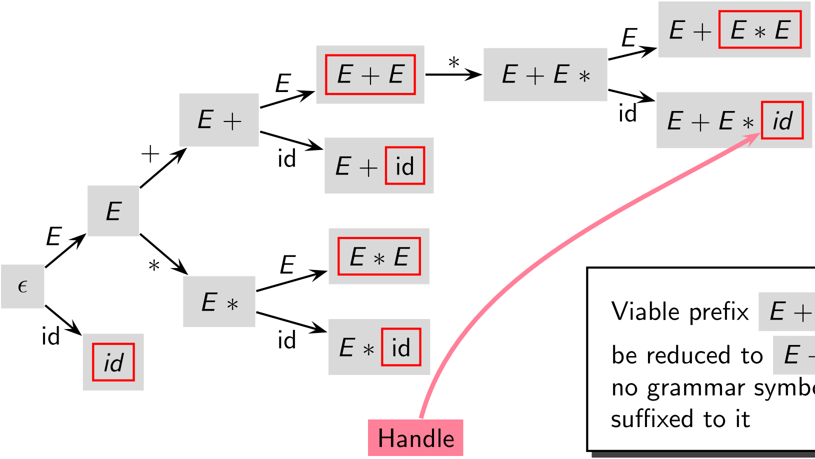


Step 3: Viable Prefixes for Our Grammar (After Incorporating Precedences and Associativities)



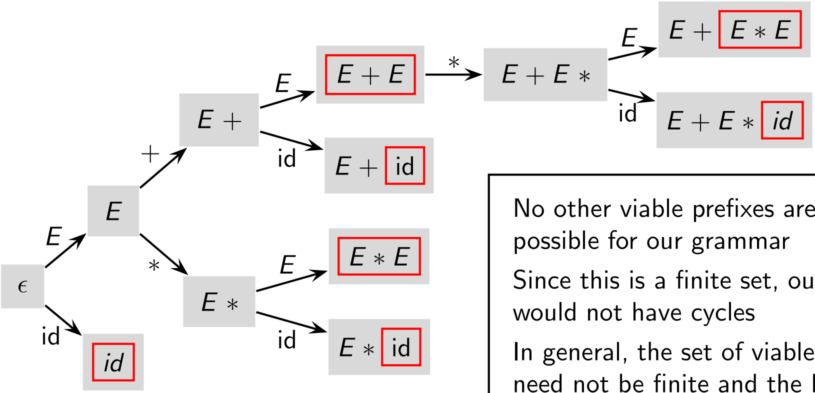
Viable prefix $E + E * E$ must be reduced to $E + E$ and no grammar symbol can be suffixed to it

Step 3: Viable Prefixes for Our Grammar (After Incorporating Precedences and Associativities)



Viable prefix $E+E*$id must be reduced to $E+E*E$ and no grammar symbol can be suffixed to it

Step 3: Viable Prefixes for Our Grammar (After Incorporating Precedences and Associativities)



No other viable prefixes are possible for our grammar

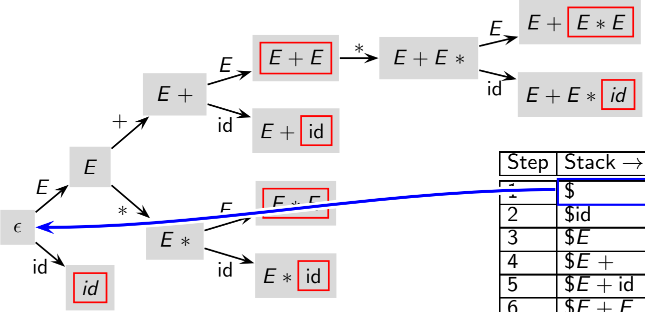
Since this is a finite set, our DFA would not have cycles

In general, the set of viable prefixes need not be finite and the DFA to recognize them may have cycles

Step 3: Viable Prefixes for Our Example

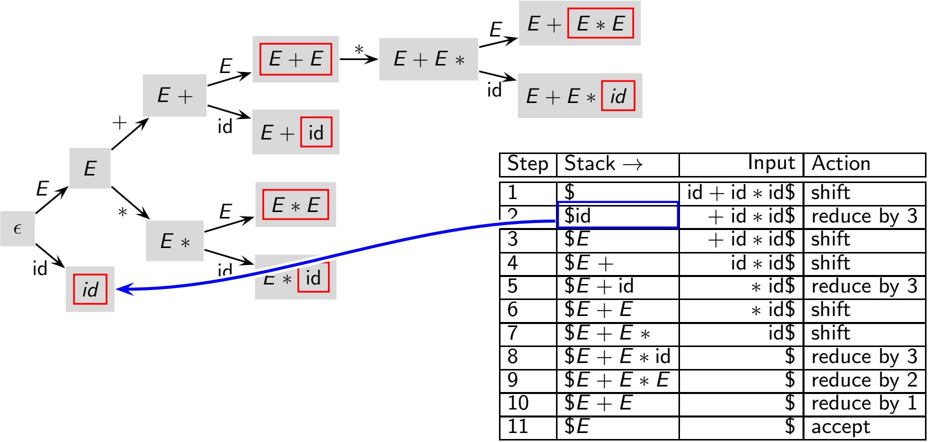
Step	Stack →	Input	Action
1	$\$$	$\text{id} + \text{id} * \text{id}\$$	shift
2	$\$\text{id}$	$+ \text{id} * \text{id}\$$	reduce by 3
3	$\$E$	$+ \text{id} * \text{id}\$$	shift
4	$\$E +$	$\text{id} * \text{id}\$$	shift
5	$\$E + \text{id}$	$* \text{id}\$$	reduce by 3
6	$\$E + E$	$* \text{id}\$$	shift
7	$\$E + E *$	$\text{id}\$$	shift
8	$\$E + E * \text{id}$	$\$$	reduce by 3
9	$\$E + E * E$	$\$$	reduce by 2
10	$\$E + E$	$\$$	reduce by 1
11	$\$E$	$\$$	accept

Step 3: Viable Prefixes for Our Example



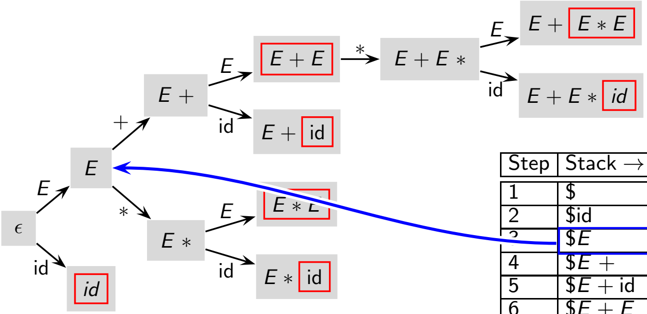
Step	Stack →	Input	Action
1	$\$$	id + id ∗ id$	shift
2	$id	+ id ∗ id$	reduce by 3
3	$\$E$	+ id ∗ id$	shift
4	$\$E+$	id ∗ id$	shift
5	$\$E+$ id	∗ id$	reduce by 3
6	$\$E+E$	∗ id$	shift
7	$\$E+E*$	id$	shift
8	$\$E+E*$ id	$	reduce by 3
9	$\$E+E*E$	$	reduce by 2
10	$\$E+E$	$	reduce by 1
11	$\$E$	$	accept

Step 3: Viable Prefixes for Our Example



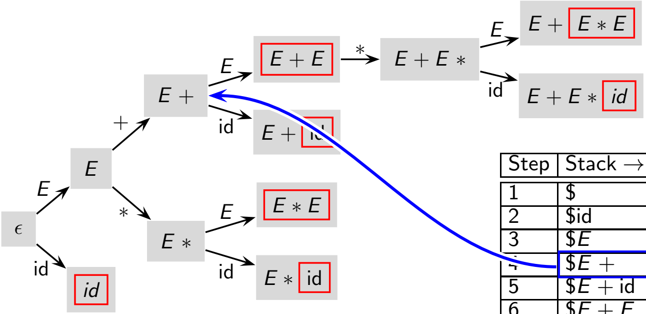
Step	Stack →	Input	Action
1	\$	$\mathrm{id} + \mathrm{id} * \mathrm{id}\$$	shift
2	\$id	$+ \mathrm{id} * \mathrm{id}\$$	reduce by 3
3	$\$E$	$+ \mathrm{id} * \mathrm{id}\$$	shift
4	$\$E +$	$\mathrm{id} * \mathrm{id}\$$	shift
5	$\$E + \mathrm{id}$	$* \mathrm{id}\$$	reduce by 3
6	$\$E + E$	$* \mathrm{id}\$$	shift
7	$\$E + E *$	id\$	shift
8	$\$E + E * \mathrm{id}$	\$	reduce by 3
9	$\$E + E * E$	\$	reduce by 2
10	$\$E + E$	\$	reduce by 1
11	$\$E$	\$	accept

Step 3: Viable Prefixes for Our Example



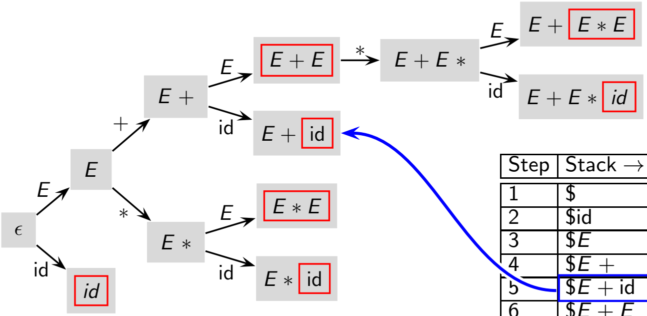
Step	Stack →	Input	Action
1	\$	id + id ∗ id\$	shift
2	\$id	+ id ∗ id\$	reduce by 3
3	\$$E$	+ id ∗ id\$	shift
4	\$$E$ +	id ∗ id\$	shift
5	\$$E$ + id	∗ id\$	reduce by 3
6	\$$E$ + E	∗ id\$	shift
7	\$$E$ + E ∗	id\$	shift
8	\$$E$ + E ∗ id	\$	reduce by 3
9	\$$E$ + E ∗ E	\$	reduce by 2
10	\$$E$ + E	\$	reduce by 1
11	\$$E$	\$	accept

Step 3: Viable Prefixes for Our Example



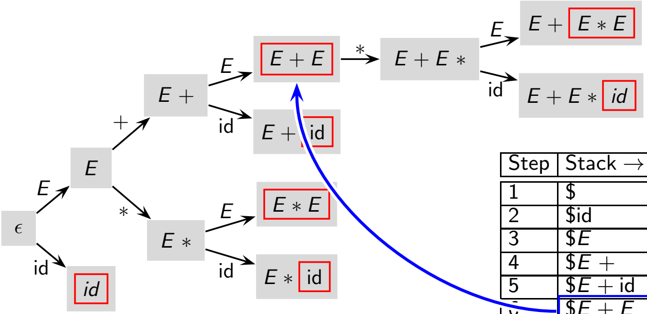
Step	Stack →	Input	Action
1	$	id + id ∗ id$	shift
2	$id	+ id ∗ id$	reduce by 3
3	$$E$	+ id ∗ id$	shift
4	$$E$ +	id ∗ id$	shift
5	$$E$ + id	∗ id$	reduce by 3
6	$$E + E$	∗ id$	shift
7	$$E + E$ ∗	id$	shift
8	$$E + E$ ∗ id	$	reduce by 3
9	$$E + E * E$	$	reduce by 2
10	$$E + E$	$	reduce by 1
11	$$E$	$	accept

Step 3: Viable Prefixes for Our Example



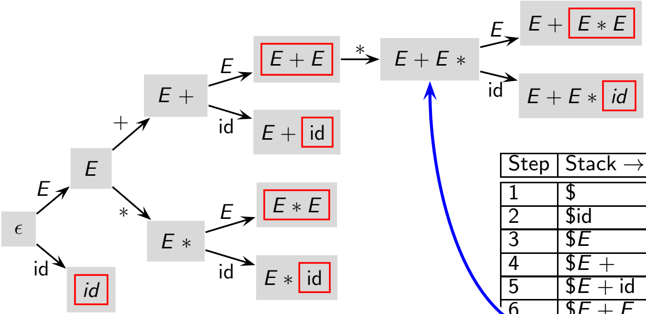
Step	Stack →	Input	Action
1	$\$$	$\mathrm{id} + \mathrm{id} * \mathrm{id}\$$	shift
2	$\$\mathrm{id}$	$+ \mathrm{id} * \mathrm{id}\$$	reduce by 3
3	$\$E$	$+ \mathrm{id} * \mathrm{id}\$$	shift
4	$\$E +$	$\mathrm{id} * \mathrm{id}\$$	shift
5	$\$E + \mathrm{id}$	$* \mathrm{id}\$$	reduce by 3
6	$\$E + E$	$* \mathrm{id}\$$	shift
7	$\$E + E *$	$\mathrm{id}\$$	shift
8	$\$E + E * \mathrm{id}$	$\$$	reduce by 3
9	$\$E + E * E$	$\$$	reduce by 2
10	$\$E + E$	$\$$	reduce by 1
11	$\$E$	$\$$	accept

Step 3: Viable Prefixes for Our Example



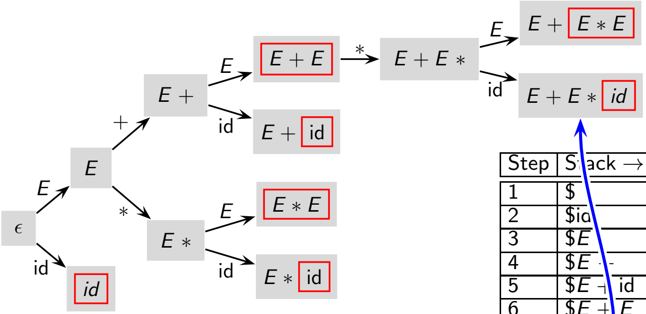
Step	Stack →	Input	Action
1	\$	id + id ∗ id\$	shift
2	\$id	+ id ∗ id\$	reduce by 3
3	$\$E$	+ id ∗ id\$	shift
4	$\$E +$	id ∗ id\$	shift
5	$\$E +$ id	∗ id\$	reduce by 3
6	$\$E + E$	∗ id\$	shift
7	$\$E + E *$	id\$	shift
8	$\$E + E *$ id	\$	reduce by 3
9	$\$E + E * E$	\$	reduce by 2
10	$\$E + E$	\$	reduce by 1
11	$\$E$	\$	accept

Step 3: Viable Prefixes for Our Example



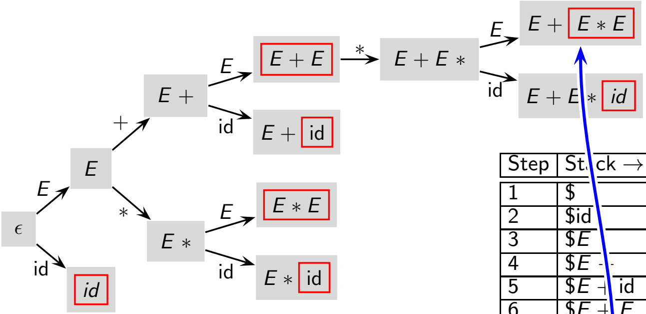
Step	Stack →	Input	Action
1	\$	id + id * id\$	shift
2	\$id	+ id * id\$	reduce by 3
3	$\$E$	+ id * id\$	shift
4	$\$E\,+$	id * id\$	shift
5	$\$E + \text{id}$	* id\$	reduce by 3
6	$\$E + E$	* id\$	shift
7	$\boxed{\$E + E\,*}$	id\$	shift
8	$\$E + E * \text{id}$	\$	reduce by 3
9	$\$E + E * E$	\$	reduce by 2
10	$\$E + E$	\$	reduce by 1
11	$\$E$	\$	accept

Step 3: Viable Prefixes for Our Example



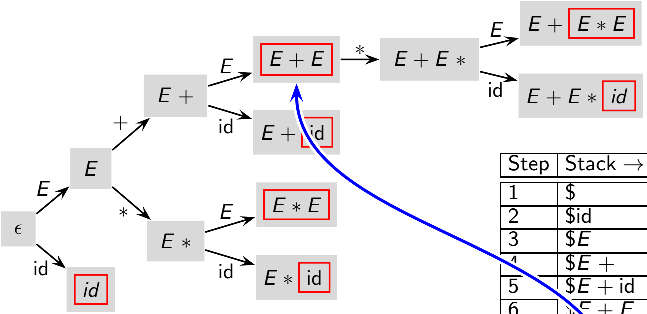
Step	Stack →	Input	Action
1	\$	id + id ∗ id\$	shift
2	\$id	+ id ∗ id\$	reduce by 3
3	\$$E$	+ id ∗ id\$	shift
4	\$$E$ +	id ∗ id\$	shift
5	\$$E$ + id	∗ id\$	reduce by 3
6	\$$E$ + E	∗ id\$	shift
7	\$$E$ + E ∗	id\$	shift
8	\$$E$ + E ∗ id	\$	reduce by 3
9	\$$E$ + E ∗ E	\$	reduce by 2
10	\$$E$ + E	\$	reduce by 1
11	\$$E$	\$	accept

Step 3: Viable Prefixes for Our Example



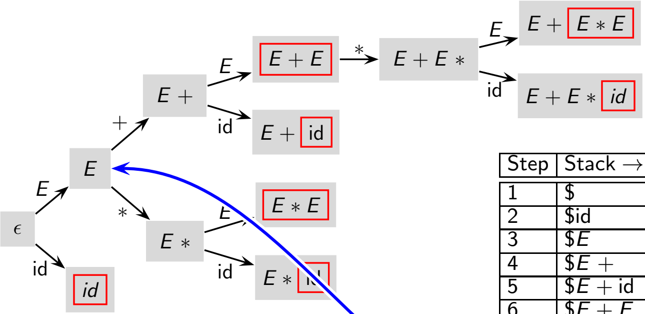
Step	Stack →	Input	Action
1	\$	id + id ∗ id\$	shift
2	\$id	+ id ∗ id\$	reduce by 3
3	\$$E$	+ id ∗ id\$	shift
4	\$$E$ +	id ∗ id\$	shift
5	\$$E$ + id	∗ id\$	reduce by 3
6	\$$E$ + E	∗ id\$	shift
7	\$$E$ + E ∗	id\$	shift
8	\$$E$ + E ∗ id	\$	reduce by 3
9	\$$E$ + E ∗ E	\$	reduce by 2
10	\$$E$ + E	\$	reduce by 1
11	\$$E$	\$	accept

Step 3: Viable Prefixes for Our Example



Step	Stack →	Input	Action
1	$	id + id ∗ id$	shift
2	$id	+ id ∗ id$	reduce by 3
3	$E	+ id ∗ id$	shift
4	$E +	id ∗ id$	shift
5	$E + id	∗ id$	reduce by 3
6	$E + E	∗ id$	shift
7	$E + E ∗	id$	shift
8	$E + E ∗ id	$	reduce by 3
9	$E + E ∗ E	$	reduce by 2
10	$E + E	$	reduce by 1
11	$E	$	accept

Step 3: Viable Prefixes for Our Example



Step	Stack →	Input	Action
1	\$	id + id ∗ id\$	shift
2	\$id	+ id ∗ id\$	reduce by 3
3	\$$E$	+ id ∗ id\$	shift
4	\$$E$ +	id ∗ id\$	shift
5	\$$E$ + id	∗ id\$	reduce by 3
6	\$$E + E$	∗ id\$	shift
7	\$$E + E$ ∗	id\$	shift
8	\$$E + E$ ∗ id	\$	reduce by 3
9	\$$E + E * E$	\$	reduce by 2
10	\$$E + E$	\$	reduce by 1
11	\$$E$	\$	accept

Step 4: Valid Items for Viable Prefixes

- An item is a grammar production with a dot (•) in it somewhere in the RHS
- The dot separates what has been seen from what may be seen in the input
- We identify a set of items for a viable prefix to form a state of the parser

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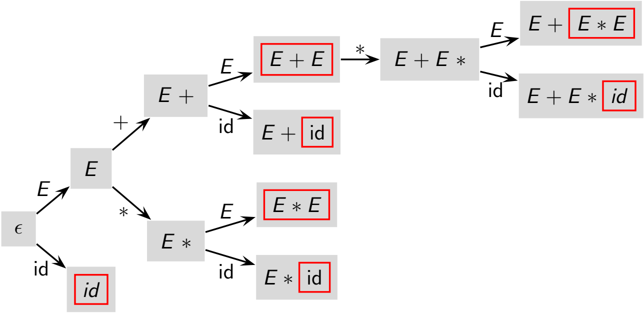

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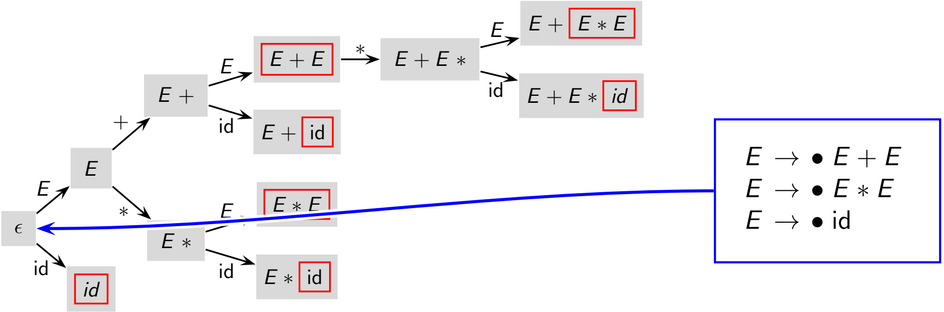

$$
\begin{aligned}
E &\rightarrow \bullet\ E + E \\
E &\rightarrow \bullet\ E * E \\
E &\rightarrow \bullet\ \text{id}
\end{aligned}
$$

Step 4: Valid Items for Viable Prefixes

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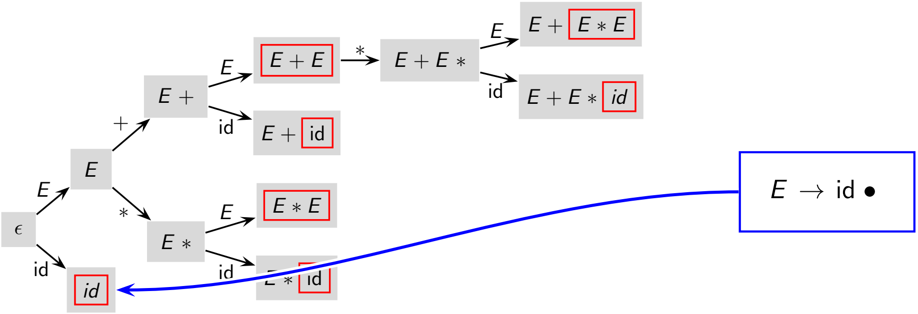

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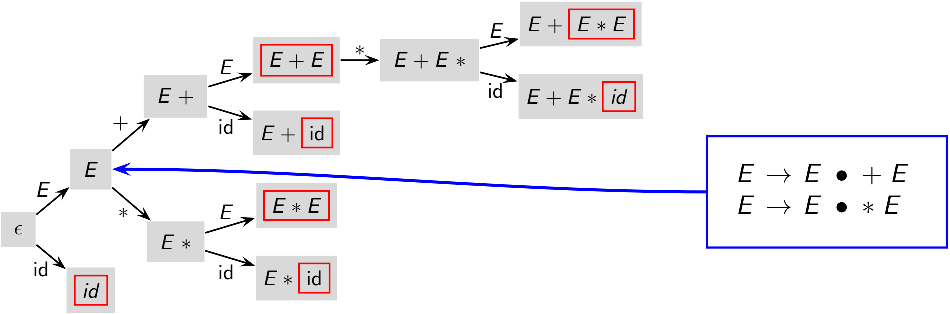

$$E \rightarrow E \bullet + E$$
$$E \rightarrow E \bullet * E$$

Step 4: Valid Items for Viable Prefixes

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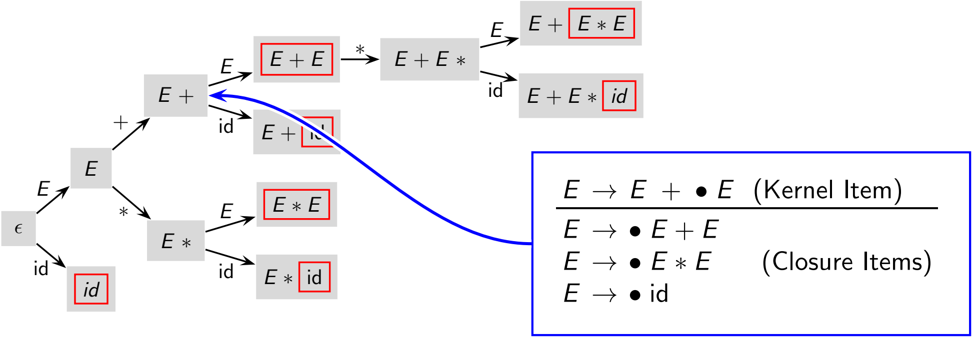

$E \rightarrow E + \bullet E$ (Kernel Item)

$E \rightarrow \bullet E + E$
$E \rightarrow \bullet E * E$ (Closure Items)
$E \rightarrow \bullet$ id

Step 4: Valid Items for Viable Prefixes

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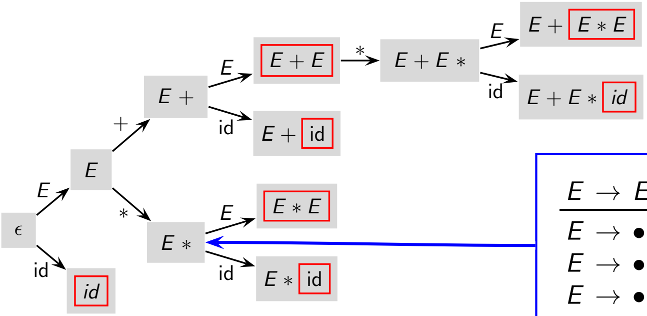

$E \rightarrow E * \bullet E$ (Kernel Item)

$E \rightarrow \bullet E + E$

$E \rightarrow \bullet E * E$ (Closure Items)

$E \rightarrow \bullet \text{id}$

Step 4: Valid Items for Viable Prefixes

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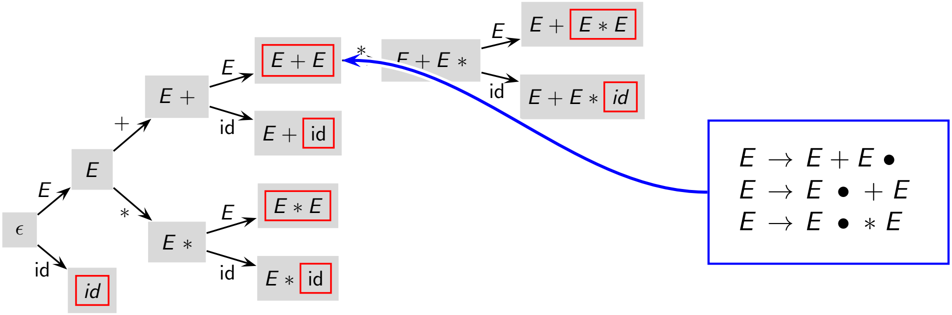

$$E \rightarrow E + E \bullet$$
$$E \rightarrow E \bullet + E$$
$$E \rightarrow E \bullet * E$$

Step 4: Valid Items for Viable Prefixes

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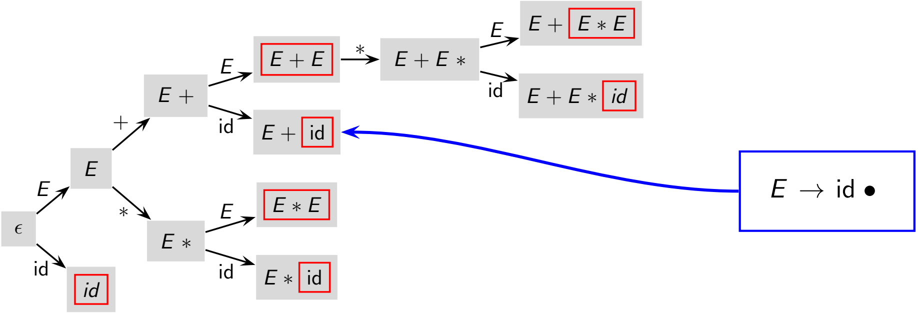

Step 4: Valid Items for Viable Prefixes

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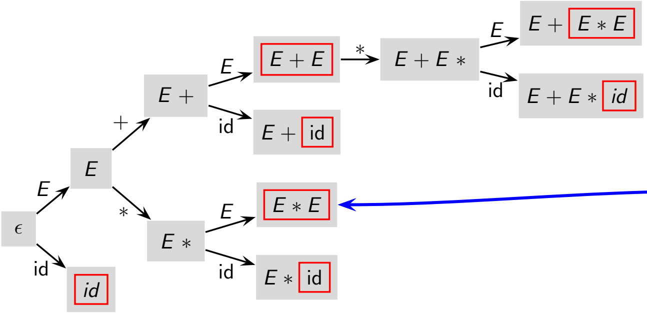

$$E \rightarrow E * E \bullet$$
$$E \rightarrow E \bullet + E$$
$$E \rightarrow E \bullet * E$$

Step 4: Valid Items for Viable Prefixes

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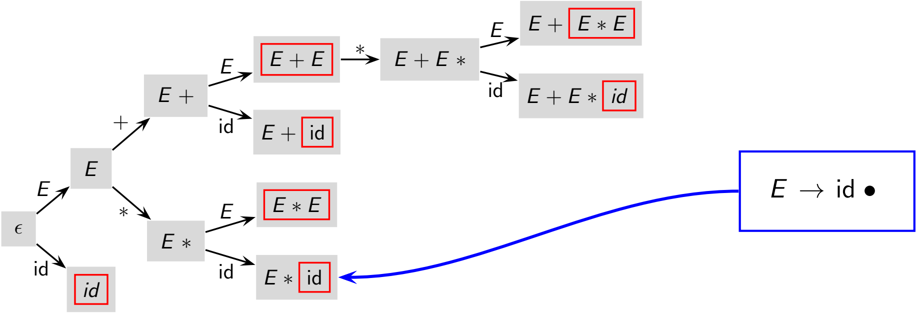

Step 4: Valid Items for Viable Prefixes

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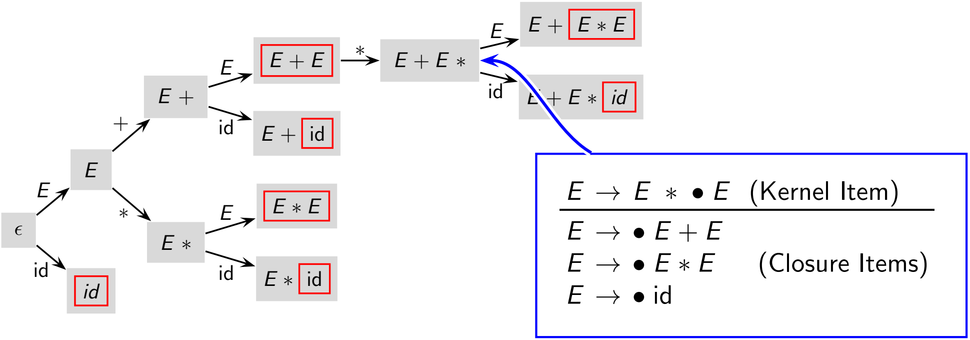

$E \rightarrow E * \bullet E$ (Kernel Item)

$E \rightarrow \bullet E + E$
$E \rightarrow \bullet E * E$ (Closure Items)
$E \rightarrow \bullet \text{id}$

Step 4: Valid Items for Viable Prefixes

- An item is a grammar production with a dot (•) in it somewhere in the RHS
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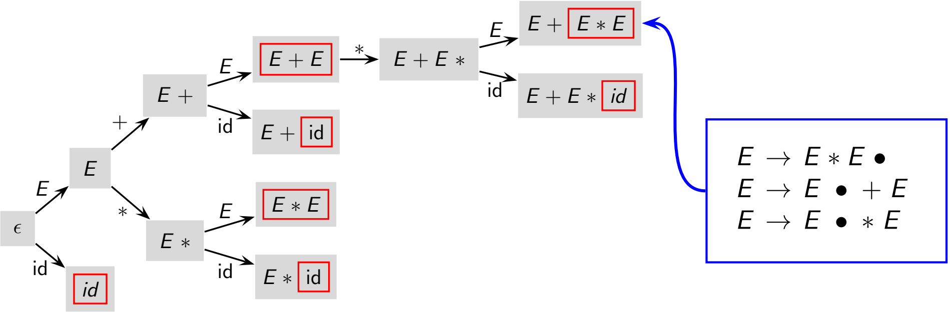
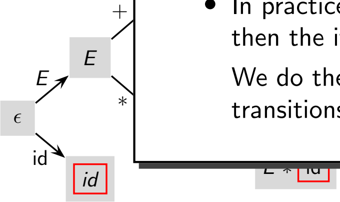


$$E \rightarrow E * E \bullet$$
$$E \rightarrow E \bullet + E$$
$$E \rightarrow E \bullet * E$$

Step 4: Valid Items for Viable Prefixes

- An item is a grammar production with a dot (•) in it somewhere in the RHS
- The dot separates what has been seen from what may be seen in the input
- We iden



- An item set may not describe a viable prefix on its own
  (Prefixes of a viable prefix may be described by other item sets)
- Item sets for different viable prefixes may be same
- In practice, we do not construct the viable prefixes and then the item sets for them

  We do the opposite: we construct the item sets and the transitions between them give us the viable prefixes

Step 5: The Last Piece of Jigsaw Puzzle: Computing FOLLOW Sets

Consider $\beta A w \overset{rm}{\Rightarrow} \beta\alpha w$ and $A \rightarrow \alpha$
When do we reduce occurrence of α in $\gamma = \beta\alpha$ using $A \rightarrow \alpha$ using LR(k) items?
(i.e., when do we decide that α and $A \rightarrow \alpha$ form a handle in γ?)

Read the input from Left to right

Trace the Rightmost derivation in Reverse

The number of lookahead symbols in the items

Step 5: The Last Piece of Jigsaw Puzzle: Computing FOLLOW Sets

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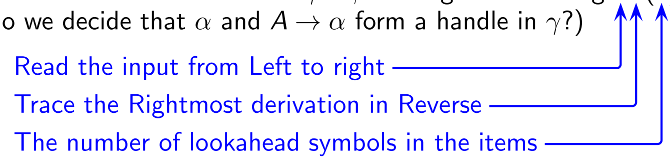
(i.e., when do we decide that α and $A \rightarrow \alpha$ form a handle in γ?)



Read the input from Left to right

Trace the Rightmost derivation in Reverse

The number of lookahead symbols in the items

- As soon as we find α in γ
- When we find α in γ and the next input token can follow A in some sentential form
- When we find α in γ and the next input token follows A in γ

Step 5: The Last Piece of Jigsaw Puzzle: Computing FOLLOW Sets

Consider $\beta A w \overset{rm}{\Rightarrow} \beta \alpha w$ and $A \to \alpha$
When do we reduce occurrence of α in $\gamma = \beta\alpha$ using $A \to \alpha$ using LR(k) items?
(i.e., when do we decide that α and $A \to \alpha$ form a handle in γ?)

Read the input from Left to right
Trace the Rightmost derivation in Reverse
The number of lookahead symbols in the items

- As soon as we find α in γ

  LR(0) items and no lookahead in the input SLR(0) Parser

- When we find α in γ and the next input token can follow A in some sentential form

- When we find α in γ and the next input token follows A in γ

Step 5: The Last Piece of Jigsaw Puzzle: Computing FOLLOW Sets

Consider $\beta A w \overset{rm}{\Rightarrow} \beta \alpha w$ and $A \rightarrow \alpha$
When do we reduce occurrence of α in $\gamma = \beta\alpha$ using $A \rightarrow \alpha$ using LR(k) items?
(i.e., when do we decide that α and $A \rightarrow \alpha$ form a handle in γ?)

Read the input from Left to right
Trace the Rightmost derivation in Reverse
The number of lookahead symbols in the items

- As soon as we find α in γ

  LR(0) items and no lookahead in the input — SLR(0) Parser

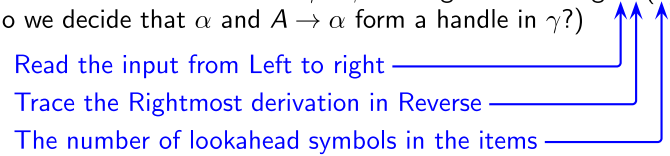
- When we find α in γ and the next input token can follow A in some sentential form

  LR(0) items and 1 lookahead in the input — SLR(1) Parser

- When we find α in γ and the next input token follows A in γ

Step 5: The Last Piece of Jigsaw Puzzle: Computing FOLLOW Sets

Consider $\beta A w \overset{rm}{\Rightarrow} \beta \alpha w$ and $A \rightarrow \alpha$
When do we reduce occurrence of α in $\gamma = \beta\alpha$ using $A \rightarrow \alpha$ using LR(k) items?
(i.e., when do we decide that α and $A \rightarrow \alpha$ form a handle in γ?)



