Assignment - I

Name :- S.V Keerthivasan

Reg. No :- 17ME065

Sub :- Finite Element Analysis

Sub code :- 17MEC21

Dept :- Mechanical Engineering.

Part - A

1) what is meant by finite element analysis?

A small unit having definite shape of geometry and nodes is called finite element.

Finite element method is a numerical method for solving problems of engineering and mathematical physics.

2) what is meant by Joint (or) node and element?

<u>Node</u> :- Each kind of finite element has a specific structural shape and is interconnected with the adjacent elements by nodal points (or) nodes. At the nodes degree of freedom are located. The forces will act only at nodes and not in elements.

<u>Elements</u> :- A small unit having definite shape of geometry and node is called element.

3.) Define discretization unit classification?

The art of sub-dividing a structure into a convenient number is called discretization. Its classification are.

1.) one dimensional element.
2.) Two - dimensional element.
3.) Three - dimensional element.
4.) Asymmetric element.

4.) state the three phases of finite element method?

1) pre prossering.
2) Analysis
3.) post - processing

5.) what is structural and non-structural problem?

Displacement at each nodal point is obtained by using these displacement solution, stress and strain in each element can be calculated.

Temperature (or) fluid pressure at each nodal point is obtained. By using values, properties of such a heat flow can be calculated.

6) Name any four FEA software, and application's?

Software :- Ansys, Nastran, coromag, visa.
Application :- Mechanical design, Aircraft structure, civil engineering, Biomedical engineering.

7.) List advantages and disadvantages of FEA?

Advantages :- 1.) model irregular shaped bodies quite easily.
2.) Include dynamic effects.

Disadvantages :-

1) FEA method is time consuming process (by time)
2.) FEA method can't produces exact results as those analytical method.

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8) Discuss about Galerkin's method of Approximation ?

$$\int R(x)\, \phi(x) .\, dx = 0$$

where,

$\phi(x) \rightarrow$ weighing function, $R(x) \rightarrow$ Residue function.

9) List application of FEA?

1.) Civil engineering - Folded plates, shear roofs, shear walls, etc ...

2.) Mechanical design - composite material, linkages, gears, etc ...

3.) Bio-medical engineering - stress analysis of crystalls, bones, teeth, etc...

4.) Air-craft structure - Analysis of aircraft wins, fins, rocket, etc ...

10.) what is the classification of co-ordinates and explain ?

1) Global co-ordinates : point in the entire structure is marked.

2.) Local co-ordinates : Each element need co make co-ordinates.

3.) Natural co-ordinates :- Any point inside element is near encads.

11.) Define boundary condition and distinguish between essential and natural boundary conditions ?

<u>Boundary Condition</u> :-

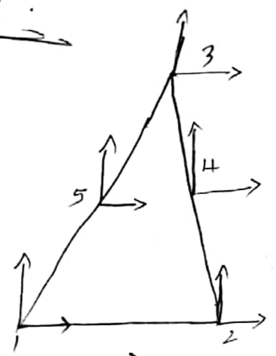
The main types of loading available in FEA include force, pressure and temperature. These can be applied to points, nodes, edges, etc.

Essential Boundary conditions	Natural Boundary conditions.
They are imposed explicity on the solution.	They are antomatically will be satisfied after solution of problem.

12) what is a higher order element? give an example?

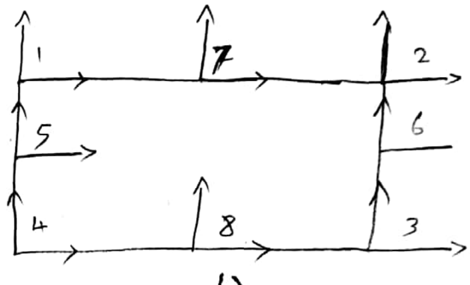
Higher - order elements capture a more complex data representation than their linear element pre-decessors and reduce the required number of element needed to decompose fluid flow, stress and other simulation type.

Example :-



a.)

6 - noded higher element



b.)

8 - noded higher element.

PART - B.

1.) List and briefly descrete the general step of the FEM?

step -1 :- Discretization structure :-

The art of subdividing a structure into a conirnent number. Elements are classified as, 1.) one - dimensional element 2.) Two - dimensional element, 3.) three - dimensional element 4.) Axisymmetric element.

step 2 :- numbering of nodes and elements :-

* maximum nodal number - minimum node number
* shorter nodal number will reduce memory size.

step 3 - selection of displacement or Interpolation functions.

polynomial function used to solve problem by using linear, quadratic and cubic function will used in polynomial.

step 4 - Define material behaviour by using strain - displacement relationship and stress - strain relationship :-

* strain (e) = $\frac{du}{dx}$ where $u \to$ displacement, $x \to$ length,

where,

stress (σ) - Ee ; where,

$E \to$ young's modulus,

$e \Rightarrow$ strain.

step 5 :- Element stiffness matrix and equation :-

$$\{F\} = [K^e] \{u^e\}.$$

where,

$F \to$ Force, $K \to$ stiffness, $u \to$ displacement, $e \to$ element.

ex :-

$$\begin{Bmatrix} F_1 \\ F_2 \\ F_n \end{Bmatrix} = \begin{bmatrix} K_{11}, & K_{12} & \ldots & K_{1n} \\ K_{21}. & K_{22} & \ldots & K_{2n} \\ K_{n1}. & K_{n2} & \ldots & K_{nn} \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_n \end{Bmatrix}$$

step 6 :- Assemble the element equation to obtain global (or) total equation.

$$\{F\} = [K] . \{u\}.$$

where, $\{F\} \to$ Global force vector, $[K] \Rightarrow$ Global stiffness matrix.

$\{u\} \to$ Global displacement matrix.

Step 7 :- Applying boundary conditions :-

Depending upon node, apply boundary conditions.

Step 8 :- Solution for the unknown displacement :-

Equation can be solved and unknown displacement $\{u\}$ are, calculated by using Gaussian elimination (or) Gauss - seidal method.

Step 9 :- computation of element strain and stress from nodal displacement $\{u\}$:-

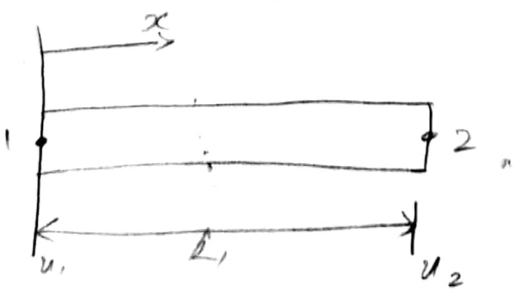
* strain $(e) = \frac{du}{dx} = \frac{u_2 - u_1}{x_2 - x_1}$ where,
  $u_1 - u_2 \rightarrow$ nodal displacement
  $x_2 - x_1 \rightarrow$ Actual length.

* stress $(\sigma) = Ee$, where,
  $E \rightarrow$ ~~young~~ young's modulus,
  $e \rightarrow$ strain.

Step 10 :- Interpret the result (post processing) :-

Analysis and evaluation of the solution results is referred to as post - processor. computer programs help's the user to interpret the result by displaying them in graphical form.

2) Derivation of stiffness matrix and shape function for one - dimensional bar element?



