

CAPACITY OF UNDERWATER WIRELESS COMMUNICATION CHANNEL WITH DIFFERENT ACOUSTIC PROPAGATION LOSS MODELS

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ABSTRACT

In this paper, we calculate the capacity of a point-to-point communication link in an underwater acoustic channel. The analysis takes into account the effects of various acoustic propagation loss models. A physical model of ambient noise power spectral density is also considered. We perform a comparative assessment of the influence of various acoustic transmission loss models on the acoustic bandwidth and the capacity.

KEYWORDS

Channel Capacity, Optimal Bandwidth, Point-to-Point Communication, Underwater Acoustic Channel,

1. INTRODUCTION

Underwater (UW) acoustic networks are generally formed by acoustically connected ocean bottom sensor nodes, autonomous UW vehicles, and surface stations that serve as gateways and provide radio communication links to on-shore stations [1]. UW acoustic sensor networks consist of sensors and vehicles deployed underwater and networked via acoustic links to perform collaborative monitoring tasks. However, the acoustic channels impose many constraints that affect the design of UW communication systems. These are characterized by a path loss that depends on both the transmission distance and the signal frequency. The signal frequency determines the absorption loss, which increases with distance as well [2, 3], eventually imposing a limit on the available bandwidth.

The Shannon capacity of a channel represents the theoretical upper bound for the maximum rate of data transmission at an arbitrarily small bit error rate, and is given by the mutual information of the channel maximized over all possible source distributions. The capacity of a time invariant additive white Gaussian noise (AWGN) channel with bandwidth B and SNR γ is

$C(\gamma) = B \log_2(1 + \gamma)$ where the capacity achieving source distribution is Gaussian [5]. Authors of [10] present numerical solution for the capacity of a very simple UW acoustic channel without considering its frequency and distance dependant attenuation characteristics. In [11], experimental results on channel capacity for a shallow water wave-guide are presented. The capacity analysis of UW acoustic OFDM based cellular network is presented in [12]. In [13], author presents the UW capacity based on an acoustic path loss model and investigates the capacity distance relation. In this paper, the results of [13] are extended. Based on the statistical and empirical acoustic path loss models available in the literature, we calculate the capacity of a UW point-to-point link. We analyze the effects of different propagation phenomena such as surface reflection, surface duct, bottom bounce, and other effects such as acoustic absorption and spreading on the capacity.

2. ACOUSTIC TRANSMISSION LOSS AND AMBIENT NOISE

In this section, analytical models for UW propagation loss and ambient noise are introduced. Acoustic transmission loss (TL) is the accumulated decrease in acoustic intensity as the sound travels from the source to the receiver.

2.1 Absorption and Spreading Loss

Acoustic path loss depends on the signal frequency and distance. This dependence is a consequence of absorption (i.e., transfer of acoustic energy into heat). In addition, signal experiences a spreading loss, which increases with distance. Spreading loss refers to the energy distributed over an increasingly larger area due to the regular weakening of a sound signal as it spreads outwards from the source. The overall transmission loss that occurs in UW channel over a transmission distance of l meters at a signal frequency f is given by [2]:

$$TL = k.10\log l + l.10\log a(f) \quad (1)$$

where k is spreading factor ($k = 2$ for spherical spreading, $k = 1$ for cylindrical spreading, and $k = 1.5$ for the so-called practical spreading). In general, for shallow water channels, cylindrical spreading is assumed ($k = 1$) while for deep water channels spherical spreading is assumed ($k = 2$). Now $10\log a(f)$ is the absorption coefficient expressed using Thorp's formula, which gives $a(f)$ in dB/km for f in kHz as follows [2]:

$$10\log a(f) = 0.11 \frac{f^2}{1+f^2} + 44 \frac{f^2}{4100+f^2} + 2.75 \cdot 10^{-4} f^2 + 0.003$$

(2)

The absorption coefficient increases rapidly with frequency, and is a major factor that limits the maximal usable frequency for an acoustic link of a given distance. The transmission loss due to absorption and spreading (we refer this case as model 1) is shown in Figure 1 for $k = 1.5$. The loss increases rapidly with frequency and distance, imposing a limit on the available acoustic bandwidth.