Read the input from Left to right
Trace the Rightmost derivation in Reverse
The number of lookahead symbols in the items

- As soon as we find α in γ

  LR(0) items and no lookahead in the input — SLR(0) Parser

- When we find α in γ and the next input token can follow A in some sentential form

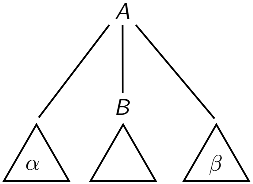
  LR(0) items and 1 lookahead in the input — SLR(1) Parser

- When we find α in γ and the next input token follows A in γ

  LR(1) items and 1 lookahead in the input — CLR(1) Parser

Step 5: FIRST and FOLLOW Sets

- FIRST(β) contains the terminals that may begin a string derivable from β
  If β derives ϵ, then $\epsilon \in$ FIRST(β)
  It is computed as the least fixed point solution of the following constraints
  For $A \rightarrow X_1X_2 \ldots X_k$, FIRST$(A) \supseteq X_i, 1 \leq i \leq k,$ provided $\forall j < i, \epsilon \in$ FIRST(X_j)
- FOLLOW(A) contains the terminals that follow A in some sentential form
  It is computed as the least fixed point solution of the following constraints

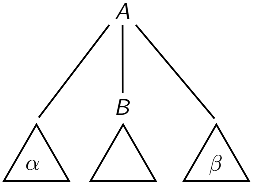


For production $A \rightarrow \alpha B \beta$

- If A is the start non-terminal
  FOLLOW$(A) \supseteq \{\$\}$
- FOLLOW$(B) \supseteq$ FIRST$(\beta) - \{\epsilon\}$
- If β is ϵ or $\epsilon \in$ FIRST(β)
  FOLLOW$(B) \supseteq$ FOLLOW(A)

Step 5: FIRST and FOLLOW Sets

- FIRST(β) contains the terminals that may begin a string derivable from β
  If β derives ϵ, then $\epsilon \in$ FIRST(β)
  It is computed as the least fixed point solution of the following constraints
  For $A \rightarrow X_1 X_2 \ldots X_k$, FIRST$(A) \supseteq X_i, 1 \leq i \leq k$, provided $\forall j < i, \epsilon \in$ FIRST(X_j)
- FOLLOW(A) contains the terminals that follow A in some sentential form
  It is computed as the least fixed point solution of the following constraints



For production $A \rightarrow \alpha B \beta$

- If A is the start non-terminal
  FOLLOW$(A) \supseteq \{\$\}$
- FOLLOW$(B) \supseteq$ FIRST$(\beta) - \{\epsilon\}$
- If β is ϵ or $\epsilon \in$ FIRST(β)
  FOLLOW$(B) \supseteq$ FOLLOW(A)

For our grammar
$E \rightarrow E + E$
$E \rightarrow E * E$
$E \rightarrow$ id

FIRST$(E) = \{$id$\}$
FOLLOW$(E) = \{\$, +, *\}$

Step 6: Computing LR(0) Item Sets for Expressions Grammar

An item does not contain any lookahead symbol

Trace the Rightmost derivation in Reverse

Read the input from Left to right

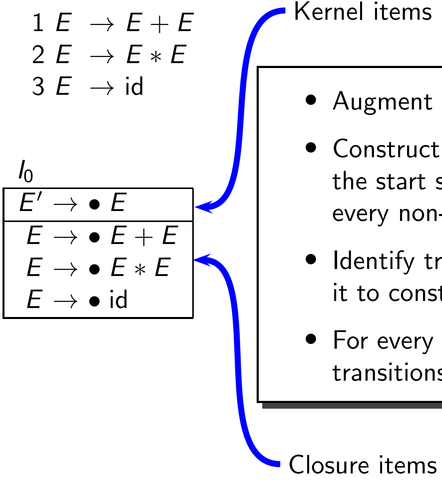
Step 6: Computing LR(0) Item Sets for Expressions Grammar

$$\begin{array}{lll} 0 & E' & \rightarrow E \\ 1 & E & \rightarrow E + E \\ 2 & E & \rightarrow E * E \\ 3 & E & \rightarrow \text{id} \end{array}$$

- Augment the grammar by adding a synthetic start symbol
- Construct the start state by putting a dot at the start of the start symbol and taking a closure (add every rule for every non-terminal that has a dot before it in some rule)
- Identify transitions on every symbol that has a dot before it to construct new states
- For every state so identified, take a closure and identify transitions on every symbol that has a dot before it

Step 6: Computing LR(0) Item Sets for Expressions Grammar

$0 \; E' \rightarrow E$
$1 \; E \rightarrow E + E$
$2 \; E \rightarrow E * E$
$3 \; E \rightarrow \text{id}$



I_0

$E' \rightarrow \bullet E$

$E \rightarrow \bullet E + E$
$E \rightarrow \bullet E * E$
$E \rightarrow \bullet \text{id}$

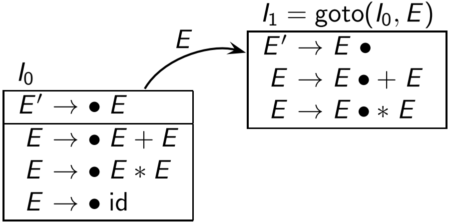
Kernel items

Closure items

- Augment the grammar by adding a synthetic start symbol
- Construct the start state by putting a dot at the start of the start symbol and taking a closure (add every rule for every non-terminal that has a dot before it in some rule)
- Identify transitions on every symbol that has a dot before it to construct new states
- For every state so identified, take a closure and identify transitions on every symbol that has a dot before it

Step 6: Computing LR(0) Item Sets for Expressions Grammar

0 $E' \rightarrow E$
1 $E \rightarrow E + E$
2 $E \rightarrow E * E$
3 $E \rightarrow \text{id}$



I_0

$E' \rightarrow \bullet\, E$
$E \rightarrow \bullet\, E + E$
$E \rightarrow \bullet\, E * E$
$E \rightarrow \bullet\, \text{id}$

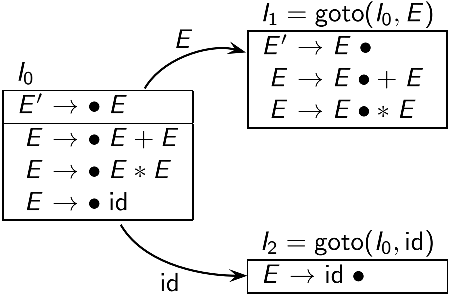
E

$I_1 = \text{goto}(I_0, E)$

$E' \rightarrow E \bullet$
$E \rightarrow E \bullet + E$
$E \rightarrow E \bullet * E$

Step 6: Computing LR(0) Item Sets for Expressions Grammar

0 $E' \rightarrow E$
1 $E \rightarrow E + E$
2 $E \rightarrow E * E$
3 $E \rightarrow \text{id}$



I_0

$E' \rightarrow \bullet\, E$
$E \rightarrow \bullet\, E + E$
$E \rightarrow \bullet\, E * E$
$E \rightarrow \bullet\, \text{id}$

E

$I_1 = \text{goto}(I_0, E)$

$E' \rightarrow E\, \bullet$
$E \rightarrow E \bullet + E$
$E \rightarrow E \bullet * E$

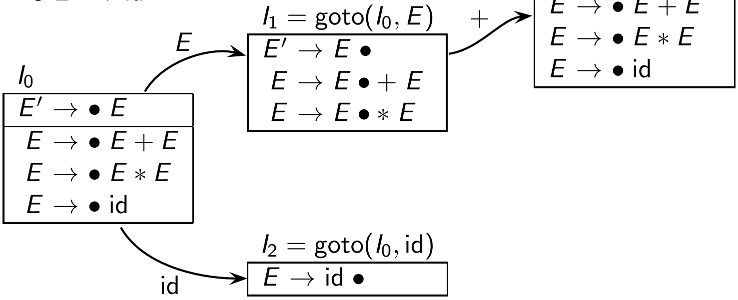
id

$I_2 = \text{goto}(I_0, \text{id})$

$E \rightarrow \text{id}\, \bullet$

Step 6: Computing LR(0) Item Sets for Expressions Grammar

$0 \; E' \rightarrow E$
$1 \; E \rightarrow E + E$
$2 \; E \rightarrow E * E$
$3 \; E \rightarrow \text{id}$



I_0
$E' \rightarrow \bullet E$
$E \rightarrow \bullet E + E$
$E \rightarrow \bullet E * E$
$E \rightarrow \bullet \text{id}$

E

$I_1 = \text{goto}(I_0, E)$
$E' \rightarrow E \bullet$
$E \rightarrow E \bullet + E$
$E \rightarrow E \bullet * E$

$+$

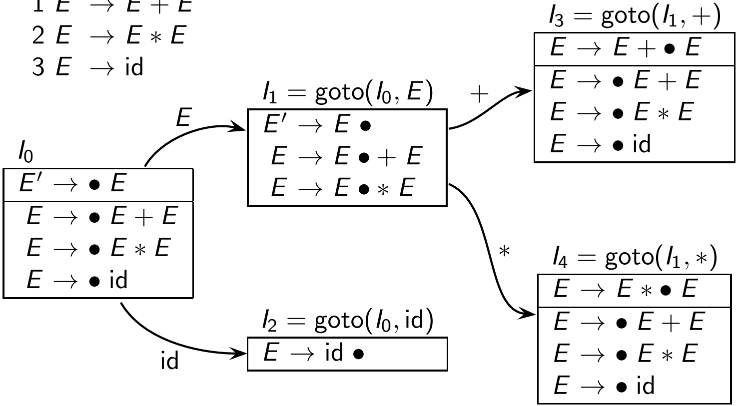
$I_3 = \text{goto}(I_1, +)$
$E \rightarrow E + \bullet E$
$E \rightarrow \bullet E + E$
$E \rightarrow \bullet E * E$
$E \rightarrow \bullet \text{id}$

id

$I_2 = \text{goto}(I_0, \text{id})$
$E \rightarrow \text{id} \bullet$

Step 6: Computing LR(0) Item Sets for Expressions Grammar

0 $E' \rightarrow E$
1 $E \rightarrow E + E$
2 $E \rightarrow E * E$
3 $E \rightarrow \text{id}$



I_0
$E' \rightarrow \bullet E$
$E \rightarrow \bullet E + E$
$E \rightarrow \bullet E * E$
$E \rightarrow \bullet \text{id}$

$I_1 = \text{goto}(I_0, E)$
$E' \rightarrow E \bullet$
$E \rightarrow E \bullet + E$
$E \rightarrow E \bullet * E$

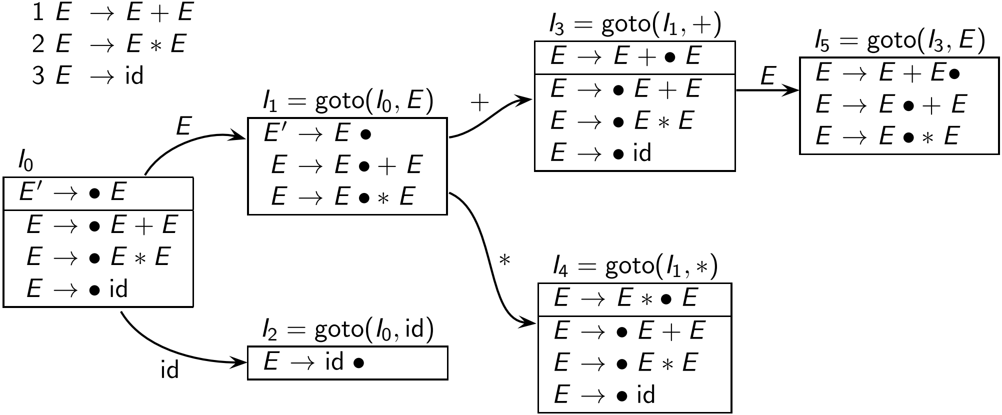
$I_2 = \text{goto}(I_0, \text{id})$
$E \rightarrow \text{id} \bullet$

$I_3 = \text{goto}(I_1, +)$
$E \rightarrow E + \bullet E$
$E \rightarrow \bullet E + E$
$E \rightarrow \bullet E * E$
$E \rightarrow \bullet \text{id}$

$I_4 = \text{goto}(I_1, *)$
$E \rightarrow E * \bullet E$
$E \rightarrow \bullet E + E$
$E \rightarrow \bullet E * E$
$E \rightarrow \bullet \text{id}$

Step 6: Computing LR(0) Item Sets for Expressions Grammar

0 $E' \rightarrow E$
1 $E \rightarrow E + E$
2 $E \rightarrow E * E$
3 $E \rightarrow \text{id}$



I_0
$E' \rightarrow \bullet E$
$E \rightarrow \bullet E + E$
$E \rightarrow \bullet E * E$
$E \rightarrow \bullet \text{id}$

$I_1 = \text{goto}(I_0, E)$
$E' \rightarrow E \bullet$
$E \rightarrow E \bullet + E$
$E \rightarrow E \bullet * E$

$I_2 = \text{goto}(I_0, \text{id})$
$E \rightarrow \text{id} \bullet$

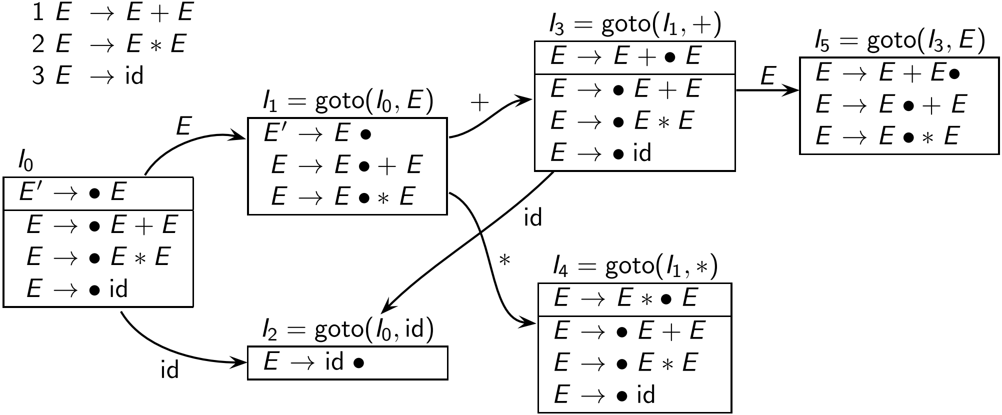
$I_3 = \text{goto}(I_1, +)$
$E \rightarrow E + \bullet E$
$E \rightarrow \bullet E + E$
$E \rightarrow \bullet E * E$
$E \rightarrow \bullet \text{id}$

$I_4 = \text{goto}(I_1, *)$
$E \rightarrow E * \bullet E$
$E \rightarrow \bullet E + E$
$E \rightarrow \bullet E * E$
$E \rightarrow \bullet \text{id}$

$I_5 = \text{goto}(I_3, E)$
$E \rightarrow E + E \bullet$
$E \rightarrow E \bullet + E$
$E \rightarrow E \bullet * E$

Step 6: Computing LR(0) Item Sets for Expressions Grammar

0 $E' \rightarrow E$
1 $E \rightarrow E + E$
2 $E \rightarrow E * E$
3 $E \rightarrow \text{id}$



I_0
$E' \rightarrow \bullet\, E$
$E \rightarrow \bullet\, E + E$
$E \rightarrow \bullet\, E * E$
$E \rightarrow \bullet\, \text{id}$

$I_1 = \text{goto}(I_0, E)$
$E' \rightarrow E\, \bullet$
$E \rightarrow E \bullet + E$
$E \rightarrow E \bullet * E$

$I_2 = \text{goto}(I_0, \text{id})$
$E \rightarrow \text{id}\, \bullet$

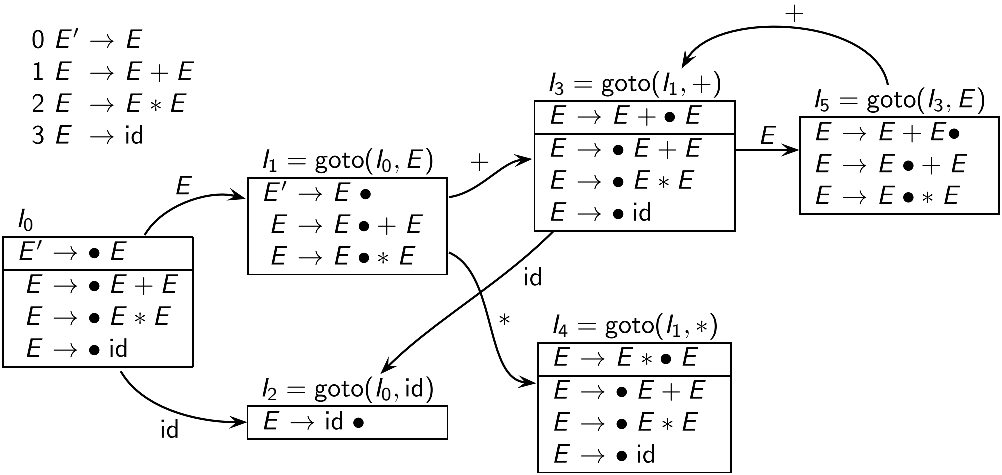
$I_3 = \text{goto}(I_1, +)$
$E \rightarrow E + \bullet\, E$
$E \rightarrow \bullet\, E + E$
$E \rightarrow \bullet\, E * E$
$E \rightarrow \bullet\, \text{id}$

$I_4 = \text{goto}(I_1, *)$
$E \rightarrow E * \bullet\, E$
$E \rightarrow \bullet\, E + E$
$E \rightarrow \bullet\, E * E$
$E \rightarrow \bullet\, \text{id}$

$I_5 = \text{goto}(I_3, E)$
$E \rightarrow E + E \bullet$
$E \rightarrow E \bullet + E$
$E \rightarrow E \bullet * E$

Step 6: Computing LR(0) Item Sets for Expressions Grammar

0 $E' \to E$
1 $E \to E + E$
2 $E \to E * E$
3 $E \to \text{id}$



I_0
$E' \to \bullet\ E$
$E \to \bullet\ E + E$
$E \to \bullet\ E * E$
$E \to \bullet\ \text{id}$

$I_1 = \text{goto}(I_0, E)$
$E' \to E\ \bullet$
$E \to E \bullet + E$
$E \to E \bullet * E$

$I_2 = \text{goto}(I_0, \text{id})$
$E \to \text{id}\ \bullet$

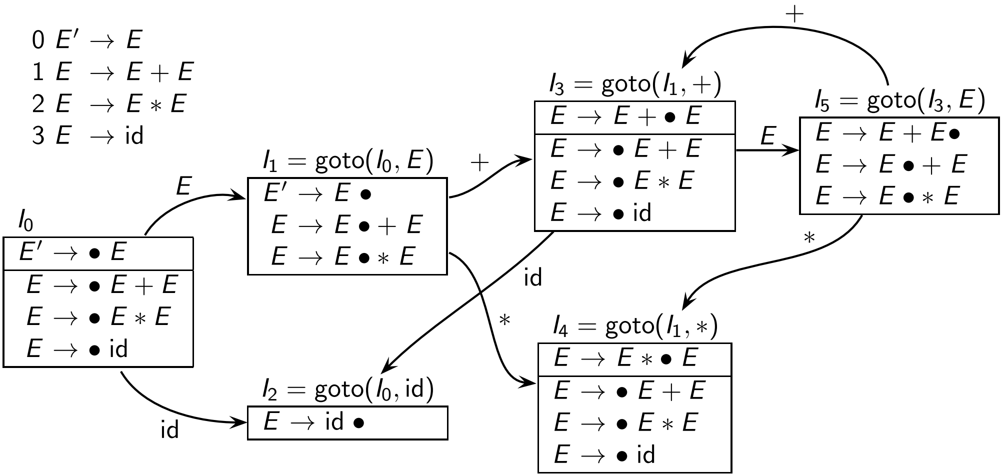
$I_3 = \text{goto}(I_1, +)$
$E \to E + \bullet\ E$
$E \to \bullet\ E + E$
$E \to \bullet\ E * E$
$E \to \bullet\ \text{id}$

$I_4 = \text{goto}(I_1, *)$
$E \to E * \bullet\ E$
$E \to \bullet\ E + E$
$E \to \bullet\ E * E$
$E \to \bullet\ \text{id}$

$I_5 = \text{goto}(I_3, E)$
$E \to E + E \bullet$
$E \to E \bullet + E$
$E \to E \bullet * E$

Step 6: Computing LR(0) Item Sets for Expressions Grammar

0 $E' \to E$
1 $E \to E + E$
2 $E \to E * E$
3 $E \to \text{id}$



I_0
$E' \to \bullet E$
$E \to \bullet E + E$
$E \to \bullet E * E$
$E \to \bullet \text{id}$

$I_1 = \text{goto}(I_0, E)$
$E' \to E \bullet$
$E \to E \bullet + E$
$E \to E \bullet * E$

$I_2 = \text{goto}(I_0, \text{id})$
$E \to \text{id} \bullet$

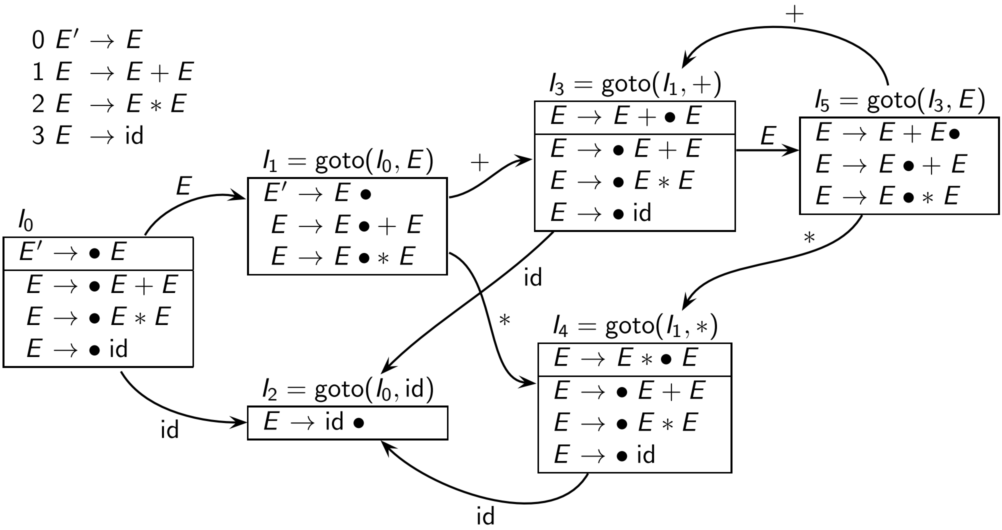
$I_3 = \text{goto}(I_1, +)$
$E \to E + \bullet E$
$E \to \bullet E + E$
$E \to \bullet E * E$
$E \to \bullet \text{id}$

$I_4 = \text{goto}(I_1, *)$
$E \to E * \bullet E$
$E \to \bullet E + E$
$E \to \bullet E * E$
$E \to \bullet \text{id}$

$I_5 = \text{goto}(I_3, E)$
$E \to E + E \bullet$
$E \to E \bullet + E$
$E \to E \bullet * E$

Step 6: Computing LR(0) Item Sets for Expressions Grammar

0 $E' \rightarrow E$
1 $E \rightarrow E + E$
2 $E \rightarrow E * E$
3 $E \rightarrow \text{id}$



I_0
$E' \rightarrow \bullet\, E$
$E \rightarrow \bullet\, E + E$
$E \rightarrow \bullet\, E * E$
$E \rightarrow \bullet\, \text{id}$

$I_1 = \text{goto}(I_0, E)$
$E' \rightarrow E\, \bullet$
$E \rightarrow E \bullet + E$
$E \rightarrow E \bullet * E$

$I_2 = \text{goto}(I_0, \text{id})$
$E \rightarrow \text{id}\, \bullet$

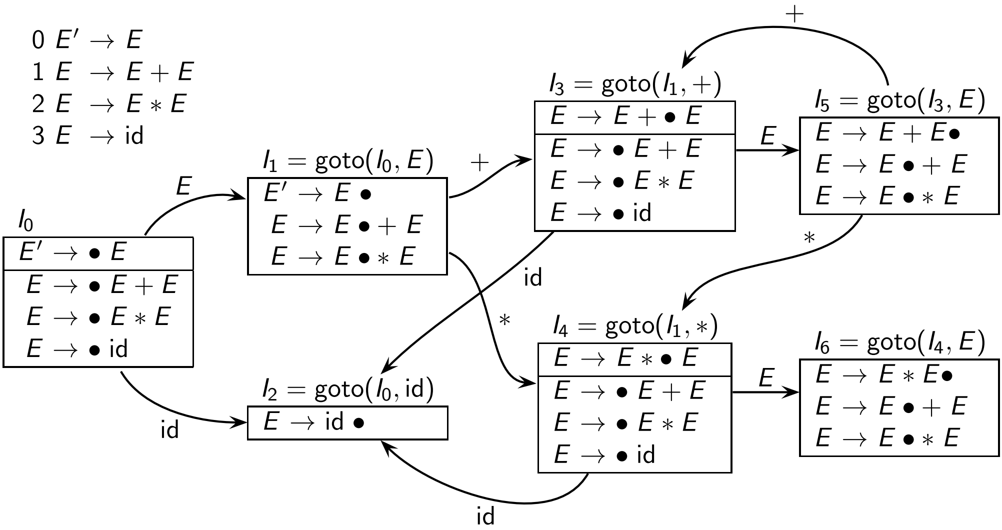
$I_3 = \text{goto}(I_1, +)$
$E \rightarrow E + \bullet\, E$
$E \rightarrow \bullet\, E + E$
$E \rightarrow \bullet\, E * E$
$E \rightarrow \bullet\, \text{id}$

$I_4 = \text{goto}(I_1, *)$
$E \rightarrow E * \bullet\, E$
$E \rightarrow \bullet\, E + E$
$E \rightarrow \bullet\, E * E$
$E \rightarrow \bullet\, \text{id}$

$I_5 = \text{goto}(I_3, E)$
$E \rightarrow E + E \bullet$
$E \rightarrow E \bullet + E$
$E \rightarrow E \bullet * E$

Step 6: Computing LR(0) Item Sets for Expressions Grammar

$0 \; E' \rightarrow E$
$1 \; E \rightarrow E + E$
$2 \; E \rightarrow E * E$
$3 \; E \rightarrow \text{id}$



I_0

$E' \rightarrow \bullet E$
$E \rightarrow \bullet E + E$
$E \rightarrow \bullet E * E$
$E \rightarrow \bullet \text{id}$

$I_1 = \text{goto}(I_0, E)$

$E' \rightarrow E \bullet$
$E \rightarrow E \bullet + E$
$E \rightarrow E \bullet * E$

$I_2 = \text{goto}(I_0, \text{id})$

$E \rightarrow \text{id} \bullet$

$I_3 = \text{goto}(I_1, +)$

$E \rightarrow E + \bullet E$
$E \rightarrow \bullet E + E$
$E \rightarrow \bullet E * E$
$E \rightarrow \bullet \text{id}$

$I_4 = \text{goto}(I_1, *)$

$E \rightarrow E * \bullet E$
$E \rightarrow \bullet E + E$
$E \rightarrow \bullet E * E$
$E \rightarrow \bullet \text{id}$

$I_5 = \text{goto}(I_3, E)$

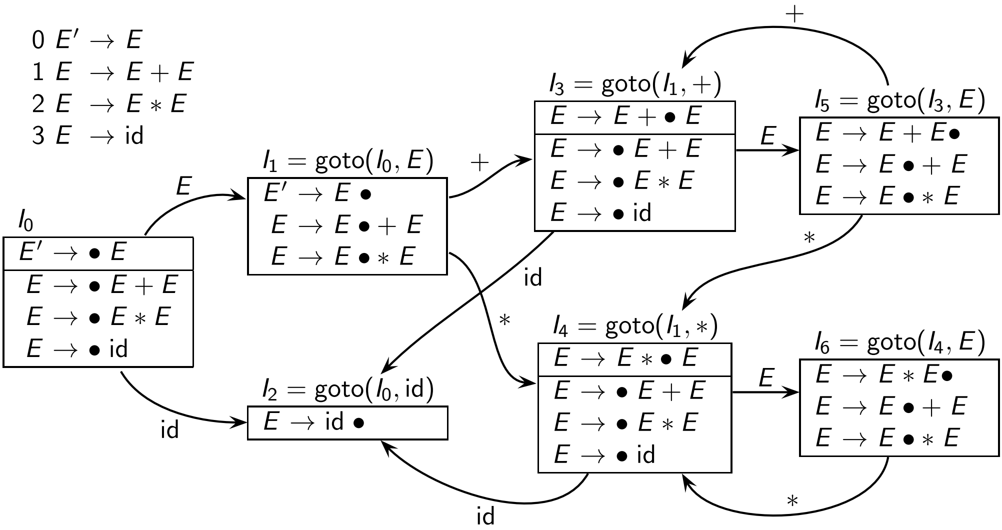
$E \rightarrow E + E \bullet$
$E \rightarrow E \bullet + E$
$E \rightarrow E \bullet * E$

$I_6 = \text{goto}(I_4, E)$

$E \rightarrow E * E \bullet$
$E \rightarrow E \bullet + E$
$E \rightarrow E \bullet * E$

Step 6: Computing LR(0) Item Sets for Expressions Grammar

$$\begin{array}{lll} 0 & E' & \rightarrow E \\ 1 & E & \rightarrow E + E \\ 2 & E & \rightarrow E * E \\ 3 & E & \rightarrow \text{id} \end{array}$$



I_0
$E' \rightarrow \bullet E$
$E \rightarrow \bullet E + E$
$E \rightarrow \bullet E * E$
$E \rightarrow \bullet \text{id}$

$I_1 = \text{goto}(I_0, E)$
$E' \rightarrow E \bullet$
$E \rightarrow E \bullet + E$
$E \rightarrow E \bullet * E$

$I_2 = \text{goto}(I_0, \text{id})$
$E \rightarrow \text{id} \bullet$

$I_3 = \text{goto}(I_1, +)$
$E \rightarrow E + \bullet E$
$E \rightarrow \bullet E + E$
$E \rightarrow \bullet E * E$
$E \rightarrow \bullet \text{id}$

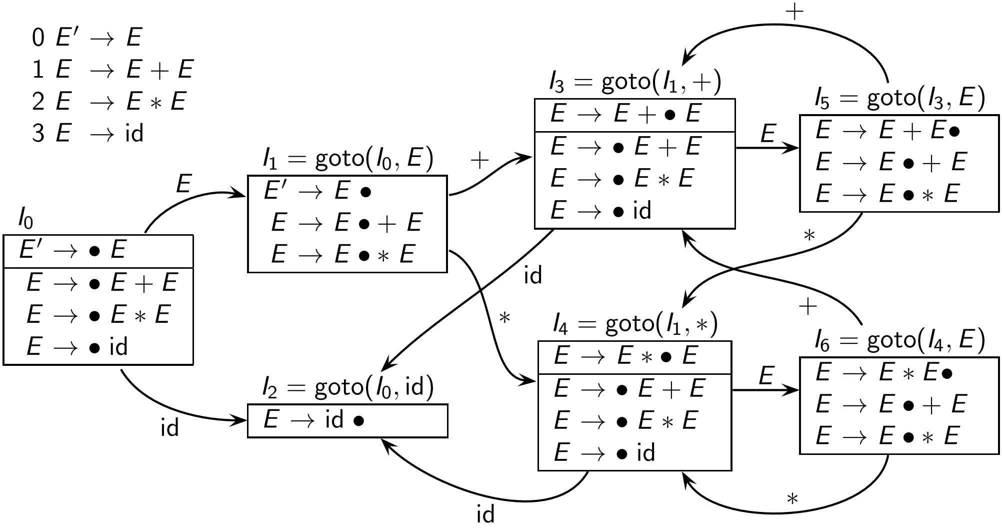
$I_4 = \text{goto}(I_1, *)$
$E \rightarrow E * \bullet E$
$E \rightarrow \bullet E + E$
$E \rightarrow \bullet E * E$
$E \rightarrow \bullet \text{id}$

$I_5 = \text{goto}(I_3, E)$
$E \rightarrow E + E \bullet$
$E \rightarrow E \bullet + E$
$E \rightarrow E \bullet * E$

$I_6 = \text{goto}(I_4, E)$
$E \rightarrow E * E \bullet$
$E \rightarrow E \bullet + E$
$E \rightarrow E \bullet * E$

Step 6: Computing LR(0) Item Sets for Expressions Grammar

0 $E' \to E$
1 $E \to E + E$
2 $E \to E * E$
3 $E \to \text{id}$



I_0
$E' \to \bullet E$
$E \to \bullet E + E$
$E \to \bullet E * E$
$E \to \bullet \text{id}$

$I_1 = \text{goto}(I_0, E)$
$E' \to E \bullet$
$E \to E \bullet + E$
$E \to E \bullet * E$

$I_2 = \text{goto}(I_0, \text{id})$
$E \to \text{id} \bullet$

$I_3 = \text{goto}(I_1, +)$
$E \to E + \bullet E$
$E \to \bullet E + E$
$E \to \bullet E * E$
$E \to \bullet \text{id}$

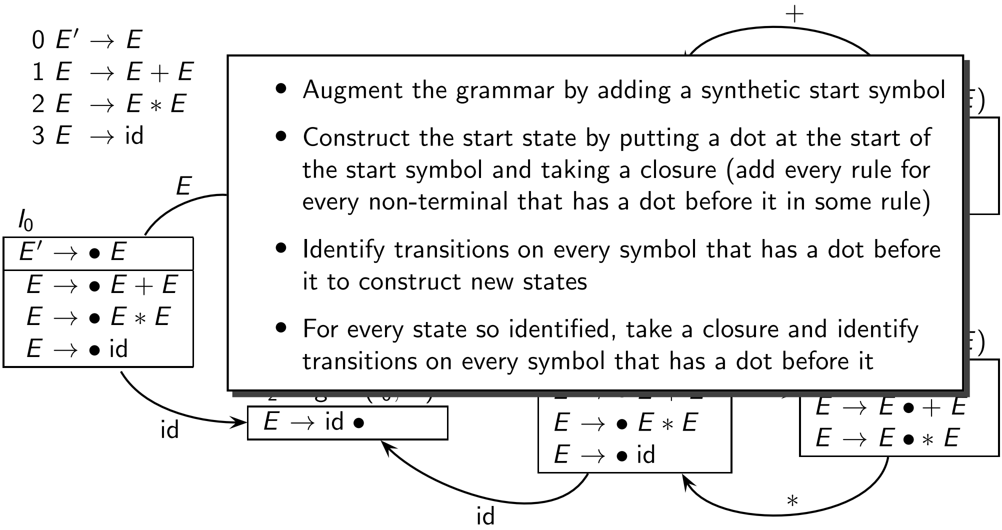
$I_4 = \text{goto}(I_1, *)$
$E \to E * \bullet E$
$E \to \bullet E + E$
$E \to \bullet E * E$
$E \to \bullet \text{id}$

$I_5 = \text{goto}(I_3, E)$
$E \to E + E \bullet$
$E \to E \bullet + E$
$E \to E \bullet * E$

$I_6 = \text{goto}(I_4, E)$
$E \to E * E \bullet$
$E \to E \bullet + E$
$E \to E \bullet * E$

Step 6: Computing LR(0) Item Sets for Expressions Grammar

0 $E' \rightarrow E$
1 $E \rightarrow E + E$
2 $E \rightarrow E * E$
3 $E \rightarrow \text{id}$

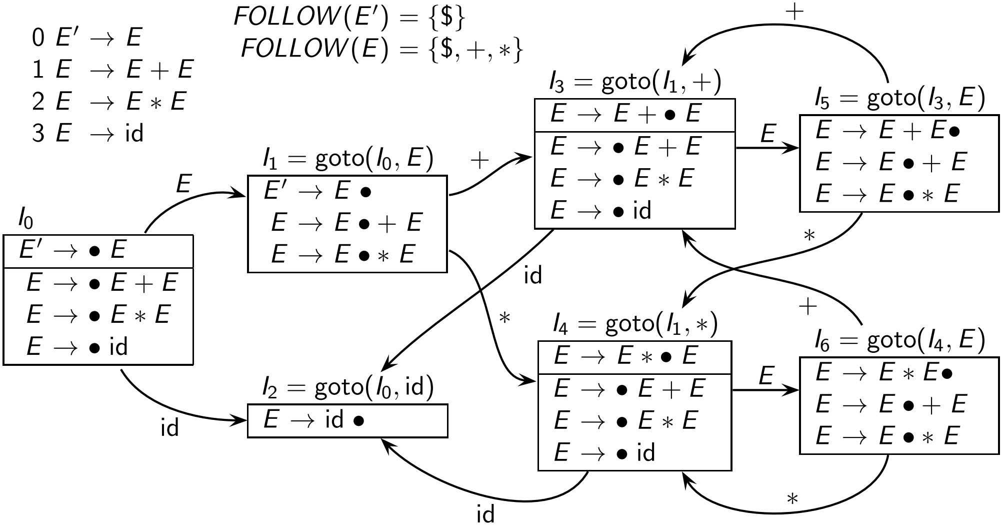


- Augment the grammar by adding a synthetic start symbol
- Construct the start state by putting a dot at the start of the start symbol and taking a closure (add every rule for every non-terminal that has a dot before it in some rule)
- Identify transitions on every symbol that has a dot before it to construct new states
- For every state so identified, take a closure and identify transitions on every symbol that has a dot before it

Step 6: Computing LR(0) Item Sets for Expressions Grammar

$0 \; E' \to E$
$1 \; E \to E + E$
$2 \; E \to E * E$
$3 \; E \to \text{id}$

$FOLLOW(E') = \{\$\}$
$FOLLOW(E) = \{\$, +, *\}$



I_0
$E' \to \bullet E$
$E \to \bullet E + E$
$E \to \bullet E * E$
$E \to \bullet \text{id}$

$I_1 = \text{goto}(I_0, E)$
$E' \to E \bullet$
$E \to E \bullet + E$
$E \to E \bullet * E$

$I_2 = \text{goto}(I_0, \text{id})$
$E \to \text{id} \bullet$

$I_3 = \text{goto}(I_1, +)$
$E \to E + \bullet E$
$E \to \bullet E + E$
$E \to \bullet E * E$
$E \to \bullet \text{id}$

$I_4 = \text{goto}(I_1, *)$
$E \to E * \bullet E$
$E \to \bullet E + E$
$E \to \bullet E * E$
$E \to \bullet \text{id}$