Shape function derivation :-

Let us consider one-dimensional equation.

$$u = a_0 + a_1 x \rightarrow ①$$

write above equation on matrix form.

$$u = \begin{bmatrix} 1 & x \end{bmatrix} \cdot \begin{Bmatrix} a_0 \\ a_1 \end{Bmatrix}$$

$$\begin{pmatrix} u = u_1 \\ x = 0 \end{pmatrix} \begin{pmatrix} u = u_2 \\ x = l \end{pmatrix} \Rightarrow ④$$

sub ③, ④ in ①,

$u_1 = a_0$

$u_2 = a_0 + a_1 l$

we can written in matrix form.

$$\begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & l \end{bmatrix} \begin{Bmatrix} a_0 \\ a_1 \end{Bmatrix}$$

where,

$u \rightarrow$ degree of freedom.

$A \rightarrow$ global co-ordinates matrix.

$C \rightarrow$ connectivity matrix.

Then, to find A,

$$\begin{Bmatrix} a_0 \\ a_1 \end{Bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & l \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix}$$

$$\begin{Bmatrix} a_0 \\ a_1 \end{Bmatrix} = \frac{1}{l-0} \begin{bmatrix} l & -0 \\ -1 & l \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix}$$

Note :-

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}^{-1} = \frac{1}{(a_{11}a_{22} - a_{21}a_{22})} \times \begin{bmatrix} a_{22} & -a_{12} \\ -a_{21} & -a_{11} \end{bmatrix}$$

sub. $\begin{Bmatrix} a_0 \\ a_1 \end{Bmatrix}$ in ② $\begin{Bmatrix} a_0 \\ a_1 \end{Bmatrix} = \frac{1}{l} \begin{bmatrix} l & 0 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix}$.

$$u = \begin{bmatrix} 1 & x \end{bmatrix} \frac{1}{l} \begin{bmatrix} l & 0 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix}.$$

$$u = \frac{1}{l}[1 \ -x]\begin{bmatrix} l & 0 \\ -1 & 1 \end{bmatrix}\begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix}$$

$$= \frac{1}{l}[l - x \quad 0 + x]\begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix}.$$

$$= \begin{bmatrix} \frac{l-x}{l} & \frac{x}{l} \end{bmatrix}\begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} \quad \left[\because N_1 = \frac{l-x}{l} \quad N_2 = \frac{x}{l}\right]$$

$$= [N_1 \ N_2]\begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix}. \quad \text{where,}$$

$$u = N_1 u_1 + N_2 u_2 \quad N_1, N_2 \to \text{shape function.}$$

<u>stiffness matrix [K] derivation :-</u>

$$[K] = \int_v [B]^T \cdot [D][B] . dv \to ①$$

For one - dimensional equation :-

$$u = N_1 u_1 + N_2 u_2 \quad \left[\because N_1 = \frac{l-x}{l}, N_2 = \frac{x}{l}\right]$$

<u>Then diff w.r.to 'x' :-</u>

$$[B] = \frac{dN_1}{dx} \cdot \frac{dN_2}{dx}$$

$$= \left[\frac{d}{dx}\left(\frac{l-x}{l}\right) \cdot \frac{d}{dx}\left(\frac{x}{l}\right)\right] \Rightarrow \frac{1}{l}\left[\frac{d}{dx}(l-x)\frac{d}{dx^2}\right]$$

$$= \frac{1}{l}[(0-1) \ 1]$$

$$[B] = \left[\frac{-1}{l} \ \frac{1}{l}\right]$$

Then,

$$[B]^T = \begin{Bmatrix} -1/l \\ 1/l \end{Bmatrix}.$$

where,

$$[D] = [E] = E$$

sub. $[B], [B]^T; [D]$ in ① $\Rightarrow$

$$[K] = \int_0^l \begin{Bmatrix} -1/l \\ 1/l \end{Bmatrix} . E \begin{bmatrix} \frac{-1}{l} & \frac{1}{l} \end{bmatrix} . dv$$

$$= \int_0^l \begin{bmatrix} -1/l \times -1/l & 1/l \times -1/l \\ -1/l \times \frac{1}{l} & 1/l \times \frac{1}{l} \end{bmatrix} E.dv$$

$$= \int_0^l \begin{bmatrix} 1/l^2 & -1/l^2 \\ -1/l^2 & 1/l^2 \end{bmatrix} . E dv \quad [\because dv = A.dx].$$

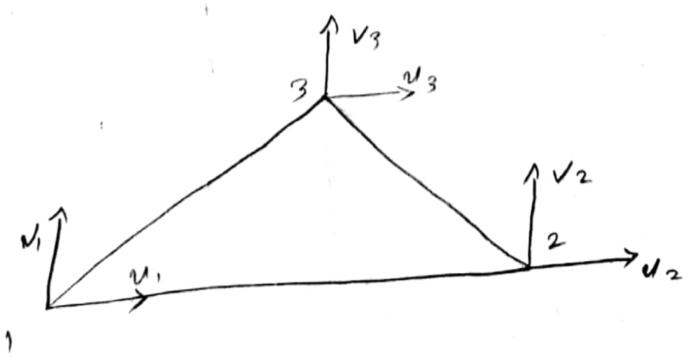
$$= \int_0^l \begin{bmatrix} 1/l^2 & -1/l^2 \\ -1/l^2 & 1/l^2 \end{bmatrix} E.A.dx = \begin{bmatrix} 1/l^2 & -1/l^2 \\ -1/l^2 & 1/l^2 \end{bmatrix} .E.A \int_0^l dx$$

$$= \begin{bmatrix} 1/l^2 & -1/l^2 \\ -1/l^2 & 1/l^2 \end{bmatrix} E.A(x)_0^l = \begin{bmatrix} 1/l^2 & -1/l^2 \\ -1/l^2 & 1/l^2 \end{bmatrix} .E.A(l-0).$$

$$= l.E.A \begin{bmatrix} 1/l^2 & -1/l^2 \\ -1/l^2 & 1/l^2 \end{bmatrix} = \frac{l.E.A}{l^2} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$[K] = \frac{E.A}{l} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \text{(or)} [K] = \frac{A.E}{l} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

3) Derivation of stiffness matrix and shape functions for two dimensional beam elements.



shape function derivation :-

displacement $\{u\} = \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \\ v_1 \\ v_2 \\ v_3 \end{Bmatrix}$

For, six degree of fradom,

$u = a_1 + a_2 x + a_3 y \rightarrow ①$, $v = a_4 + a_3 x + a_6 y \rightarrow ②$.

From ①,

let us consider, $u_1 = a + a_2 x_1 + a_3 y_1$

$u_2 = a_1 + a_2 x_2 + a_3 y_2$

$u_3 = a_2 + a_2 x_3 + a_3 y_3$

Now, written above equation is matrix form :-

$$\begin{Bmatrix} u_1 \\ u_2 \\ u_3 \end{Bmatrix} = \begin{bmatrix} 1 & x_1 & y_1 \\ 1 & x_2 & y_2 \\ 1 & x_3 & y_3 \end{bmatrix} \begin{Bmatrix} a_1 \\ a_2 \\ a_3 \end{Bmatrix}$$

To find (a_1, a_2, a_3)

$$\begin{Bmatrix} a_1 \\ a_2 \\ a_3 \end{Bmatrix} = \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \end{Bmatrix} \begin{bmatrix} 1 & x_1 & y_1 \\ 1 & x_2 & y_2 \\ 1 & x_3 & y_3 \end{bmatrix}^{-1} \rightarrow ③$$

consider,

$$D = \begin{bmatrix} 1 & x_1 & y_1 \\ 1 & x_2 & y_2 \\ 1 & x_3 & y_3 \end{bmatrix}$$

For finding inverse matrix :-

$$D^{-1} = \frac{C^T}{|D|} \rightarrow ④$$

Find co-factor (c) of matrix D,

$C_{11} = \begin{vmatrix} x_2 & y_2 \\ x_3 & y_3 \end{vmatrix} = (x_2 y_3 - x_3 y_2)$, $C_{12} = \begin{vmatrix} 1 & y_2 \\ 1 & y_3 \end{vmatrix} = y_2 - y_3$