2.2 Loss Due to Sound Propagation Characteristics

Sound propagates in the sea through many different paths, which depend upon the sound-speed structure in the water as well as the source and receiver locations. Further, multipath propagation is affected by depth, frequency and transmission range. In the next sub-sections, we present the transmission loss expressions corresponding to three basic propagation paths between a source-receiver pair: surface reflection, surface duct, and bottom bounce.

2.2.1 Surface Reflection

Surface reflection describes the reflection of sound from the sea surface, and is affected by the roughness or smoothness of the sea. When the sea is rough, the transmission loss on reflection can be found using the Beckmann-Spizzichino surface reflection model [2]:

$$TL_{SR} = 10\log \left(\frac{1 + \left(\frac{f}{f_1} \right)^2}{1 + \left(\frac{f}{f_2} \right)^2} \right) - \left(1 + \frac{(90-w)}{60} \right) \left(\frac{\theta}{30} \right)^2 \quad (3)$$

where $f_1 = \sqrt{10}f_2$ and $f_2 = 378w^{-2}$, where w is the wind speed in knots, and θ is the angle of incidence to the horizontal measured in degrees. The total acoustic path loss is computed (we refer this case as model 2) using Eqn. (4) below and is shown in Figure 1 ($w = 0$ m/sec, $\theta = 50^\circ$).

$$TL = k \cdot 10\log l + l \cdot 10\log a(f) + TL_{SR} \quad (4)$$

2.2.2 Surface duct

In a surface duct, sound propagates to long ranges by successive reflections from the sea surface along ray paths that are long arcs of circles and the corresponding transmission loss,

including loss due to absorption and spreading is given as follows (we refer this case as model 3) [2]:

$$TL = k.10\log l + l.(10\log a(f) + \alpha_L) \quad (5)$$

where H is the layer depth in meters and $\alpha_L = \frac{26.6f(1.4)^S}{[(1452 + 3.5t)H]^{1/2}}$. Here S stands for the sea state number, and t is the temperature. The resulting transmission loss is plotted in Figure 1 (assumed parameters are $S = 0$, $H = 91$ meters, and $t = 22^\circ C$).

2.2.3 Bottom bounce

This corresponds to the reflection of sound from the sea floor. The reflection loss of sound incident at a grazing angle θ_1 to a plain boundary between two fluids of density ρ_1 and ρ_2 and of sound velocity c_1 and c_2 is given by the ratio of intensity of the reflected wave I_r related to the intensity of the incident wave I_i [2]:

$$TL_{bottom} = 10\log \left[\frac{I_r}{I_i} \right] = 10\log \left(\left[\frac{m \sin \theta - (n^2 - \cos^2 \theta)^{1/2}}{m \sin \theta + (n^2 - \cos^2 \theta)^{1/2}} \right]^2 \right) \quad (6)$$

where $m = \rho_2 / \rho_1$ and $n = c_1 / c_2$. The attenuation coefficient α_s due to the presence of sediments at the sea floor is $\alpha_s = \beta f^v$ where v is an empirical constant (typically 1 for many measurements on sands and clays) and β (dB/m-kHz) depends upon porosity and is approximately equal to 0.5. The total transmission loss is computed as (we refer this case as model 4) [2]:

$$TL = (k.10\log l) + (l.10\log a(f)) + (\alpha_s l) + |TL_{bottom}| \quad (7)$$

The attenuation corresponding to this loss model is also shown in Figure 2 (parameters assumed are: $m = 1.95$, $n = 0.86$, $\theta = 35^\circ$, $\beta = 0.5$, and $v = 1$). The graph corresponding to model 5 in Figure 1 considers the combined effect of loss models 1-4.