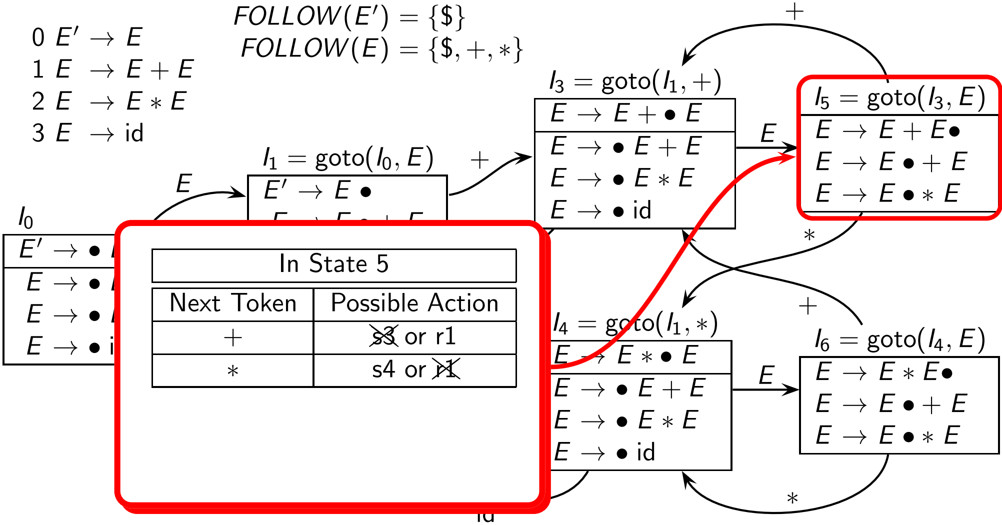
$I_5 = \text{goto}(I_3, E)$
$E \to E + E \bullet$
$E \to E \bullet + E$
$E \to E \bullet * E$

$I_6 = \text{goto}(I_4, E)$
$E \to E * E \bullet$
$E \to E \bullet + E$
$E \to E \bullet * E$

Step 6: Computing LR(0) Item Sets for Expressions Grammar

$0\ E' \rightarrow E$
$1\ E \rightarrow E + E$
$2\ E \rightarrow E * E$
$3\ E \rightarrow \text{id}$

$FOLLOW(E') = \{\$\}$
$FOLLOW(E) = \{\$, +, *\}$



In State 5	
Next Token	Possible Action
$+$	~~s3~~ or r1
$*$	s4 or ~~r1~~

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cs302: Implementation of Programming Languages

Topic:

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Conceptual Issues in Parsing

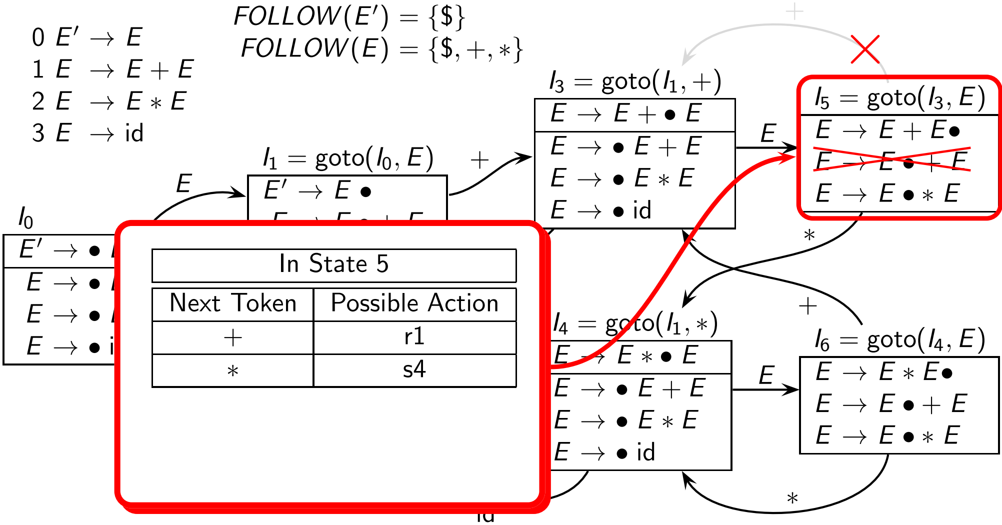
CLR(1) Parsing

LALR(1) Parsing

Step 6: Computing LR(0) Item Sets for Expressions Grammar

0 $E' \to E$
1 $E \to E + E$
2 $E \to E * E$
3 $E \to \text{id}$

$FOLLOW(E') = \{\$\}$
$FOLLOW(E) = \{\$, +, *\}$



I_0
$E' \to \bullet$
$E \to \bullet$
$E \to \bullet$
$E \to \bullet$ i

$I_1 = \text{goto}(I_0, E)$
$E' \to E \bullet$
$E \to E \bullet + E$

$I_3 = \text{goto}(I_1, +)$
$E \to E + \bullet E$
$E \to \bullet E + E$
$E \to \bullet E * E$
$E \to \bullet \text{id}$

$I_5 = \text{goto}(I_3, E)$
$E \to E + E \bullet$
~~$E \to E \bullet + E$~~
$E \to E \bullet * E$

$I_4 = \text{goto}(I_1, *)$
$E \to E * \bullet E$
$E \to \bullet E + E$
$E \to \bullet E * E$
$E \to \bullet \text{id}$

$I_6 = \text{goto}(I_4, E)$
$E \to E * E \bullet$
$E \to E \bullet + E$
$E \to E \bullet * E$

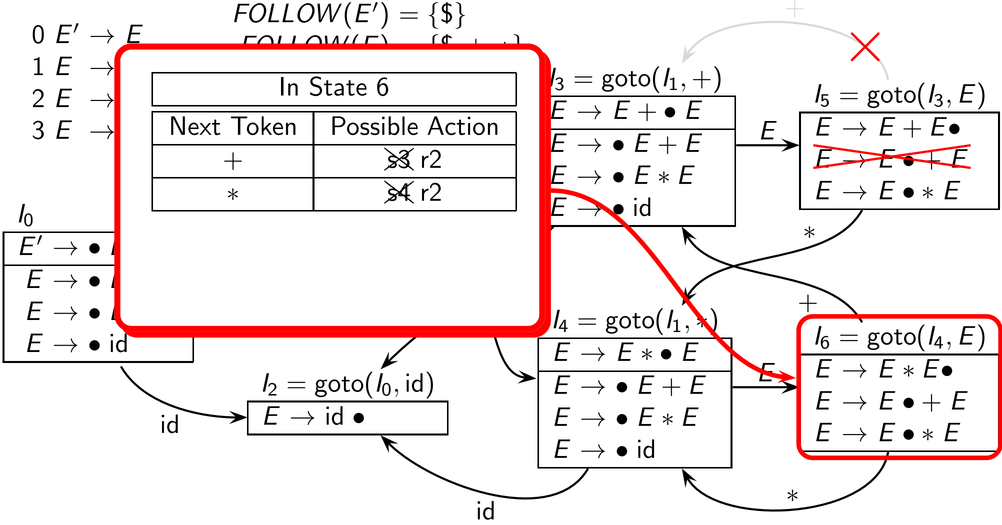
In State 5	
Next Token	Possible Action
$+$	r1
$*$	s4

Step 6: Computing LR(0) Item Sets for Expressions Grammar

0 $E' \rightarrow E$
1 $E \rightarrow$
2 $E \rightarrow$
3 $E \rightarrow$

$FOLLOW(E') = \{\$\}$
$FOLLOW(E) = \{\$, +, *\}$

In State 6	
Next Token	Possible Action
$+$	~~s3~~ r2
$*$	~~s4~~ r2



I_0
$E' \rightarrow \bullet$
$E \rightarrow \bullet$
$E \rightarrow \bullet$
$E \rightarrow \bullet\ \text{id}$

$I_2 = \text{goto}(I_0, \text{id})$
$E \rightarrow \text{id} \bullet$

$I_3 = \text{goto}(I_1, +)$
$E \rightarrow E + \bullet E$
$E \rightarrow \bullet E + E$
$E \rightarrow \bullet E * E$
$E \rightarrow \bullet\ \text{id}$

$I_4 = \text{goto}(I_1, *)$
$E \rightarrow E * \bullet E$
$E \rightarrow \bullet E + E$
$E \rightarrow \bullet E * E$
$E \rightarrow \bullet\ \text{id}$

$I_5 = \text{goto}(I_3, E)$
$E \rightarrow E + E \bullet$
~~$E \rightarrow E \bullet + E$~~
$E \rightarrow E \bullet * E$

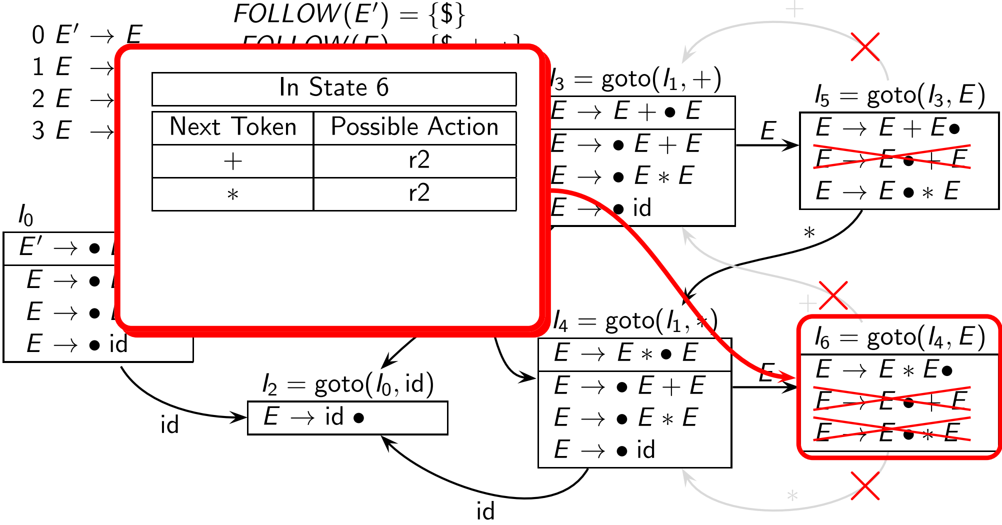
$I_6 = \text{goto}(I_4, E)$
$E \rightarrow E * E \bullet$
$E \rightarrow E \bullet + E$
$E \rightarrow E \bullet * E$

Step 6: Computing LR(0) Item Sets for Expressions Grammar

0 $E' \rightarrow E$
1 $E \rightarrow$
2 $E \rightarrow$
3 $E \rightarrow$

$FOLLOW(E') = \{\$\}$
$FOLLOW(E) = \{\$, +, *\}$

In State 6	
Next Token	Possible Action
$+$	r2
$*$	r2



I_0
$E' \rightarrow \bullet E$
$E \rightarrow \bullet E$
$E \rightarrow \bullet E$
$E \rightarrow \bullet \text{id}$

$I_2 = \text{goto}(I_0, \text{id})$
$E \rightarrow \text{id} \bullet$

$I_3 = \text{goto}(I_1, +)$
$E \rightarrow E + \bullet E$
$E \rightarrow \bullet E + E$
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$E \rightarrow \bullet \text{id}$

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~~$E \rightarrow E \bullet + E$~~
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~~$E \rightarrow E \bullet + E$~~
~~$E \rightarrow E \bullet * E$~~

Step 6: Computing LR(0) Item Sets for Expressions Grammar

0 $E' \rightarrow E$
1 $E \rightarrow E + E$
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$FOLLOW(E') = \{\$\}$
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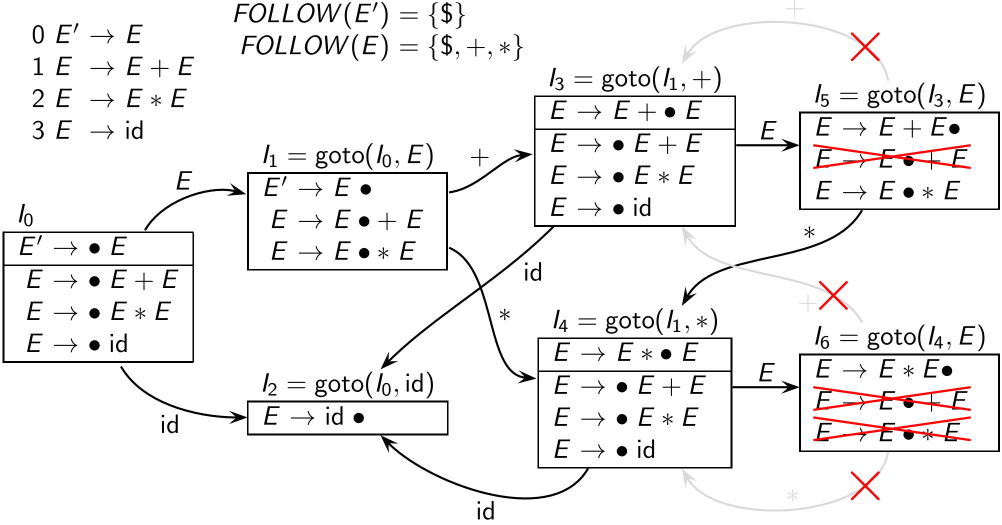

I_0
$E' \rightarrow \bullet E$
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$I_1 = \text{goto}(I_0, E)$
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$E \rightarrow E \bullet * E$

$I_2 = \text{goto}(I_0, \text{id})$
$E \rightarrow \text{id} \bullet$

$I_3 = \text{goto}(I_1, +)$
$E \rightarrow E + \bullet E$
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~~$E \rightarrow E \bullet + E$~~
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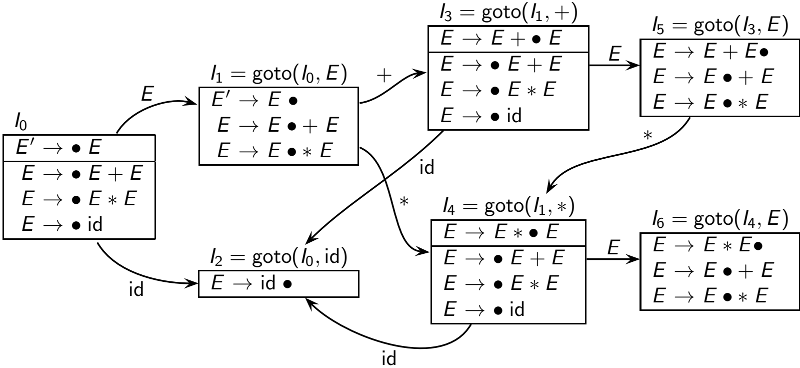
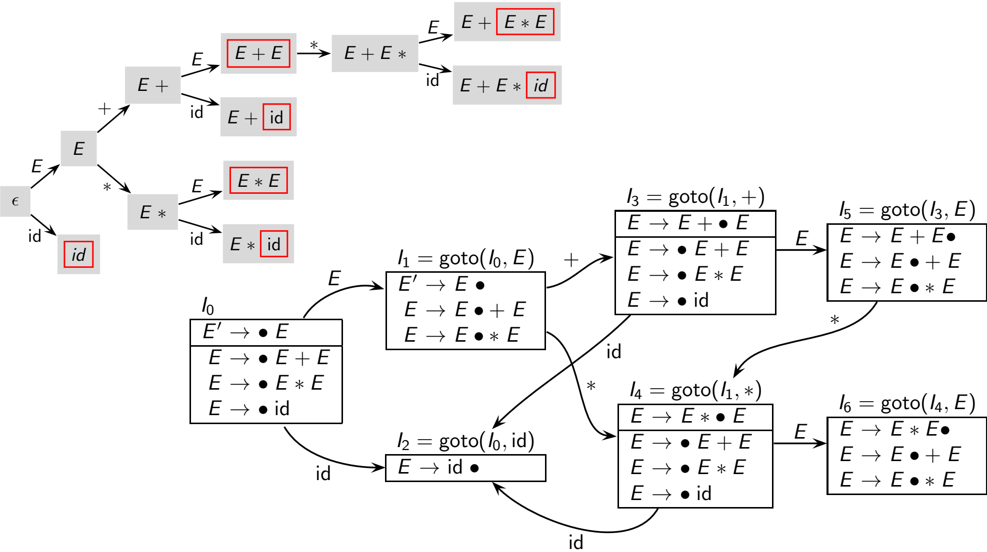
$I_6 = \text{goto}(I_4, E)$
$E \rightarrow E * E \bullet$
~~$E \rightarrow E \bullet + E$~~
~~$E \rightarrow E \bullet * E$~~

The DFA of Item Sets Accepts Viable Prefixes



The DFA of Item Sets Accepts Viable Prefixes



I_0
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$I_1 = \text{goto}(I_0, E)$
$E' \rightarrow E \bullet$
$E \rightarrow E \bullet + E$
$E \rightarrow E \bullet * E$

$I_2 = \text{goto}(I_0, \text{id})$
$E \rightarrow \text{id} \bullet$

$I_3 = \text{goto}(I_1, +)$
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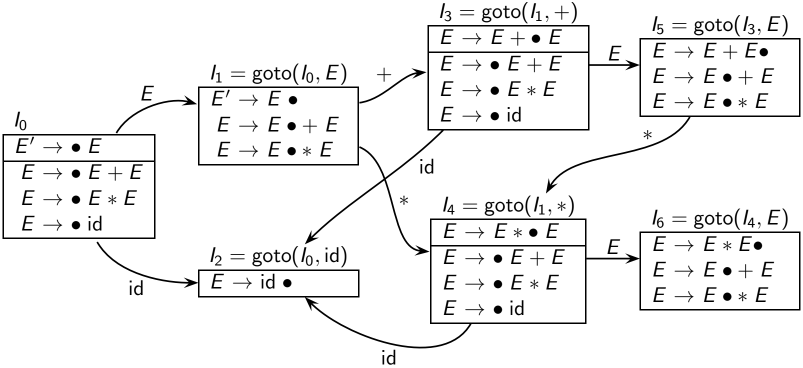
Putting it All Together: Constructing the Parsing Table

0 $E' \rightarrow E$
1 $E \rightarrow E + E$
2 $E \rightarrow E * E$
3 $E \rightarrow \text{id}$

$FOLLOW(E') = \{\$\}$
$FOLLOW(E) = \{\$, +, *\}$

	id	+	*	\$	E
0	$s2$				$c1$
1		$s3$	$s4$	acc	
2		$r3$	$r3$	$r3$	
3	$s2$				$c5$
4	$s2$				$c6$
5		$r1$	$s4$	$r1$	
6		$r2$	$r2$	$r2$	



I_0
$E' \rightarrow \bullet E$
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$E \rightarrow \bullet E * E$
$E \rightarrow \bullet \text{id}$

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$E' \rightarrow E \bullet$
$E \rightarrow E \bullet + E$
$E \rightarrow E \bullet * E$

$I_2 = \text{goto}(I_0, \text{id})$
$E \rightarrow \text{id} \bullet$

$I_3 = \text{goto}(I_1, +)$
$E \rightarrow E + \bullet E$
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cs302: Implementation of Programming Languages

Topic:

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Putting it All Together: Constructing the Parsing Table

	id	+	*	$	E
0	s2				c1
1		s3	s4	acc	
2		r3	r3	r3	
3	s2				c5
4	s2				c6
5		r1	s4	r1	
6		r2	r2	r2	

$0 \; E' \rightarrow E$
$1 \; E \rightarrow E + E$
$2 \; E \rightarrow E * E$
$3 \; E \rightarrow \text{id}$

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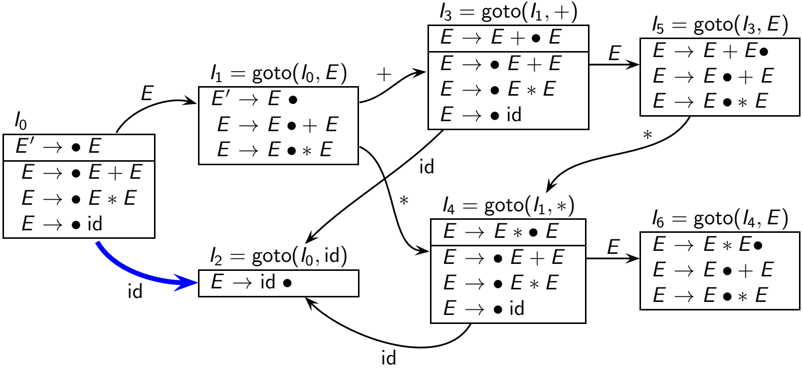

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Putting it All Together: Constructing the Parsing Table

	id	$+$	$*$	\$	E
0	$s2$				$c1$
1		$s3$	$s4$	acc	
2		$r3$	$r3$	$r3$	
3	$s2$				$c5$
4	$s2$				$c6$
5		$r1$	$s4$	$r1$	
6		$r2$	$r2$	$r2$	

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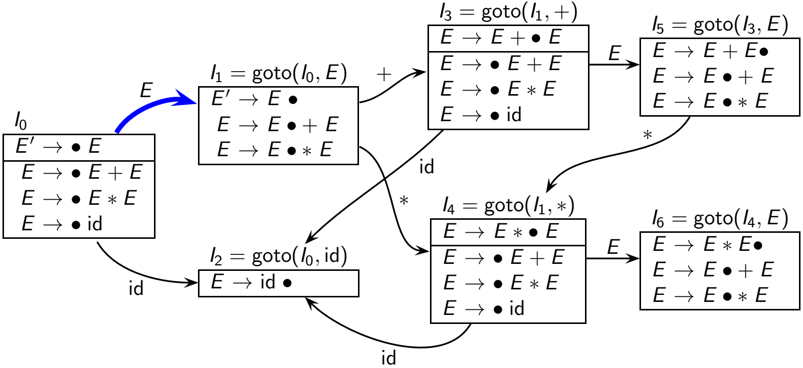

Putting it All Together: Constructing the Parsing Table

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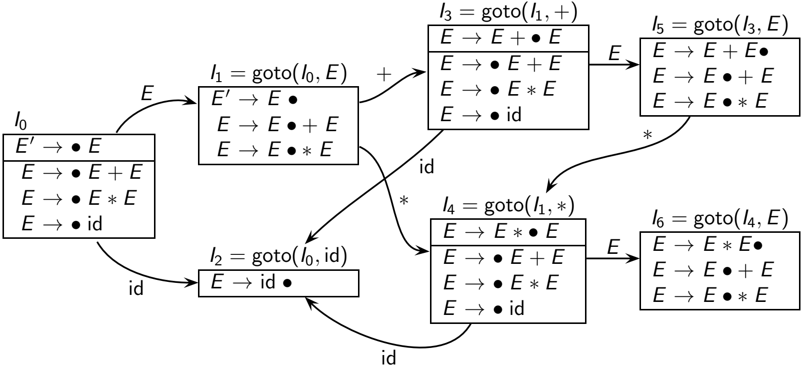
	id	+	*	$	E
0	s2				c1
1		s3	s4	acc	
2		r3	r3	r3	
3	s2				c5
4	s2				c6
5		r1	s4	r1	
6		r2	r2	r2	



I_0
$E' \rightarrow \bullet E$
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$E \rightarrow \bullet E * E$
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$I_1 = \text{goto}(I_0, E)$
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$E \rightarrow E \bullet * E$

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$E \rightarrow \text{id} \bullet$

$I_3 = \text{goto}(I_1, +)$
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$E \rightarrow \bullet \text{id}$

$I_4 = \text{goto}(I_1, *)$
$E \rightarrow E * \bullet E$
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$I_5 = \text{goto}(I_3, E)$
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$E \rightarrow E * E \bullet$
$E \rightarrow E \bullet + E$
$E \rightarrow E \bullet * E$

Putting it All Together: Constructing the Parsing Table

	id	+	*	$	E
0	$s2$				$c1$
1		$s3$	$\boxed{s4}$	acc	
2		$r3$	$r3$	$r3$	
3	$s2$				$c5$
4	$s2$				$c6$
5		$r1$	$s4$	$r1$	
6		$r2$	$r2$	$r2$	

0 $E' \rightarrow E$
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$FOLLOW(E') = \{\$\}$
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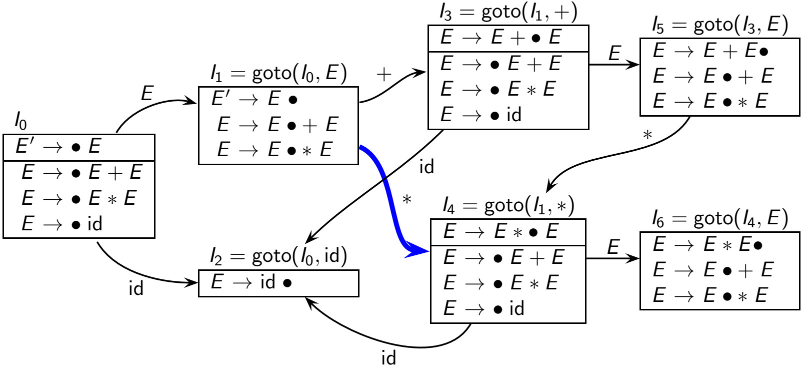

Putting it All Together: Constructing the Parsing Table

	id	$+$	$*$	\$	E
0	$s2$				$c1$
1		$\boxed{s3}$	$s4$	acc	
2		$r3$	$r3$	$r3$	
3	$s2$				$c5$
4	$s2$				$c6$
5		$r1$	$s4$	$r1$	
6		$r2$	$r2$	$r2$	

0 $E' \rightarrow E$
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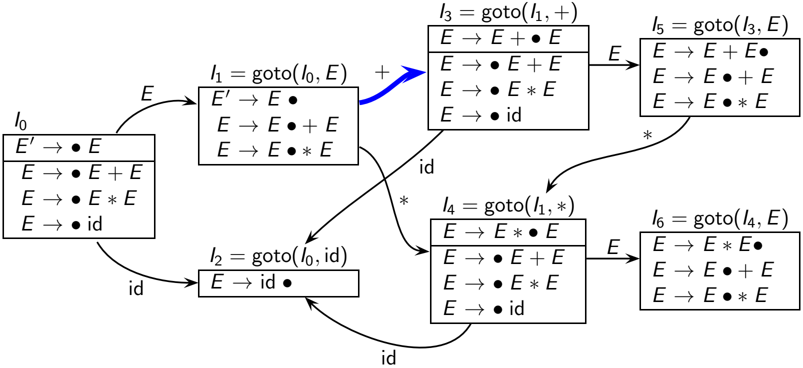

I_0
$E' \rightarrow \bullet E$
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$E \rightarrow \bullet \text{id}$

$I_1 = \text{goto}(I_0, E)$
$E' \rightarrow E \bullet$
$E \rightarrow E \bullet + E$
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$I_2 = \text{goto}(I_0, \text{id})$
$E \rightarrow \text{id} \bullet$

$I_3 = \text{goto}(I_1, +)$
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Putting it All Together: Constructing the Parsing Table

	id	$+$	$*$	\$	E
0	$s2$				$c1$
1		$s3$	$s4$	acc	
2		$r3$	$r3$	$r3$	
3	$s2$				$c5$
4	$s2$				$c6$
5		$r1$	$s4$	$r1$	
6		$r2$	$r2$	$r2$	

0 $E' \rightarrow E$
1 $E \rightarrow E + E$
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$$FOLLOW(E') = \{\$\}$$
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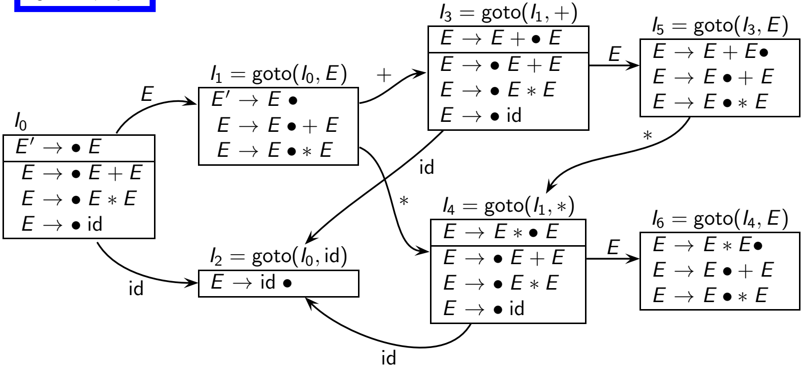

I_0: $E' \rightarrow \bullet E$ | $E \rightarrow \bullet E + E$, $E \rightarrow \bullet E * E$, $E \rightarrow \bullet \text{id}$

$I_1 = \text{goto}(I_0, E)$: $E' \rightarrow E \bullet$ | $E \rightarrow E \bullet + E$, $E \rightarrow E \bullet * E$

$I_2 = \text{goto}(I_0, \text{id})$: $E \rightarrow \text{id} \bullet$

$I_3 = \text{goto}(I_1, +)$: $E \rightarrow E + \bullet E$ | $E \rightarrow \bullet E + E$, $E \rightarrow \bullet E * E$, $E \rightarrow \bullet \text{id}$

$I_4 = \text{goto}(I_1, *)$: $E \rightarrow E * \bullet E$ | $E \rightarrow \bullet E + E$, $E \rightarrow \bullet E * E$, $E \rightarrow \bullet \text{id}$

$I_5 = \text{goto}(I_3, E)$: $E \rightarrow E + E \bullet$, $E \rightarrow E \bullet + E$, $E \rightarrow E \bullet * E$

$I_6 = \text{goto}(I_4, E)$: $E \rightarrow E * E \bullet$, $E \rightarrow E \bullet + E$, $E \rightarrow E \bullet * E$

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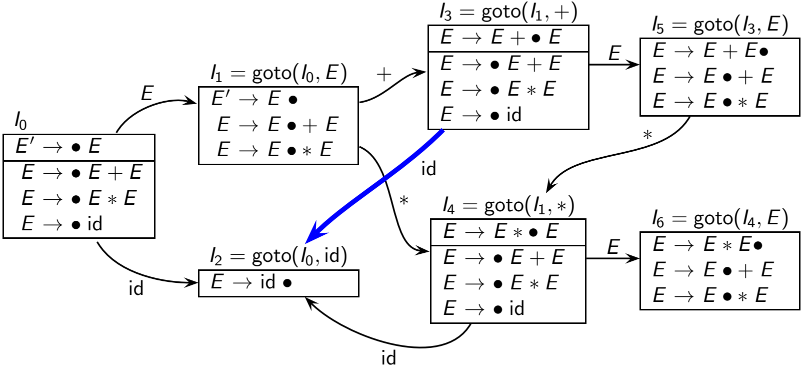
$FOLLOW(E) = \{\$, +, *\}$

	id	+	*	$	E
0	s2				c1
1		s3	s4	acc	
2		r3	r3	r3	
3	$\boxed{s2}$				c5
4	s2				c6
5		r1	s4	r1	
6		r2	r2	r2	



I_0
$E' \rightarrow \bullet E$
$E \rightarrow \bullet E + E$
$E \rightarrow \bullet E * E$
$E \rightarrow \bullet \text{id}$

$I_1 = \text{goto}(I_0, E)$
$E' \rightarrow E \bullet$
$E \rightarrow E \bullet + E$
$E \rightarrow E \bullet * E$

$I_2 = \text{goto}(I_0, \text{id})$
$E \rightarrow \text{id} \bullet$

$I_3 = \text{goto}(I_1, +)$
$E \rightarrow E + \bullet E$
$E \rightarrow \bullet E + E$
$E \rightarrow \bullet E * E$
$E \rightarrow \bullet \text{id}$

$I_4 = \text{goto}(I_1, *)$
$E \rightarrow E * \bullet E$
$E \rightarrow \bullet E + E$
$E \rightarrow \bullet E * E$
$E \rightarrow \bullet \text{id}$

$I_5 = \text{goto}(I_3, E)$
$E \rightarrow E + E \bullet$
$E \rightarrow E \bullet + E$
$E \rightarrow E \bullet * E$

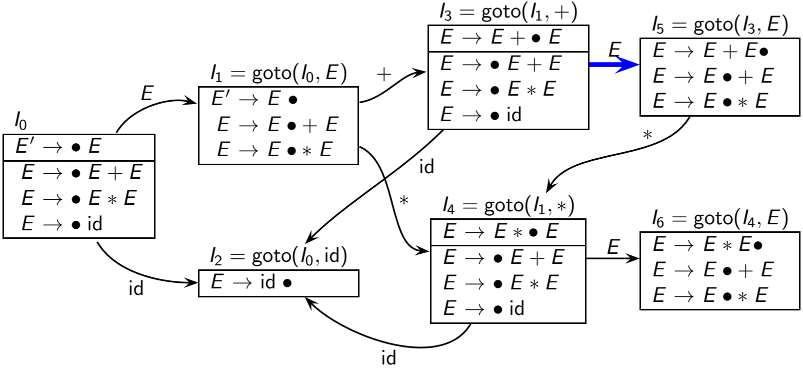
$I_6 = \text{goto}(I_4, E)$
$E \rightarrow E * E \bullet$
$E \rightarrow E \bullet + E$
$E \rightarrow E \bullet * E$

Putting it All Together: Constructing the Parsing Table

$0 \; E' \rightarrow E$
$1 \; E \rightarrow E + E$
$2 \; E \rightarrow E * E$
$3 \; E \rightarrow \text{id}$

$FOLLOW(E') = \{\$\}$
$FOLLOW(E) = \{\$, +, *\}$

	id	+	*	$	E
0	s2				c1
1		s3	s4	acc	
2		r3	r3	r3	
3	s2				c5
4	s2				c6
5		r1	s4	r1	
6		r2	r2	r2	



I_0
$E' \rightarrow \bullet E$
$E \rightarrow \bullet E + E$
$E \rightarrow \bullet E * E$
$E \rightarrow \bullet \text{id}$

$I_1 = \text{goto}(I_0, E)$
$E' \rightarrow E \bullet$
$E \rightarrow E \bullet + E$
$E \rightarrow E \bullet * E$

$I_2 = \text{goto}(I_0, \text{id})$
$E \rightarrow \text{id} \bullet$

$I_3 = \text{goto}(I_1, +)$
$E \rightarrow E + \bullet E$
$E \rightarrow \bullet E + E$
$E \rightarrow \bullet E * E$
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$E \rightarrow E * E \bullet$
$E \rightarrow E \bullet + E$
$E \rightarrow E \bullet * E$

IIT Bombay
cs302: Implementation of Programming Languages

Topic:

Syntax Analysis

Section:

Grammars, Derivations, and Parse Trees

Shift Reduce Parsing

SLR(1) Parsing

Conceptual Issues in Parsing

CLR(1) Parsing

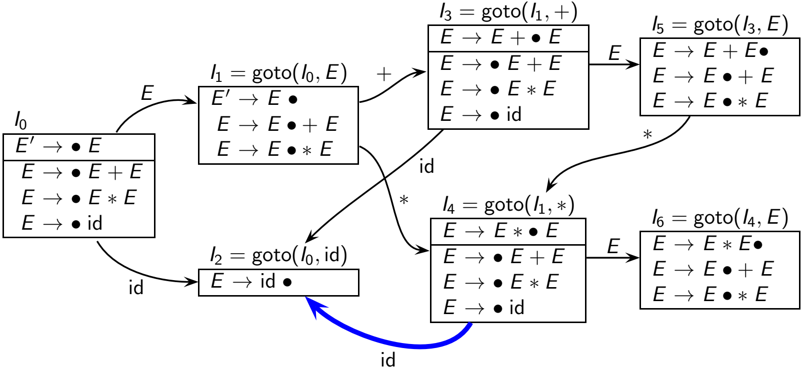
LALR(1) Parsing

Putting it All Together: Constructing the Parsing Table

	id	$+$	$*$	\$	E
0	$s2$				$c1$
1		$s3$	$s4$	acc	
2		$r3$	$r3$	$r3$	
3	$s2$				$c5$
4	$\boxed{s2}$				$c6$
5		$r1$	$s4$	$r1$	
6		$r2$	$r2$	$r2$	

0 $E' \rightarrow E$
1 $E \rightarrow E + E$
2 $E \rightarrow E * E$
3 $E \rightarrow \text{id}$

$FOLLOW(E') = \{\$\}$
$FOLLOW(E) = \{\$, +, *\}$



I_0
$E' \rightarrow \bullet E$
$E \rightarrow \bullet E + E$
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$E \rightarrow \bullet \text{id}$

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$E \rightarrow E \bullet * E$

$I_2 = \text{goto}(I_0, \text{id})$
$E \rightarrow \text{id} \bullet$

$I_3 = \text{goto}(I_1, +)$
$E \rightarrow E + \bullet E$
$E \rightarrow \bullet E + E$
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$I_5 = \text{goto}(I_3, E)$
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$E \rightarrow E \bullet + E$
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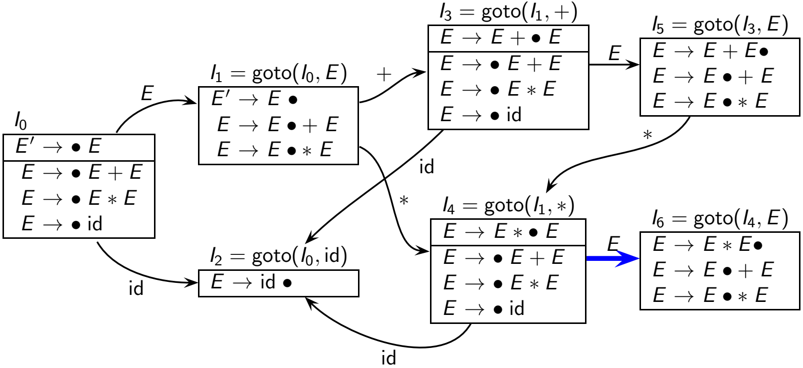
$E \rightarrow E * E \bullet$
$E \rightarrow E \bullet + E$
$E \rightarrow E \bullet * E$

Putting it All Together: Constructing the Parsing Table

	id	$+$	$*$	\$	E
0	$s2$				$c1$
1		$s3$	$s4$	acc	
2		$r3$	$r3$	$r3$	
3	$s2$				$c5$
4	$s2$				$\boxed{c6}$
5		$r1$	$s4$	$r1$	
6		$r2$	$r2$	$r2$	