$C_{13} = \begin{vmatrix} 1 & x_2 \\ 1 & x_3 \end{vmatrix} = x_3 - x_2$, $C_{21} = \begin{vmatrix} x_1 & y_1 \\ x_3 & y_3 \end{vmatrix} = x_3 y_1 - x_2 y_3$

$C_{22} = \begin{vmatrix} 1 & y_1 \\ 1 & y_3 \end{vmatrix} = y_3 - y_1$, $C_{23} = \begin{vmatrix} 1 & x_1 \\ 1 & x_3 \end{vmatrix} = x_1 - x_3$, $C_{31} = \begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \end{vmatrix} = x_1 y_2 - x_2 y_2$.

$$C_{32} = -\begin{vmatrix} 1 & y_1 \\ 1 & y_2 \end{vmatrix} = y_1 - y_2 \quad , \quad C_{33} = \begin{vmatrix} 1 & x_1 \\ 1 & x_2 \end{vmatrix} = x_2 - x_1$$

$$C = \begin{bmatrix} x_2y_3 - x_3y_2 & y_2 - y_3 & x_3 - x_2 \\ x_3y_1 - x_1y_3 & y_3 - y_1 & x_1 - x_3 \\ x_1y_2 - x_2y_1 & y_1 - y_2 & x_2 - x_1 \end{bmatrix}$$

$$C^T = \begin{bmatrix} x_2y_3 - x_3y_2 & x_3y_3 - x_3y_2 & x_1y_2 - x_2y_1 \\ y_2 - y_3 & y_3 - y_1 & y_1 - y_2 \\ x_3 - x_2 & x_1 - x_3 & x_2 - x_1 \end{bmatrix} \rightarrow (5)$$

$$|D| = \begin{vmatrix} 1 & x_1 & y_1 \\ 1 & x_2 & y_2 \\ 1 & x_3 & y_3 \end{vmatrix} \Rightarrow |D| = (x_2y_3 - x_3y_2) - x(y_3 - y_2) + y_1(x_3 - x_2) \rightarrow (6)$$

sub (5) & (6) in (4):-

$$D^{-1} = \frac{\begin{bmatrix} (x_2y_3 - x_3y_2) & (x_2y_1 - x_1y_2) & (x_1y_2 - x_3y_1) \\ (y_2 - y_3) & (y_3 - y_1) & (y_1 - y_2) \\ (x_3 - x_2) & (x_1 - x_3) & (x_2 - x_1) \end{bmatrix}}{(x_2y_3 - x_3y_2) - x(y_3 - y_2) + y_1(x_3 - x_2)}$$

sub D^{-1} value in (3):-

$$\begin{Bmatrix} a_1 \\ a_2 \\ a_3 \end{Bmatrix} = \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \end{Bmatrix} \frac{\begin{bmatrix} (x_2y_3 - x_3y_2) & (x_3y_1 - x_1y_3) & (x_1y_2 - x_2y_1) \\ (y_2 - y_3) & (y_3 - y_1) & (y_1 - y_2) \\ (x_3 - x_2) & (x_1 - x_3) & (x_2 - x_1) \end{bmatrix}}{(x_2y_3 - x_3y_2) - x(y_3 - y_2) + y_1(x_3 - x_2)}$$

Let, Area of triangle:-

$$A = \frac{1}{2}\begin{bmatrix} 1 & x_1 & y_1 \\ 1 & x_2 & y_2 \\ 1 & x_3 & y_3 \end{bmatrix} = \frac{1}{2}\left[(x_2y_3 - x_3y_2) - x_1(y_3 - y_2) + y_1(x_3 - x_2)\right]$$

$$2A = (x_2y_3 - x_3y_2) - x_1(y_3 - y_2) + y_1(x_3 - x_2) \rightarrow (8)$$

sub ⑧ in ⑦:-

$$\begin{Bmatrix} a_1 \\ a_2 \\ a_3 \end{Bmatrix} = \frac{1}{2A} \begin{bmatrix} (x_2y_3 - x_3y_2) & (x_3y_1 - x_1y_3) & (x_1y_2 - x_2y_1) \\ (y_2 - y_3) & (y_3 - y_1) & (y_1 - y_2) \\ (x_3 - x_2) & (x_1 - x_3) & (x_2 - x_1) \end{bmatrix} \cdot \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \end{Bmatrix} \rightarrow ⑨$$

Then,

$$\begin{Bmatrix} a_1 \\ a_2 \\ a_3 \end{Bmatrix} = \frac{1}{2A} \begin{bmatrix} p_1 & p_2 & p_3 \\ q_1 & q_2 & q_3 \\ r_1 & r_2 & r_3 \end{bmatrix} \cdot \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \end{Bmatrix} \rightarrow ⑩$$

where,

$p_1 \rightarrow (x_2y_3 - x_3y_2)$, $p_2 \rightarrow (x_3y_1 - x_1y_3)$. $p_3 = (x_1y_2 - x_2y_1)$

$q_1 \rightarrow (y_2 - y_1)$, $q_2 \rightarrow (y_3 - y_1)$, $q_3 \rightarrow (y_1 - y_2)$.

$r_1 \rightarrow (x_3 - x_2)$, $r_2 \rightarrow (x_1 - x_3)$, $r_3 = (x_2 - x_1)$.

From ①,

$$u = a_1 + a_2x + a_3y.$$

plane equation written as matrix form.

$$u = [1 \quad x \quad y] \begin{Bmatrix} a_1 \\ a_2 \\ a_3 \end{Bmatrix}$$

sub ⑩ in $\begin{Bmatrix} u_1 \\ u_2 \\ u_3 \end{Bmatrix}$

$$a = [1 \quad x \quad y] \cdot \frac{1}{2A} \begin{bmatrix} p_1 & p_2 & p_3 \\ q_1 & q_2 & q_3 \\ r_1 & r_2 & r_3 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \end{Bmatrix}$$

$$u = \frac{1}{2A} \left[(p_1 + q_1x + r_1y) \cdot (p_2 + q_2x + r_2y) \cdot (p_3 + q_3x + r_3y) \right] \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \end{Bmatrix}$$

$$u = \left[\frac{(p_1 + q_1x + r_1y)}{2A} \cdot \frac{(p_2 + q_2x + r_2y)}{2A} \cdot \frac{(p_3 + q_3x + r_3y)}{2A} \right] \cdot \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \end{Bmatrix}$$

Let us take,

$$N_1 = \frac{p_1 + q_1x + r_1y}{2A}, \quad N_2 = \frac{p_2 + q_2x + r_2y}{2A}, \quad N_3 = \frac{p_3 + q_3x + r_3y}{2A}.$$

Then, ($N_1, N_2, N_3 \rightarrow$ shape function's).

$$u = [N_1 \; N_2 \; N_3] \cdot \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \end{Bmatrix} \rightarrow (11)$$

Similarly for (2) equation

$$v = [N_1 \; N_2 \; N_3] \cdot \begin{Bmatrix} v_1 \\ v_2 \\ v_3 \end{Bmatrix} \rightarrow (12)$$

Then assemble (11) & (12),

$$\text{Displacement } (u) = \begin{Bmatrix} u \\ v \end{Bmatrix} \begin{bmatrix} N_1 & 0 & N_2 & 0 & N_3 & 0 \\ 0 & N_1 & 0 & N_2 & 0 & N_3 \end{bmatrix}$$

Stiffness matrix [K] derivation.