2.3 Ambient Noise

The ambient noise in ocean is modeled using four sources: turbulence, shipping, waves, and thermal noise. Most of the ambient noise sources can be described by Gaussian statistics and a continuous power spectral density (PSD). The following empirical formula gives the PSD of the four noise components in dB re μ Pa per Hz as a function of frequency in kHz [2]:

$$\begin{aligned}
10\log N_t(f) &= 17 - (30\log f) \\
10\log N_s(f) &= 40 + 20(s - 0.5) + 26\log f - 60\log(f + 0.03) \\
10\log N_w(f) &= 50 + 7.5w^{1/2} + 20\log f - 40\log(f + 0.4) \\
10\log N_{th}(f) &= -15 + 20\log f
\end{aligned}
\tag{8}$$

where w is the wind speed in m/s and s is the shipping activity factor. Figure 2 shows the overall PSD calculated as $N(f) = N_t(f) + N_s(f) + N_w(f) + N_{th}(f)$, The PSD decays with frequency.

2.4 Underwater Signal-to-Noise Ratio

Since the transmission loss in a UW channel depends both on frequency as well as the transmission distance, let it be represented by $A(l, f)$. Using $A(l, f)$ and noise PSD $N(f)$, the signal to noise ratio (SNR) at the receiver at a distance l and frequency f for a transmitted power of P and receiver noise bandwidth Δf is given by $\gamma(l, f) = \frac{P / A(l, f)}{N(f) \Delta f}$. Considering

absorption and spreading loss alone, the frequency dependent factor in the SNR $1/[A(l, f)N(f)]$ is plotted in Figure 3 for different propagation loss models. It may be noted that the optimum transmission band depends on link distance. Further, for each l , there exists an optimal frequency $f_0(l)$ for which maximum SNR is obtained. This is the frequency for which the term $1/[A(l, f)N(f)]$ becomes maximum [13]. The optimal frequency is shown in Figure 4 for various loss models.

3. UNDERWATER CHANNEL CAPACITY

In this section, we rely on the UW capacity model given in [13]. The channel is assumed to be time invariant for some interval of time and the ambient noise is assumed to be Gaussian. Two definitions are used for the capacity: the 3dB acoustic bandwidth and the optimal bandwidth.

3.1 Capacity Based on 3dB Bandwidth

The acoustic 3dB bandwidth $B_3(l)$ is the range of frequencies around $f_0(l)$ for which $\gamma(l, f) \succ \gamma(l, f_0(l))/2$. We choose the transmission bandwidth to be equal to $B_3(l)$. The transmitted signal power spectral density (PSD) $S_l(f)$ is assumed to be flat over the transmission

bandwidth, i.e., $S_l(f) = S_l$ for $f \in B_3(l)$ and 0 elsewhere. The total transmission power is then $P_3(l) = S_l B_3(l)$. The corresponding capacity expression is given as [13]

$$C_3(l) = \int_{B_3(l)} \log_2 \left(1 + \frac{P_3(l)/B_3(l)}{A(l, f)N(f)} \right) df \quad (9)$$

where $P_3(l)$ is the minimum transmission power required to ensure that the received SNR is equal to a target value γ_0 and is computed as

$$P_3(l) = \gamma_0 B_3(l) \frac{\int_{B_3(l)} N(f) df}{\int_{B_3(l)} A^{-1}(l, f) df}$$

(10)

3.2 Capacity Based on Optimal Bandwidth

In this section, we consider the computation of capacity based on the notion of an optimal bandwidth [13]. A case in which the transmitted signal PSD $S_l(f)$ is adjusted in accordance with the given channel and noise characteristics was analyzed in [13]. This adjustment is equivalent to allocating power through ‘water pouring’. In the absence of multipath and channel fading, the optimal capacity of a point-to-point link is given by [13]

$$C(l) = \int_{B(l)} \log_2 \left(\frac{K_l}{A(l, f)N(f)} \right) df$$

(11)

where $B(l)$ is the optimum band of operation and K_l is a constant. Here $B(l)$ is the frequency range over which $A(l, f)N(f) \leq K_l$ and $S_l(f) \geq 0$. The corresponding transmitted power is given by $P(l) = \int_{B(l)} S_l(f) df$ where the signal PSD should satisfy the water filling principle

$$S_l(f) = K_l - A(l, f)N(f), f \in B(l)$$

(12)

The transmission power $P(l)$ is selected as the minimum power required such that the received SNR equals a target value γ_0 and is computed as

$$P(l) = K_l B(l) - \int_{B(l)} A(l, f) N(f) df$$

(13)

The optimal PSD is then determined through the numerical algorithm in [13].