0 $E' \to E$
1 $E \to E + E$
2 $E \to E * E$
3 $E \to \text{id}$

$FOLLOW(E') = \{\$\}$
$FOLLOW(E) = \{\$, +, *\}$



I_0
$E' \to \bullet E$
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$E \to \bullet E * E$
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$I_1 = \text{goto}(I_0, E)$
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$I_2 = \text{goto}(I_0, \text{id})$
$E \to \text{id} \bullet$

$I_3 = \text{goto}(I_1, +)$
$E \to E + \bullet E$
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$E \to E \bullet * E$

Putting it All Together: Constructing the Parsing Table

	id	$+$	$*$	\$	E
0	$s2$				$c1$
1		$s3$	$s4$	acc	
2		$r3$	$r3$	$r3$	
3	$s2$				$c5$
4	$s2$				$c6$
5		$r1$	$s4$	$r1$	
6		$r2$	$r2$	$r2$	

0 $E' \rightarrow E$
1 $E \rightarrow E + E$
2 $E \rightarrow E * E$
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$FOLLOW(E') = \{\$\}$
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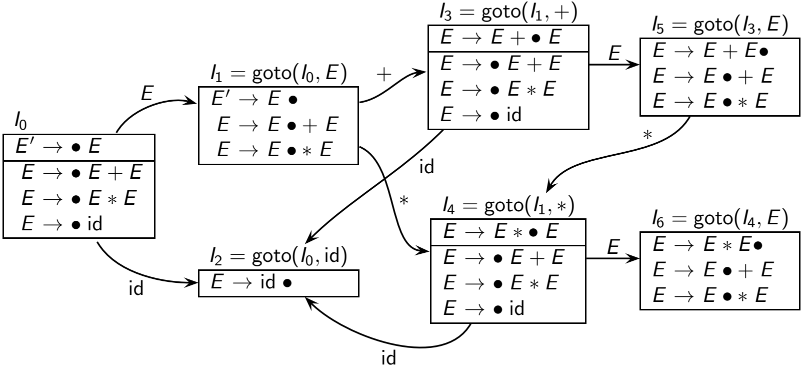

I_0
$E' \rightarrow \bullet E$
$E \rightarrow \bullet E + E$
$E \rightarrow \bullet E * E$
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$I_1 = \text{goto}(I_0, E)$
$E' \rightarrow E \bullet$
$E \rightarrow E \bullet + E$
$E \rightarrow E \bullet * E$

$I_2 = \text{goto}(I_0, \text{id})$
$E \rightarrow \text{id} \bullet$

$I_3 = \text{goto}(I_1, +)$
$E \rightarrow E + \bullet E$
$E \rightarrow \bullet E + E$
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$E \rightarrow E * \bullet E$
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$E \rightarrow E + E \bullet$
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$I_6 = \text{goto}(I_4, E)$
$E \rightarrow E * E \bullet$
$E \rightarrow E \bullet + E$
$E \rightarrow E \bullet * E$

Putting it All Together: Constructing the Parsing Table

	id	+	*	$	E
0	s2				c1
1		s3	s4	acc	
2		r3	r3	r3	
3	s2				c5
4	s2				c6
5		r1	s4	r1	
6		r2	r2	r2	

0 $E' \rightarrow E$
1 $E \rightarrow E + E$
2 $E \rightarrow E * E$
3 $E \rightarrow \text{id}$

$FOLLOW(E') = \{\$\}$
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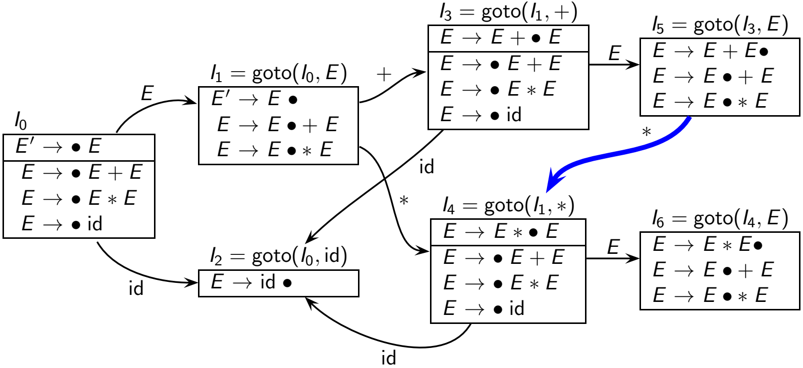

I_0
$E' \rightarrow \bullet E$
$E \rightarrow \bullet E + E$
$E \rightarrow \bullet E * E$
$E \rightarrow \bullet \text{id}$

$I_1 = \text{goto}(I_0, E)$
$E' \rightarrow E \bullet$
$E \rightarrow E \bullet + E$
$E \rightarrow E \bullet * E$

$I_2 = \text{goto}(I_0, \text{id})$
$E \rightarrow \text{id} \bullet$

$I_3 = \text{goto}(I_1, +)$
$E \rightarrow E + \bullet E$
$E \rightarrow \bullet E + E$
$E \rightarrow \bullet E * E$
$E \rightarrow \bullet \text{id}$

$I_4 = \text{goto}(I_1, *)$
$E \rightarrow E * \bullet E$
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$I_5 = \text{goto}(I_3, E)$
$E \rightarrow E + E \bullet$
$E \rightarrow E \bullet + E$
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$I_6 = \text{goto}(I_4, E)$
$E \rightarrow E * E \bullet$
$E \rightarrow E \bullet + E$
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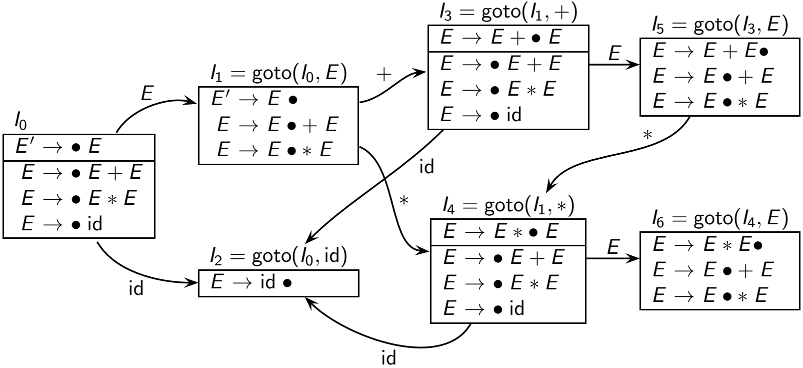
Putting it All Together: Constructing the Parsing Table

0 $E' \rightarrow E$
1 $E \rightarrow E + E$
2 $E \rightarrow E * E$
3 $E \rightarrow \text{id}$

$FOLLOW(E') = \{\$\}$
$FOLLOW(E) = \{\$, +, *\}$

	id	+	*	$	E
0	s2				c1
1		s3	s4	acc	
2		r3	r3	r3	
3	s2				c5
4	s2				c6
5		r1	s4	r1	
6		r2	r2	r2	



Destination Reached: From Intuitions to Formal Algorithms in Shift Reduce Parsing

1 $E \rightarrow E + E$
2 $E \rightarrow E * E$
3 $E \rightarrow \text{id}$

	id	+	*	$\$$	E
0	$s2$				$c1$
1		$s3$	$s4$	acc	
2		$r3$	$r3$	$r3$	
3	$s2$				$c5$
4	$s2$				$c6$
5		$\cancel{s3}/r1$	$s4/\cancel{r1}$	$r1$	
6		$\cancel{s3}/r2$	$\cancel{s4}/r2$	$r2$	

Step	Stack →	Input	Action
1	$\$$	$\text{id} + \text{id} * \text{id}\$$	shift
2	$\$\text{id}$	$+ \text{id} * \text{id}\$$	reduce by 3
3	$\$E$	$+ \text{id} * \text{id}\$$	shift
4	$\$E +$	$\text{id} * \text{id}\$$	shift
5	$\$E + \text{id}$	$* \text{id}\$$	reduce by 3
6	$\$E + E$	$* \text{id}\$$	shift
7	$\$E + E *$	$\text{id}\$$	shift
8	$\$E + E * \text{id}$	$\$$	reduce by 3
9	$\$E + E * E$	$\$$	reduce by 2
10	$\$E + E$	$\$$	reduce by 1
11	$\$E$	$\$$	accept

Step	Stack →	Input	Action
1	$\$0$	$\text{id} + \text{id} * \text{id}\$$	$s2$
2	$\$0\ \text{id}\ 2$	$+ \text{id} * \text{id}\$$	$r3$ and $c1$
3	$\$0\ E\ 1$	$+ \text{id} * \text{id}\$$	$s3$
4	$\$0\ E\ 1 + 3$	$\text{id} * \text{id}\$$	$s2$
5	$\$0\ E\ 1 + 3\ \text{id}\ 2$	$* \text{id}\$$	$r3$ and $c5$
6	$\$0\ E\ 1 + 3\ E\ 5$	$* \text{id}\$$	$s4$
7	$\$0\ E\ 1 + 3\ E\ 5 * 4$	$\text{id}\$$	$s2$
8	$\$0\ E\ 1 + 3\ E\ 5 * 4\ \text{id}\ 2$	$\$$	$r3$ and $c6$
9	$\$0\ E\ 1 + 3\ E\ 5 * 4\ E6$	$\$$	$r2$ and $c5$
10	$\$0\ E\ 1 + 3\ E\ 5$	$\$$	$r1$ and $c1$
11	$\$0\ E\ 1$	$\$$	$accept$

Conceptual Issues in Parsing

Conceptual Issues in Parsing

parsing-slides-sanyal-part4.pdf

CLR(1) Parsing

Limitation of SLR(1) Parsing

- We illustrate the limitations of SLR(1) parsing by using the pointer assignment grammar given below

$$\begin{array}{rcl} S & \rightarrow & L = R \mid R \\ L & \rightarrow & *R \mid \text{id} \\ R & \rightarrow & L \end{array}$$

- We compute the FOLLOW sets and sets of LR(0) items to demonstrate the problem
- We explain the cause of the problem
- This explanation leads us to a more precise method of CLR(1) parsing (Canonical LR(1) parsing that uses the LR(1) items)

Computing the FOLLOW Sets for Pointer Assignment Grammar

$$S' \rightarrow S$$

$$S \rightarrow L = R \mid R$$

$$\begin{array}{rcl} L & \rightarrow & *R \mid \text{id} \\ R & \rightarrow & L \end{array}$$

Computing the FOLLOW Sets for Pointer Assignment Grammar

$$
\begin{array}{lcl}
S' & \rightarrow & S \qquad \Rightarrow \begin{array}{lcl} \text{FOLLOW}(S') & \supseteq & \{\$\} \\ \text{FOLLOW}(S) & \supseteq & \text{FOLLOW}(S') \end{array} \\
S & \rightarrow & L = R \mid R \\
L & \rightarrow & *R \mid \text{id} \\
R & \rightarrow & L
\end{array}
$$

Computing the FOLLOW Sets for Pointer Assignment Grammar

$$\begin{array}{rcl} S' & \rightarrow & S \\ & & \\ S & \rightarrow & L = R \mid R \\ & & \\ L & \rightarrow & *R \mid \text{id} \\ R & \rightarrow & L \end{array} \quad \begin{array}{c} \Rightarrow \\ \\ \Rightarrow \\ \\ \\ \\ \end{array} \quad \begin{array}{lcl} \text{FOLLOW}(S') & \supseteq & \{\$\} \\ \text{FOLLOW}(S) & \supseteq & \text{FOLLOW}(S') \\ \text{FOLLOW}(L) & \supseteq & \{=\} \\ \text{FOLLOW}(R) & \supseteq & \text{FOLLOW}(S) \\ & & \\ & & \end{array}$$

Computing the FOLLOW Sets for Pointer Assignment Grammar

$$
\begin{array}{rcll@{\quad}rcl}
S' & \rightarrow & S & \Rightarrow & \text{FOLLOW}(S') & \supseteq & \{\$\} \\
 & & & & \text{FOLLOW}(S) & \supseteq & \text{FOLLOW}(S') \\
S & \rightarrow & L = R \mid R & \Rightarrow & \text{FOLLOW}(L) & \supseteq & \{=\} \\
 & & & & \text{FOLLOW}(R) & \supseteq & \text{FOLLOW}(S) \\
L & \rightarrow & *R \mid \text{id} & \Rightarrow & \text{FOLLOW}(R) & \supseteq & \text{FOLLOW}(L) \\
R & \rightarrow & L & & & &
\end{array}
$$

Computing the FOLLOW Sets for Pointer Assignment Grammar

$$
\begin{array}{rcllcl}
S' & \rightarrow & S & \Rightarrow & \text{FOLLOW}(S') & \supseteq \{\$\} \\
 & & & & \text{FOLLOW}(S) & \supseteq \text{FOLLOW}(S') \\
S & \rightarrow & L = R \mid R & \Rightarrow & \text{FOLLOW}(L) & \supseteq \{=\} \\
 & & & & \text{FOLLOW}(R) & \supseteq \text{FOLLOW}(S) \\
L & \rightarrow & *R \mid \text{id} & \Rightarrow & \text{FOLLOW}(R) & \supseteq \text{FOLLOW}(L) \\
R & \rightarrow & L & \Rightarrow & \text{FOLLOW}(L) & \supseteq \text{FOLLOW}(R)
\end{array}
$$

Computing the FOLLOW Sets for Pointer Assignment Grammar

$$
\begin{array}{rcl}
S' \rightarrow S & \Rightarrow & \text{FOLLOW}(S') \supseteq \{\$\} \\
 & & \text{FOLLOW}(S) \supseteq \text{FOLLOW}(S') \\
S \rightarrow L = R \mid R & \Rightarrow & \text{FOLLOW}(L) \supseteq \{=\} \\
 & & \text{FOLLOW}(R) \supseteq \text{FOLLOW}(S) \\
L \rightarrow *R \mid \text{id} & \Rightarrow & \text{FOLLOW}(R) \supseteq \text{FOLLOW}(L) \\
R \rightarrow L & \Rightarrow & \text{FOLLOW}(L) \supseteq \text{FOLLOW}(R)
\end{array}
$$

	FOLLOW
S'	$\{\$\}$
S	$\{\$\}$
R	$\{=, \$\}$
L	$\{=, \$\}$

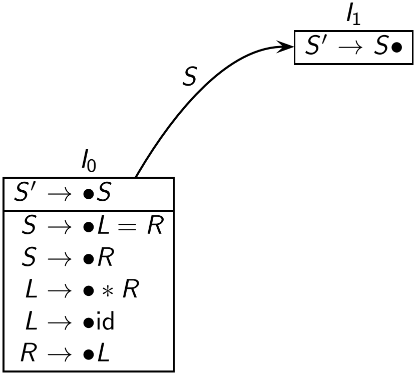
LR(0) Item Sets for Pointer Assignment Grammar

I_0

$S' \rightarrow \bullet S$

$S \rightarrow \bullet L = R$
$S \rightarrow \bullet R$
$L \rightarrow \bullet * R$
$L \rightarrow \bullet \text{id}$
$R \rightarrow \bullet L$

LR(0) Item Sets for Pointer Assignment Grammar



I_1

$S' \rightarrow S\bullet$

S

I_0

$S' \rightarrow \bullet S$

$S \rightarrow \bullet L = R$

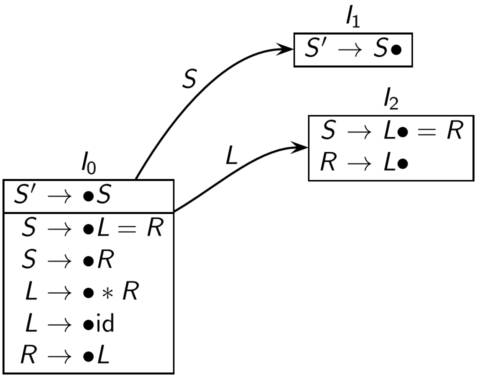
$S \rightarrow \bullet R$

$L \rightarrow \bullet * R$

$L \rightarrow \bullet \text{id}$

$R \rightarrow \bullet L$

LR(0) Item Sets for Pointer Assignment Grammar



I_0

$S' \rightarrow \bullet S$

$S \rightarrow \bullet L = R$
$S \rightarrow \bullet R$
$L \rightarrow \bullet * R$
$L \rightarrow \bullet \text{id}$
$R \rightarrow \bullet L$

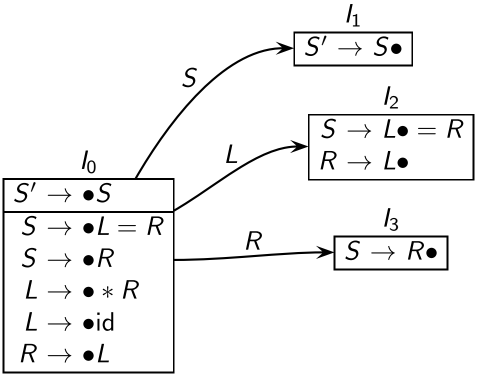
I_1

$S' \rightarrow S \bullet$

I_2

$S \rightarrow L \bullet = R$
$R \rightarrow L \bullet$

LR(0) Item Sets for Pointer Assignment Grammar



I_0

$S' \to \bullet S$

$S \to \bullet L = R$

$S \to \bullet R$

$L \to \bullet * R$

$L \to \bullet \text{id}$

$R \to \bullet L$

I_1

$S' \to S\bullet$

I_2

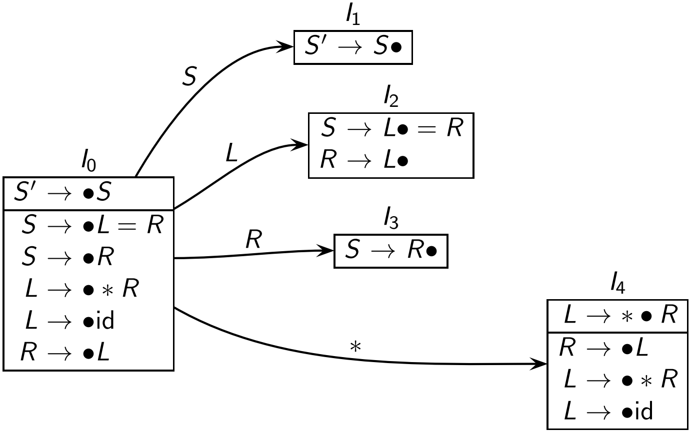
$S \to L\bullet = R$

$R \to L\bullet$

I_3

$S \to R\bullet$

LR(0) Item Sets for Pointer Assignment Grammar



I_0

$S' \to \bullet S$

$S \to \bullet L = R$
$S \to \bullet R$
$L \to \bullet * R$
$L \to \bullet \text{id}$
$R \to \bullet L$

I_1

$S' \to S \bullet$

I_2

$S \to L \bullet = R$
$R \to L \bullet$

I_3

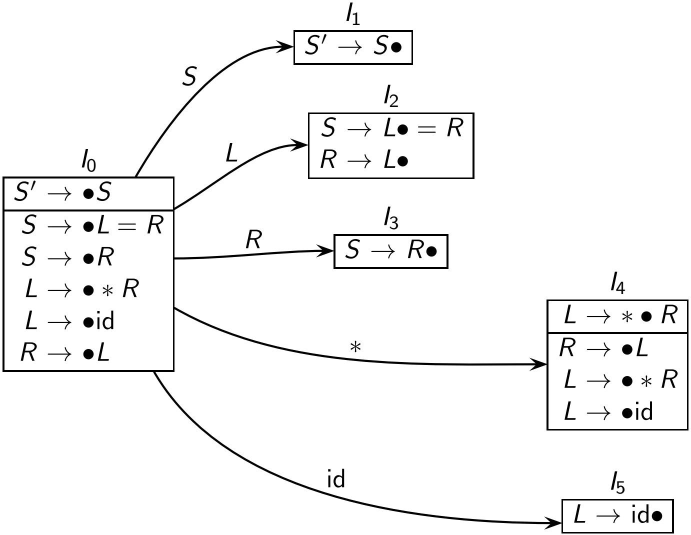
$S \to R \bullet$

I_4

$L \to * \bullet R$

$R \to \bullet L$
$L \to \bullet * R$
$L \to \bullet \text{id}$

LR(0) Item Sets for Pointer Assignment Grammar



I_0

$S' \rightarrow \bullet S$

$S \rightarrow \bullet L = R$

$S \rightarrow \bullet R$

$L \rightarrow \bullet * R$

$L \rightarrow \bullet \text{id}$

$R \rightarrow \bullet L$

I_1

$S' \rightarrow S \bullet$

I_2

$S \rightarrow L \bullet = R$

$R \rightarrow L \bullet$

I_3

$S \rightarrow R \bullet$

I_4

$L \rightarrow * \bullet R$

$R \rightarrow \bullet L$

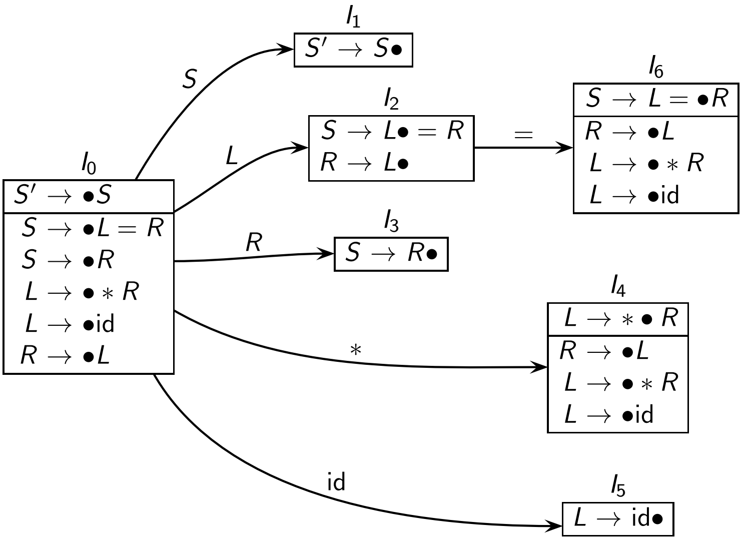
$L \rightarrow \bullet * R$

$L \rightarrow \bullet \text{id}$

I_5

$L \rightarrow \text{id} \bullet$

LR(0) Item Sets for Pointer Assignment Grammar



I_0

$S' \rightarrow \bullet S$

$S \rightarrow \bullet L = R$
$S \rightarrow \bullet R$
$L \rightarrow \bullet * R$
$L \rightarrow \bullet \text{id}$
$R \rightarrow \bullet L$

I_1

$S' \rightarrow S \bullet$

I_2

$S \rightarrow L \bullet = R$
$R \rightarrow L \bullet$

I_3

$S \rightarrow R \bullet$

I_4

$L \rightarrow * \bullet R$

$R \rightarrow \bullet L$
$L \rightarrow \bullet * R$
$L \rightarrow \bullet \text{id}$

I_5

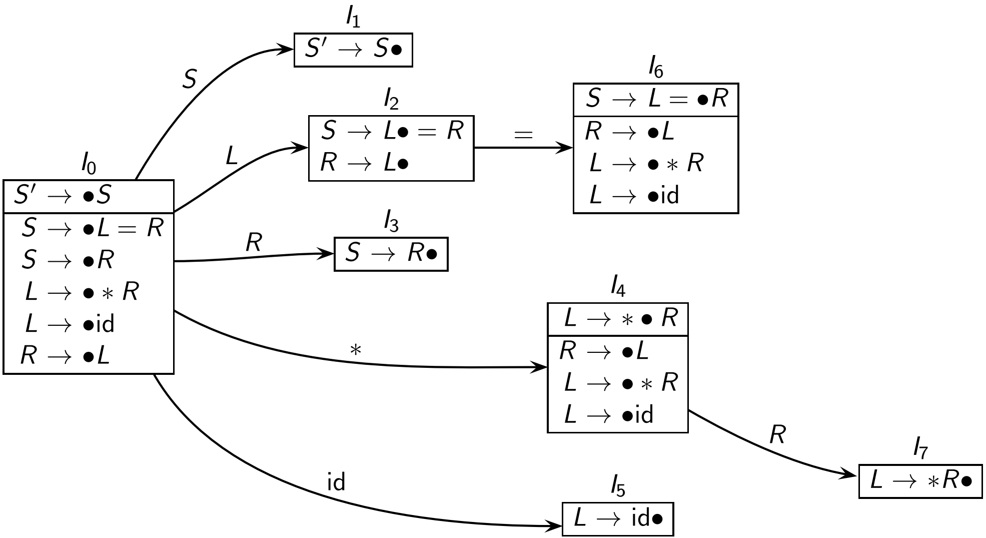
$L \rightarrow \text{id} \bullet$

I_6

$S \rightarrow L = \bullet R$

$R \rightarrow \bullet L$
$L \rightarrow \bullet * R$
$L \rightarrow \bullet \text{id}$

LR(0) Item Sets for Pointer Assignment Grammar



I_0
$S' \rightarrow \bullet S$
$S \rightarrow \bullet L = R$
$S \rightarrow \bullet R$
$L \rightarrow \bullet * R$
$L \rightarrow \bullet \text{id}$
$R \rightarrow \bullet L$

I_1
$S' \rightarrow S \bullet$

I_2
$S \rightarrow L \bullet = R$
$R \rightarrow L \bullet$

I_3
$S \rightarrow R \bullet$

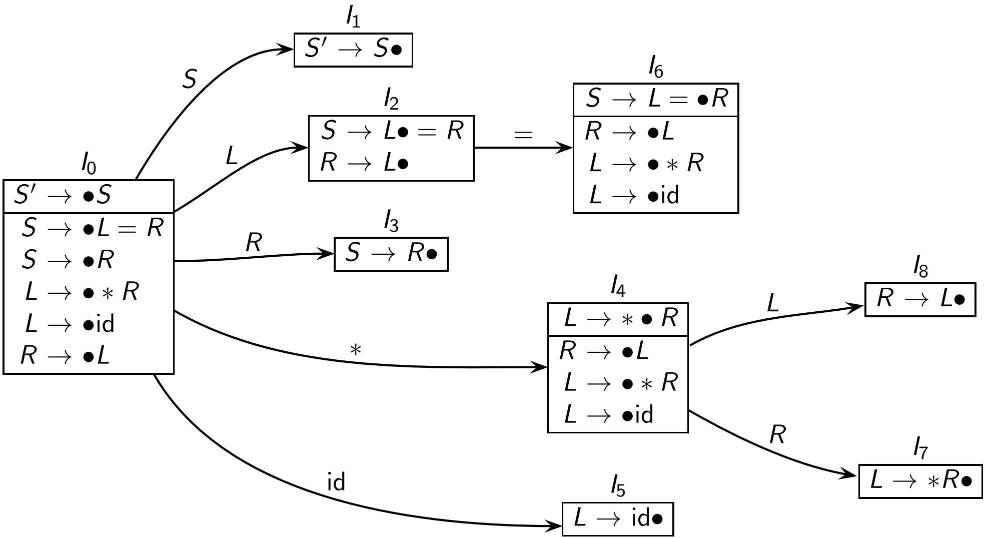
I_4
$L \rightarrow * \bullet R$
$R \rightarrow \bullet L$
$L \rightarrow \bullet * R$
$L \rightarrow \bullet \text{id}$

I_5
$L \rightarrow \text{id} \bullet$

I_6
$S \rightarrow L = \bullet R$
$R \rightarrow \bullet L$
$L \rightarrow \bullet * R$
$L \rightarrow \bullet \text{id}$

I_7
$L \rightarrow * R \bullet$

LR(0) Item Sets for Pointer Assignment Grammar



I_0
$S' \to \bullet S$
$S \to \bullet L = R$
$S \to \bullet R$
$L \to \bullet * R$
$L \to \bullet \text{id}$
$R \to \bullet L$

I_1
$S' \to S \bullet$

I_2
$S \to L \bullet = R$
$R \to L \bullet$

I_3
$S \to R \bullet$

I_4
$L \to * \bullet R$
$R \to \bullet L$
$L \to \bullet * R$
$L \to \bullet \text{id}$

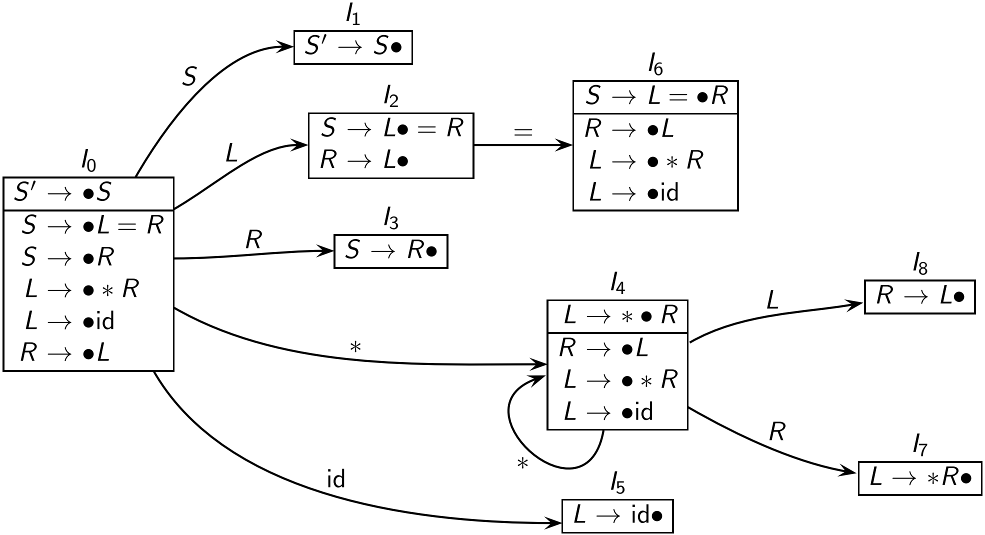
I_5
$L \to \text{id} \bullet$

I_6
$S \to L = \bullet R$
$R \to \bullet L$
$L \to \bullet * R$
$L \to \bullet \text{id}$

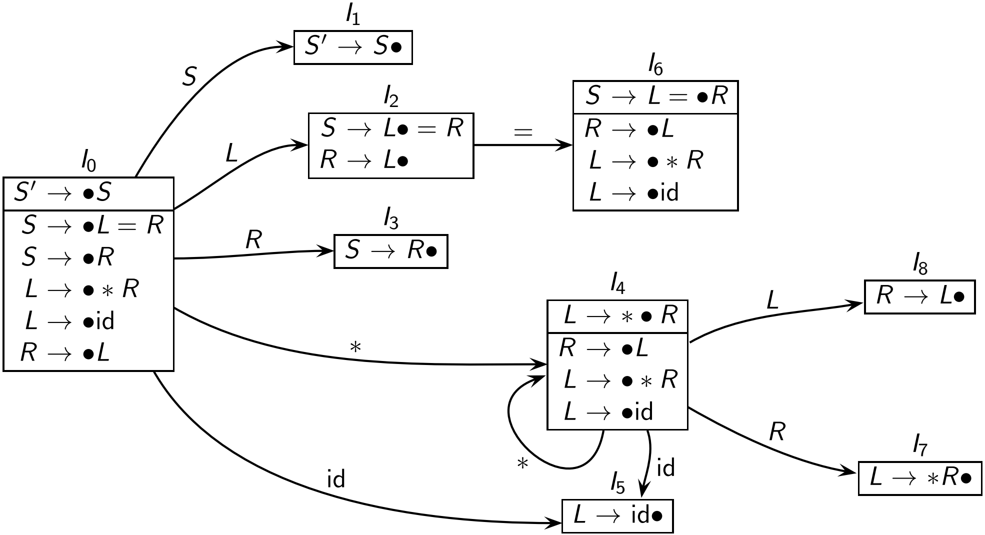
I_7
$L \to * R \bullet$

I_8
$R \to L \bullet$

LR(0) Item Sets for Pointer Assignment Grammar



LR(0) Item Sets for Pointer Assignment Grammar



I_0

$S' \rightarrow \bullet S$

$S \rightarrow \bullet L = R$

$S \rightarrow \bullet R$

$L \rightarrow \bullet * R$

$L \rightarrow \bullet \text{id}$

$R \rightarrow \bullet L$

I_1

$S' \rightarrow S \bullet$

I_2

$S \rightarrow L \bullet = R$

$R \rightarrow L \bullet$

I_3

$S \rightarrow R \bullet$

I_4

$L \rightarrow * \bullet R$

$R \rightarrow \bullet L$

$L \rightarrow \bullet * R$

$L \rightarrow \bullet \text{id}$

I_5

$L \rightarrow \text{id} \bullet$

I_6

$S \rightarrow L = \bullet R$

$R \rightarrow \bullet L$

$L \rightarrow \bullet * R$

$L \rightarrow \bullet \text{id}$

I_7

$L \rightarrow * R \bullet$

I_8

$R \rightarrow L \bullet$

IIT Bombay

cs302: Implementation of Programming Languages

Topic:

Syntax Analysis

Section:

Grammars, Derivations, and Parse Trees

Shift Reduce Parsing

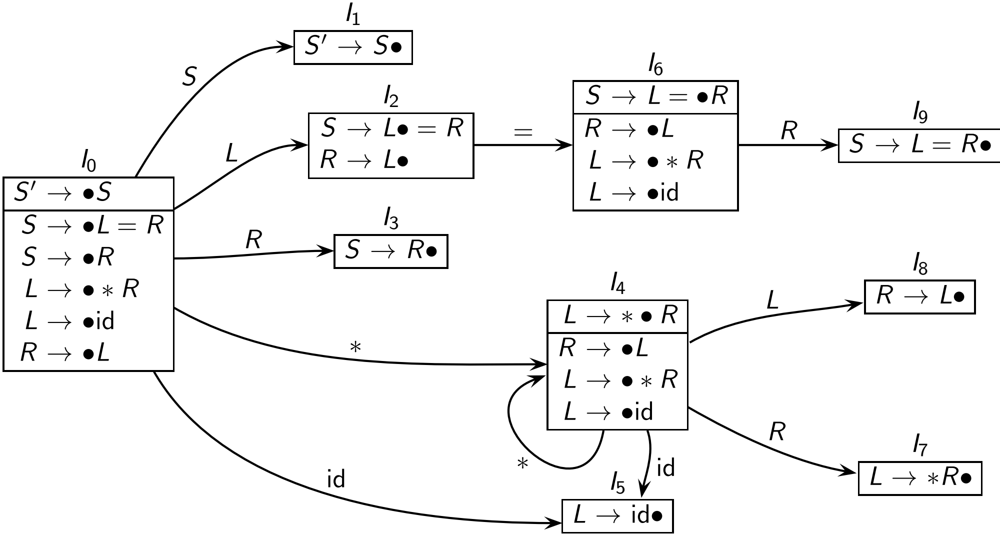
SLR(1) Parsing

Conceptual Issues in Parsing

CLR(1) Parsing

LALR(1) Parsing

LR(0) Item Sets for Pointer Assignment Grammar



I_0
$S' \to \bullet S$
$S \to \bullet L = R$
$S \to \bullet R$
$L \to \bullet * R$
$L \to \bullet \text{id}$
$R \to \bullet L$

I_1
$S' \to S\bullet$

I_2
$S \to L\bullet = R$
$R \to L\bullet$

I_3
$S \to R\bullet$

I_4
$L \to * \bullet R$
$R \to \bullet L$
$L \to \bullet * R$
$L \to \bullet \text{id}$

I_5
$L \to \text{id}\bullet$

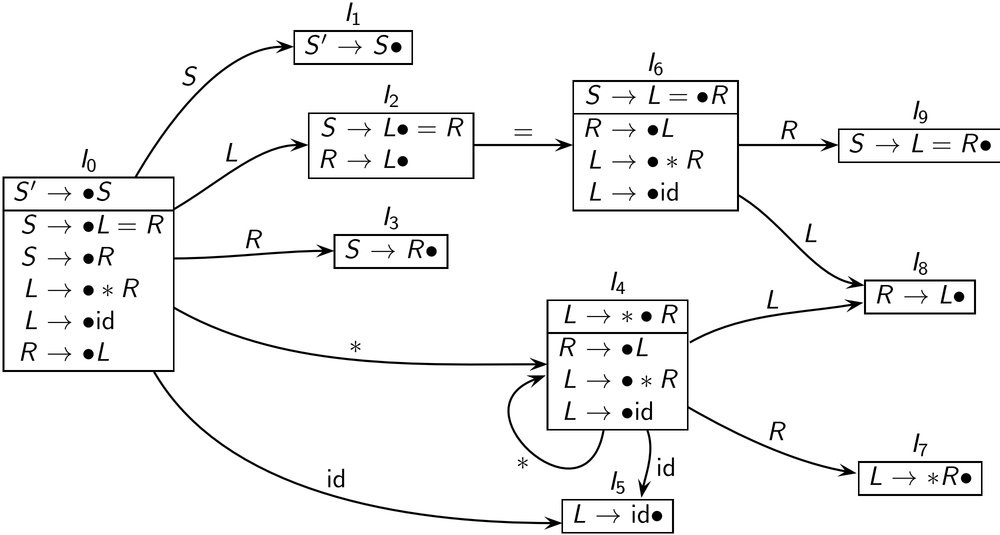
I_6
$S \to L = \bullet R$
$R \to \bullet L$
$L \to \bullet * R$
$L \to \bullet \text{id}$

I_7
$L \to * R\bullet$

I_8
$R \to L\bullet$

I_9
$S \to L = R\bullet$

LR(0) Item Sets for Pointer Assignment Grammar



I_0
$S' \to \bullet S$
$S \to \bullet L = R$
$S \to \bullet R$
$L \to \bullet * R$
$L \to \bullet \text{id}$
$R \to \bullet L$

I_1
$S' \to S \bullet$

I_2
$S \to L \bullet = R$
$R \to L \bullet$

I_3
$S \to R \bullet$

I_4
$L \to * \bullet R$
$R \to \bullet L$
$L \to \bullet * R$
$L \to \bullet \text{id}$