W.K.T,

$$[K] = \int_v [B]^T [D] [B] \cdot dv \quad [\text{From one dimensional equation}]$$

$$[\because V = A r^T]$$

$$[K] = [B]^T [D] [B] v$$

$$[K] = [B]^T [D] [B] A.t$$

where,

★ $A \rightarrow$ Area of triangle $= \frac{1}{2} \begin{bmatrix} 1 & x_1 & y_1 \\ 1 & x_2 & y_2 \\ 1 & x_3 & y_3 \end{bmatrix}$

★ $t \rightarrow$ thickness of element.

★ $[B] \rightarrow$ strain displacement matrix $= \frac{1}{2A} \begin{bmatrix} q_1 & 0 & q_2 & 0 & q_3 \\ 0 & r_1 & 0 & r_2 & r_3 \\ r_1 & q_1 & r_2 q_2 & r_3 q_3 \end{bmatrix}$

where,

$$q_1 = y_2 - y_3, \quad q_2 = y_3 - y_2, \quad q_3 = y_1 - y_2$$

$$r_1 = x_3 - x_2, \quad r_2 = x_1 - x_3, \quad r_3 = x_2 - x_1$$

★ $[D] \rightarrow$ stress strain relationship matrix. for plane stress problem

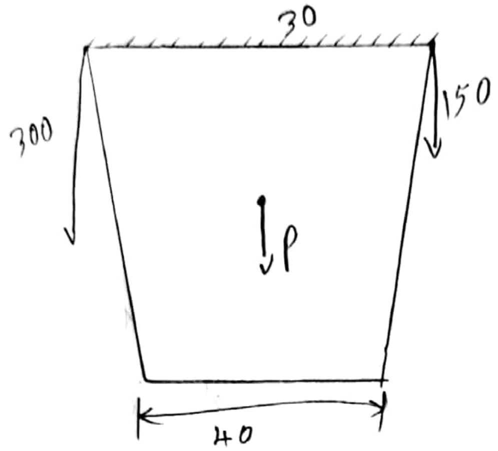
$$[D] = \frac{E}{1-2} \begin{bmatrix} 1 & v & 0 \\ v & 1 & 0-v \end{bmatrix}$$

where,

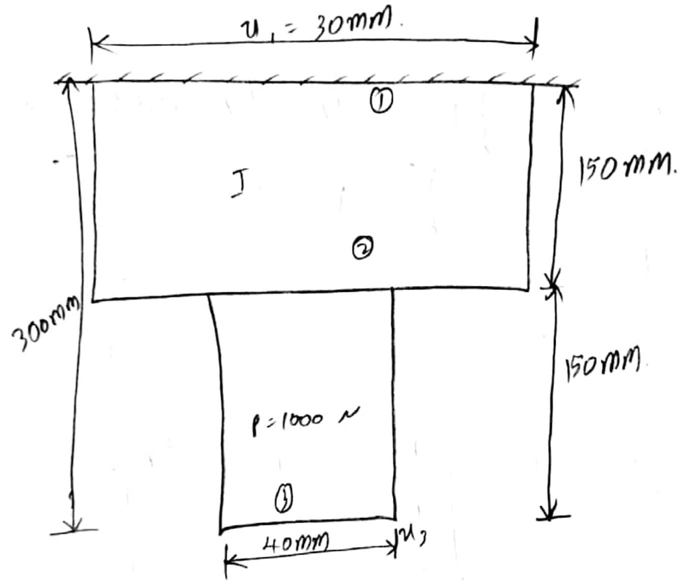
E = young's modulus

ν = poission's ratio

4.)



gn :



Thickness (t) = 10 mm, Density (ρ) = 7800 kg/m³

(ρ) = 76518 N/m³

= 76518×10^{-4} N/mm³

Load (P) = 1 KN = 1×10^3 N ; young's Modulus (E) = 2×10^5 mN/m²

= $2 \times 10^5 \times 10^6 \times 10^6$ N/mm.

E = 2×10^5 N/mm²

To find :

1) Displacement (u_1, u_2, u_3) = ?

2.) Reaction force (R_1, R_2, R_3) = ?

Soln :

* Area of node at each node points [1, 2, 3].

1.) Area (A_1) = width$_1$ × thickness $= 80 \times 10 = 800\ mm^2$

2.) Area (A_2) = width$_2$ × thickness$_2$ $= w_2 \times 10$.

To find w_2 :-

$$w_2 = \frac{w_1 + w_2}{2} = \frac{80+40}{2} = \frac{120}{2} = 60\ mm.$$

$$A_2 = 60 \times 10 = 600\ mm^2$$

3 Area $(A_3) = w_3 \times t_3 = 40 \times 10 = 400\ mm^2$

★ Area of each element (I, II) :-

Element I :- (node 1, 2).

$$A_1 = \frac{A_1 + A_2}{2} = \frac{800+600}{2} = \frac{1400}{2} = 700\ mm^2$$

Element II :- (node 2, 3) :-

$$\bar{A_2} = \frac{A_2 + A_3}{2} = \frac{600+400}{2} = \frac{1000}{2} = 500\ mm^2$$

① displacement :- $\{u\}$:-

WKT :-

$$\{F\} = [K] \cdot \{u\}.$$

To find, $\{F\} \cdot [K]$.

★ [F]

W.K.T => $\{F\} = \frac{\rho . A l}{2} \begin{Bmatrix} 1 \\ 1 \end{Bmatrix}$.

Element I :- (node (1, 2))

$$\begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix} = \frac{\rho \bar{A_1} l_1}{2} \begin{Bmatrix} 1 \\ 1 \end{Bmatrix} = \frac{76518 \times 10^{-9} \times 700 \times 150}{2} \begin{Bmatrix} 1 \\ 1 \end{Bmatrix}.$$

$$= 4017 \begin{Bmatrix} 1 \\ 1 \end{Bmatrix}$$

$$\begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix} = \begin{Bmatrix} 4.017 \\ 4.017 \end{Bmatrix} \rightarrow ①$$

Element II :- (node (2, 3)).

$$\begin{Bmatrix} F_2 \\ F_3 \end{Bmatrix} = \frac{\rho \bar{A}_2 l_2}{2} \begin{Bmatrix} 1 \\ 1 \end{Bmatrix} - \frac{76518 \times 10^{-9} \times 500 \times 150}{2} \begin{Bmatrix} 1 \\ 1 \end{Bmatrix}.$$

$$= 2869 \begin{Bmatrix} 1 \\ 1 \end{Bmatrix}$$

$$\begin{Bmatrix} F_2 \\ F_3 \end{Bmatrix} = \begin{Bmatrix} 2.869 \\ 2.869 \end{Bmatrix} \rightarrow ②$$

Assemble ① ② :-

$$\begin{Bmatrix} F_1 \\ F_2 \\ F_3 \end{Bmatrix} = \begin{Bmatrix} 4017 \\ 4.017 + 2869 \\ 2.869 \end{Bmatrix} = \begin{Bmatrix} 4.017 \\ 6.886 \\ 2.869 \end{Bmatrix} = \begin{Bmatrix} 4.017 \\ 6886 + 1000 \\ 2.869 \end{Bmatrix} = \begin{Bmatrix} 4.017 \\ 1000 + 6886 \\ 2.869 \end{Bmatrix}.$$

Add load (P) on F_2.

* [K].