4. NUMERICAL RESULTS

The numerical results for the capacity and the bandwidth are obtained using MATLAB. The parameters used are wind speed $w = 0$ m/s, moderate shipping activity $s = 0.5$, and spreading factor $k = 1.5$. The SNR threshold is set to $\gamma_0 = 20$ dB. Figures 5 & 6 respectively show the bandwidth and the capacity versus distance based on 3dB bandwidth definition. The resulting bandwidth efficiency is 6.65bps/Hz. Table 1 shows the comparison of capacity and bandwidth for different loss models. It may be noted that both the capacity and the bandwidth decreases drastically as the transmission distance increases. Assuming absorption and spreading alone, channel capacity is almost equal to 27.3kbps for $l = 40$ km while for the combined loss model 5, the capacity is 1.33kbps which is equivalent to almost 95% reduction in capacity.

For the case of optimal bandwidth, the transmitted signal PSD for each distance and for the desired threshold SNR γ_0 is determined using the numerical algorithm mentioned earlier. Figures 7 & 8 respectively show the bandwidth, and the capacity obtained based on the notion of optimal bandwidth. The resulting bandwidth efficiency is approximately equal to 8.25bps/Hz and is listed in Table 1 ($l = 40$ km). The capacity improves by approximately 178% as compared to that achievable based on 3 dB bandwidth definition. The numerical results also reveal that for all the loss models described in this paper, both bandwidth and capacity decays almost linearly with distance on a logarithmic scale.

5. CONCLUSION

In this paper, numerical results for the capacity of time invariant UW point-to-point link were presented, considering the effects of various acoustic path loss models and a specific model of ambient noise PSD. The path loss corresponding to different acoustic propagation phenomena such as surface reflection, surface ducts, and bottom bounce, were considered for the capacity calculation. A comparative assessment of the influence of these loss models on the capacity and achievable bandwidth were presented.

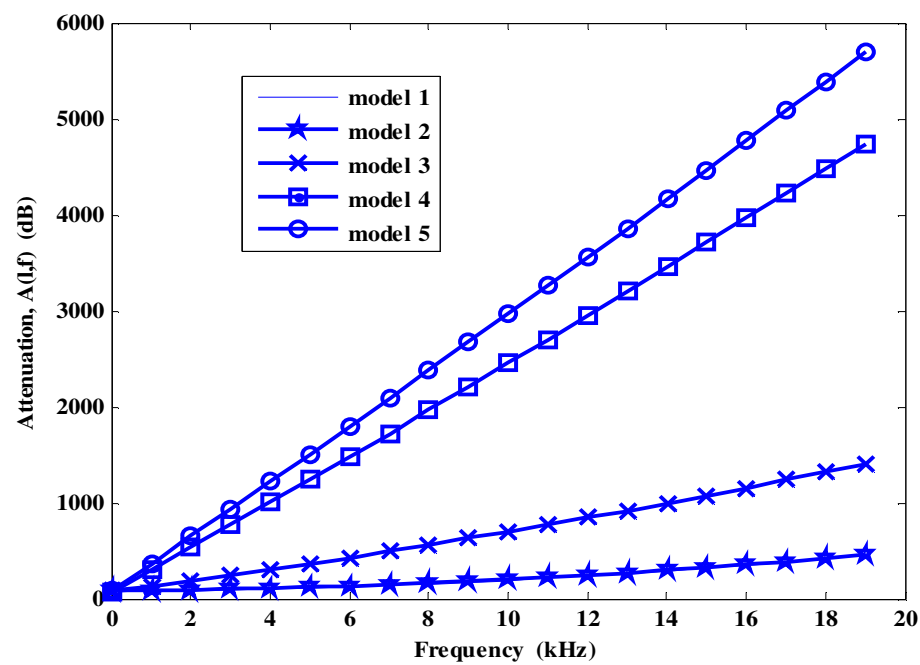


Fig. 1. Attenuation for different propagation loss models ($l = 100km$)