I_5
$L \to \text{id} \bullet$

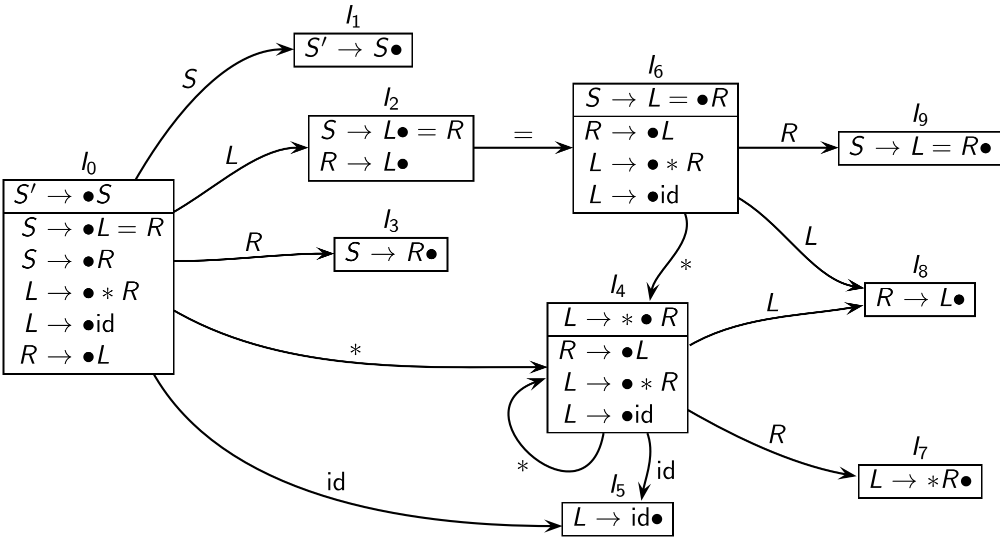
I_6
$S \to L = \bullet R$
$R \to \bullet L$
$L \to \bullet * R$
$L \to \bullet \text{id}$

I_7
$L \to * R \bullet$

I_8
$R \to L \bullet$

I_9
$S \to L = R \bullet$

LR(0) Item Sets for Pointer Assignment Grammar



I_0

$S' \rightarrow \bullet S$

$S \rightarrow \bullet L = R$
$S \rightarrow \bullet R$
$L \rightarrow \bullet * R$
$L \rightarrow \bullet \text{id}$
$R \rightarrow \bullet L$

I_1

$S' \rightarrow S \bullet$

I_2

$S \rightarrow L \bullet = R$
$R \rightarrow L \bullet$

I_3

$S \rightarrow R \bullet$

I_4

$L \rightarrow * \bullet R$

$R \rightarrow \bullet L$
$L \rightarrow \bullet * R$
$L \rightarrow \bullet \text{id}$

I_5

$L \rightarrow \text{id} \bullet$

I_6

$S \rightarrow L = \bullet R$

$R \rightarrow \bullet L$
$L \rightarrow \bullet * R$
$L \rightarrow \bullet \text{id}$

I_7

$L \rightarrow * R \bullet$

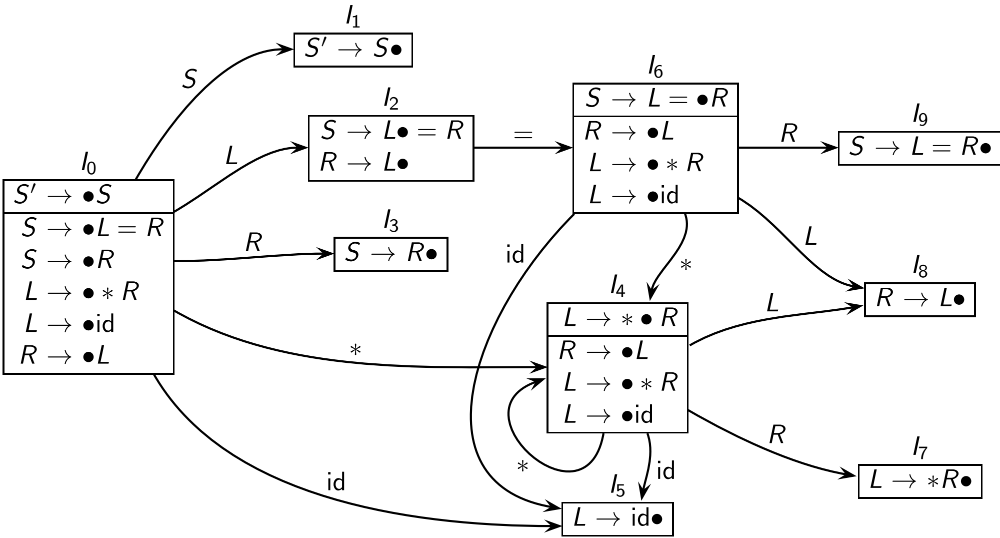
I_8

$R \rightarrow L \bullet$

I_9

$S \rightarrow L = R \bullet$

LR(0) Item Sets for Pointer Assignment Grammar



I_0
$S' \to \bullet S$
$S \to \bullet L = R$
$S \to \bullet R$
$L \to \bullet * R$
$L \to \bullet \text{id}$
$R \to \bullet L$

I_1
$S' \to S \bullet$

I_2
$S \to L \bullet = R$
$R \to L \bullet$

I_3
$S \to R \bullet$

I_4
$L \to * \bullet R$
$R \to \bullet L$
$L \to \bullet * R$
$L \to \bullet \text{id}$

I_5
$L \to \text{id} \bullet$

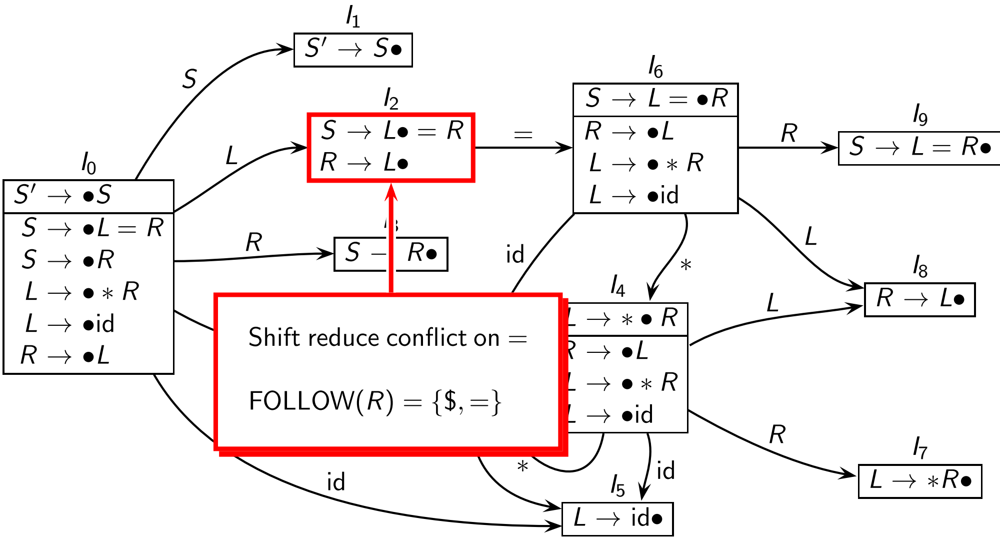
I_6
$S \to L = \bullet R$
$R \to \bullet L$
$L \to \bullet * R$
$L \to \bullet \text{id}$

I_7
$L \to * R \bullet$

I_8
$R \to L \bullet$

I_9
$S \to L = R \bullet$

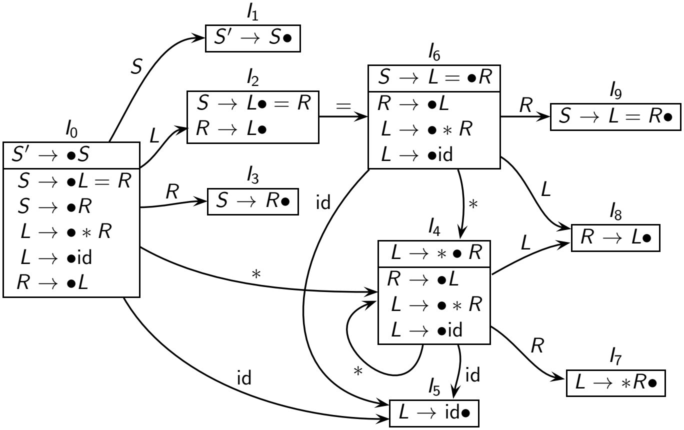
LR(0) Item Sets for Pointer Assignment Grammar



Shift reduce conflict on $=$

$\text{FOLLOW}(R) = \{\$, =\}$

Limitation of SLR(1) Parsing



I_0: $S' \to \bullet S$; $S \to \bullet L = R$; $S \to \bullet R$; $L \to \bullet * R$; $L \to \bullet \text{id}$; $R \to \bullet L$

I_1: $S' \to S\bullet$

I_2: $S \to L\bullet = R$; $R \to L\bullet$

I_3: $S \to R\bullet$

I_4: $L \to * \bullet R$; $R \to \bullet L$; $L \to \bullet * R$; $L \to \bullet \text{id}$

I_5: $L \to \text{id}\bullet$

I_6: $S \to L = \bullet R$; $R \to \bullet L$; $L \to \bullet * R$; $L \to \bullet \text{id}$

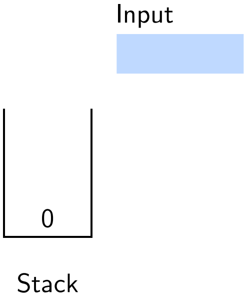
I_7: $L \to *R\bullet$

I_8: $R \to L\bullet$

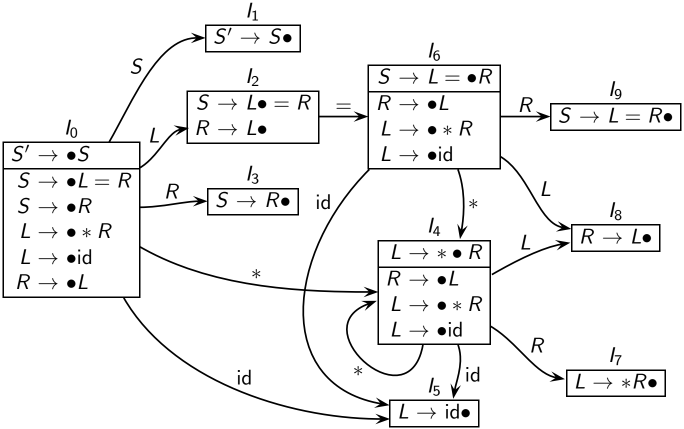
I_9: $S \to L = R\bullet$

1 $S \to L = R$
2 $S \to R$
3 $L \to *R$
4 $L \to \text{id}$
5 $R \to L$

	FOLLOW
S'	$\{\$\}$
S	$\{\$\}$
R	$\{=, \$\}$
L	$\{=, \$\}$



Limitation of SLR(1) Parsing



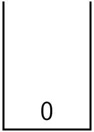
1	S	$\to$	$L = R$
2	S	$\to$	R
3	L	$\to$	$*R$
4	L	$\to$	id
5	R	$\to$	L

	FOLLOW
S'	$\{\$\}$
S	$\{\$\}$
R	$\{=, \$\}$
L	$\{=, \$\}$

Shift 5

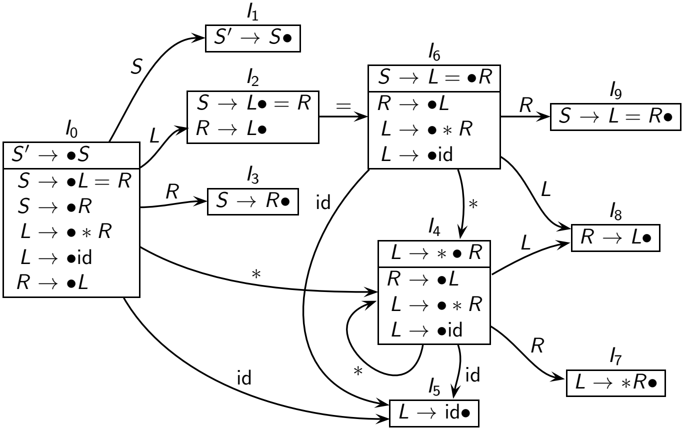
Input

id = id\$



Stack

Limitation of SLR(1) Parsing



1	S	$\rightarrow$	$L = R$
2	S	$\rightarrow$	R
3	L	$\rightarrow$	$*R$
4	L	$\rightarrow$	id
5	R	$\rightarrow$	L

	FOLLOW
S'	$\{\$\}$
S	$\{\$\}$
R	$\{=, \$\}$
L	$\{=, \$\}$

Reduce by 4

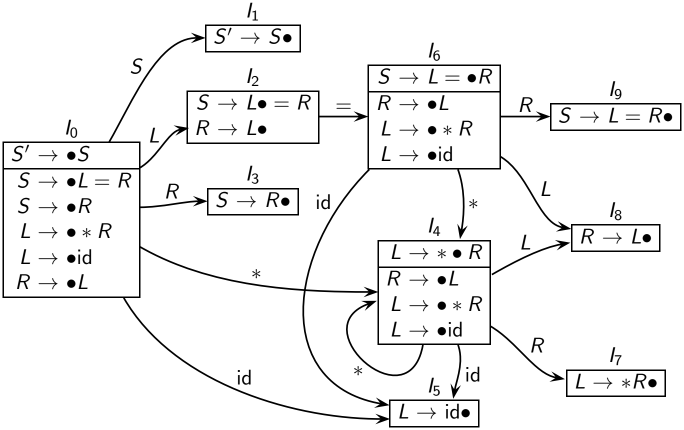
Input

$= \text{id}\$$

5
id
0

Stack

Limitation of SLR(1) Parsing



Cover by 2

Input

$= \text{id}\$$

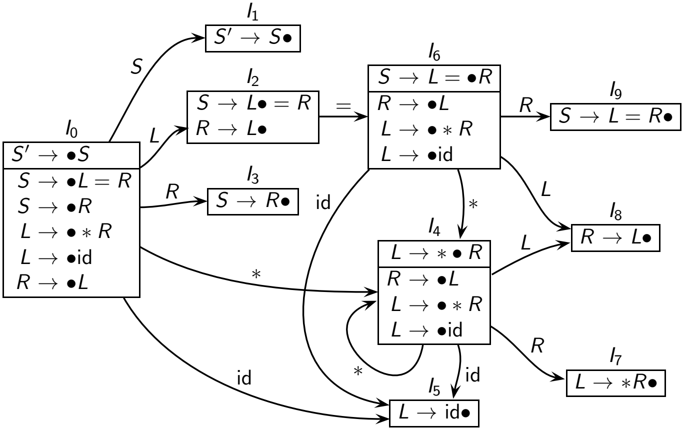
L
0

Stack

1 $S \rightarrow L = R$
2 $S \rightarrow R$
3 $L \rightarrow *R$
4 $L \rightarrow \text{id}$
5 $R \rightarrow L$

	FOLLOW
S'	$\{\$\}$
S	$\{\$\}$
R	$\{=, \$\}$
L	$\{=, \$\}$

Limitation of SLR(1) Parsing



I_0
$S' \to \bullet S$
$S \to \bullet L = R$
$S \to \bullet R$
$L \to \bullet * R$
$L \to \bullet \text{id}$
$R \to \bullet L$

I_1
$S' \to S \bullet$

I_2
$S \to L \bullet = R$
$R \to L \bullet$

I_3
$S \to R \bullet$

I_4
$L \to * \bullet R$
$R \to \bullet L$
$L \to \bullet * R$
$L \to \bullet \text{id}$

I_5
$L \to \text{id} \bullet$

I_6
$S \to L = \bullet R$
$R \to \bullet L$
$L \to \bullet * R$
$L \to \bullet \text{id}$

I_7
$L \to * R \bullet$

I_8
$R \to L \bullet$

I_9
$S \to L = R \bullet$

1	S	$\to$	$L = R$
2	S	$\to$	R
3	L	$\to$	$*R$
4	L	$\to$	id
5	R	$\to$	L

	FOLLOW
S'	$\{\$\}$
S	$\{\$\}$
R	$\{=, \$\}$
L	$\{=, \$\}$

Reduce by 5

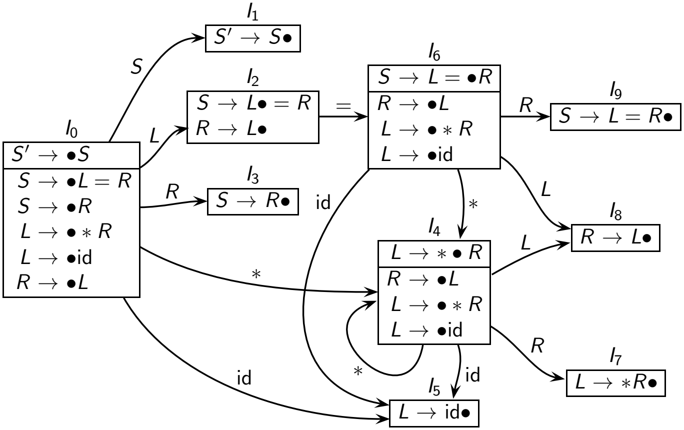
Input

$= \text{id}\$$

2
L
0

Stack

Limitation of SLR(1) Parsing



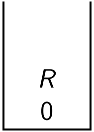
1	S	$\rightarrow$	$L = R$
2	S	$\rightarrow$	R
3	L	$\rightarrow$	$*R$
4	L	$\rightarrow$	id
5	R	$\rightarrow$	L

	FOLLOW
S'	$\{\$\}$
S	$\{\$\}$
R	$\{=, \$\}$
L	$\{=, \$\}$

Cover by 3

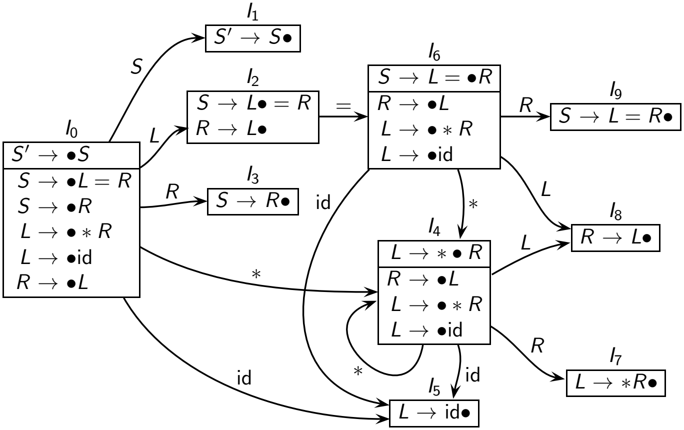
Input

$= \text{id}\$$



Stack

Limitation of SLR(1) Parsing



I_0: $S' \rightarrow \bullet S$, $S \rightarrow \bullet L = R$, $S \rightarrow \bullet R$, $L \rightarrow \bullet * R$, $L \rightarrow \bullet \text{id}$, $R \rightarrow \bullet L$

I_1: $S' \rightarrow S \bullet$

I_2: $S \rightarrow L \bullet = R$, $R \rightarrow L \bullet$

I_3: $S \rightarrow R \bullet$

I_4: $L \rightarrow * \bullet R$, $R \rightarrow \bullet L$, $L \rightarrow \bullet * R$, $L \rightarrow \bullet \text{id}$

I_5: $L \rightarrow \text{id} \bullet$

I_6: $S \rightarrow L = \bullet R$, $R \rightarrow \bullet L$, $L \rightarrow \bullet * R$, $L \rightarrow \bullet \text{id}$

I_7: $L \rightarrow * R \bullet$

I_8: $R \rightarrow L \bullet$

I_9: $S \rightarrow L = R \bullet$

1 $S \rightarrow L = R$
2 $S \rightarrow R$
3 $L \rightarrow *R$
4 $L \rightarrow \text{id}$
5 $R \rightarrow L$

	FOLLOW
S'	$\{\$\}$
S	$\{\$\}$
R	$\{=, \$\}$
L	$\{=, \$\}$

Error
No action on =

Input
$= \text{id}\$$

3
R
0

Stack

Limitation of SLR(1) Parsing: Use of FOLLOW Information

- Let FOLLOW$(A) = \{b, c\}$. Then b may follow A in some right sentential forms whereas in some other right sentential form, c may follow A

  A symbol in follow set need not follow A in *every* right sentential form

Limitation of SLR(1) Parsing: Use of FOLLOW Information

- Let FOLLOW(A) $= \{b, c\}$. Then b may follow A in some right sentential forms whereas in some other right sentential form, c may follow A

  A symbol in follow set need not follow A in *every* right sentential form
- We should declare handle $A \rightarrow \alpha$ in a viable prefix γ only if the follow symbols actually follows A in the right sentential form containing γ

Limitation of SLR(1) Parsing: Use of FOLLOW Information

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- In our grammar, there is no right sentential form with a prefix '$R =$'

Limitation of SLR(1) Parsing: Use of FOLLOW Information

- Let FOLLOW(A) $= \{b, c\}$. Then b may follow A in some right sentential forms whereas in some other right sentential form, c may follow A

  A symbol in follow set need not follow A in *every* right sentential form
- We should declare handle $A \rightarrow \alpha$ in a viable prefix γ only if the follow symbols actually follows A in the right sentential form containing γ
- In our grammar, there is no right sentential form with a prefix '$R =$'
  - Every right sentential form containing '$R =$' begins with a '$*$' and has a viable prefix '$*R$'

    We will never see '$=$' after an R without seeing a '$*$' before the 'R'

Limitation of SLR(1) Parsing: Use of FOLLOW Information

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    We will never see '$=$' after an R without seeing a '$*$' before the 'R'
  - $S \overset{rm}{\Rightarrow} L = R \overset{rm}{\Rightarrow} L = L \overset{rm}{\Rightarrow} L = \text{id} \overset{rm}{\Rightarrow} \text{id} = \text{id}$

    $S \overset{rm}{\Rightarrow} L = R \overset{rm}{\Rightarrow} L = \text{id} \overset{rm}{\Rightarrow} *R = \text{id} \overset{rm}{\Rightarrow} *\text{id} = \text{id}$

    $S \overset{rm}{\Rightarrow} L = R \overset{rm}{\Rightarrow} L = \text{id} \overset{rm}{\Rightarrow} *R = \text{id} \overset{rm}{\Rightarrow} *L = \text{id} \overset{rm}{\Rightarrow} *\text{id} = \text{id}$

    $\ldots$

Limitation of SLR(1) Parsing: Use of FOLLOW Information

- Let FOLLOW(A) $= \{b, c\}$. Then b may follow A in some right sentential forms whereas in some other right sentential form, c may follow A

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- We should declare handle $A \rightarrow \alpha$ in a viable prefix γ only if the follow symbols actually follows A in the right sentential form containing γ
- In our grammar, there is no right sentential form with a prefix '$R =$'
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    We will never see '$=$' after an R without seeing a '$*$' before the 'R'
  - $S \overset{rm}{\Rightarrow} L = R \overset{rm}{\Rightarrow} L = L \overset{rm}{\Rightarrow} L = \text{id} \overset{rm}{\Rightarrow} \text{id} = \text{id}$

    $S \overset{rm}{\Rightarrow} L = R \overset{rm}{\Rightarrow} L = \text{id} \overset{rm}{\Rightarrow} *R = \text{id} \overset{rm}{\Rightarrow} *\text{id} = \text{id}$

    $S \overset{rm}{\Rightarrow} L = R \overset{rm}{\Rightarrow} L = \text{id} \overset{rm}{\Rightarrow} *R = \text{id} \overset{rm}{\Rightarrow} *L = \text{id} \overset{rm}{\Rightarrow} *\text{id} = \text{id}$

    $\ldots$
  - '$=$' is in FOLLOW(R) only for the right sentential forms that begin with a '$*$'

    Input 'id $=$ id' does not begin with a '$*$' so L cannot be reduced to R on '$=$'

LR(1) Item Sets

Two changes from LR(0) construction

- Items are of the form $A \rightarrow \alpha \bullet \beta, a$ consisting of
  - the *core* $A \rightarrow \alpha \bullet \beta$ and
  - the *lookahead* a

  If S is the start symbol, then I_0 contains $S' \rightarrow \bullet S, \$$
- Closure of an item $A \rightarrow \alpha \bullet B\beta, a$ contains the items of the form $B \rightarrow \bullet\gamma, \text{FIRST}(\beta a)$

Transition of an item $A \rightarrow \alpha \bullet B\beta, a$ on B gives an item $A \rightarrow \alpha B \bullet \beta, a$

The lookahead does not change during a transition

LR(1) Item Sets

Two changes from LR(0) construction

- Items are of the form $A \rightarrow \alpha \bullet \beta, a$ consisting of
    - the *core* $A \rightarrow \alpha \bullet \beta$ and
    -

If S i

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The goal is to compute different subsets of FOLLOW(A) for $A \rightarrow \alpha$ in different right sentential forms

Since the construction of sets of items creates a DFA to recognize all viable prefixes, the subsets of FOLLOW can be computed for the productions in sets of items

Transition of an item $A \rightarrow \alpha \bullet B\beta, a$ on B gives an item $A \rightarrow \alpha B \bullet \beta, a$

The lookahead does not change during a transition

LR(1) Item Sets

Two changes from LR(0) construction

- Items are of the form $A \rightarrow \alpha \bullet \beta, a$ consisting of
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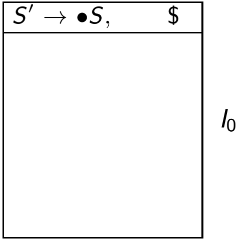
Transition of an item $A \rightarrow \alpha \bullet B\beta, a$ on B gives an item $A \rightarrow \alpha B \bullet \beta, a$

The lookahead does not change during a transition



LR(1) Item Sets

Two changes from LR(0) construction

- Items are of the form $A \rightarrow \alpha \bullet \beta, a$ consisting of
    - the *core* $A \rightarrow \alpha \bullet \beta$ and
    - the *lookahead* a

  If S is the start symbol, then I_0 contains $S' \rightarrow \bullet S, \$$
- Closure of an item $A \rightarrow \alpha \bullet B\beta, a$ contains the items of the form $B \rightarrow \bullet\gamma, \text{FIRST}(\beta a)$

Transition of an item $A \rightarrow \alpha \bullet B\beta, a$ on B gives an item $A \rightarrow \alpha B \bullet \beta, a$

The lookahead does not change during a transition

$S' \rightarrow \bullet S, \quad \$$

$S \rightarrow \bullet L = R, \$$
$S \rightarrow \bullet R, \quad \$$

I_0

LR(1) Item Sets

Two changes from LR(0) construction

- Items are of the form $A \rightarrow \alpha \bullet \beta, a$ consisting of
    - the *core* $A \rightarrow \alpha \bullet \beta$ and
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Transition of an item $A \rightarrow \alpha \bullet B\beta, a$ on B gives an item $A \rightarrow \alpha B \bullet \beta, a$

The lookahead does not change during a transition

$S' \rightarrow \bullet S,$	$\$$
$S \rightarrow \bullet L = R,$	$\$$
$S \rightarrow \bullet R,$	$\$$
$L \rightarrow \bullet * R,$	$=$
$L \rightarrow \bullet \text{id},$	$=$

I_0

LR(1) Item Sets

Two changes from LR(0) construction

- Items are of the form $A \rightarrow \alpha \bullet \beta, a$ consisting of
  - the *core* $A \rightarrow \alpha \bullet \beta$ and
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Transition of an item $A \rightarrow \alpha \bullet B\beta, a$ on B gives an item $A \rightarrow \alpha B \bullet \beta, a$

The lookahead does not change during a transition

$S' \rightarrow \bullet S,$	$\$$
$S \rightarrow \bullet L = R,$	$\$$
$S \rightarrow \bullet R,$	$\$$
$L \rightarrow \bullet * R,$	$=$
$L \rightarrow \bullet \text{id},$	$=$
$R \rightarrow \bullet L,$	$\$$
$L \rightarrow \bullet * R,$	$\$$

I_0

LR(1) Item Sets

Two changes from LR(0) construction

- Items are of the form $A \rightarrow \alpha \bullet \beta, a$ consisting of
  - the *core* $A \rightarrow \alpha \bullet \beta$ and
  - the *lookahead* a

  If S is the start symbol, then I_0 contains $S' \rightarrow \bullet S, \$$
- Closure of an item $A \rightarrow \alpha \bullet B\beta, a$ contains the items of the form $B \rightarrow \bullet\gamma, \text{FIRST}(\beta a)$

Transition of an item $A \rightarrow \alpha \bullet B\beta, a$ on B gives an item $A \rightarrow \alpha B \bullet \beta, a$

The lookahead does not change during a transition

$S' \rightarrow \bullet S,$	$\$$
$S \rightarrow \bullet L = R,$	$\$$
$S \rightarrow \bullet R,$	$\$$
$L \rightarrow \bullet * R,$	$=$
$L \rightarrow \bullet \text{id},$	$=$
$R \rightarrow \bullet L,$	$\$$
$L \rightarrow \bullet * R,$	$\$$
$L \rightarrow \bullet \text{id},$	$\$$

I_0

LR(1) Item Sets

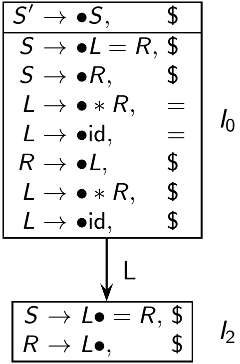
Two changes from LR(0) construction

- Items are of the form $A \to \alpha \bullet \beta, a$ consisting of
    - the *core* $A \to \alpha \bullet \beta$ and
    - the *lookahead* a

  If S is the start symbol, then I_0 contains $S' \to \bullet S, \$$
- Closure of an item $A \to \alpha \bullet B\beta, a$ contains the items of the form $B \to \bullet\gamma, \text{FIRST}(\beta a)$

Transition of an item $A \to \alpha \bullet B\beta, a$ on B gives an item $A \to \alpha B \bullet \beta, a$

The lookahead does not change during a transition



$S' \to \bullet S, \quad \$$

$S \to \bullet L = R, \$$

$S \to \bullet R, \quad \$$

$L \to \bullet * R, \quad =$

$L \to \bullet \text{id}, \quad =$

$R \to \bullet L, \quad \$$

$L \to \bullet * R, \quad \$$

$L \to \bullet \text{id}, \quad \$$

I_0

L

$S \to L \bullet = R, \$$

$R \to L \bullet, \quad \$$

I_2

LR(1) Item Sets

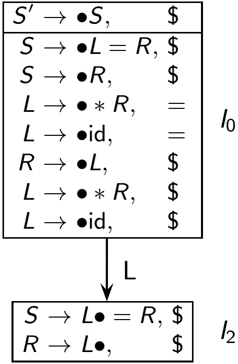
Two changes from LR(0) construction

- Items are of the form $A \rightarrow \alpha \bullet \beta, a$ consisting of
  - the *core* $A \rightarrow \alpha \bullet \beta$ and
  - the *lookahead* a

  If S is the start symbol, then I_0 contains $S' \rightarrow \bullet S, \$$
- Closure of an item $A \rightarrow \alpha \bullet B\beta, a$ contains the items of the form $B \rightarrow \bullet\gamma, \text{FIRST}(\beta a)$

Transition of an item $A \rightarrow \alpha \bullet B\beta, a$ on B gives an item $A \rightarrow \alpha B \bullet \beta, a$

The lookahead does not change during a transition



$S' \rightarrow \bullet S, \ \$$

$S \rightarrow \bullet L = R, \ \$$
$S \rightarrow \bullet R, \ \$$
$L \rightarrow \bullet * R, \ =$
$L \rightarrow \bullet \text{id}, \ =$
$R \rightarrow \bullet L, \ \$$
$L \rightarrow \bullet * R, \ \$$
$L \rightarrow \bullet \text{id}, \ \$$

I_0

$S \rightarrow L \bullet = R, \ \$$
$R \rightarrow L \bullet, \ \$$

I_2

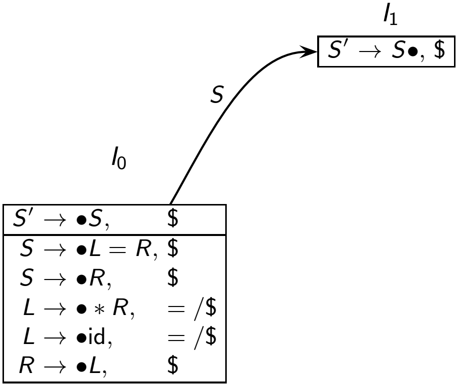
Reduction by $R \rightarrow L\bullet$ only on $\$$ and not on $=$
No shift reduce conflict

LR(1) Sets of Items for Pointer Assignment Grammar

I_0

$S' \rightarrow \bullet S,$	$\$$
$S \rightarrow \bullet L = R,$	$\$$
$S \rightarrow \bullet R,$	$\$$
$L \rightarrow \bullet * R,$	$= /\$$
$L \rightarrow \bullet \text{id},$	$= /\$$
$R \rightarrow \bullet L,$	$\$$

LR(1) Sets of Items for Pointer Assignment Grammar



I_1

$S' \rightarrow S\bullet,\ \$$

S

I_0

$S' \rightarrow \bullet S,\quad \$$

$S \rightarrow \bullet L = R,\ \$$

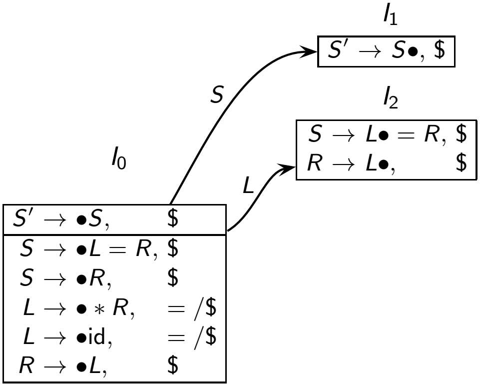
$S \rightarrow \bullet R,\quad \$$

$L \rightarrow \bullet * R,\quad =/\$$

$L \rightarrow \bullet \text{id},\quad =/\$$

$R \rightarrow \bullet L,\quad \$$

LR(1) Sets of Items for Pointer Assignment Grammar



I_0

$S' \rightarrow \bullet S, \quad \$$

$S \rightarrow \bullet L = R, \ \$$

$S \rightarrow \bullet R, \quad \$$

$L \rightarrow \bullet * R, \quad = /\$$

$L \rightarrow \bullet \text{id}, \quad = /\$$

$R \rightarrow \bullet L, \quad \$$

I_1

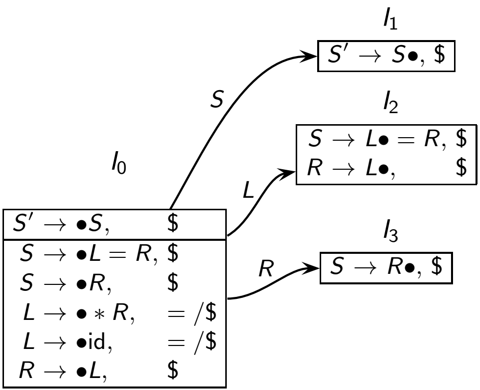
$S' \rightarrow S\bullet, \ \$$

I_2

$S \rightarrow L\bullet = R, \ \$$

$R \rightarrow L\bullet, \quad \$$

LR(1) Sets of Items for Pointer Assignment Grammar



I_0

$S' \rightarrow \bullet S, \quad \$$

$S \rightarrow \bullet L = R, \quad \$$

$S \rightarrow \bullet R, \quad \$$

$L \rightarrow \bullet * R, \quad =/\$$

$L \rightarrow \bullet \text{id}, \quad =/\$$

$R \rightarrow \bullet L, \quad \$$

I_1

$S' \rightarrow S\bullet, \ \$$

I_2

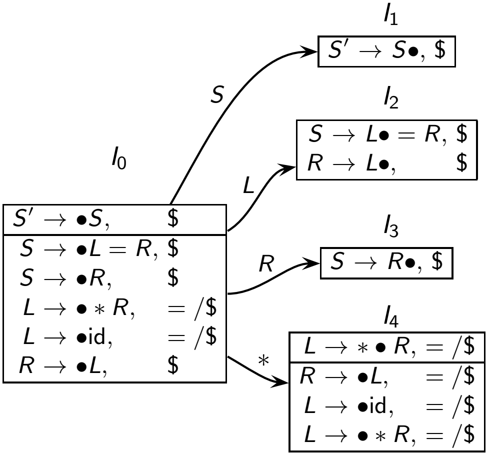
$S \rightarrow L\bullet = R, \ \$$

$R \rightarrow L\bullet, \quad \$$

I_3

$S \rightarrow R\bullet, \ \$$

LR(1) Sets of Items for Pointer Assignment Grammar



I_0

$S' \rightarrow \bullet S, \quad \$$

$S \rightarrow \bullet L = R, \ \$$

$S \rightarrow \bullet R, \quad \$$

$L \rightarrow \bullet * R, \quad = /\$$

$L \rightarrow \bullet \text{id}, \quad = /\$$

$R \rightarrow \bullet L, \quad \$$

I_1

$S' \rightarrow S\bullet, \ \$$

I_2

$S \rightarrow L\bullet = R, \ \$$

$R \rightarrow L\bullet, \quad \$$

I_3

$S \rightarrow R\bullet, \ \$$

I_4

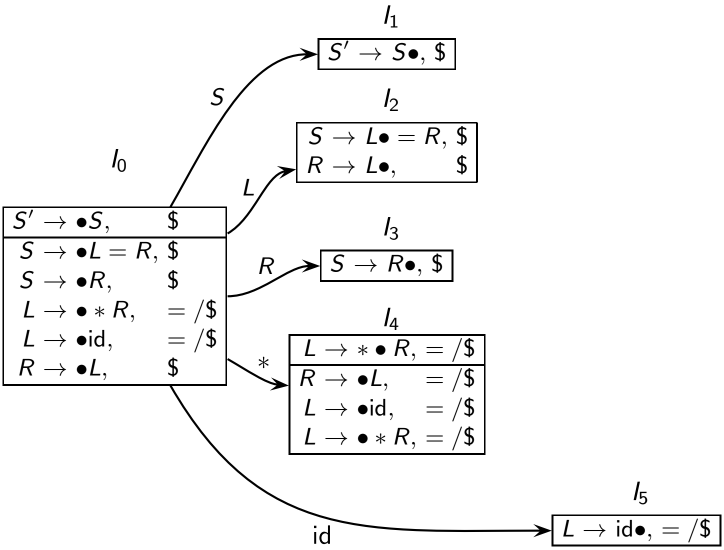
$L \rightarrow * \bullet R, \ = /\$$

$R \rightarrow \bullet L, \quad = /\$$

$L \rightarrow \bullet \text{id}, \quad = /\$$

$L \rightarrow \bullet * R, \ = /\$$

LR(1) Sets of Items for Pointer Assignment Grammar



I_0

$S' \rightarrow \bullet S, \quad \$$

$S \rightarrow \bullet L = R, \quad \$$

$S \rightarrow \bullet R, \quad \$$

$L \rightarrow \bullet * R, \quad =/\$$

$L \rightarrow \bullet \text{id}, \quad =/\$$

$R \rightarrow \bullet L, \quad \$$

I_1

$S' \rightarrow S\bullet, \ \$$

I_2

$S \rightarrow L\bullet = R, \ \$$

$R \rightarrow L\bullet, \quad \$$

I_3

$S \rightarrow R\bullet, \ \$$

I_4

$L \rightarrow * \bullet R, =/\$$

$R \rightarrow \bullet L, \quad =/\$$

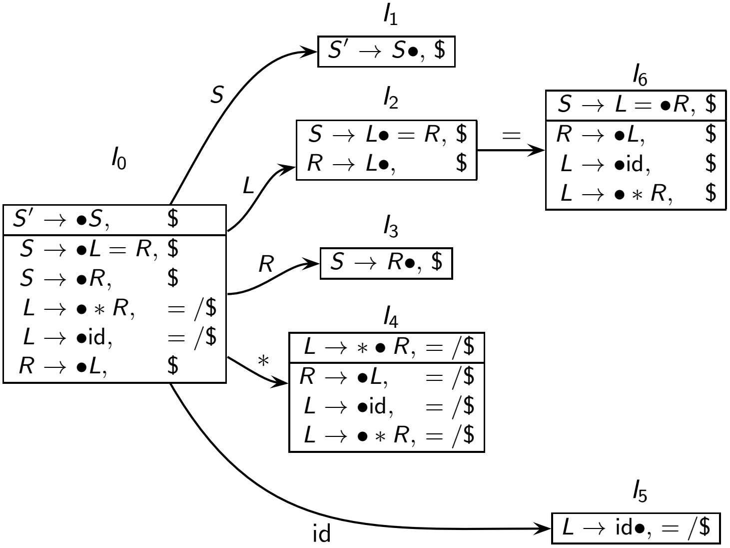
$L \rightarrow \bullet \text{id}, \quad =/\$$