W.K.T :- $\begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix} = \frac{AE}{l} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix}.$

element I (node 1, 2) :-

$$\begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix} = \frac{\bar{A}_1 E}{l_1} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} = \frac{700 \times (2 \times 10^5)}{150} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix}.$$

$$\begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix} = 9.33 \times 10^5 \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix}.$$

$$\begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix} = 1 \times 10^3 \begin{bmatrix} 9.33 & -9.33 \\ -9.33 & 9.33 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} \rightarrow ③.$$

Element - II (node 2, 3) :-

$$\begin{Bmatrix} F_2 \\ F_3 \end{Bmatrix} = \frac{\bar{A}_2 E}{l_2} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} u_2 \\ u_3 \end{Bmatrix} = \frac{500 \times (2 \times 10^5)}{150} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} u_2 \\ u_3 \end{Bmatrix}.$$

$$\begin{Bmatrix} F_2 \\ F_3 \end{Bmatrix} = 6.66 \times 10^5 \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} \begin{Bmatrix} u_2 \\ u_3 \end{Bmatrix}.$$

$$\begin{Bmatrix} F_2 \\ F_3 \end{Bmatrix} = 1\times10^5 \begin{bmatrix} 6.66 & -6.66 \\ -6.66 & 6.66 \end{bmatrix} \begin{Bmatrix} u_2 \\ u_3 \end{Bmatrix} \rightarrow (4).$$

Assemble (3) & (4),

$$\begin{Bmatrix} F_1 \\ F_2 \\ F_3 \end{Bmatrix} = 1\times10^5 \begin{bmatrix} 9.33 & 9.33 & 0 \\ -9.33 & 9.33+6.66 & -6.66 \\ 0 & -6.66 & 6.66 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \end{Bmatrix} \rightarrow (5)$$

Then to find u_1, u_2, u_3 :-

Now applying boundary conditions in (5),

1.) $u_1 = 0$, (2) $F_1 = 4.017\,N$, $F_2 = 1006.886\,N$, $F_3 = 2869\,N$

Then,

$$\begin{Bmatrix} 4.017 \\ 1006.886 \\ 2869 \end{Bmatrix} = 1\times10^3 \begin{bmatrix} 9.33 & -9.33 & 0 \\ -9.33 & 15.99 & -6.66 \\ 0 & -6.66 & 6.66 \end{bmatrix} \begin{Bmatrix} 0 \\ u_2 \\ u_3 \end{Bmatrix}$$

$$\begin{Bmatrix} 1006.886 \\ 2.869 \end{Bmatrix} = 1\times10^3 \begin{bmatrix} 15.99 & -6.66 \\ -6.66 & 6.66 \end{bmatrix} \begin{Bmatrix} u_2 \\ u_3 \end{Bmatrix}.$$

$$\begin{Bmatrix} 1006.886 \\ 2.869 \end{Bmatrix} = \begin{matrix} 1\times10^5\,(15.99\,u_2 - 6.66\,u_3) \rightarrow (6). \\ 1\times10^5\,(-6.66\,u_2 + 6.66\,u_3) \rightarrow (7). \end{matrix}$$

By using elemination method solve, (6) & (7),

$u_2 = 1.082\times10^{-3}\,mm.$

$u_3 = 1.086\times10^{-3}\,mm.$

(2) Reaction force :-

W.K.T $= \{R\} = [K]\cdot\{u\} - \{F\}.$

Then,

$$\begin{Bmatrix} R_1 \\ R_2 \\ R_3 \end{Bmatrix} = 1\times10^3 \begin{bmatrix} 9.33 & -9.33 & 0 \\ -9.33 & 15.99 & -6.66 \\ 0 & -6.66 & 6.66 \end{bmatrix} \begin{Bmatrix} 0 \\ 1.082\times10^{-3} \\ 1.086\times10^{-3} \end{Bmatrix} - \begin{Bmatrix} 4.017 \\ 1006.886 \\ 2.869 \end{Bmatrix}$$

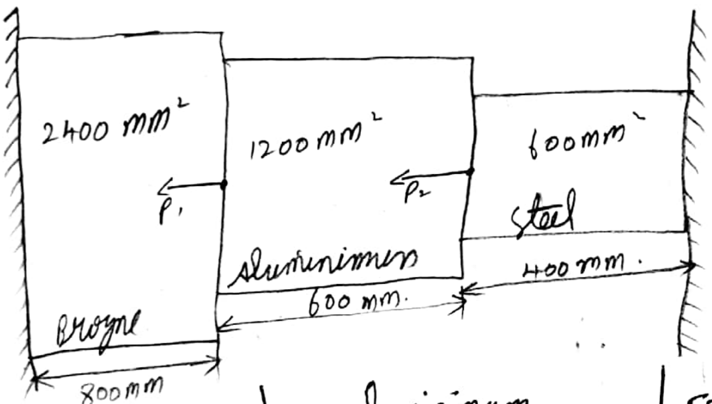
$$= \begin{bmatrix} 0 - 1009506 + 0 \\ 0 + 1694.412 - 723.276 \\ 0 - 720.612 + 723.276 \end{bmatrix} - \begin{Bmatrix} 4.017 \\ 1006.886 \\ 2.869 \end{Bmatrix}$$

$$\begin{Bmatrix} R_1 \\ R_2 \\ R_3 \end{Bmatrix} = \begin{Bmatrix} -1013.523 \\ -35.75 \\ -0.205 \end{Bmatrix}$$

Result :-

1.) $U_1 = 0$, $U_2 = 1.082 \times 10^{-3}$ mm, $U_3 = 7086 \times 10^{-3}$ mm.

2.) $R_1 = 1013.523$ N, $R_2 = 35.75$ N, $R_3 = 0.205$ N.



given data :-

For Bronze.

$A_1 = 2400\ mm^2$

$E_1 = 80 GPa = 80 \times 10^3\ N/mm^2$

$\alpha_1 = 19 \times 10^{-6} / c°$

$P_1 = -60 \times 10^3 N$

$l_1 = 800 mm$

For aluminimum

$A_2 = 1200\ mm^2$

$E_2 = 70 GPa = 70 \times 10^3\ N/mm^2$

$\alpha_2 = 23 \times 10^{-6} / c°$

$P_2 = -75 \times 10^3 N$

$l_2 = 600 mm$.

For steel,

$A_3 = 600\ mm^2$

$E_3 = 200 GPa = 200 \times 10^3\ N/mm^2$

$\alpha_3 = 18 \times 10^{-6} / c°$

$l_3 = 400 mm$.

$\Delta T = 80°C$.

To find :-

1) Element stress $(\sigma_1, \sigma_2, \sigma_3) = ?$

Solution :-

1.) stress,

W.K.T :-

$$\sigma = E \frac{\partial u}{\partial x} = E\alpha . \Delta T.$$

To find, u_1, u_2, u_3, u_4 :-

W.K.T FEA equation's.