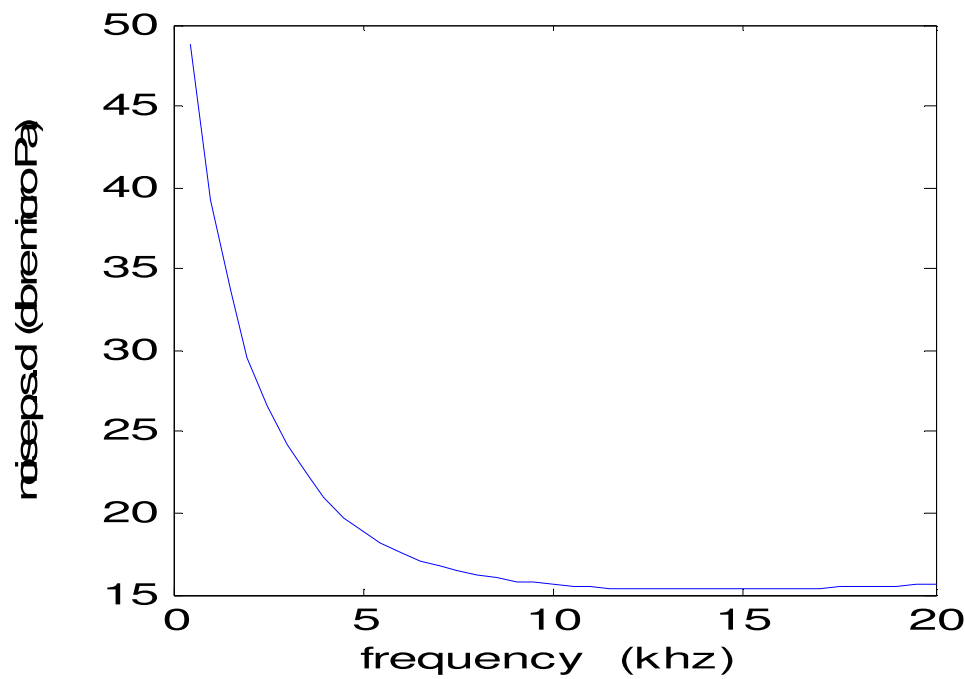


Fig. 2. Ambient Noise PSD

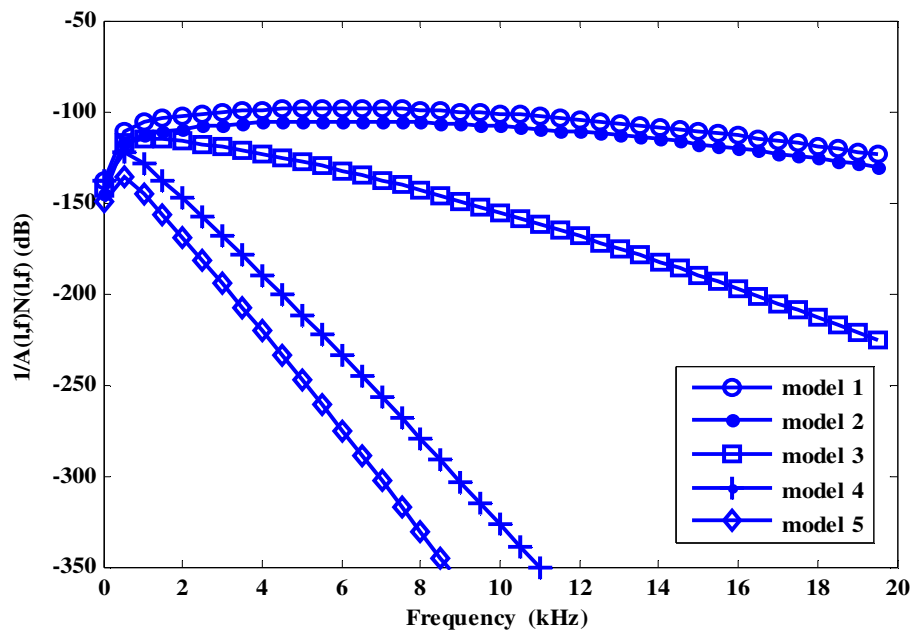


Fig. 3. $1/A(l,f)N(f)$ for different loss models