$L \rightarrow \bullet * R, =/\$$

I_5

$L \rightarrow \text{id}\bullet, =/\$$

LR(1) Sets of Items for Pointer Assignment Grammar



I_0

$S' \rightarrow \bullet S, \quad \$$

$S \rightarrow \bullet L = R, \ \$$

$S \rightarrow \bullet R, \quad \$$

$L \rightarrow \bullet * R, \quad =/\$$

$L \rightarrow \bullet \text{id}, \quad =/\$$

$R \rightarrow \bullet L, \quad \$$

I_1

$S' \rightarrow S \bullet, \ \$$

I_2

$S \rightarrow L \bullet = R, \ \$$

$R \rightarrow L \bullet, \quad \$$

I_3

$S \rightarrow R \bullet, \ \$$

I_4

$L \rightarrow * \bullet R, \ =/\$$

$R \rightarrow \bullet L, \quad =/\$$

$L \rightarrow \bullet \text{id}, \quad =/\$$

$L \rightarrow \bullet * R, \ =/\$$

I_5

$L \rightarrow \text{id} \bullet, \ =/\$$

I_6

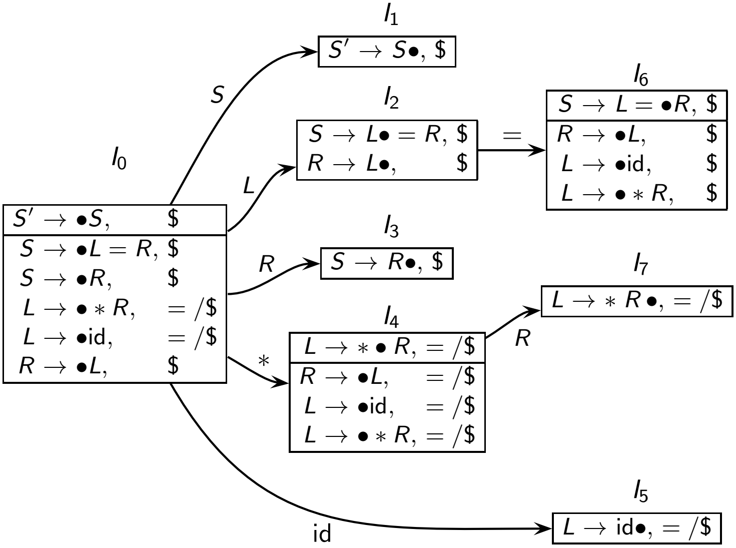
$S \rightarrow L = \bullet R, \ \$$

$R \rightarrow \bullet L, \quad \$$

$L \rightarrow \bullet \text{id}, \quad \$$

$L \rightarrow \bullet * R, \quad \$$

LR(1) Sets of Items for Pointer Assignment Grammar



I_0

$S' \rightarrow \bullet S, \quad \$$

$S \rightarrow \bullet L = R, \ \$$

$S \rightarrow \bullet R, \quad \$$

$L \rightarrow \bullet * R, \quad = / \$$

$L \rightarrow \bullet \text{id}, \quad = / \$$

$R \rightarrow \bullet L, \quad \$$

I_1

$S' \rightarrow S \bullet, \ \$$

I_2

$S \rightarrow L \bullet = R, \ \$$

$R \rightarrow L \bullet, \quad \$$

I_3

$S \rightarrow R \bullet, \ \$$

I_4

$L \rightarrow * \bullet R, \ = / \$$

$R \rightarrow \bullet L, \quad = / \$$

$L \rightarrow \bullet \text{id}, \quad = / \$$

$L \rightarrow \bullet * R, \ = / \$$

I_5

$L \rightarrow \text{id} \bullet, \ = / \$$

I_6

$S \rightarrow L = \bullet R, \ \$$

$R \rightarrow \bullet L, \quad \$$

$L \rightarrow \bullet \text{id}, \quad \$$

$L \rightarrow \bullet * R, \quad \$$

I_7

$L \rightarrow * R \bullet, \ = / \$$

IIT Bombay
cs302: Implementation of Programming Languages

Topic:

Syntax Analysis

Section:

Grammars, Derivations, and Parse Trees

Shift Reduce Parsing

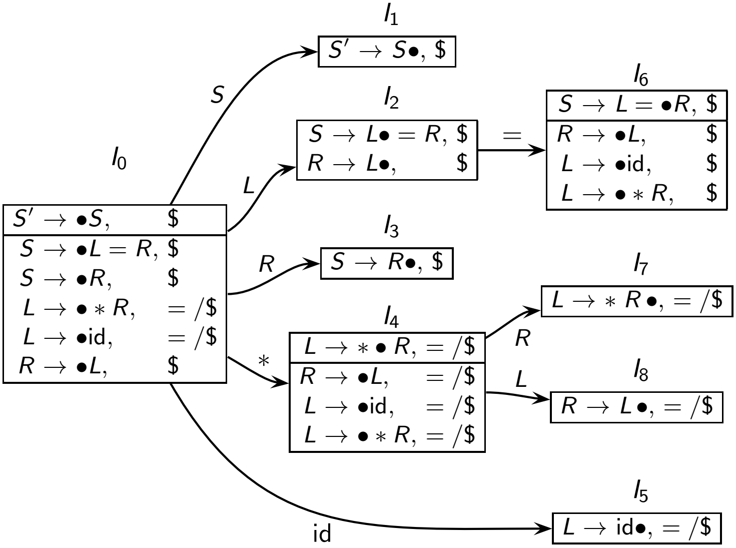
SLR(1) Parsing

Conceptual Issues in Parsing

CLR(1) Parsing

LALR(1) Parsing

LR(1) Sets of Items for Pointer Assignment Grammar



I_0
$S' \rightarrow \bullet S, \quad \$$
$S \rightarrow \bullet L = R, \quad \$$
$S \rightarrow \bullet R, \quad \$$
$L \rightarrow \bullet * R, \quad = /\$$
$L \rightarrow \bullet \text{id}, \quad = /\$$
$R \rightarrow \bullet L, \quad \$$

I_1
$S' \rightarrow S \bullet, \ \$$

I_2
$S \rightarrow L \bullet = R, \ \$$
$R \rightarrow L \bullet, \quad \$$

I_3
$S \rightarrow R \bullet, \ \$$

I_4
$L \rightarrow * \bullet R, \ = /\$$
$R \rightarrow \bullet L, \quad = /\$$
$L \rightarrow \bullet \text{id}, \quad = /\$$
$L \rightarrow \bullet * R, \ = /\$$

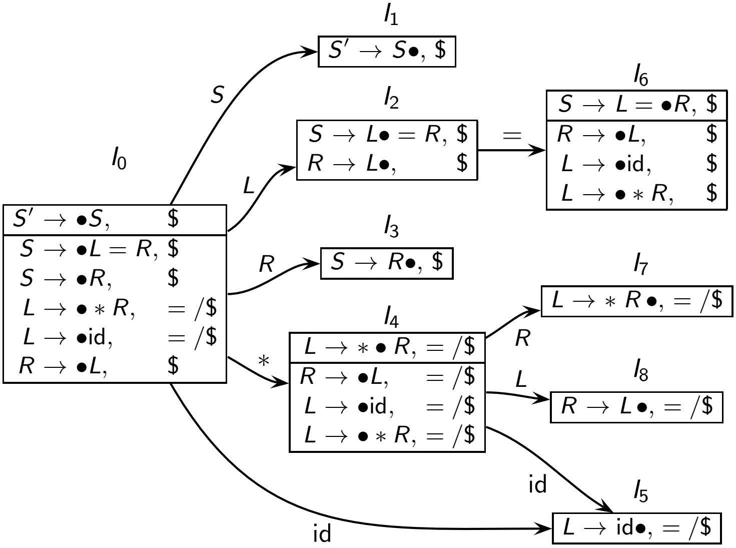
I_5
$L \rightarrow \text{id} \bullet, \ = /\$$

I_6
$S \rightarrow L = \bullet R, \ \$$
$R \rightarrow \bullet L, \quad \$$
$L \rightarrow \bullet \text{id}, \quad \$$
$L \rightarrow \bullet * R, \quad \$$

I_7
$L \rightarrow * R \bullet, \ = /\$$

I_8
$R \rightarrow L \bullet, \ = /\$$

LR(1) Sets of Items for Pointer Assignment Grammar



I_0

$S' \rightarrow \bullet S,\ \$$

$S \rightarrow \bullet L = R,\ \$$

$S \rightarrow \bullet R,\ \$$

$L \rightarrow \bullet * R,\ =/\$$

$L \rightarrow \bullet \text{id},\ =/\$$

$R \rightarrow \bullet L,\ \$$

I_1

$S' \rightarrow S \bullet,\ \$$

I_2

$S \rightarrow L \bullet = R,\ \$$

$R \rightarrow L \bullet,\ \$$

I_3

$S \rightarrow R \bullet,\ \$$

I_4

$L \rightarrow * \bullet R,\ =/\$$

$R \rightarrow \bullet L,\ =/\$$

$L \rightarrow \bullet \text{id},\ =/\$$

$L \rightarrow \bullet * R,\ =/\$$

I_5

$L \rightarrow \text{id} \bullet,\ =/\$$

I_6

$S \rightarrow L = \bullet R,\ \$$

$R \rightarrow \bullet L,\ \$$

$L \rightarrow \bullet \text{id},\ \$$

$L \rightarrow \bullet * R,\ \$$

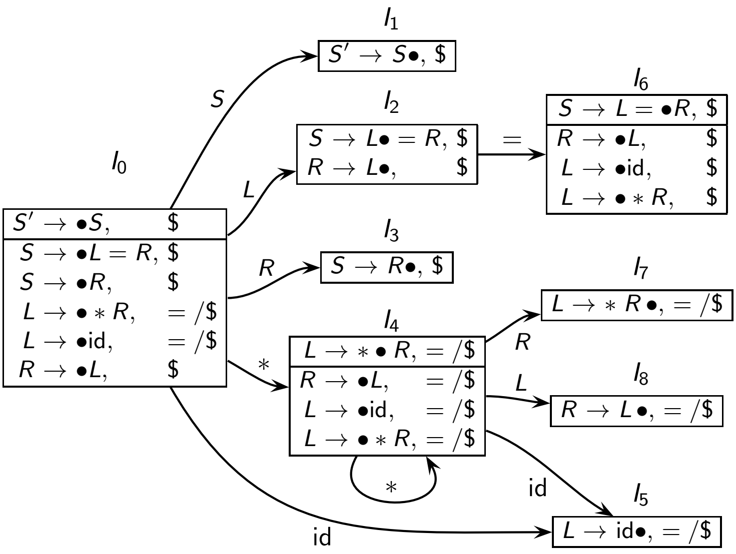
I_7

$L \rightarrow * R \bullet,\ =/\$$

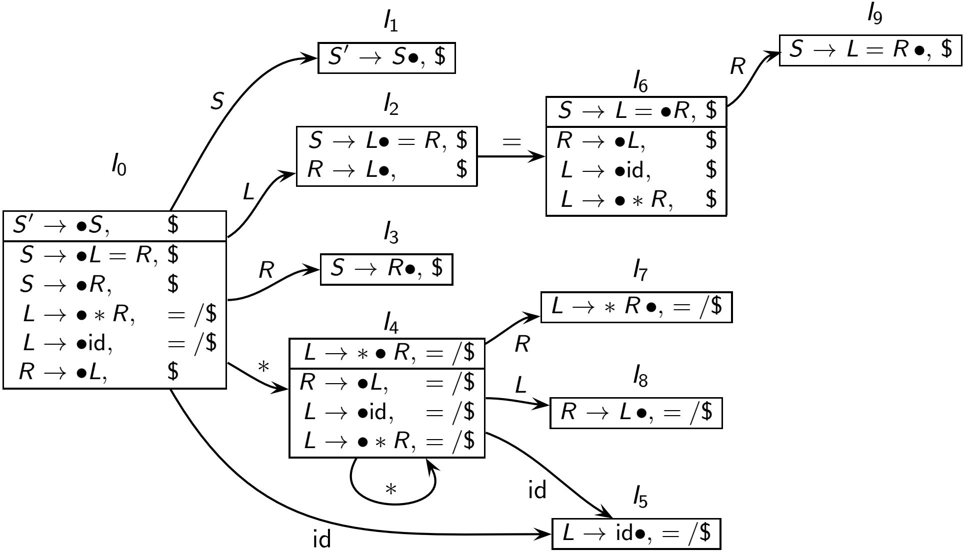
I_8

$R \rightarrow L \bullet,\ =/\$$

LR(1) Sets of Items for Pointer Assignment Grammar



LR(1) Sets of Items for Pointer Assignment Grammar



I_0
$S' \to \bullet S, \quad \$$
$S \to \bullet L = R, \quad \$$
$S \to \bullet R, \quad \$$
$L \to \bullet * R, \quad =/\$$
$L \to \bullet \text{id}, \quad =/\$$
$R \to \bullet L, \quad \$$

I_1
$S' \to S\bullet, \ \$$

I_2
$S \to L\bullet = R, \ \$$
$R \to L\bullet, \quad \$$

I_3
$S \to R\bullet, \ \$$

I_4
$L \to * \bullet R, \ =/\$$
$R \to \bullet L, \quad =/\$$
$L \to \bullet \text{id}, \quad =/\$$
$L \to \bullet * R, \ =/\$$

I_5
$L \to \text{id}\bullet, \ =/\$$

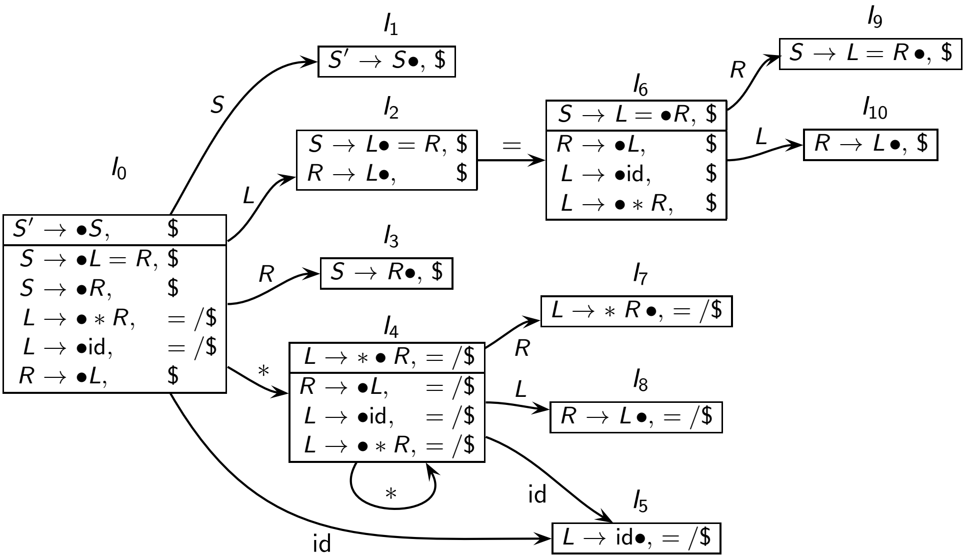
I_6
$S \to L = \bullet R, \ \$$
$R \to \bullet L, \quad \$$
$L \to \bullet \text{id}, \quad \$$
$L \to \bullet * R, \quad \$$

I_7
$L \to * R \bullet, \ =/\$$

I_8
$R \to L \bullet, \ =/\$$

I_9
$S \to L = R \bullet, \ \$$

LR(1) Sets of Items for Pointer Assignment Grammar



I_0

$S' \rightarrow \bullet S, \quad \$$

$S \rightarrow \bullet L = R, \ \$$

$S \rightarrow \bullet R, \quad \$$

$L \rightarrow \bullet * R, \quad =/\$$

$L \rightarrow \bullet \text{id}, \quad =/\$$

$R \rightarrow \bullet L, \quad \$$

I_1

$S' \rightarrow S\bullet, \ \$$

I_2

$S \rightarrow L\bullet = R, \ \$$

$R \rightarrow L\bullet, \quad \$$

I_3

$S \rightarrow R\bullet, \ \$$

I_4

$L \rightarrow * \bullet R, \ =/\$$

$R \rightarrow \bullet L, \quad =/\$$

$L \rightarrow \bullet \text{id}, \quad =/\$$

$L \rightarrow \bullet * R, \ =/\$$

I_5

$L \rightarrow \text{id}\bullet, \ =/\$$

I_6

$S \rightarrow L = \bullet R, \ \$$

$R \rightarrow \bullet L, \quad \$$

$L \rightarrow \bullet \text{id}, \quad \$$

$L \rightarrow \bullet * R, \quad \$$

I_7

$L \rightarrow * R\bullet, \ =/\$$

I_8

$R \rightarrow L\bullet, \ =/\$$

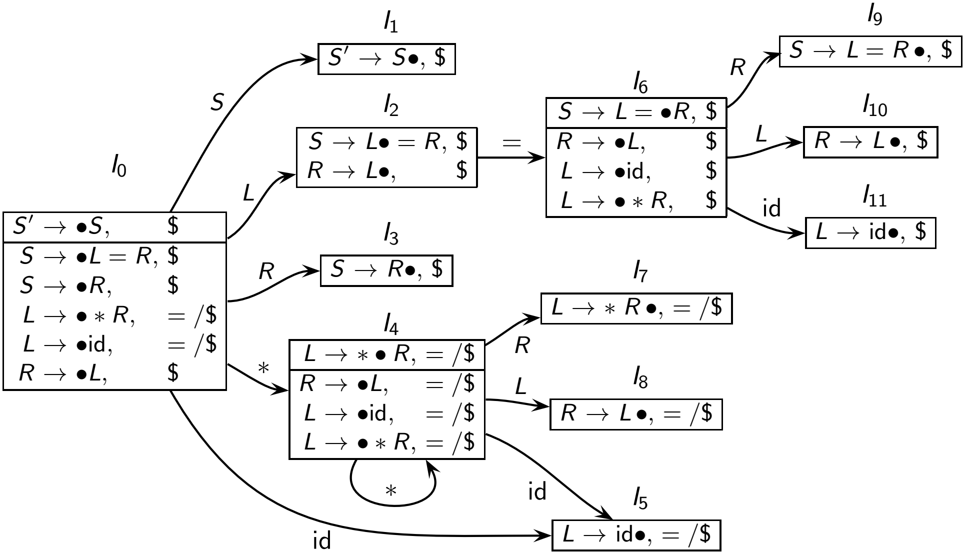
I_9

$S \rightarrow L = R\bullet, \ \$$

I_{10}

$R \rightarrow L\bullet, \ \$$

LR(1) Sets of Items for Pointer Assignment Grammar



I_0

$S' \rightarrow \bullet S,\ \$$
$S \rightarrow \bullet L = R,\ \$$
$S \rightarrow \bullet R,\ \$$
$L \rightarrow \bullet * R,\ =/\$$
$L \rightarrow \bullet \text{id},\ =/\$$
$R \rightarrow \bullet L,\ \$$

I_1

$S' \rightarrow S\bullet,\ \$$

I_2

$S \rightarrow L\bullet = R,\ \$$
$R \rightarrow L\bullet,\ \$$

I_3

$S \rightarrow R\bullet,\ \$$

I_4

$L \rightarrow * \bullet R,\ =/\$$
$R \rightarrow \bullet L,\ =/\$$
$L \rightarrow \bullet \text{id},\ =/\$$
$L \rightarrow \bullet * R,\ =/\$$

I_5

$L \rightarrow \text{id}\bullet,\ =/\$$

I_6

$S \rightarrow L = \bullet R,\ \$$
$R \rightarrow \bullet L,\ \$$
$L \rightarrow \bullet \text{id},\ \$$
$L \rightarrow \bullet * R,\ \$$

I_7

$L \rightarrow * R\bullet,\ =/\$$

I_8

$R \rightarrow L\bullet,\ =/\$$

I_9

$S \rightarrow L = R\bullet,\ \$$

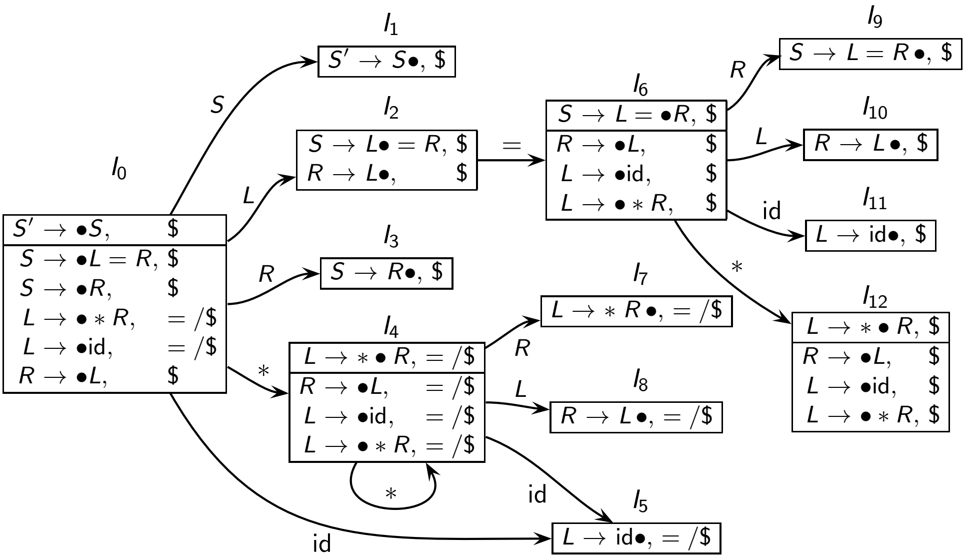
I_{10}

$R \rightarrow L\bullet,\ \$$

I_{11}

$L \rightarrow \text{id}\bullet,\ \$$

LR(1) Sets of Items for Pointer Assignment Grammar



I_0
$S' \rightarrow \bullet S, \quad \$$
$S \rightarrow \bullet L = R, \quad \$$
$S \rightarrow \bullet R, \quad \$$
$L \rightarrow \bullet * R, \quad =/\$$
$L \rightarrow \bullet \text{id}, \quad =/\$$
$R \rightarrow \bullet L, \quad \$$

I_1
$S' \rightarrow S \bullet, \ \$$

I_2
$S \rightarrow L \bullet = R, \ \$$
$R \rightarrow L \bullet, \quad \$$

I_3
$S \rightarrow R \bullet, \ \$$

I_4
$L \rightarrow * \bullet R, \ =/\$$
$R \rightarrow \bullet L, \quad =/\$$
$L \rightarrow \bullet \text{id}, \quad =/\$$
$L \rightarrow \bullet * R, \ =/\$$

I_5
$L \rightarrow \text{id} \bullet, \ =/\$$

I_6
$S \rightarrow L = \bullet R, \ \$$
$R \rightarrow \bullet L, \quad \$$
$L \rightarrow \bullet \text{id}, \quad \$$
$L \rightarrow \bullet * R, \quad \$$

I_7
$L \rightarrow * R \bullet, \ =/\$$

I_8
$R \rightarrow L \bullet, \ =/\$$

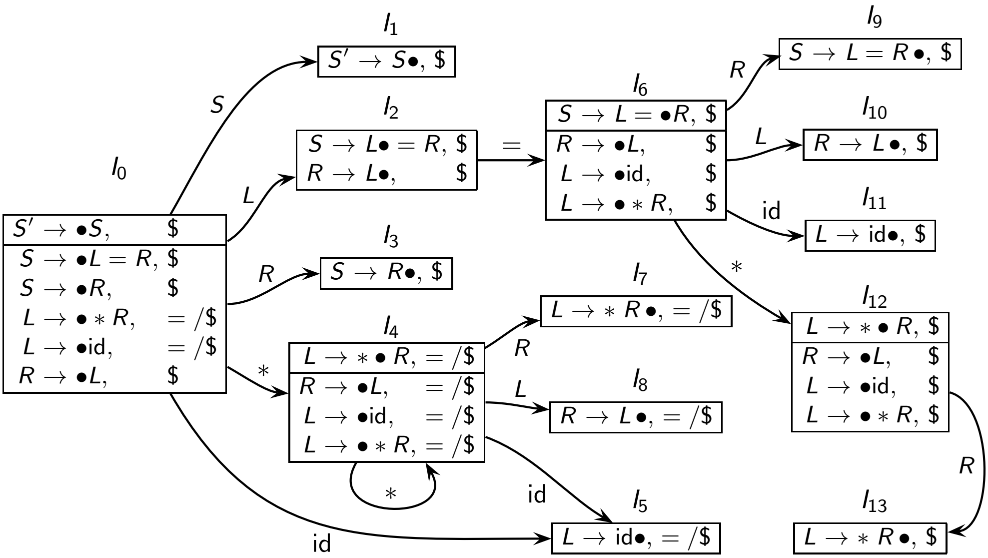
I_9
$S \rightarrow L = R \bullet, \ \$$

I_{10}
$R \rightarrow L \bullet, \ \$$

I_{11}
$L \rightarrow \text{id} \bullet, \ \$$

I_{12}
$L \rightarrow * \bullet R, \ \$$
$R \rightarrow \bullet L, \quad \$$
$L \rightarrow \bullet \text{id}, \quad \$$
$L \rightarrow \bullet * R, \ \$$

LR(1) Sets of Items for Pointer Assignment Grammar



I_0

$S' \rightarrow \bullet S, \quad \$$
$S \rightarrow \bullet L = R, \ \$$
$S \rightarrow \bullet R, \quad \$$
$L \rightarrow \bullet * R, \quad =/\$$
$L \rightarrow \bullet \text{id}, \quad =/\$$
$R \rightarrow \bullet L, \quad \$$

I_1

$S' \rightarrow S\bullet, \ \$$

I_2

$S \rightarrow L\bullet = R, \ \$$
$R \rightarrow L\bullet, \quad \$$

I_3

$S \rightarrow R\bullet, \ \$$

I_4

$L \rightarrow * \bullet R, \ =/\$$
$R \rightarrow \bullet L, \quad =/\$$
$L \rightarrow \bullet \text{id}, \quad =/\$$
$L \rightarrow \bullet * R, \ =/\$$

I_5

$L \rightarrow \text{id}\bullet, \ =/\$$

I_6

$S \rightarrow L = \bullet R, \ \$$
$R \rightarrow \bullet L, \quad \$$
$L \rightarrow \bullet \text{id}, \quad \$$
$L \rightarrow \bullet * R, \quad \$$

I_7

$L \rightarrow * R\bullet, \ =/\$$

I_8

$R \rightarrow L\bullet, \ =/\$$

I_9

$S \rightarrow L = R\bullet, \ \$$

I_{10}

$R \rightarrow L\bullet, \ \$$

I_{11}

$L \rightarrow \text{id}\bullet, \ \$$

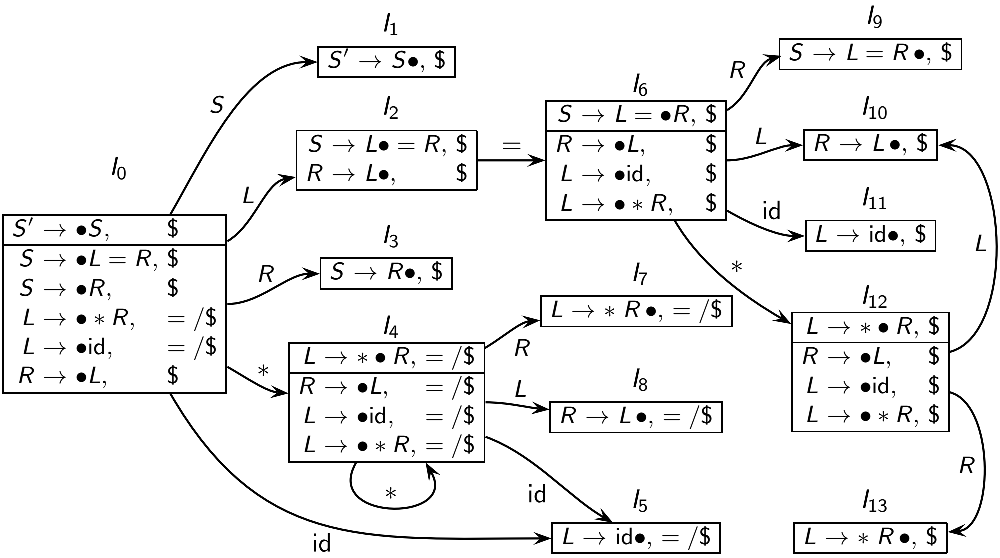
I_{12}

$L \rightarrow * \bullet R, \ \$$
$R \rightarrow \bullet L, \quad \$$
$L \rightarrow \bullet \text{id}, \quad \$$
$L \rightarrow \bullet * R, \ \$$

I_{13}

$L \rightarrow * R\bullet, \ \$$

LR(1) Sets of Items for Pointer Assignment Grammar



I_0
$S' \rightarrow \bullet S,\ \$$
$S \rightarrow \bullet L = R,\ \$$
$S \rightarrow \bullet R,\ \$$
$L \rightarrow \bullet * R,\ =/\$$
$L \rightarrow \bullet \text{id},\ =/\$$
$R \rightarrow \bullet L,\ \$$

I_1
$S' \rightarrow S \bullet,\ \$$

I_2
$S \rightarrow L \bullet = R,\ \$$
$R \rightarrow L \bullet,\ \$$

I_3
$S \rightarrow R \bullet,\ \$$

I_4
$L \rightarrow * \bullet R,\ =/\$$
$R \rightarrow \bullet L,\ =/\$$
$L \rightarrow \bullet \text{id},\ =/\$$
$L \rightarrow \bullet * R,\ =/\$$

I_5
$L \rightarrow \text{id} \bullet,\ =/\$$

I_6
$S \rightarrow L = \bullet R,\ \$$
$R \rightarrow \bullet L,\ \$$
$L \rightarrow \bullet \text{id},\ \$$
$L \rightarrow \bullet * R,\ \$$

I_7
$L \rightarrow * R \bullet,\ =/\$$

I_8
$R \rightarrow L \bullet,\ =/\$$

I_9
$S \rightarrow L = R \bullet,\ \$$

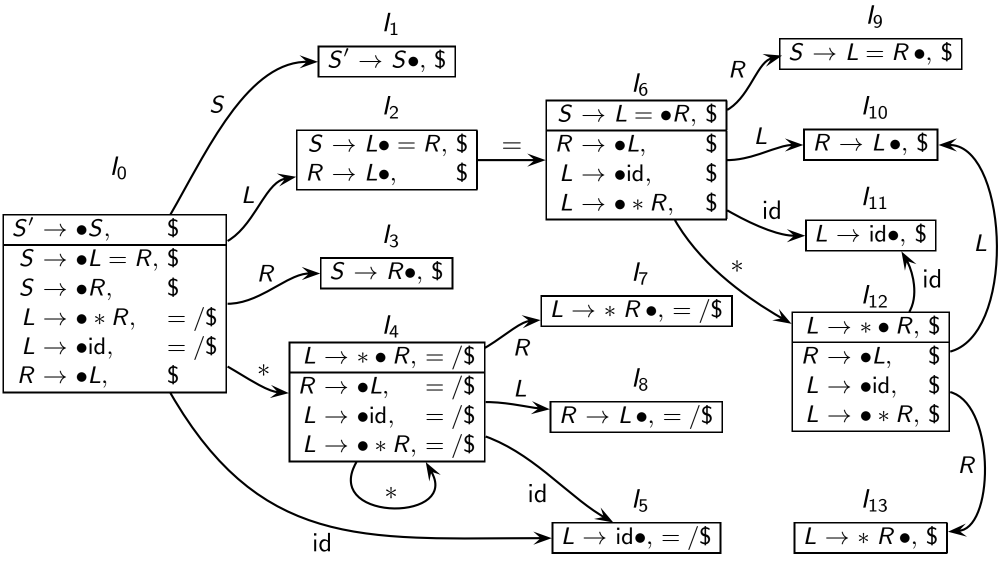
I_{10}
$R \rightarrow L \bullet,\ \$$

I_{11}
$L \rightarrow \text{id} \bullet,\ \$$

I_{12}
$L \rightarrow * \bullet R,\ \$$
$R \rightarrow \bullet L,\ \$$
$L \rightarrow \bullet \text{id},\ \$$
$L \rightarrow \bullet * R,\ \$$

I_{13}
$L \rightarrow * R \bullet,\ \$$

LR(1) Sets of Items for Pointer Assignment Grammar



I_0
$S' \to \bullet S, \quad \$$
$S \to \bullet L = R, \quad \$$
$S \to \bullet R, \quad \$$
$L \to \bullet * R, \quad =/\$$
$L \to \bullet \text{id}, \quad =/\$$
$R \to \bullet L, \quad \$$

I_1
$S' \to S\bullet, \ \$$

I_2
$S \to L\bullet = R, \quad \$$
$R \to L\bullet, \quad \$$

I_3
$S \to R\bullet, \ \$$

I_4
$L \to * \bullet R, \quad =/\$$
$R \to \bullet L, \quad =/\$$
$L \to \bullet \text{id}, \quad =/\$$
$L \to \bullet * R, \quad =/\$$

I_5
$L \to \text{id}\bullet, \ =/\$$

I_6
$S \to L = \bullet R, \quad \$$
$R \to \bullet L, \quad \$$
$L \to \bullet \text{id}, \quad \$$
$L \to \bullet * R, \quad \$$

I_7
$L \to * R\bullet, \ =/\$$

I_8
$R \to L\bullet, \ =/\$$

I_9
$S \to L = R\bullet, \ \$$

I_{10}
$R \to L\bullet, \ \$$

I_{11}
$L \to \text{id}\bullet, \ \$$

I_{12}
$L \to * \bullet R, \quad \$$
$R \to \bullet L, \quad \$$
$L \to \bullet \text{id}, \quad \$$
$L \to \bullet * R, \quad \$$

I_{13}
$L \to * R\bullet, \ \$$

IIT Bombay

cs302: Implementation of Programming Languages

Topic:

Syntax Analysis

Section:

Grammars, Derivations, and Parse Trees

Shift Reduce Parsing

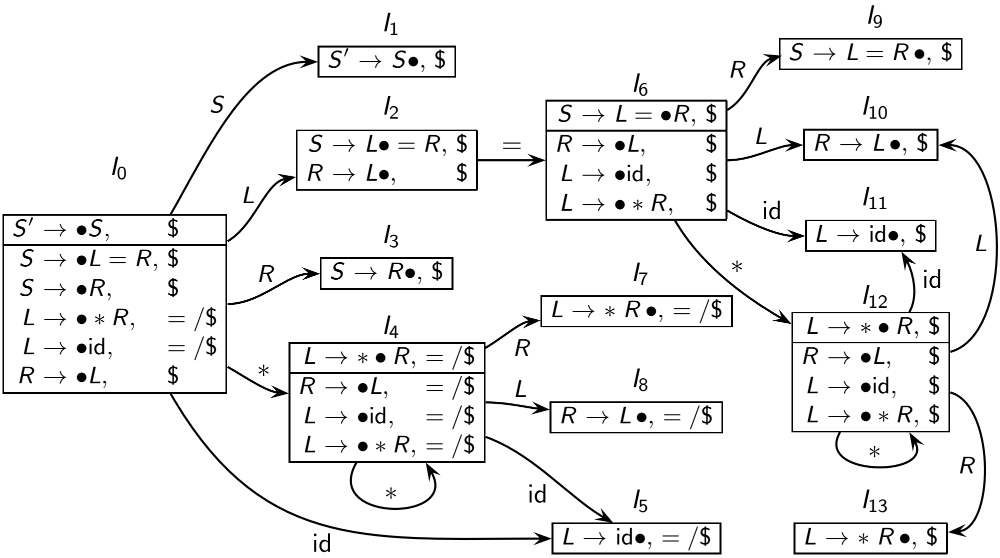
SLR(1) Parsing

Conceptual Issues in Parsing

CLR(1) Parsing

LALR(1) Parsing

LR(1) Sets of Items for Pointer Assignment Grammar



I_0

$S' \to \bullet S,\ \$$

$S \to \bullet L = R,\ \$$

$S \to \bullet R,\ \$$

$L \to \bullet * R,\ =/\$$

$L \to \bullet \text{id},\ =/\$$

$R \to \bullet L,\ \$$

I_1

$S' \to S\bullet,\ \$$

I_2

$S \to L\bullet = R,\ \$$

$R \to L\bullet,\ \$$

I_3

$S \to R\bullet,\ \$$

I_4

$L \to * \bullet R,\ =/\$$

$R \to \bullet L,\ =/\$$

$L \to \bullet \text{id},\ =/\$$

$L \to \bullet * R,\ =/\$$

I_5

$L \to \text{id}\bullet,\ =/\$$

I_6

$S \to L = \bullet R,\ \$$

$R \to \bullet L,\ \$$

$L \to \bullet \text{id},\ \$$

$L \to \bullet * R,\ \$$

I_7

$L \to * R\bullet,\ =/\$$

I_8

$R \to L\bullet,\ =/\$$

I_9

$S \to L = R\bullet,\ \$$

I_{10}

$R \to L\bullet,\ \$$

I_{11}

$L \to \text{id}\bullet,\ \$$

I_{12}

$L \to * \bullet R,\ \$$

$R \to \bullet L,\ \$$

$L \to \bullet \text{id},\ \$$

$L \to \bullet * R,\ \$$

I_{13}

$L \to * R\bullet,\ \$$

LR(1) (aka CLR(1)) Parsing Table for Pointer Assignment Grammar

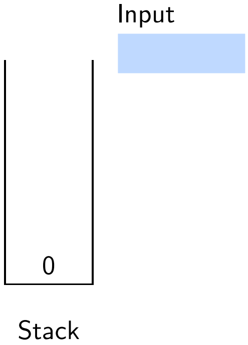
0 $S' \rightarrow S$
1 $S \rightarrow L = R$
2 $S \rightarrow R$
3 $L \rightarrow * R$
4 $L \rightarrow \text{id}$
5 $R \rightarrow L$

State	Action				Goto		
	`id`	`*`	`=`	`$`	S	L	R
0	$s5$	$s4$			$c1$	$c2$	$c3$
1				acc			
2			$s6$	$r5$			
3				$r2$			
4	$s5$	$s4$				$c8$	$c7$
5			$r4$	$r4$			
6	$s11$	$s12$				$c10$	$c9$
7			$r3$	$r3$			
8			$r5$	$r5$			
9				$r1$			
10				$r5$			
11				$r4$			
12	$s11$	$s12$				c10	c13
13				$r3$			

LR(1) (aka CLR(1)) Parsing for the Pointer Assignment Grammar

State	Action				Goto		
	`id`	*	=	$	S	L	R
0	$s5$	$s4$			$c1$	$c2$	$c3$
1				acc			
2			$s6$	$r5$			
3				$r2$			
4	$s5$	$s4$				$c8$	$c7$
5			$r4$	$r4$			
6	$s11$	$s12$				$c10$	$c9$
7			$r3$	$r3$			
8			$r5$	$r5$			
9				$r1$			
10				$r5$			
11				$r4$			
12	$s11$	$s12$				c10	c13
13				$r3$			

0 $S' \rightarrow S$
1 $S \rightarrow L = R$
2 $S \rightarrow R$
3 $L \rightarrow * R$
4 $L \rightarrow \text{id}$
5 $R \rightarrow L$



LR(1) (aka CLR(1)) Parsing for the Pointer Assignment Grammar

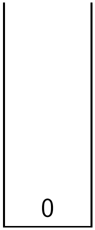
State	Action				Goto		
	`id`	`*`	`=`	$	S	L	R
0	$s5$	$s4$			$c1$	$c2$	$c3$
1				acc			
2			$s6$	$r5$			
3				$r2$			
4	$s5$	$s4$				$c8$	$c7$
5			$r4$	$r4$			
6	$s11$	$s12$				$c10$	$c9$
7			$r3$	$r3$			
8			$r5$	$r5$			
9				$r1$			
10				$r5$			
11				$r4$			
12	$s11$	$s12$				c10	c13
13				$r3$			

0 $S' \rightarrow S$
1 $S \rightarrow L = R$
2 $S \rightarrow R$
3 $L \rightarrow * R$
4 $L \rightarrow \text{id}$
5 $R \rightarrow L$

Input

id = id$

Shift 5



Stack

LR(1) (aka CLR(1)) Parsing for the Pointer Assignment Grammar

State	Action				Goto		
	`id`	$*$	=	\$	S	L	R
0	$s5$	$s4$			$c1$	$c2$	$c3$
1				acc			
2			$s6$	$r5$			
3				$r2$			
4	$s5$	$s4$				$c8$	$c7$
5			$r4$	$r4$			
6	$s11$	$s12$				$c10$	$c9$
7			$r3$	$r3$			
8			$r5$	$r5$			
9				$r1$			
10				$r5$			
11				$r4$			
12	$s11$	$s12$				c10	c13
13				$r3$			

0 $S' \rightarrow S$
1 $S \rightarrow L = R$
2 $S \rightarrow R$
3 $L \rightarrow * R$
4 $L \rightarrow \text{id}$
5 $R \rightarrow L$

Input

$= \text{id}\$$

Reduce by 4

5
id
0

Stack

LR(1) (aka CLR(1)) Parsing for the Pointer Assignment Grammar

State	Action				Goto		
	`id`	$*$	`=`	\$	S	L	R
0	$s5$	$s4$			$c1$	$c2$	$c3$
1				acc			
2			$s6$	$r5$			
3				$r2$			
4	$s5$	$s4$				$c8$	$c7$
5			$r4$	$r4$			
6	$s11$	$s12$				$c10$	$c9$
7			$r3$	$r3$			
8			$r5$	$r5$			
9				$r1$			
10				$r5$			
11				$r4$			
12	$s11$	$s12$				c10	c13
13				$r3$			

0 $S' \rightarrow S$
1 $S \rightarrow L = R$
2 $S \rightarrow R$
3 $L \rightarrow * R$
4 $L \rightarrow \text{id}$
5 $R \rightarrow L$

Input

$= \text{id}\$$

Cover by 2

L
0

Stack

LR(1) (aka CLR(1)) Parsing for the Pointer Assignment Grammar

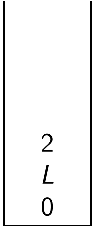
State	Action				Goto		
	`id`	$*$	`=`	\$	S	L	R
0	$s5$	$s4$			$c1$	$c2$	$c3$
1				acc			
2			$s6$	$r5$			
3				$r2$			
4	$s5$	$s4$				$c8$	$c7$
5			$r4$	$r4$			
6	$s11$	$s12$				$c10$	$c9$
7			$r3$	$r3$			
8			$r5$	$r5$			
9				$r1$			
10				$r5$			
11				$r4$			
12	$s11$	$s12$				c10	c13
13				$r3$			

0 $S' \rightarrow S$
1 $S \rightarrow L = R$
2 $S \rightarrow R$
3 $L \rightarrow * R$
4 $L \rightarrow \text{id}$
5 $R \rightarrow L$

Input

$= \text{id}\$$

Shift 6



Stack

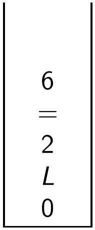
LR(1) (aka CLR(1)) Parsing for the Pointer Assignment Grammar

State	Action				Goto		
	`id`	$*$	`=`	\$	S	L	R
0	$s5$	$s4$			$c1$	$c2$	$c3$
1				acc			
2			$s6$	$r5$			
3				$r2$			
4	$s5$	$s4$				$c8$	$c7$
5			$r4$	$r4$			
6	$s11$	$s12$				$c10$	$c9$
7			$r3$	$r3$			
8			$r5$	$r5$			
9				$r1$			
10				$r5$			
11				$r4$			
12	$s11$	$s12$				c10	c13
13				$r3$			

0 $S' \rightarrow S$
1 $S \rightarrow L = R$
2 $S \rightarrow R$
3 $L \rightarrow * R$
4 $L \rightarrow \text{id}$
5 $R \rightarrow L$

Input

id\$



Shift 11

Stack

LR(1) (aka CLR(1)) Parsing for the Pointer Assignment Grammar

State	Action				Goto		
	`id`	$*$	`=`	\$	S	L	R
0	$s5$	$s4$			$c1$	$c2$	$c3$
1				acc			
2			$s6$	$r5$			
3				$r2$			
4	$s5$	$s4$				$c8$	$c7$
5			$r4$	$r4$			
6	$s11$	$s12$				$c10$	$c9$
7			$r3$	$r3$			
8			$r5$	$r5$			
9				$r1$			
10				$r5$			
11				$r4$			
12	$s11$	$s12$				c10	c13
13				$r3$			

0 $S' \to S$
1 $S \to L = R$
2 $S \to R$
3 $L \to * R$
4 $L \to \text{id}$
5 $R \to L$

Input

\$

Reduce by 4

11
id
6
$=$
2
L
0

Stack

LR(1) (aka CLR(1)) Parsing for the Pointer Assignment Grammar

State	Action				Goto		
	`id`	$*$	`=`	\$	S	L	R
0	$s5$	$s4$			$c1$	$c2$	$c3$
1				acc			
2			$s6$	$r5$			
3				$r2$			
4	$s5$	$s4$				$c8$	$c7$
5			$r4$	$r4$			
6	$s11$	$s12$				$c10$	$c9$
7			$r3$	$r3$			
8			$r5$	$r5$			
9				$r1$			
10				$r5$			
11				$r4$			
12	$s11$	$s12$				c10	c13
13				$r3$			

0 $S' \rightarrow S$
1 $S \rightarrow L = R$
2 $S \rightarrow R$
3 $L \rightarrow * R$
4 $L \rightarrow \text{id}$
5 $R \rightarrow L$

Input

\$

Cover by 10

L
6
$=$
2
L
0

Stack

LR(1) (aka CLR(1)) Parsing for the Pointer Assignment Grammar

State	Action				Goto		
	`id`	$*$	`=`	\$	S	L	R
0	$s5$	$s4$			$c1$	$c2$	$c3$
1				acc			
2			$s6$	$r5$			
3				$r2$			
4	$s5$	$s4$				$c8$	$c7$
5			$r4$	$r4$			
6	$s11$	$s12$				$c10$	$c9$
7			$r3$	$r3$			
8			$r5$	$r5$			
9				$r1$			
10				$r5$			
11				$r4$			
12	$s11$	$s12$				c10	c13
13				$r3$			

0 $S' \rightarrow S$
1 $S \rightarrow L = R$
2 $S \rightarrow R$
3 $L \rightarrow * R$
4 $L \rightarrow \text{id}$
5 $R \rightarrow L$

Input

\$

10
L
6
$=$
2
L
0

Stack

Reduce by 5

LR(1) (aka CLR(1)) Parsing for the Pointer Assignment Grammar

State	Action				Goto		
	`id`	$*$	`=`	\$	S	L	R
0	$s5$	$s4$			$c1$	$c2$	$c3$
1				acc			
2			$s6$	$r5$			
3				$r2$			
4	$s5$	$s4$				$c8$	$c7$
5			$r4$	$r4$			
6	$s11$	$s12$				$c10$	$c9$
7			$r3$	$r3$			
8			$r5$	$r5$			
9				$r1$			
10				$r5$			
11				$r4$			
12	$s11$	$s12$				c10	c13
13				$r3$			

0 $S' \to S$
1 $S \to L = R$
2 $S \to R$
3 $L \to * R$
4 $L \to \text{id}$
5 $R \to L$

Input

\$

Cover by 9

R
6
$=$
2
L
0

Stack

IIT Bombay
cs302: Implementation of Programming Languages

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Shift Reduce Parsing

SLR(1) Parsing

Conceptual Issues in Parsing

CLR(1) Parsing

LALR(1) Parsing

LR(1) (aka CLR(1)) Parsing for the Pointer Assignment Grammar

State	Action				Goto		
	`id`	$*$	`=`	\$	S	L	R
0	$s5$	$s4$			$c1$	$c2$	$c3$
1				acc			
2			$s6$	$r5$			
3				$r2$			
4	$s5$	$s4$				$c8$	$c7$
5			$r4$	$r4$			
6	$s11$	$s12$				$c10$	$c9$
7			$r3$	$r3$			
8			$r5$	$r5$			
9				$r1$			
10				$r5$			
11				$r4$			
12	$s11$	$s12$				c10	c13
13				$r3$			

0 $S' \to S$
1 $S \to L = R$
2 $S \to R$
3 $L \to * R$
4 $L \to \text{id}$
5 $R \to L$

Input

\$

9
R
6
$=$
2
L
0

Stack

Reduce by 1

LR(1) (aka CLR(1)) Parsing for the Pointer Assignment Grammar

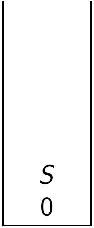
State	Action				Goto		
	`id`	`*`	`=`	$	S	L	R
0	$s5$	$s4$			$c1$	$c2$	$c3$
1				acc			
2			$s6$	$r5$			
3				$r2$			
4	$s5$	$s4$				$c8$	$c7$
5			$r4$	$r4$			
6	$s11$	$s12$				$c10$	$c9$
7			$r3$	$r3$			
8			$r5$	$r5$			
9				$r1$			
10				$r5$			
11				$r4$			
12	$s11$	$s12$				c10	c13
13				$r3$			

0 $S' \rightarrow S$
1 $S \rightarrow L = R$
2 $S \rightarrow R$
3 $L \rightarrow * R$
4 $L \rightarrow \text{id}$
5 $R \rightarrow L$

Input

$

Cover by 1



Stack

LR(1) (aka CLR(1)) Parsing for the Pointer Assignment Grammar

State	Action				Goto		
	`id`	`*`	`=`	`$`	S	L	R
0	$s5$	$s4$			$c1$	$c2$	$c3$
1				acc			
2			$s6$	$r5$			
3				$r2$			
4	$s5$	$s4$				$c8$	$c7$
5			$r4$	$r4$			
6	$s11$	$s12$				$c10$	$c9$
7			$r3$	$r3$			
8			$r5$	$r5$			
9				$r1$			
10				$r5$			
11				$r4$			
12	$s11$	$s12$				c10	c13
13				$r3$			

0 $S' \rightarrow S$
1 $S \rightarrow L = R$
2 $S \rightarrow R$
3 $L \rightarrow * R$
4 $L \rightarrow \text{id}$
5 $R \rightarrow L$

Input

$

Accept

1
S
0

Stack

Another Example of LR(1) (aka CLR(1)) Parsing

$A \rightarrow aBe$
$A \rightarrow aCd$
$A \rightarrow bBd$
$A \rightarrow bCe$
$B \rightarrow f$
$C \rightarrow f$

Another Example of LR(1) (aka CLR(1)) Parsing

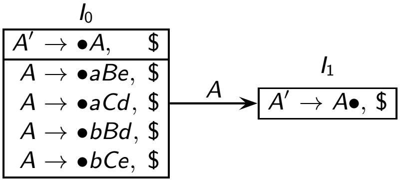
$$\begin{array}{l}
A \rightarrow aBe \\
A \rightarrow aCd \\
A \rightarrow bBd \\
A \rightarrow bCe \\
B \rightarrow f \\
C \rightarrow f
\end{array}$$

I_0

$$\begin{array}{|rcll|}
\hline
A' & \rightarrow & \bullet A, & \$ \\
\hline
A & \rightarrow & \bullet aBe, & \$ \\
A & \rightarrow & \bullet aCd, & \$ \\
A & \rightarrow & \bullet bBd, & \$ \\
A & \rightarrow & \bullet bCe, & \$ \\
\hline
\end{array}$$

Another Example of LR(1) (aka CLR(1)) Parsing

$A \rightarrow aBe$
$A \rightarrow aCd$
$A \rightarrow bBd$
$A \rightarrow bCe$
$B \rightarrow f$
$C \rightarrow f$



I_0

$A' \rightarrow \bullet A, \quad \$$
$A \rightarrow \bullet aBe, \ \$$
$A \rightarrow \bullet aCd, \ \$$
$A \rightarrow \bullet bBd, \ \$$
$A \rightarrow \bullet bCe, \ \$$

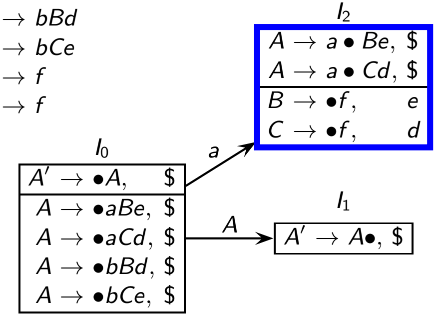
$\xrightarrow{A}$

I_1

$A' \rightarrow A\bullet, \ \$$

Another Example of LR(1) (aka CLR(1)) Parsing

$A \to aBe$
$A \to aCd$
$A \to bBd$
$A \to bCe$
$B \to f$
$C \to f$



I_0

$A' \to \bullet A,\ \$$
$A \to \bullet aBe,\ \$$
$A \to \bullet aCd,\ \$$
$A \to \bullet bBd,\ \$$
$A \to \bullet bCe,\ \$$

a

I_2

$A \to a \bullet Be,\ \$$
$A \to a \bullet Cd,\ \$$
$B \to \bullet f,\ e$
$C \to \bullet f,\ d$

A

I_1

$A' \to A \bullet,\ \$$

Closure of $P \to \alpha \bullet Q\beta, p$ contains items of the form $Q \to \bullet \gamma,\ \text{FIRST}(\beta p)$

In our example

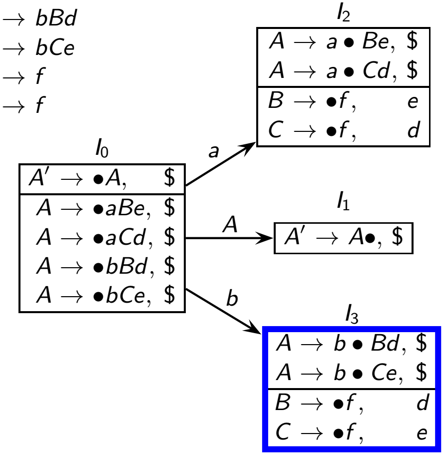
- For $Q = B$, β is e and p is \$

  If we expect to see a string derivable from B in this state, the string must be followed by
  $\text{FIRST}(\beta p) = \text{FIRST}(e\$) = e$
- For $Q = C$, β is d and p is \$

  If we expect to see a string derivable from C in this state, the string must be followed by
  $\text{FIRST}(\beta p) = \text{FIRST}(d\$) = d$

Another Example of LR(1) (aka CLR(1)) Parsing

$A \rightarrow aBe$
$A \rightarrow aCd$
$A \rightarrow bBd$
$A \rightarrow bCe$
$B \rightarrow f$
$C \rightarrow f$



I_0

$A' \rightarrow \bullet A, \ \$$

$A \rightarrow \bullet aBe, \ \$$
$A \rightarrow \bullet aCd, \ \$$
$A \rightarrow \bullet bBd, \ \$$
$A \rightarrow \bullet bCe, \ \$$

$I_0 \xrightarrow{a} I_2$, $I_0 \xrightarrow{A} I_1$, $I_0 \xrightarrow{b} I_3$

I_2

$A \rightarrow a \bullet Be, \ \$$
$A \rightarrow a \bullet Cd, \ \$$

$B \rightarrow \bullet f, \ e$
$C \rightarrow \bullet f, \ d$

I_1

$A' \rightarrow A \bullet, \ \$$

I_3

$A \rightarrow b \bullet Bd, \ \$$
$A \rightarrow b \bullet Ce, \ \$$

$B \rightarrow \bullet f, \ d$
$C \rightarrow \bullet f, \ e$

Closure of $\boxed{P \rightarrow \alpha \bullet Q\beta, p}$ contains items of the form $\boxed{Q \rightarrow \bullet \gamma, \ \text{FIRST}(\beta p)}$

In our example

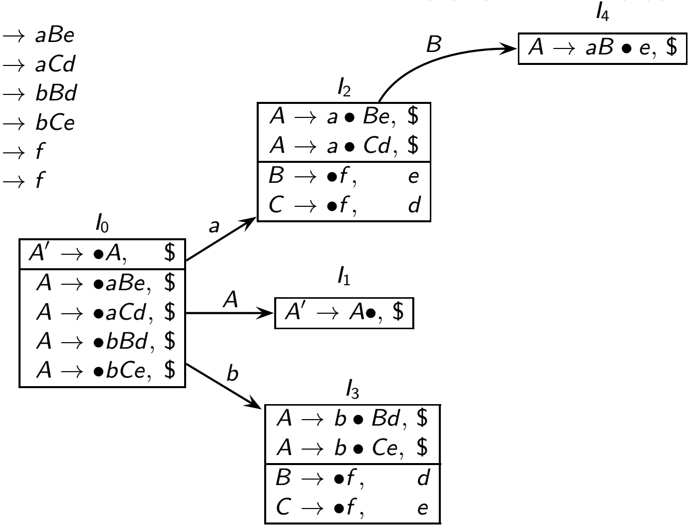
- For $Q = B$, β is d and p is \$

  If we expect to see a string derivable from B in this state, the string must be followed by
  $\text{FIRST}(\beta p) = \text{FIRST}(d\$) = d$
- For $Q = C$, β is e and p is \$

  If we expect to see a string derivable from C in this state, the string must be followed by
  $\text{FIRST}(\beta p) = \text{FIRST}(e\$) = e$

Another Example of LR(1) (aka CLR(1)) Parsing

$A \rightarrow aBe$
$A \rightarrow aCd$
$A \rightarrow bBd$
$A \rightarrow bCe$
$B \rightarrow f$
$C \rightarrow f$



I_0
$A' \rightarrow \bullet A, \$$
$A \rightarrow \bullet aBe, \$$
$A \rightarrow \bullet aCd, \$$
$A \rightarrow \bullet bBd, \$$
$A \rightarrow \bullet bCe, \$$

I_1
$A' \rightarrow A\bullet, \$$

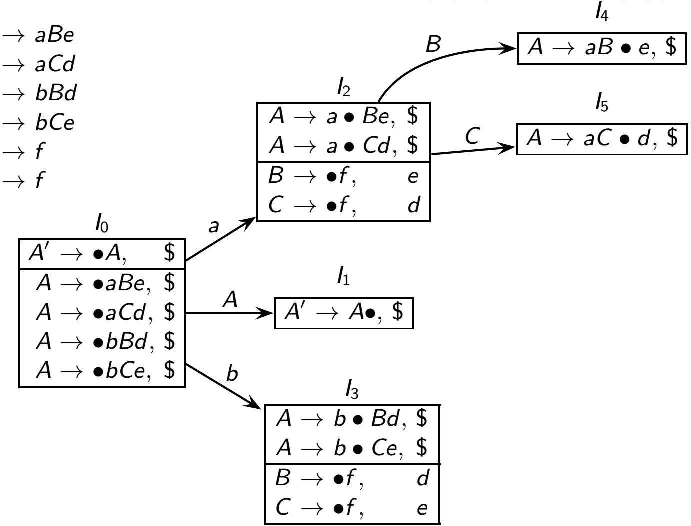
I_2
$A \rightarrow a \bullet Be, \$$
$A \rightarrow a \bullet Cd, \$$
$B \rightarrow \bullet f, e$
$C \rightarrow \bullet f, d$

I_3
$A \rightarrow b \bullet Bd, \$$
$A \rightarrow b \bullet Ce, \$$
$B \rightarrow \bullet f, d$
$C \rightarrow \bullet f, e$

I_4
$A \rightarrow aB \bullet e, \$$

Another Example of LR(1) (aka CLR(1)) Parsing

$A \rightarrow aBe$
$A \rightarrow aCd$
$A \rightarrow bBd$
$A \rightarrow bCe$
$B \rightarrow f$
$C \rightarrow f$



I_0

$A' \rightarrow \bullet A,\ \$$
$A \rightarrow \bullet aBe,\ \$$
$A \rightarrow \bullet aCd,\ \$$
$A \rightarrow \bullet bBd,\ \$$
$A \rightarrow \bullet bCe,\ \$$

I_1

$A' \rightarrow A \bullet,\ \$$

I_2

$A \rightarrow a \bullet Be,\ \$$
$A \rightarrow a \bullet Cd,\ \$$
$B \rightarrow \bullet f,\ e$
$C \rightarrow \bullet f,\ d$

I_3

$A \rightarrow b \bullet Bd,\ \$$
$A \rightarrow b \bullet Ce,\ \$$
$B \rightarrow \bullet f,\ d$
$C \rightarrow \bullet f,\ e$

I_4

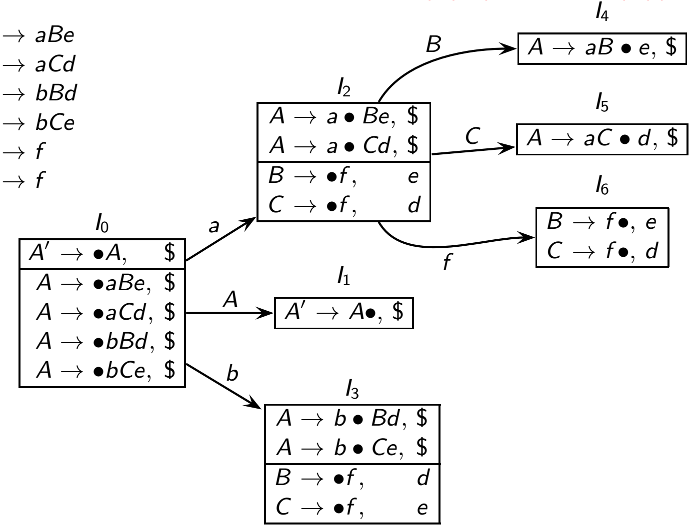
$A \rightarrow aB \bullet e,\ \$$

I_5

$A \rightarrow aC \bullet d,\ \$$

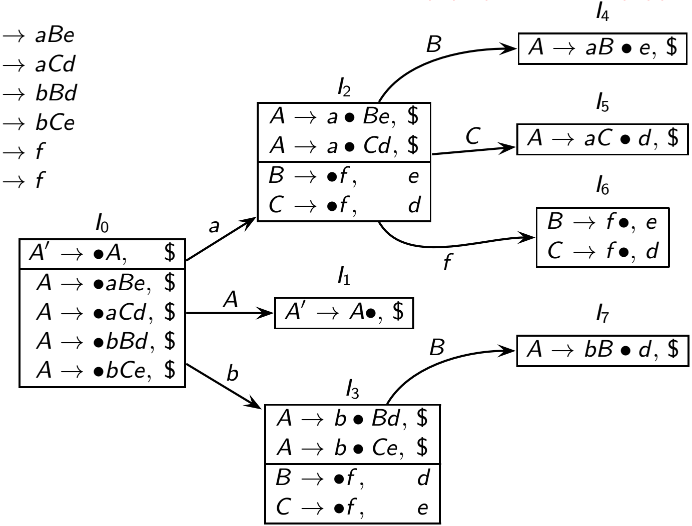
Another Example of LR(1) (aka CLR(1)) Parsing

$A \rightarrow aBe$
$A \rightarrow aCd$
$A \rightarrow bBd$
$A \rightarrow bCe$
$B \rightarrow f$
$C \rightarrow f$



Another Example of LR(1) (aka CLR(1)) Parsing

$$
\begin{aligned}
A &\rightarrow aBe\\
A &\rightarrow aCd\\
A &\rightarrow bBd\\
A &\rightarrow bCe\\
B &\rightarrow f\\
C &\rightarrow f
\end{aligned}
$$



I_0

$A' \rightarrow \bullet A,\ \$$

$A \rightarrow \bullet aBe,\ \$$
$A \rightarrow \bullet aCd,\ \$$
$A \rightarrow \bullet bBd,\ \$$
$A \rightarrow \bullet bCe,\ \$$

I_1

$A' \rightarrow A\bullet,\ \$$

I_2

$A \rightarrow a \bullet Be,\ \$$
$A \rightarrow a \bullet Cd,\ \$$

$B \rightarrow \bullet f,\ e$
$C \rightarrow \bullet f,\ d$

I_3

$A \rightarrow b \bullet Bd,\ \$$
$A \rightarrow b \bullet Ce,\ \$$

$B \rightarrow \bullet f,\ d$
$C \rightarrow \bullet f,\ e$

I_4

$A \rightarrow aB \bullet e,\ \$$

I_5

$A \rightarrow aC \bullet d,\ \$$

I_6

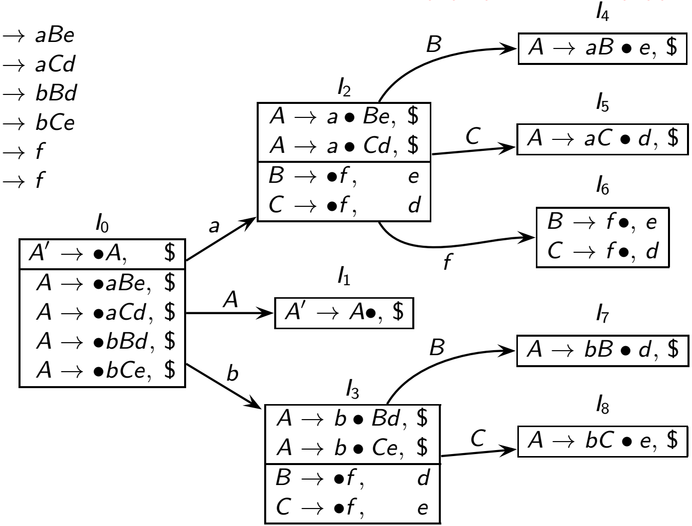
$B \rightarrow f\bullet,\ e$
$C \rightarrow f\bullet,\ d$

I_7

$A \rightarrow bB \bullet d,\ \$$

Another Example of LR(1) (aka CLR(1)) Parsing

$$
\begin{aligned}
A &\rightarrow aBe\\
A &\rightarrow aCd\\
A &\rightarrow bBd\\
A &\rightarrow bCe\\
B &\rightarrow f\\
C &\rightarrow f
\end{aligned}
$$



I_0
$A' \rightarrow \bullet A, \quad \$$
$A \rightarrow \bullet aBe, \ \$$
$A \rightarrow \bullet aCd, \ \$$
$A \rightarrow \bullet bBd, \ \$$
$A \rightarrow \bullet bCe, \ \$$

I_1
$A' \rightarrow A\bullet, \ \$$

I_2
$A \rightarrow a \bullet Be, \ \$$
$A \rightarrow a \bullet Cd, \ \$$
$B \rightarrow \bullet f, \quad e$
$C \rightarrow \bullet f, \quad d$

I_3
$A \rightarrow b \bullet Bd, \ \$$
$A \rightarrow b \bullet Ce, \ \$$
$B \rightarrow \bullet f, \quad d$
$C \rightarrow \bullet f, \quad e$

I_4
$A \rightarrow aB \bullet e, \ \$$

I_5
$A \rightarrow aC \bullet d, \ \$$

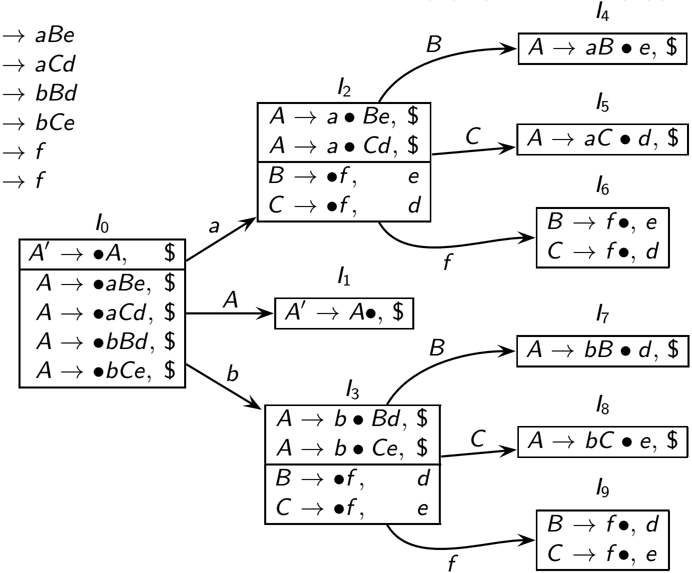
I_6
$B \rightarrow f\bullet, \ e$
$C \rightarrow f\bullet, \ d$

I_7
$A \rightarrow bB \bullet d, \ \$$

I_8
$A \rightarrow bC \bullet e, \ \$$

Another Example of LR(1) (aka CLR(1)) Parsing

$A \rightarrow aBe$
$A \rightarrow aCd$
$A \rightarrow bBd$
$A \rightarrow bCe$
$B \rightarrow f$
$C \rightarrow f$



I_0: $A' \rightarrow \bullet A,\ \$$ | $A \rightarrow \bullet aBe,\ \$$; $A \rightarrow \bullet aCd,\ \$$; $A \rightarrow \bullet bBd,\ \$$; $A \rightarrow \bullet bCe,\ \$$

$I_0 \xrightarrow{a} I_2$, $I_0 \xrightarrow{A} I_1$, $I_0 \xrightarrow{b} I_3$

I_1: $A' \rightarrow A\bullet,\ \$$

I_2: $A \rightarrow a \bullet Be,\ \$$; $A \rightarrow a \bullet Cd,\ \$$ | $B \rightarrow \bullet f,\ e$; $C \rightarrow \bullet f,\ d$

$I_2 \xrightarrow{B} I_4$, $I_2 \xrightarrow{C} I_5$, $I_2 \xrightarrow{f} I_6$

I_3: $A \rightarrow b \bullet Bd,\ \$$; $A \rightarrow b \bullet Ce,\ \$$ | $B \rightarrow \bullet f,\ d$; $C \rightarrow \bullet f,\ e$

$I_3 \xrightarrow{B} I_7$, $I_3 \xrightarrow{C} I_8$, $I_3 \xrightarrow{f} I_9$

I_4: $A \rightarrow aB \bullet e,\ \$$

I_5: $A \rightarrow aC \bullet d,\ \$$

I_6: $B \rightarrow f\bullet,\ e$; $C \rightarrow f\bullet,\ d$

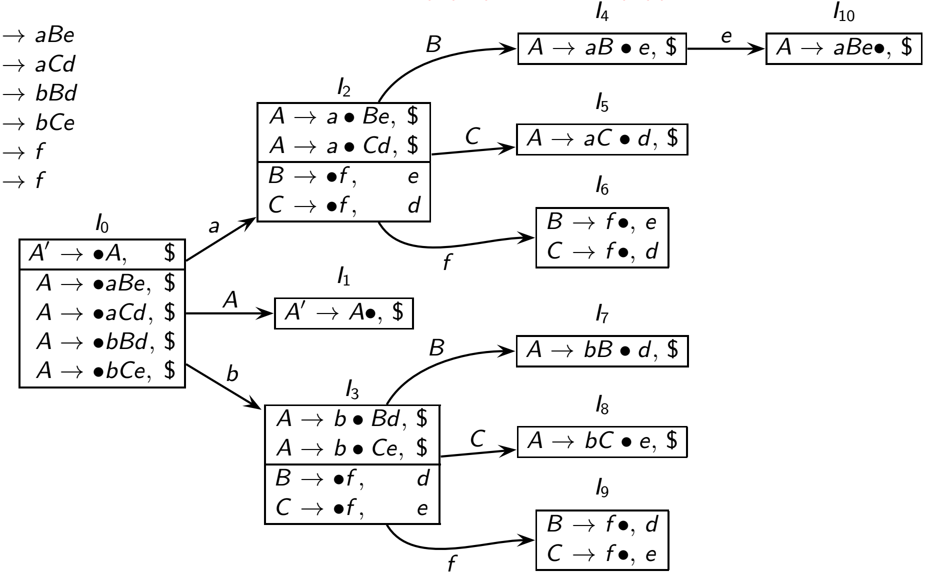
I_7: $A \rightarrow bB \bullet d,\ \$$

I_8: $A \rightarrow bC \bullet e,\ \$$

I_9: $B \rightarrow f\bullet,\ d$; $C \rightarrow f\bullet,\ e$

Another Example of LR(1) (aka CLR(1)) Parsing

$A \rightarrow aBe$
$A \rightarrow aCd$
$A \rightarrow bBd$
$A \rightarrow bCe$
$B \rightarrow f$
$C \rightarrow f$



I_0
$A' \rightarrow \bullet A, \ \$$
$A \rightarrow \bullet aBe, \ \$$
$A \rightarrow \bullet aCd, \ \$$
$A \rightarrow \bullet bBd, \ \$$
$A \rightarrow \bullet bCe, \ \$$

I_1
$A' \rightarrow A\bullet, \ \$$

I_2
$A \rightarrow a \bullet Be, \ \$$
$A \rightarrow a \bullet Cd, \ \$$
$B \rightarrow \bullet f, \ e$
$C \rightarrow \bullet f, \ d$