Element - I (node 1, 2) :-

$$\begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix} = \frac{A_1 E_1}{l_1} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} = \frac{2400 \times (80 \times 10^3)}{800} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix}$$

$$= 2.4 \times 10^5 \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix}$$

$$\begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix} = 1 \times 10^5 \begin{bmatrix} 2.4 & -2.4 \\ -2.4 & 2.4 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} \rightarrow (1)$$

Element - II (node 2, 3) :-

$$\begin{Bmatrix} F_2 \\ F_3 \end{Bmatrix} = \frac{A_2 E_2}{l_2} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} u_2 \\ u_3 \end{Bmatrix} = \frac{1200 \times (70 \times 10^3)}{600} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} u_2 \\ u_3 \end{Bmatrix}$$

$$\begin{Bmatrix} F_2 \\ F_3 \end{Bmatrix} = 1.4 \times 10^5 \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} u_2 \\ u_3 \end{Bmatrix} \Rightarrow \begin{Bmatrix} F_2 \\ F_3 \end{Bmatrix} = 1 \times 10^3 \begin{bmatrix} 1.4 & -1.4 \\ -1.4 & 1.4 \end{bmatrix} \begin{Bmatrix} u_2 \\ u_3 \end{Bmatrix} \rightarrow (2)$$

Element - III (node - 3, 4) :-

$$\begin{Bmatrix} F_3 \\ F_4 \end{Bmatrix} = \frac{A_3 E_3}{l_3} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} u_3 \\ u_4 \end{Bmatrix} = \frac{600 \times (200 \times 10^3)}{400} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} u_3 \\ u_4 \end{Bmatrix}$$

$$\begin{Bmatrix} F_3 \\ F_4 \end{Bmatrix} = 1 \times 10^5 \begin{bmatrix} 3 & -3 \\ -3 & 3 \end{bmatrix} \begin{Bmatrix} u_3 \\ u_4 \end{Bmatrix} \rightarrow (3)$$

Assemble (1), (2), (3) :-

$$\begin{Bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \end{Bmatrix} = 1 \times 10^3 \begin{bmatrix} 2.4 & -2.4 & 0 & 0 \\ -2.4 & 2.4+1.4 & -1.4 & 0 \\ 0 & -1.4 & 1.4+3 & -3 \\ 0 & 0 & -3 & 3 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{Bmatrix} \rightarrow (4)$$

To find F_1, F_2, F_3, F_4 :-

For thermal co-efficient, $\{F\} = EA\alpha \cdot \Delta T \begin{Bmatrix} -1 \\ 1 \end{Bmatrix}$.

Element-I. (node 1,2) :-

$$\begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix} = E_1 A_1 \alpha_1 \Delta T \begin{Bmatrix} -1 \\ 1 \end{Bmatrix} = (80\times10^3) \times 2400 \times 19 \times 10^{-6} \times 80 \begin{Bmatrix} -1 \\ 1 \end{Bmatrix}$$

$$= 2.91 \times 10^5 \begin{Bmatrix} -1 \\ 1 \end{Bmatrix}.$$

$$\begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix} = 1\times10^5 \begin{Bmatrix} -2.91 \\ 2.91 \end{Bmatrix} \rightarrow (5)$$

Element-II (node 2,3) :-

$$\begin{Bmatrix} F_2 \\ F_3 \end{Bmatrix} = E_2 A_2 \alpha_2 \Delta T \begin{Bmatrix} -1 \\ 1 \end{Bmatrix} = (70\times10^3)\times1000 \times (23\times10^{-6}) \times 80) \begin{Bmatrix} -1 \\ 1 \end{Bmatrix}.$$

$$= 1.54 \times 10^5 \begin{Bmatrix} -1 \\ 1 \end{Bmatrix}.$$

$$\begin{Bmatrix} F_2 \\ F_3 \end{Bmatrix} = 1\times10^5 \begin{Bmatrix} -1.54 \\ 1.54 \end{Bmatrix} \rightarrow (6)$$

Element-III (node 3,4) :-

$$\begin{Bmatrix} F_3 \\ F_4 \end{Bmatrix} = E_3 A_3 \alpha_3 \Delta T \begin{Bmatrix} -1 \\ 1 \end{Bmatrix} = (200\times10^3)\times600\times(18\times10^{-6})\times \begin{Bmatrix} -1 \\ 1 \end{Bmatrix}.$$

$$= 1.72 \times 10^5 \begin{Bmatrix} -1 \\ 1 \end{Bmatrix}.$$

$$\begin{Bmatrix} F_3 \\ F_4 \end{Bmatrix} = 1\times10^3 \begin{Bmatrix} -1.72 \\ 1.72 \end{Bmatrix}. \rightarrow (7)$$

Assemble (5), (6) & (7) :-

$$\begin{Bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \end{Bmatrix} = 1\times10^3 \begin{Bmatrix} -2.91 \\ 2.91-1.54 \\ 1.54-1.72 \\ 1.72 \end{Bmatrix} \Rightarrow \begin{Bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \end{Bmatrix} = 1\times10^3 \begin{Bmatrix} -2.91 \\ 1.37 \\ -0.18 \\ 1.72 \end{Bmatrix} \rightarrow (8)$$

sub. (8) in (4) and apply boundary conditions are,

1.) $u_1 = 0$; $u_4 = 0$ (2) $F_2 = 0.60\times10^5 N$, $F_3 = 0.79\times10^5 N$.

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Then,

$$1\times10^5\begin{Bmatrix}-2.91\\1.37-0.60\\-0.18-0.75\\1.72\end{Bmatrix} = 1\times10^3\begin{bmatrix}2.4 & -2.4 & 0 & 0\\-2.4 & 3.8 & -1.4 & 0\\0 & -1.4 & 4.4 & -3\\0 & 0 & -3 & 3\end{bmatrix}\begin{Bmatrix}0\\u_2\\u_3\\0\end{Bmatrix}$$

$$\begin{Bmatrix}0.77\\-0.93\end{Bmatrix} = \begin{Bmatrix}3.8 & -1.4\\-1.4 & 4.4\end{Bmatrix}\cdot\begin{Bmatrix}u_2\\u_3\end{Bmatrix}.$$

$$0.77 = 3.8u_2 - 1.4u_3 \rightarrow (9)$$

$$-0.93 = -1.4u_2 + 4.4u_3 \rightarrow (10).$$

By using elimination method solve (9), (10)

$$u_2 = 0.141\ mm \quad , \quad u_3 = 0.166\ mm.$$

Then, stress.

For element - I

$$\sigma_1 = E_1\frac{(u_2-u_1)}{l} - E_1\alpha_1\Delta T = 80\times10^3\frac{(0.141-0)}{800} - \left((180\times10^3)\times(19\times10^{-6})\times80\right)$$

$\sigma_1 = -107.5\ N/mm^2$ [Compressive stress].

For element - II

$$\sigma_2 = \frac{E_2(u_3-u_2)}{l_2} - E_2\alpha_2\Delta T = 70\times10^3\frac{(-0.166-0.141)}{600} - \left((70\times10^3)\times(23\times10^{-6})\times80\right)$$

$$\sigma_2 = -164.6\ N/mm^2$$

For element - III

$$\sigma_3 = \frac{E_3(u_4-u_3)}{l_3} - E_3\alpha_3\Delta T = \frac{2\times10^5(0+0.166)}{400}\left((12\times10^5)\times(18\times10^{-6})\times80\right).$$

$$\sigma_3 = -205\ N/mm^2.$$

Result.

stress,

$\sigma_1 = -101.3\ N/mm^2$, $\sigma_2 = -164.6\ N/mm^2$; $\sigma_3 = 205\ N/mm^2$

---

b) given data :-

$u_1 = 2\ mm$ $\quad v_1 = 1\ mm.$

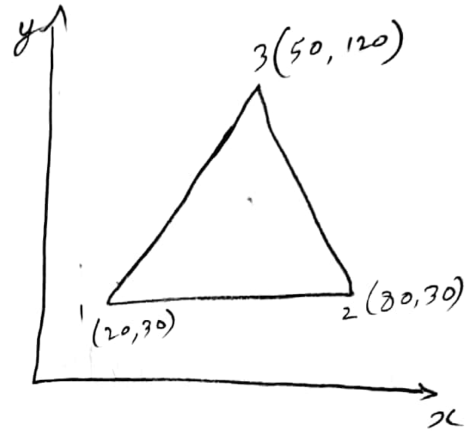
$u_2 = 0.5\ mm.$ $\quad v_2 = 0\ mm.$

$u_3 = 3\ mm$ $\quad v_3 = 1\ mm.$

$x_1 = 20\ mm.$ $\quad y_1 = 30\ mm.$

$x_2 = 80\ mm.$ $\quad y_2 = 30\ mm.$

$x_3 = 50\ mm.$ $\quad y_3 = 120\ mm.$



youngs modulus $E = 210\ GPa = 210 \times 10^9\ Pa = 210 \times 10^9\ N/m^2$

$E = 2.1 \times 10^5\ N/mm^2$

$u = 0.25$

$E = 10\ mm.$

To find,

1) $\sigma_x, \sigma_y, \tau_{xy}$ | ② θ_p ③ $\sigma_1, \sigma_2.$

solution :-

1) $\sigma_x, \sigma_y, \tau_{xy}.$

$$\sigma = [D]\ [B]\ [C]$$

$$\sigma = [D]\ [B] \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \\ v_1 \\ v_2 \\ v_3 \end{Bmatrix}$$

To find :- [D] [B]

[D] : condition is plane stress element.