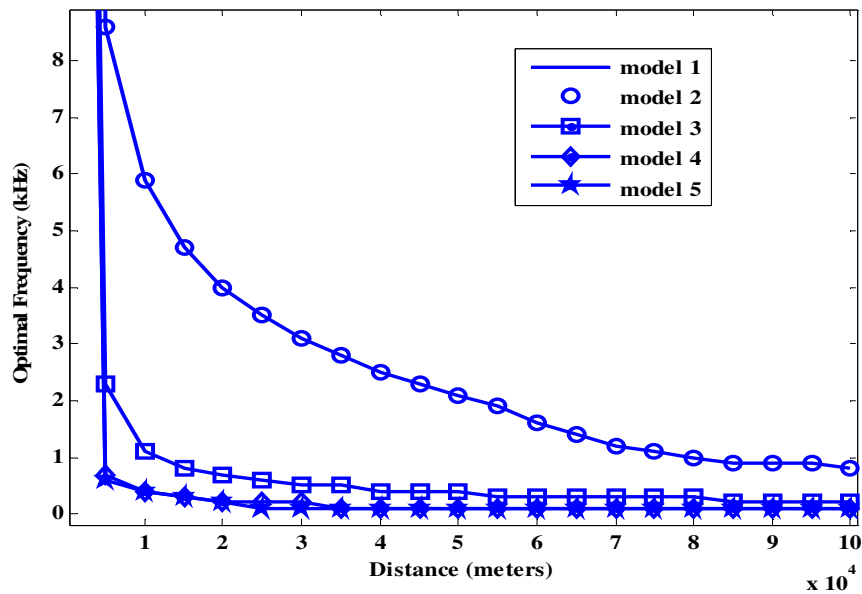


Fig. 4. Optimal frequency $f_0(l)$ vs distance

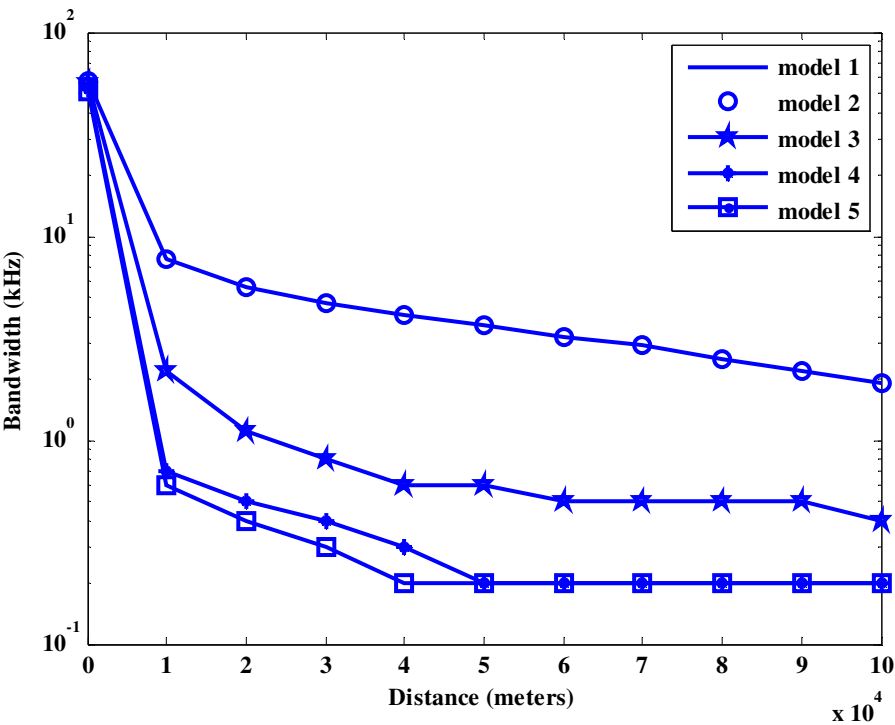