I_3
$A \rightarrow b \bullet Bd, \ \$$
$A \rightarrow b \bullet Ce, \ \$$
$B \rightarrow \bullet f, \ d$
$C \rightarrow \bullet f, \ e$

I_4
$A \rightarrow aB \bullet e, \ \$$

I_5
$A \rightarrow aC \bullet d, \ \$$

I_6
$B \rightarrow f\bullet, \ e$
$C \rightarrow f\bullet, \ d$

I_7
$A \rightarrow bB \bullet d, \ \$$

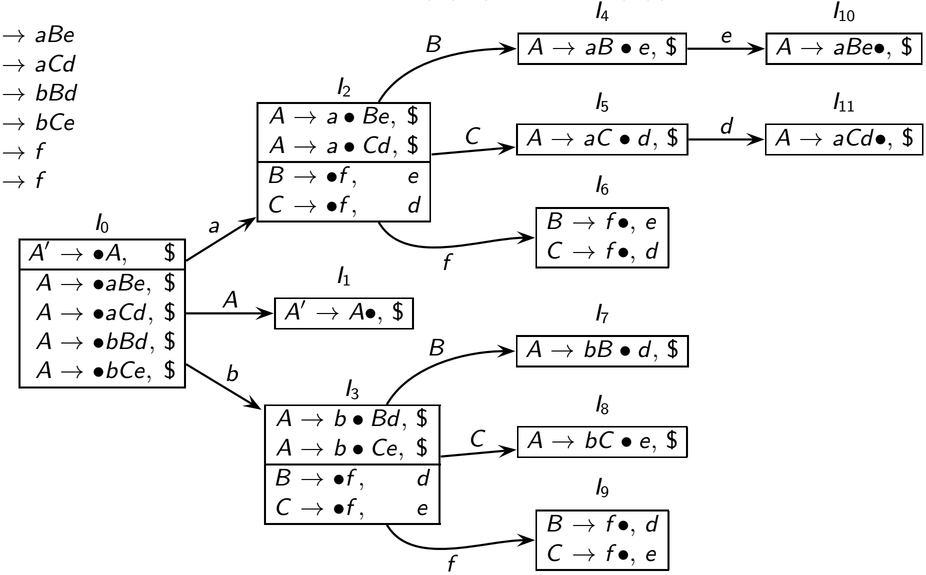
I_8
$A \rightarrow bC \bullet e, \ \$$

I_9
$B \rightarrow f\bullet, \ d$
$C \rightarrow f\bullet, \ e$

I_{10}
$A \rightarrow aBe\bullet, \ \$$

Another Example of LR(1) (aka CLR(1)) Parsing

$A \rightarrow aBe$
$A \rightarrow aCd$
$A \rightarrow bBd$
$A \rightarrow bCe$
$B \rightarrow f$
$C \rightarrow f$



I_0: $A' \rightarrow \bullet A,\ \$$ | $A \rightarrow \bullet aBe,\ \$$; $A \rightarrow \bullet aCd,\ \$$; $A \rightarrow \bullet bBd,\ \$$; $A \rightarrow \bullet bCe,\ \$$

I_1: $A' \rightarrow A\bullet,\ \$$

I_2: $A \rightarrow a \bullet Be,\ \$$; $A \rightarrow a \bullet Cd,\ \$$ | $B \rightarrow \bullet f,\ e$; $C \rightarrow \bullet f,\ d$

I_3: $A \rightarrow b \bullet Bd,\ \$$; $A \rightarrow b \bullet Ce,\ \$$ | $B \rightarrow \bullet f,\ d$; $C \rightarrow \bullet f,\ e$

I_4: $A \rightarrow aB \bullet e,\ \$$

I_5: $A \rightarrow aC \bullet d,\ \$$

I_6: $B \rightarrow f\bullet,\ e$; $C \rightarrow f\bullet,\ d$

I_7: $A \rightarrow bB \bullet d,\ \$$

I_8: $A \rightarrow bC \bullet e,\ \$$

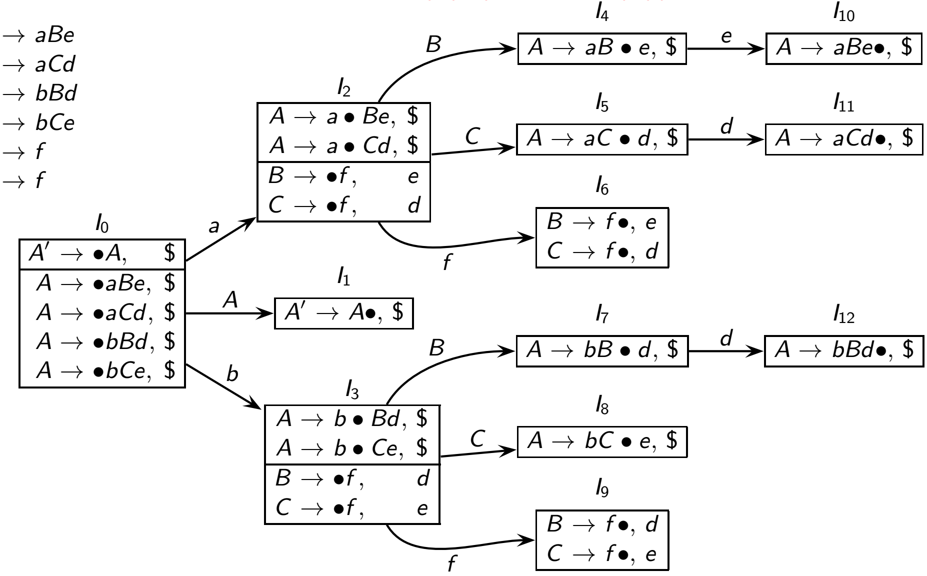
I_9: $B \rightarrow f\bullet,\ d$; $C \rightarrow f\bullet,\ e$

I_{10}: $A \rightarrow aBe\bullet,\ \$$

I_{11}: $A \rightarrow aCd\bullet,\ \$$

Another Example of LR(1) (aka CLR(1)) Parsing

$A \rightarrow aBe$
$A \rightarrow aCd$
$A \rightarrow bBd$
$A \rightarrow bCe$
$B \rightarrow f$
$C \rightarrow f$



I_0
$A' \rightarrow \bullet A,\ \$$
$A \rightarrow \bullet aBe,\ \$$
$A \rightarrow \bullet aCd,\ \$$
$A \rightarrow \bullet bBd,\ \$$
$A \rightarrow \bullet bCe,\ \$$

I_1
$A' \rightarrow A\bullet,\ \$$

I_2
$A \rightarrow a \bullet Be,\ \$$
$A \rightarrow a \bullet Cd,\ \$$
$B \rightarrow \bullet f,\ e$
$C \rightarrow \bullet f,\ d$

I_3
$A \rightarrow b \bullet Bd,\ \$$
$A \rightarrow b \bullet Ce,\ \$$
$B \rightarrow \bullet f,\ d$
$C \rightarrow \bullet f,\ e$

I_4
$A \rightarrow aB \bullet e,\ \$$

I_5
$A \rightarrow aC \bullet d,\ \$$

I_6
$B \rightarrow f\bullet,\ e$
$C \rightarrow f\bullet,\ d$

I_7
$A \rightarrow bB \bullet d,\ \$$

I_8
$A \rightarrow bC \bullet e,\ \$$

I_9
$B \rightarrow f\bullet,\ d$
$C \rightarrow f\bullet,\ e$

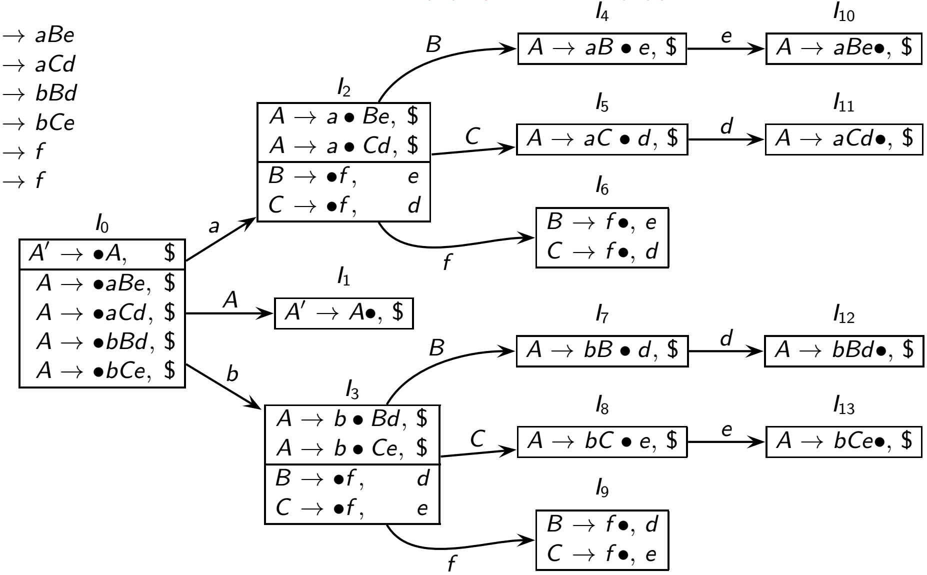
I_{10}
$A \rightarrow aBe\bullet,\ \$$

I_{11}
$A \rightarrow aCd\bullet,\ \$$

I_{12}
$A \rightarrow bBd\bullet,\ \$$

Another Example of LR(1) (aka CLR(1)) Parsing

$A \rightarrow aBe$
$A \rightarrow aCd$
$A \rightarrow bBd$
$A \rightarrow bCe$
$B \rightarrow f$
$C \rightarrow f$



I_0
$A' \rightarrow \bullet A, \quad \$$
$A \rightarrow \bullet aBe, \ \$$
$A \rightarrow \bullet aCd, \ \$$
$A \rightarrow \bullet bBd, \ \$$
$A \rightarrow \bullet bCe, \ \$$

I_1
$A' \rightarrow A\bullet, \ \$$

I_2
$A \rightarrow a \bullet Be, \ \$$
$A \rightarrow a \bullet Cd, \ \$$
$B \rightarrow \bullet f, \quad e$
$C \rightarrow \bullet f, \quad d$

I_3
$A \rightarrow b \bullet Bd, \ \$$
$A \rightarrow b \bullet Ce, \ \$$
$B \rightarrow \bullet f, \quad d$
$C \rightarrow \bullet f, \quad e$

I_4
$A \rightarrow aB \bullet e, \ \$$

I_5
$A \rightarrow aC \bullet d, \ \$$

I_6
$B \rightarrow f\bullet, \ e$
$C \rightarrow f\bullet, \ d$

I_7
$A \rightarrow bB \bullet d, \ \$$

I_8
$A \rightarrow bC \bullet e, \ \$$

I_9
$B \rightarrow f\bullet, \ d$
$C \rightarrow f\bullet, \ e$

I_{10}
$A \rightarrow aBe\bullet, \ \$$

I_{11}
$A \rightarrow aCd\bullet, \ \$$

I_{12}
$A \rightarrow bBd\bullet, \ \$$

I_{13}
$A \rightarrow bCe\bullet, \ \$$

LALR(1) Parsing

LALR(1) Parsing

- Merge item sets with identical cores (may have different lookaheads)

  States I_i : $A \rightarrow \alpha \bullet \beta, a$ and I_j : $A \rightarrow \alpha \bullet \beta, b$

  can be merged to create a new state I_{ij} : $A \rightarrow \alpha \bullet \beta, a/b$

- In practice, we do not construct LR(1) items to construct LALR(1) parser

  We construct LR(0) items and use a look-ahead propagation algorithm

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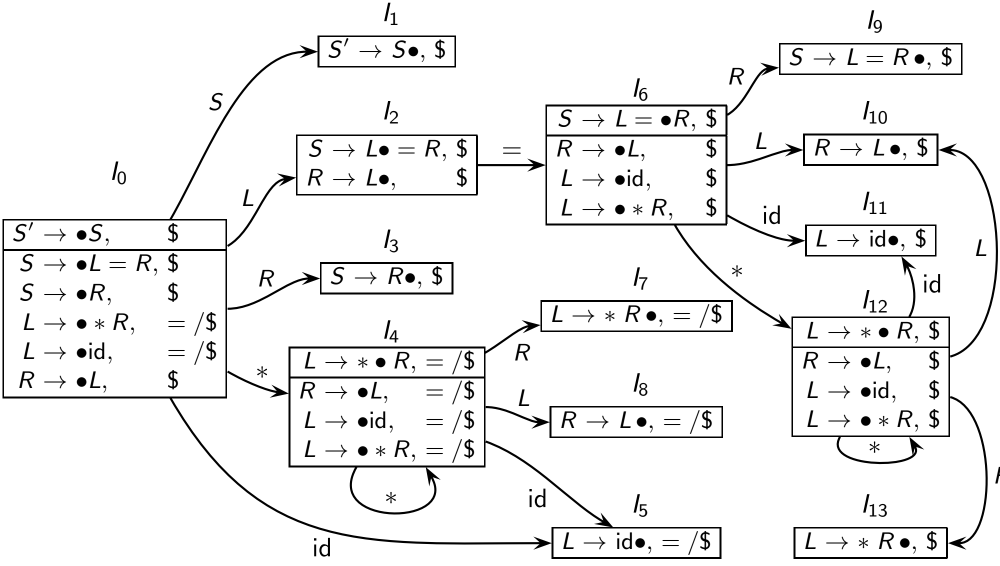
Topic:

Syntax Analysis

Section:

- Grammars, Derivations, and Parse Trees
- Shift Reduce Parsing
- SLR(1) Parsing
- Conceptual Issues in Parsing
- CLR(1) Parsing
- LALR(1) Parsing

LALR(1) Parsing for Pointer Assignment Grammar



I_0
$S' \rightarrow \bullet S,\ \$$
$S \rightarrow \bullet L = R,\ \$$
$S \rightarrow \bullet R,\ \$$
$L \rightarrow \bullet * R,\ =/\$$
$L \rightarrow \bullet \text{id},\ =/\$$
$R \rightarrow \bullet L,\ \$$

I_1
$S' \rightarrow S \bullet,\ \$$

I_2
$S \rightarrow L \bullet = R,\ \$$
$R \rightarrow L \bullet,\ \$$

I_3
$S \rightarrow R \bullet,\ \$$

I_4
$L \rightarrow * \bullet R,\ =/\$$
$R \rightarrow \bullet L,\ =/\$$
$L \rightarrow \bullet \text{id},\ =/\$$
$L \rightarrow \bullet * R,\ =/\$$

I_5
$L \rightarrow \text{id} \bullet,\ =/\$$

I_6
$S \rightarrow L = \bullet R,\ \$$
$R \rightarrow \bullet L,\ \$$
$L \rightarrow \bullet \text{id},\ \$$
$L \rightarrow \bullet * R,\ \$$

I_7
$L \rightarrow * R \bullet,\ =/\$$

I_8
$R \rightarrow L \bullet,\ =/\$$

I_9
$S \rightarrow L = R \bullet,\ \$$

I_{10}
$R \rightarrow L \bullet,\ \$$

I_{11}
$L \rightarrow \text{id} \bullet,\ \$$

I_{12}
$L \rightarrow * \bullet R,\ \$$
$R \rightarrow \bullet L,\ \$$
$L \rightarrow \bullet \text{id},\ \$$
$L \rightarrow \bullet * R,\ \$$

I_{13}
$L \rightarrow * R \bullet,\ \$$

LALR(1) Parsing for Pointer Assignment Grammar

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Topic:

Syntax Analysis

Section:

Grammars, Derivations, and Parse Trees

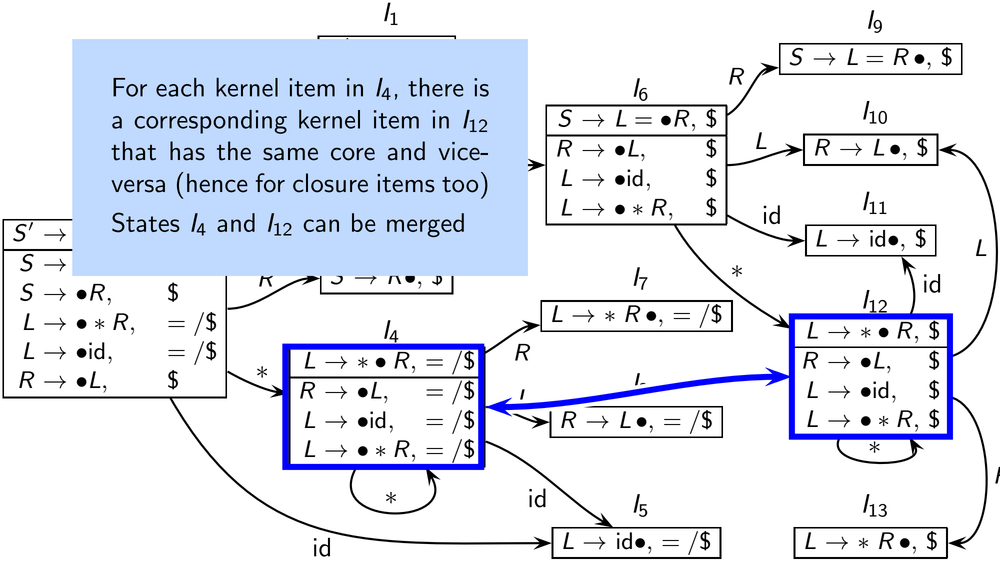
Shift Reduce Parsing

SLR(1) Parsing

Conceptual Issues in Parsing

CLR(1) Parsing

LALR(1) Parsing



For each kernel item in I_4, there is a corresponding kernel item in I_{12} that has the same core and vice-versa (hence for closure items too)

States I_4 and I_{12} can be merged

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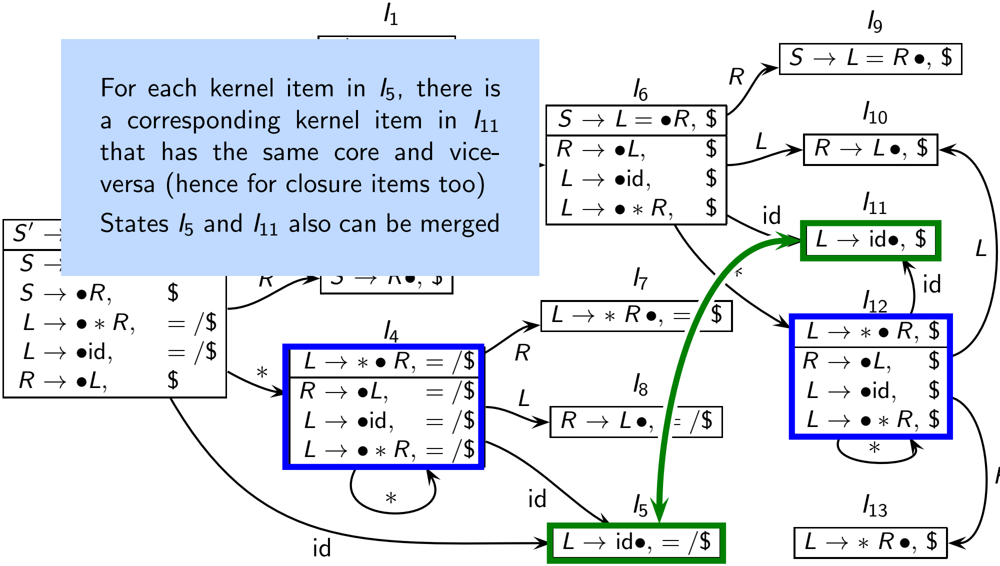
Topic:

Syntax Analysis

Section:

- Grammars, Derivations, and Parse Trees
- Shift Reduce Parsing
- SLR(1) Parsing
- Conceptual Issues in Parsing
- CLR(1) Parsing
- LALR(1) Parsing

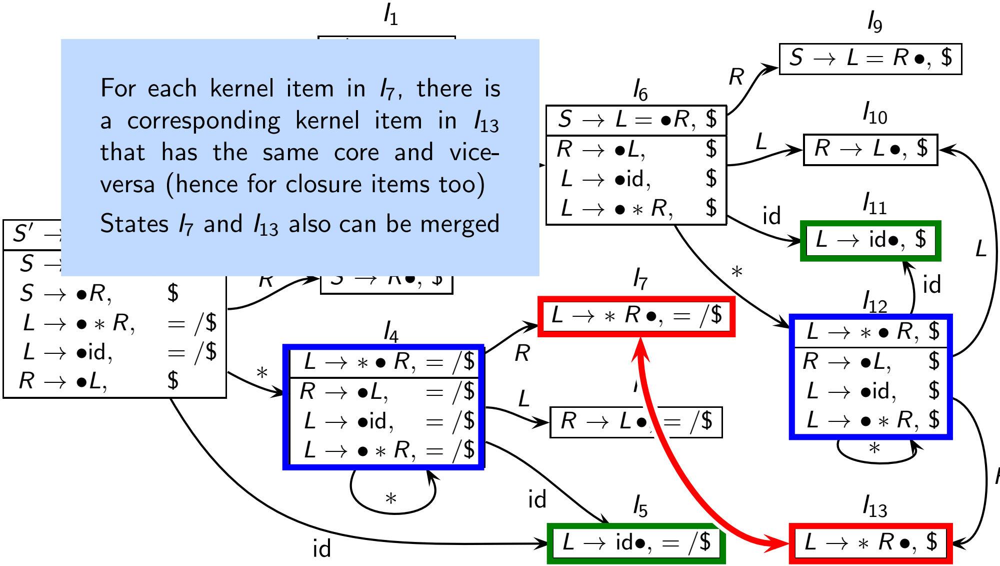
LALR(1) Parsing for Pointer Assignment Grammar



For each kernel item in I_5, there is a corresponding kernel item in I_{11} that has the same core and vice-versa (hence for closure items too)

States I_5 and I_{11} also can be merged

LALR(1) Parsing for Pointer Assignment Grammar



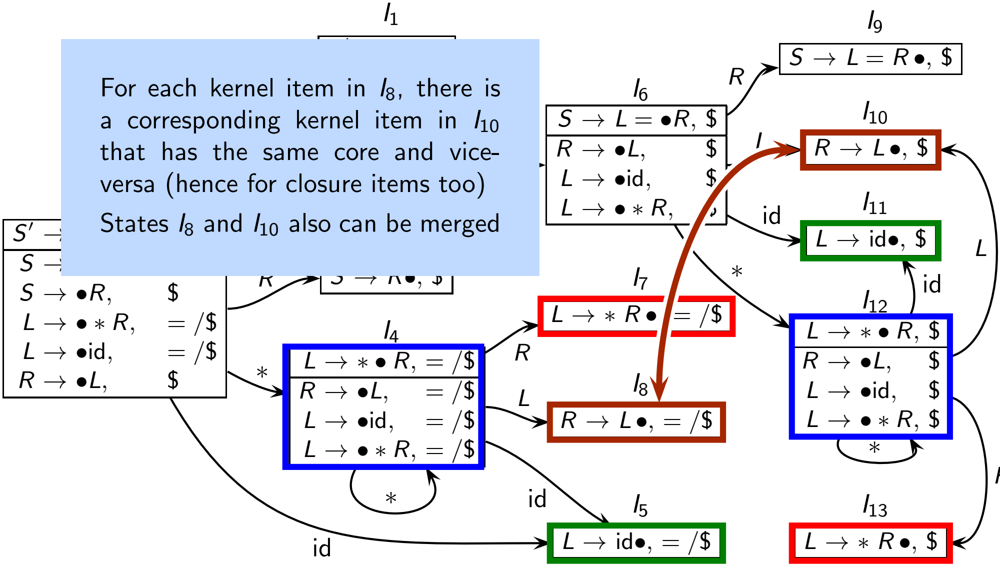
For each kernel item in I_7, there is a corresponding kernel item in I_{13} that has the same core and vice-versa (hence for closure items too)

States I_7 and I_{13} also can be merged

LALR(1) Parsing for Pointer Assignment Grammar

For each kernel item in I_8, there is a corresponding kernel item in I_{10} that has the same core and vice-versa (hence for closure items too)

States I_8 and I_{10} also can be merged



$S' \to$ …
$S \to$ …
$S \to \bullet R,\quad \$$
$L \to \bullet * R,\quad =/\$$
$L \to \bullet \text{id},\quad =/\$$
$R \to \bullet L,\quad \$$

I_1

$S \to R\bullet,\ \$$

I_4
$L \to * \bullet R,\ =/\$$
$R \to \bullet L,\quad =/\$$
$L \to \bullet \text{id},\quad =/\$$
$L \to \bullet * R,\ =/\$$

I_5
$L \to \text{id}\bullet,\ =/\$$

I_6
$S \to L = \bullet R,\ \$$
$R \to \bullet L,\quad \$$
$L \to \bullet \text{id},\quad \$$
$L \to \bullet * R,\quad \$$

I_7
$L \to * R\bullet,\ =/\$$

I_8
$R \to L\bullet,\ =/\$$

I_9
$S \to L = R\bullet,\ \$$

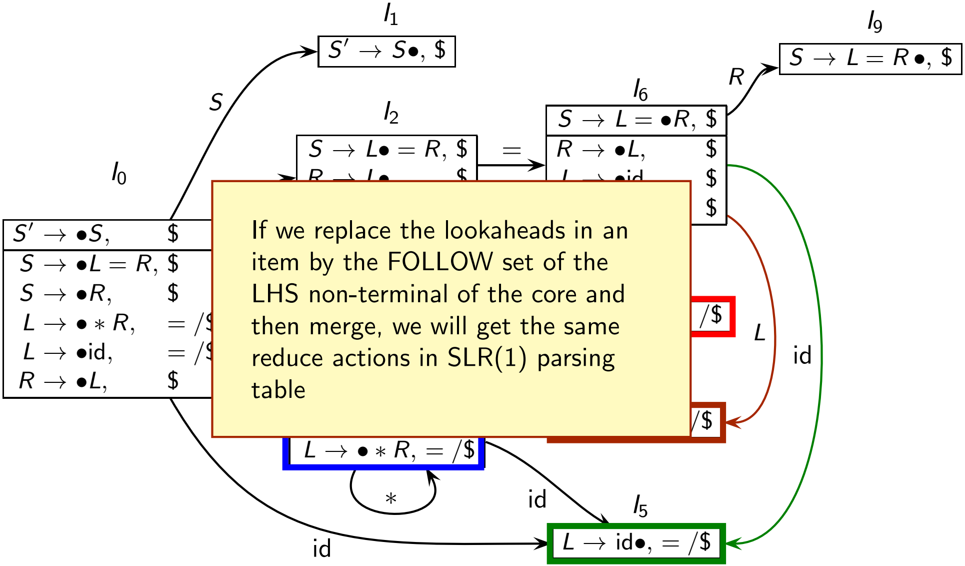
I_{10}
$R \to L\bullet,\ \$$

I_{11}
$L \to \text{id}\bullet,\ \$$

I_{12}
$L \to * \bullet R,\ \$$
$R \to \bullet L,\quad \$$
$L \to \bullet \text{id},\quad \$$
$L \to \bullet * R,\ \$$

I_{13}
$L \to * R\bullet,\ \$$

LALR(1) Parsing for Pointer Assignment Grammar



If we replace the lookaheads in an item by the FOLLOW set of the LHS non-terminal of the core and then merge, we will get the same reduce actions in SLR(1) parsing table

LALR(1) Parsing Table for Pointer Assignment Grammar

0 $S' \rightarrow S$
1 $S \rightarrow L = R$
2 $S \rightarrow R$
3 $L \rightarrow * R$
4 $L \rightarrow \text{id}$
5 $R \rightarrow L$

State	Action				Goto		
	id	*	=	$	S	L	R
0	$s5$	$s4$			$c1$	$c2$	$c3$
1				acc			
2			$s6$	$r5$			
3				$r2$			
4	$s5$	$s4$				$c8$	$c7$
5			$r4$	$r4$			
6	$s5$	$s4$				$c8$	$c9$
7			$r3$	$r3$			
8			$r5$	$r5$			
9				$r1$			

LALR(1) Vs CLR(1) Parsing

- Can merging of LR(1) states introduce shift-reduce conflict?
- Can merging of LR(1) states introduce reduce-reduce conflict?

Can Merging LR(1) Sets of Items Introduce Shift-Reduce Conflict?

- To merge states I_i and I_j, they should have identical cores but different lookaheads (if the lookaheads are same then the states will not be distinct)

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  So that the merged state is I_{ij} : $\begin{array}{ll} A \rightarrow \alpha \bullet a\beta, & p/r \\ B \rightarrow \gamma\bullet, & q/s \end{array}$

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  - If q is -reduce conflict

    $B \rightarrow \gamma\bullet, \quad a$

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A set I_{ij} of items in an LALR(1) parser can have a shift-reduce conflict *if and only if* a set I_i of LR(1) items merged to form I_{ij} has the same shift-reduce conflict

This is because a shift-reduce conflict depends both on a lookahead and a terminal in the core of an item

Can Merging LR(1) Sets of Items Introduce Reduce-Reduce Conflict?

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- For a reduce-reduce conflict in I_{ij} such that there is no reduce-reduce conflict in I_i or I_j,
  - $p = s$. This is possible without a reduce-reduce conflict in I_i and I_j
  - $r = q$. This is also possible without a reduce-reduce conflict in I_i and I_j

Can Merging LR(1) Sets of Items Introduce Reduce-Reduce Conflict?

- Let I_i : A B

So that the

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  - ○ $p = s.$ and I_j
  - ○ $r = q.$ This is also possible without a reduce-reduce conflict in I_i and I_j

Merging LR(1) sets of items can introduce reduce-reduce conflicts *even* if the original sets do not have a reduce-reduce conflict

This is because a reduce-reduce conflict depends only on lookaheads and a complete item. The terminals in a core do not play any role

LALR(1) Vs LR(1) Parsing

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- Consider a grammar $G \in \mathbb{G}(LALR(1))$
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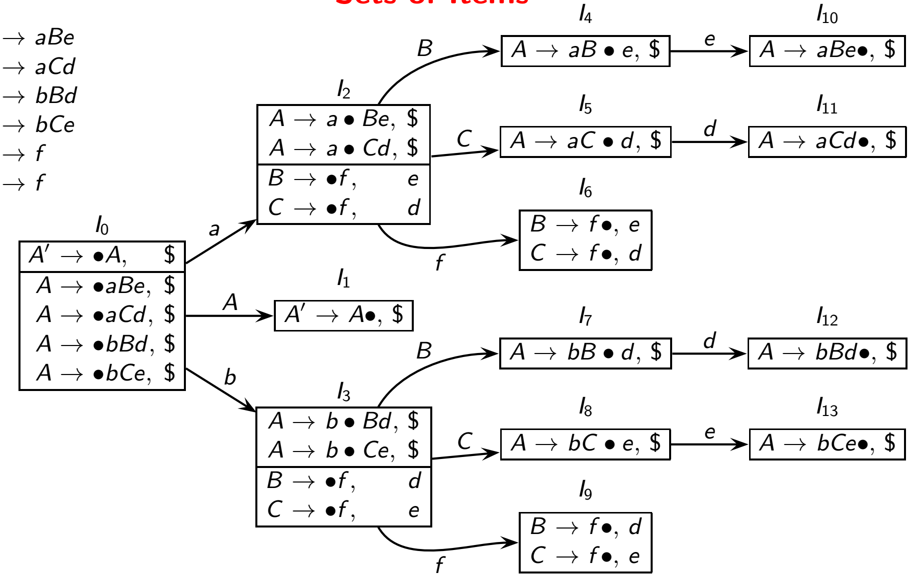
  If a parsing method admits a grammar G then the corresponding parser for G accepts all sentences in $L(G)$ and rejects all sentences not in $L(G)$
- Consider a grammar $G \notin \mathbb{G}(LALR(1))$

  An LALR(1) parser may still accept $L(G)$ because it may admit G' such that $L(G) = L(G')$

Example of Reduce-Reduce Conflict Caused by Merging LR(1) Sets of Items

$A \rightarrow aBe$
$A \rightarrow aCd$
$A \rightarrow bBd$
$A \rightarrow bCe$
$B \rightarrow f$
$C \rightarrow f$



I_0
$A' \rightarrow \bullet A,\ \$$
$A \rightarrow \bullet aBe,\ \$$
$A \rightarrow \bullet aCd,\ \$$
$A \rightarrow \bullet bBd,\ \$$
$A \rightarrow \bullet bCe,\ \$$

I_1
$A' \rightarrow A\bullet,\ \$$

I_2
$A \rightarrow a \bullet Be,\ \$$
$A \rightarrow a \bullet Cd,\ \$$
$B \rightarrow \bullet f,\ e$
$C \rightarrow \bullet f,\ d$

I_3
$A \rightarrow b \bullet Bd,\ \$$
$A \rightarrow b \bullet Ce,\ \$$
$B \rightarrow \bullet f,\ d$
$C \rightarrow \bullet f,\ e$

I_4
$A \rightarrow aB \bullet e,\ \$$

I_5
$A \rightarrow aC \bullet d,\ \$$

I_6
$B \rightarrow f\bullet,\ e$
$C \rightarrow f\bullet,\ d$

I_7
$A \rightarrow bB \bullet d,\ \$$

I_8
$A \rightarrow bC \bullet e,\ \$$

I_9
$B \rightarrow f\bullet,\ d$
$C \rightarrow f\bullet,\ e$

I_{10}
$A \rightarrow aBe\bullet,\ \$$

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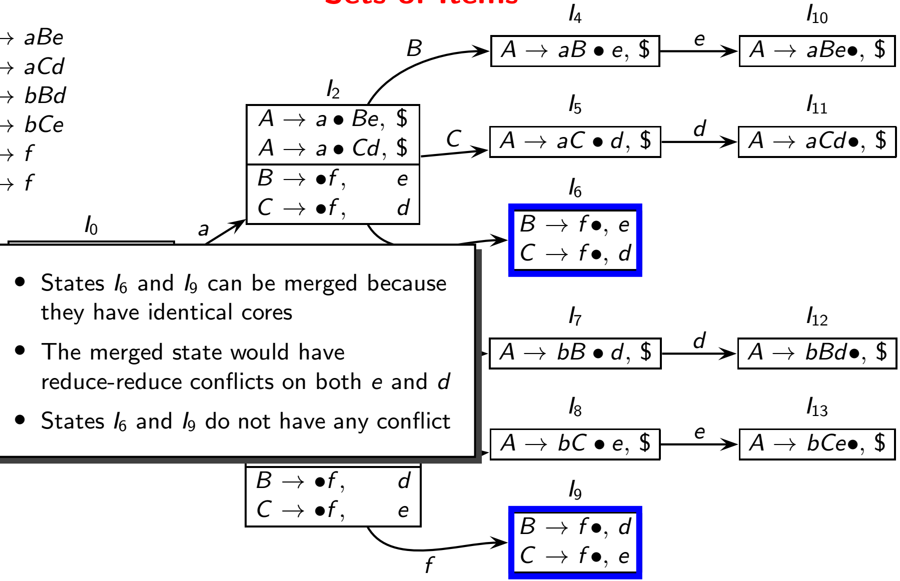

- States I_6 and I_9 can be merged because they have identical cores
- The merged state would have reduce-reduce conflicts on both e and d
- States I_6 and I_9 do not have any conflict

A Practical Example of Reduce-Reduce Conflict in LR(1) Parsing

program → func_decl var_decl
program → var_decl func_decl
var_decl → data_type ID ;
data_type → INT
func_decl → return_type ID ( )
return_type → INT
return_type → VOID

For the input “int f . . .”, when we see the token INT, the next token is ID

In this situation, the parser does not know if it should reduce INT to return_type or data_type

State I_0 contains the following items

data_type → • INT, ID
return_type → • INT, ID

The transition on INT gives the following set of items showing a reduce-reduce conflict on ID

data_type → INT •, ID
return_type → INT •, ID

A Practical Example of Reduce-Reduce Conflict in LR(1) Parsing

In this particular case, the conflict can be removed by replacing every occurrence of the non-terminals data_type and return_type by every RHS of the non-terminal

Original Grammar	Transformed Grammar
program → func_decl var_decl	program → func_decl var_decl
program → var_decl func_decl	program → var_decl func_decl
var_decl → data_type ID ;	var_decl → INT ID ;
data_type → INT	func_decl → INT ID ( )
func_decl → return_type ID ( )	func_decl → VOID ID ( )
return_type → INT	
return_type → VOID	

A Summary of Bottom Up Parsing Methods

Parsing Method	Items Used	Reduction by $A \to \alpha$	Remarks
SLR(0)	LR(0)	On any terminal	
SLR(1)	LR(0)	On the terminals in FOLLOW(A)	
LR(1), also known as Canonical LR(1) or CLR(1)	LR(1)	On lookahead a in the item "$A \to \alpha \bullet, a$"	
LALR(1)	LR(1)	On lookahead a in the item "$A \to \alpha \bullet, a$"	Conceptually, the sets of items are obtained by merging LR(1) item sets that differ only in the lookahead symbols Practically, lookaheads are propagated starting from \$ on LR(0) items

Comparison of Bottom-Up Methods and Corresponding Grammars

- A grammar G is accepted by a parsing method P if a conflict-free parser can be constructed for G using P
- An ambiguous grammar is not accepted by any parsing method
- A grammar is called SLR(0), SLR(1), LR(1), or LALR(1) if it is accepted respectively, by the the SLR(0), SLR(1), LR(1), or LALR(1) parsing method
  - Every SLR(0) grammar is also SLR(1) grammar but not vice-versa
  - Every SLR(1) grammar is also LALR(1) grammar but not vice-versa
  - Every LALR(1) grammar is also LR(1) grammar but not vice-versa
- The expressions grammar ($E \to E + E \mid E * E \mid \text{id}$) is not accepted by any parsing method because it is ambiguous

  (without post-facto instrumentation of parsing tables using precedences and associativities)