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$$[D] = \frac{E}{1-\nu^2}\begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix} = \frac{2.1\times10^5}{1-(0.25)^2}\begin{bmatrix} 1 & 0.25 & 0 \\ 0.25 & 1 & 0 \\ 0 & 0 & \frac{1-0.25}{2} \end{bmatrix}$$

$$= \frac{2.1\times10^5}{0.9375}\begin{bmatrix} 1 & 0.25 & 0 \\ 0.25 & 1 & 0 \\ 0 & 0 & 0.375 \end{bmatrix}$$

$$[D] = 56\times10^3\begin{bmatrix} 4 & 1 & 0 \\ 1 & 4 & 0 \\ 0 & 0 & 1.5 \end{bmatrix}$$

$[B]$, W.K.T ;-

$$[B] = \frac{1}{2A}\begin{bmatrix} q_1 & 0 & q_2 & 0 & q_3 & 0 \\ 0 & r_1 & 0 & r_2 & 0 & r_3 \\ r_1 & q_1 & r_2 & q_2 & r_3 & q_3 \end{bmatrix}$$

where,

$q_1 = y_2 - y_3 = 30 - 120 \Rightarrow -90$, $q_2 = y_3 - y_1 = 120 - 30 = 90$

$q_3 = y_1 - y_2 \Rightarrow 0$

$r_1 = x_3 - x_2 = 50 - 80 \Rightarrow -30$, $r_2 = x_1 - x_3 = 20 - 50 \Rightarrow -30$

$r_3 = x_2 - x_1 \Rightarrow 60$

Then,

$$A = \frac{1}{2}\begin{vmatrix} 1 & x_1 & y_1 \\ 1 & x_2 & y_2 \\ 1 & x_3 & y_3 \end{vmatrix} \Rightarrow \frac{1}{2}\begin{vmatrix} 1 & 20 & 30 \\ 1 & 80 & 30 \\ 1 & 50 & 120 \end{vmatrix} \Rightarrow \frac{1}{2}\left[1(9600-1500) - 10(120-30) + 30(50-80)\right]$$

$$\boxed{A = 2700\ mm^2}$$

Then,

$$[B] = \frac{1}{2 \times 2700}\begin{bmatrix} -90 & 0 & 90 & 0 & 0 & 0 \\ 0 & -30 & 0 & -30 & 0 & 60 \\ -30 & -90 & -30 & 90 & 60 & 0 \end{bmatrix}$$

$$= \frac{1 \times 30}{2 \times 2700}\begin{bmatrix} -3 & 0 & 3 & 0 & 0 & 0 \\ 0 & -1 & 0 & -1 & 0 & 2 \\ -1 & -3 & -1 & 3 & 2 & 0 \end{bmatrix}$$

$$[B] = 5.55 \times 10^{-3}\begin{bmatrix} -3 & 0 & 3 & 0 & 0 & 0 \\ 0 & -1 & 0 & -1 & 0 & 2 \\ -1 & -3 & -1 & 3 & 2 & 0 \end{bmatrix}$$

Then,

$$[D][B] = 0.56 \times 10^{5}\begin{bmatrix} 4 & 1 & 0 \\ 1 & 4 & 0 \\ 0 & 0 & 1.5 \end{bmatrix} \cdot 5.55 \times 10^{-3}\begin{bmatrix} -3 & 0 & 3 & 0 & 0 & 0 \\ 0 & -1 & 0 & -1 & 0 & 2 \\ -1 & -3 & -1 & 3 & 2 & 0 \end{bmatrix}$$

$$= 310.8\begin{bmatrix} -12 & -1 & 12 & -1 & 0 & 2 \\ -3 & -4 & 3 & -4 & 0 & 8 \\ -1.5 & -4.5 & -1.5 & 4.5 & 3 & 0 \end{bmatrix}$$

Then,

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = 310.8\begin{bmatrix} -12 & -1 & 12 & -1 & 0 & 2 \\ -3 & -4 & 3 & -4 & 0 & 8 \\ -1.5 & -4.5 & -1.5 & 4.5 & 3 & 0 \end{bmatrix}\begin{Bmatrix} 2 \\ 1 \\ 0.5 \\ 0 \\ 3 \\ 1 \end{Bmatrix}$$

$$= 310.8\begin{Bmatrix} -24 - 1 + 6 + 0 + 0 + 2 \\ -6 - 4 + 1.5 + 0 + 0 + 8 \\ -3 - 4.5 - 0.75 + 0 + 9 + 0 \end{Bmatrix} \Rightarrow 310.8\begin{Bmatrix} -17 \\ -0.5 \\ 0.75 \end{Bmatrix}$$

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_{xy} \end{Bmatrix} = \begin{Bmatrix} -5283.6 \\ -155.4 \\ 233.1 \end{Bmatrix}$$

where, $\sigma_x = -5283.6\ N/mm^2$

$\sigma_y = 155.4\ N/mm^2$

$\tau_{xy} = 233.1\ N/mm^2$.

2.) σ_1, σ_2.

W.K.T :-

$$\star\ \sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + (\tau_{xy})^2} = \frac{-5283.6 + 155.4}{2} + \sqrt{\left(\frac{-5283.6 + 135.4}{2}\right)^2 + (233.1)^2}$$

$$\sigma_1 = -144.826\ N/mm^2$$

$$\star\ \sigma_2 = \frac{\sigma_x + \sigma_y}{2} - \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + (\tau_{xy})^2} = \frac{-5283.6 - 155.4}{2} + \sqrt{\left(\frac{-5283.6 + 155.4}{2}\right)^2 + (233.1)^2}$$

$$\sigma_2 = -5294.17\ N/mm^2$$

3.) θ_p :-

W.K.T :- $\tan 2\theta_p = \dfrac{2\tau_{xy}}{\sigma_x - \sigma_y}$

$$2\theta_p = \tan^{-1}\left(\frac{2 + 233.1}{-5283.6 + 155.4}\right)$$

$$\boxed{\theta_p = -2.59^\circ}$$

Result :-

1.) $\sigma_x = -5287.6\ N/mm^2$

2.) $\sigma_y = -155.4\ N/mm^2$

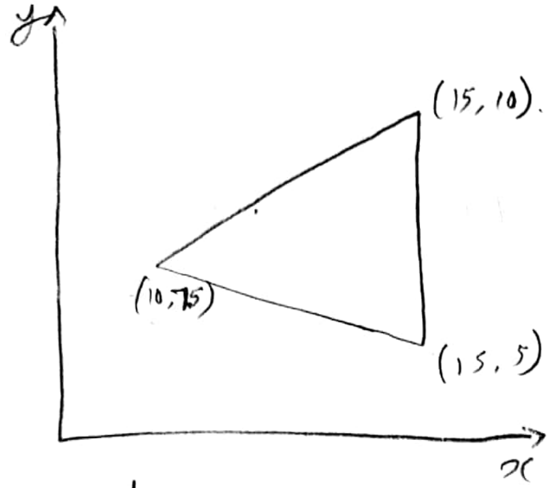
3.) $\tau_{xy} = -233.1\ N/mm^2$

2.) $\sigma_1 = -144.826\ N/mm^2$

$\sigma_2 = -5294.17\ N/mm^2$

3.) $\theta_p = -2.59^\circ$

7.)