Fig. 5. Bandwidth vs distance (3dB bandwidth definition)

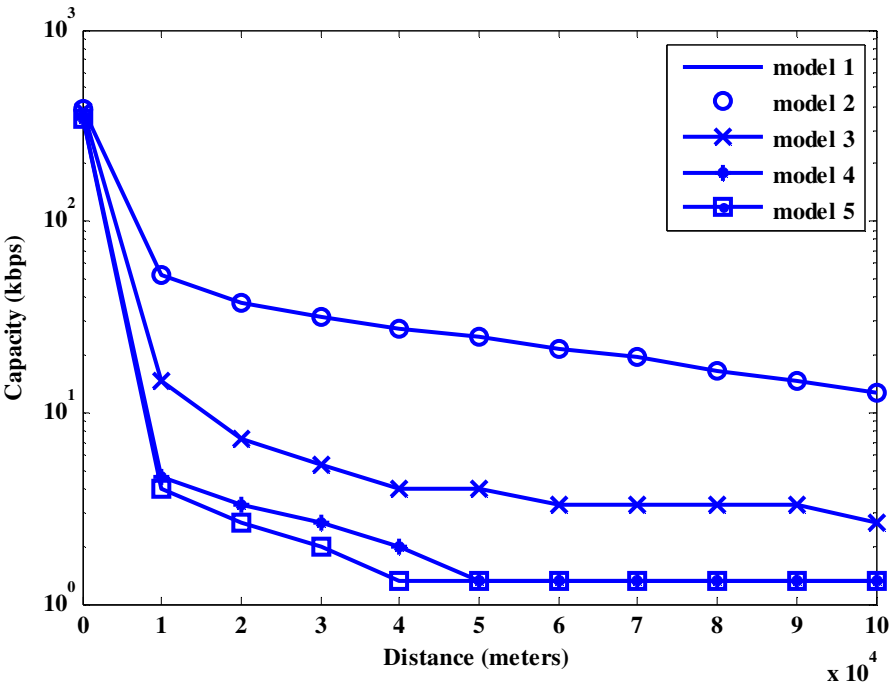


Fig. 6. Capacity vs distance (3dB bandwidth definition)

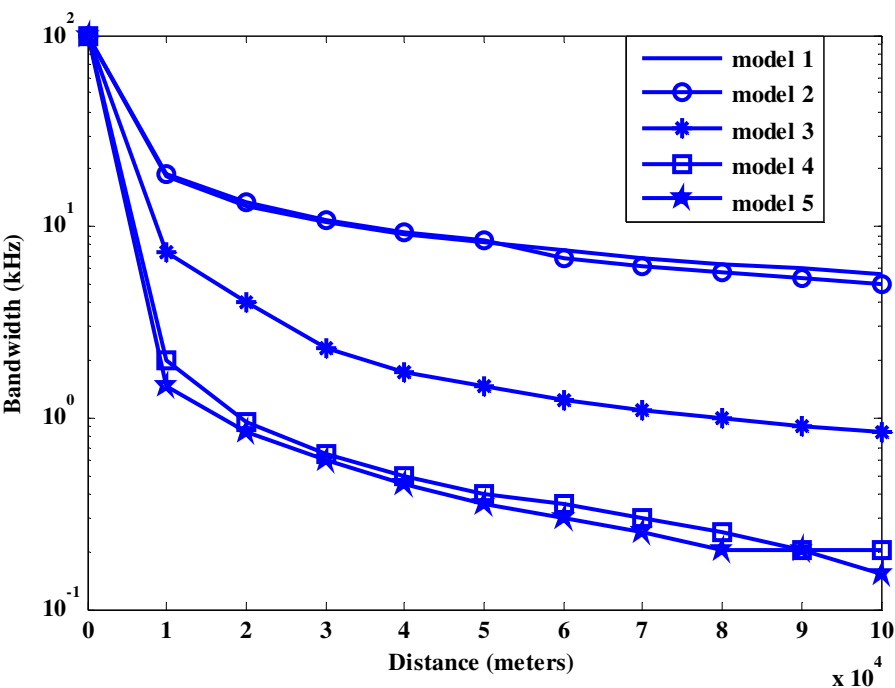


Fig. 7. Bandwidth vs distance (optimal bandwidth definition)

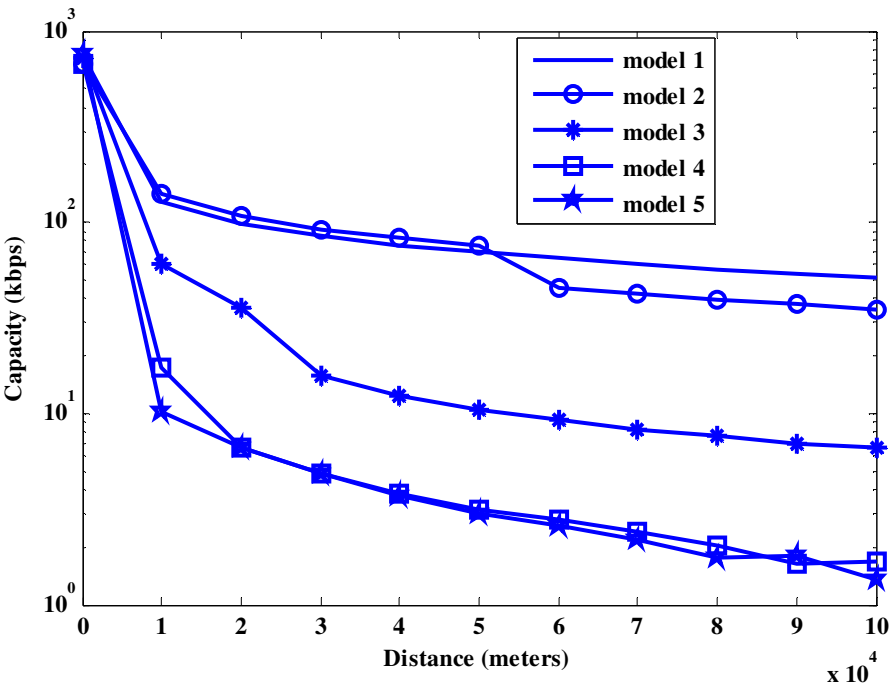


Fig. 8. Capacity vs distance (optimal bandwidth definition)

Table 1. Capacity & Bandwidth ($l = 40$ km)

Loss model	Bandwidth(B kHz)		Capacity(C kbps)		Spectral Efficiency (C / B b/s/Hz)	
	3 dB bandwidth definition	Optimal bandwidth definition	3 dB bandwidth definition	Optimal bandwidth definition	3 dB bandwidth definition	Optimal bandwidth definition
Absorption & Spreading	4.1	9.2	27.2987	75.983	6.6582	8.2581
Absorption, Spreading & Surface Reflection	4.1	9.4	27.2987	82.3	6.6582	8.7553
Absorption, Spreading & Bottom Bounce	0.6	1.75	3.995	12.345	6.6582	7.05
Absorption, Spreading & Surface Duct	0.3	0.5	1.9975	3.8154	6.6582	7.6156
Combined model	0.2	0.451	1.3316	3.728	6.6582	8.2663

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