Nodel points :-

$x_1 = 10, \ y_1 = 7.5$

$x_2 = 15, \ y_2 = 5$

$x_3 = 15, \ y_3 = 10$

$E = 2.1 \times 10^5 \ N/mm^2, \ \vartheta = 0.25$

$t = 10$ mm, condition plane stress.

To find :-

1.) stiffness matrix $[K] = ?$

solution :-

1.) $[K]$,

W.K.T :-

$$[K] = [B]^T . [D] . [B] . A t$$

To find, $[D] . [B] . [B]^T$

$\therefore$ $[D]$ conditions are plane stress.

$$[D] = \frac{E}{1-\nu} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix} \Rightarrow \frac{2.1 \times 10^5}{1-(0.25)^2} \begin{bmatrix} 1 & 0.25 & 0 \\ 0.25 & 1 & 0 \\ 0 & 0 & \frac{1-0.25}{2} \end{bmatrix}$$

$$[D] = 2.24 \times 10^5 \begin{bmatrix} 1 & 0.25 & 0 \\ 0.25 & 1 & 0 \\ 0 & 0 & 0.375 \end{bmatrix} \Rightarrow [D] = 0.56 \times 10^5 \begin{bmatrix} 4 & 1 & 0 \\ 1 & 4 & 0 \\ 0 & 0 & 1.5 \end{bmatrix}$$

W.K.T :-

$$[B] = \frac{1}{2A} \begin{bmatrix} q_1 & 0 & q_2 & 0 & q_3 & 0 \\ 0 & r_1 & 0 & r_2 & 0 & r_3 \\ r_1 & q_1 & r_2 & q_2 & r_3 & q_3 \end{bmatrix}$$

where,

$q_1 = y_2 - y_3 = 5 - 10 \Rightarrow -5$, $q_2 = y_3 - y_1 = 10 - 7.5 \Rightarrow 2.5$

$q_3 = y_1 - y_2 = 7.5 - 5 \Rightarrow 2.5$, $r_1 = x_3 - x_2 = 15 - 15 = 0$

$r_2 = x_1 - x_3 = 10 - 15 \Rightarrow -5$, $r_3 = x_2 - x_1 = 15 - 10 = -5$.

$$A = \frac{1}{2} \begin{bmatrix} 1 & x_1 & y_1 \\ 1 & x_2 & y_2 \\ 1 & x_3 & y_3 \end{bmatrix} \Rightarrow \frac{1}{2} \begin{vmatrix} 1 & 10 & 7.5 \\ 1 & 15 & 5 \\ 1 & 15 & 10 \end{vmatrix}$$

$$\Rightarrow \frac{1}{2} \left[1(150 - 75) - 10(10 - 5) + 7.5(15 - 15) \right]$$

$$A = \frac{1}{2} \left[\cancel{75}\,75 - 50 + 0 \right]$$

$$A = \frac{1}{2} \times 25$$

$$\boxed{A = 12.5\, mm^2}$$

Then,

$$[B] = \frac{1}{25} \begin{bmatrix} -5 & 0 & 2.5 & 0 & 2.5 & 0 \\ 0 & 0 & 0 & -5 & 0 & 5 \\ 0 & -5 & -5 & 2.5 & 5 & 25 \end{bmatrix}$$

$$[B] = 0.1 \begin{bmatrix} -2 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & -2 & 0 & 2 \\ 0 & -2 & -2 & 1 & 2 & 1 \end{bmatrix}$$

$$[B]^T = 0.1 \begin{bmatrix} -2 & 0 & 0 \\ 0 & 0 & -2 \\ 1 & 0 & -2 \\ 0 & -2 & 1 \\ 0 & 2 & 1 \end{bmatrix}$$

$$[D][B] = 0.56 \times 10^5 \begin{bmatrix} 4 & 1 & 0 \\ 1 & 4 & 0 \\ 0 & 0 & 1.5 \end{bmatrix} \times 0.1 \begin{bmatrix} -2 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 2 & 0 & 2 \\ 0 & -2 & -2 & 1 & 2 & 1 \end{bmatrix}$$

$$[D][B] = 5600 \begin{bmatrix} -8 & 0 & 4 & -2 & 4 & 2 \\ -2 & 0 & 1 & -5 & 1 & 5 \\ 0 & -3 & -3 & 1.5 & 3 & 1.5 \end{bmatrix}$$

$$[B]^T . [D] . [B] = 0.1 \begin{bmatrix} -2 & 0 & 3 \\ 0 & 0 & -2 \\ 1 & 0 & -2 \\ 0 & -2 & 1 \\ 1 & 0 & 2 \\ 0 & 2 & 1 \end{bmatrix} . 5600 \begin{bmatrix} -8 & 0 & 4 & -2 & 4 & 2 \\ -2 & 0 & 1 & -8 & 1 & 8 \\ 0 & -3 & -3 & 1.5 & 3 & 1.5 \end{bmatrix}$$

$$[B]^T . [D] . [B] = 560 \begin{bmatrix} 16 & 0 & -8 & 4 & -8 & -4 \\ 0 & +6 & 6 & -3 & -6 & 3 \\ -8 & 6 & 10 & -1 & -2 & -1 \\ 4 & -3 & -5 & 17.5 & 1 & -14.5 \\ -8 & -6 & -2 & 1 & 10 & 5 \\ -4 & -3 & -1 & -14.5 & 5 & 17.5 \end{bmatrix}$$

Then,

$$[K] = 560 \begin{bmatrix} 16 & 0 & -8 & 4 & -8 & -4 \\ 0 & 6 & 6 & -3 & -6 & 3 \\ -8 & 6 & 10 & -1 & -2 & -1 \\ 4 & -3 & -5 & 17.5 & 1 & -14.5 \\ -8 & -6 & -2 & 1 & 10 & 5 \\ -4 & -3 & -1 & -14.5 & 5 & 17.5 \end{bmatrix} \times 12.5 \times 10$$

$$[K] = 7 \times 10^4 \begin{bmatrix} 16 & 0 & -5 & 4 & -5 & -4 \\ 0 & 6 & 6 & -3 & -6 & 3 \\ -8 & 6 & 10 & -1 & -2 & -1 \\ 4 & -3 & -5 & 17.5 & 1 & -14.5 \\ -8 & -6 & -2 & 1 & 10 & 5 \\ -4 & -3 & -1 & -14.5 & 5 & 17.5 \end{bmatrix}$$

Result :-

1.) Stiffness matrix [K] :-

$$[K] = 7 \times 10^4 \begin{bmatrix} 16 & 0 & -8 & 4 & -8 & -4 \\ 0 & 6 & 6 & -3 & -6 & 3 \\ -8 & 6 & 10 & -1 & -2 & -1 \\ 4 & -3 & -5 & 17.5 & 1 & -14.3 \\ -8 & -6 & -2 & 1 & 10 & 5 \\ -4 & -3 & -1 & -14.5 & 5 & 17.5 \end{bmatrix}.$$