CSSC 432 **Probability and Statistics - Internal 1**

Marks: 20

I. Answer any 5 **5 x 4 = 20**

1. Discuss the difference between joint probability, marginal probability, and conditional probability

Joint probability, marginal probability, and conditional probability are related concepts in probability theory that describe the relationship between two or more events.

Joint probability is the probability of two or more events occurring simultaneously. It is denoted by P(A and B) and can be calculated as P(A and B) = P(A | B) * P(B), where P(A | B) is the conditional probability of event A given that event B has occurred, and P(B) is the marginal probability of event B.

Marginal probability is the probability of an event occurring, independent of any other events. It is denoted by P(B) and can be calculated as P(B) = P(A and B) + P(not A and B).

Conditional probability is the probability of an event occurring given that another event has already occurred. It is denoted by P(A | B) and can be calculated as P(A | B) = P(A and B) / P(B).

In summary, joint probability is the probability of two or more events occurring together, marginal probability is the probability of an event occurring independently, and conditional probability is the probability of an event occurring given that another event has already occurred.

2. (a). State Addition and Multiplication principles of counting.

Addition Principle: Events A and B are mutually exclusive - Total possible outcomes of event A or B - Sum of outcomes of events - n(A) + n(B)
Multiplication principle: Events A and B are independent - Total possible outcomes of events A and B - Product of outcomes of events - n(A) * n(B)

(b). List all the formulae of Permutation and Combination categorised under with- and without-replacement

ordered sampling with replacement	n^k
ordered sampling without replacement	$P_k^n = \frac{n!}{(n-k)!}$
unordered sampling without replacement	$\binom{n}{k} = \frac{n!}{k!(n-k)!}$
unordered sampling with replacement	$\binom{n+k-1}{k}$

3. (a). State Baye’s Theorem

Bayes' Theorem is a mathematical formula used in probability and statistics to calculate the conditional probability of an event based on prior knowledge of related events. It provides a way of updating the probability of an event as new information becomes available.

The theorem is stated as follows:

$P(A \mid B) = P(B \mid A) * P(A) / P(B)$

where:

$P(A \mid B)$ is the conditional probability of event A given that event B has occurred
$P(B \mid A)$ is the probability of event B given that event A has occurred
$P(A)$ is the prior probability of event A
$P(B)$ is the prior probability of event B

(b). Suppose a medical test for a certain disease is 99% accurate. That is, if a person has the disease, the test result will be positive 99% of the time, and if a person does not have the disease, the test result will be negative 99% of the time. Further, suppose that the disease affects 1% of the population. Now, if a person tests positive for the disease, what is the probability that the person actually has the disease?

Let A be the event that a person has the disease, and B be the event that the person tests positive for the disease. Then, we have:

$P(A|B) = P(B|A) * P(A) / P(B)$

$P(B|A) = 0.99$ (the test result is positive given that the person has the disease)

$P(A) = 0.01$ (the prior probability that the person has the disease)

$P(B) = P(B|A) * P(A) + P(B|\text{not } A) * P(\text{not } A)$
$= 0.99 * 0.01 + 0.01 * 0.99$
$= 0.0298$

So, $P(A|B) = (0.99 * 0.01) / 0.0298 = 0.33$.

Therefore, the probability that a person actually has the disease, given that they tested positive, is 33%.

4. (a). State the axiomatic definitions of probability.

4·6·1. Probability Function. $P(A)$ **is the probability function defined on a σ-field B of events if the following properties or axioms hold :**

1. For each $A \in$ B, $P(A)$ is defined, is real and $P(A) \geq 0$

2. $P(S) = 1$

3. If $\{A_n\}$ is any finite or infinite sequence of disjoint events in B, then

$$P\left(\bigcup_{i=1}^{n} A_n\right) = \sum_{i=1}^{n} P(A_i) \qquad \text{...(4·4)}$$

(b). Suppose that in a school, 30% of the students play chess, 40% play carrom, and 25% play both. What is the probability that a student selected at random from the school plays chess given that he plays carrom?

Let A be the event that a student plays chess and B be the event that a student plays carrom. We are given the following information:

P(A) = 0.30 (30% of the students play chess)
P(B) = 0.40 (40% of the students play carrom)
P(A and B) = 0.25 (25% of the students play both)

We want to find P(A|B), the probability that a student selected at random from the school plays chess given that he plays carrom. This can be calculated using the formula:

P(A|B) = P(A and B) / P(B)
= 0.25 / 0.40
= 0.625

Therefore, the probability that a student selected at random from the school plays chess given that he plays carrom is 0.625 or 62.5%.

5. (a). What is a Random Variable? Give examples of the two types of Random Variables.

Random Variable - A real valued function - maps the sample space to real numbers
Types - Discrete and Continuous - Examples -
Discrete - X: No of heads in tossing a coin
Continuous - X: Body temperature of a patient measured every minute

(b). Consider the following joint probability distribution:

X	Y	$P(X = x, Y = y)$
0	0	0.1
0	1	0.2
1	0	0.3
1	1	0.4

Find the marginal probability distributions for X and Y and compute $P(X = 1 \mid Y = 1)$.

The marginal probability distribution of X is found by summing the joint probabilities over all values of Y.

$P(X = 0) = P(X = 0, Y = 0) + P(X = 0, Y = 1) = 0.1 + 0.2 = 0.3$
$P(X = 1) = P(X = 1, Y = 0) + P(X = 1, Y = 1) = 0.3 + 0.4 = 0.7$

The marginal probability distribution of Y is found by summing the joint probabilities over all values of X.

$P(Y = 0) = P(X = 0, Y = 0) + P(X = 1, Y = 0) = 0.1 + 0.3 = 0.4$
$P(Y = 1) = P(X = 0, Y = 1) + P(X = 1, Y = 1) = 0.2 + 0.4 = 0.6$

The conditional probability $P(X = 1 \mid Y = 1)$ is found by dividing the joint probability $P(X = 1, Y = 1)$ by the marginal probability of Y.

$P(X = 1 \mid Y = 1) = P(X = 1, Y = 1) / P(Y = 1) = 0.4 / 0.6 = \frac{2}{3}$

6. (a). Define your own experiment and discuss its sample space, random variable and its corresponding distribution with its probability mass or density function and its cumulative distribution function. Plot the distribution.

   (b). A box contains 8 red balls and 7 blue balls. Find the probability of getting 3 red balls and 2 blue balls if a sample of 5 balls is drawn without replacement.

The number of ways to choose 3 red balls and 2 blue balls from the box is given by the combination formula:

C(8,3) * C(7,2)

Where C(8,3) is the number of ways to choose 3 red balls from 8 and C(7,2) is the number of ways to choose 2 blue balls from 7.

Therefore, the probability of getting 3 red balls and 2 blue balls if a sample of 5 balls is drawn without replacement is given by:

C(8,3) * C(7,2) / C(15,5)

Where C(15,5) is the number of ways to choose 5 balls from a total of 15.

So, the required probability is:

C(8,3) * C(7,2) / C(15,5) = (56 * 21) / 3003 = 0.184 or 18.4%.

1. (a). A committee of seven people is to be formed from a pool of 10 men and 12 women.

    (I) What is the probability that the committee will consist of three women and four men?

    (II) What is the probability that the committee will consist of all women?

    (I) To determine the probability that the committee will consist of three women and four men, we need to find the number of ways in which three women and four men can be selected, and divide that by the total number of possible committees. The number of ways to select three women from the pool of 12 is C(12,3), and the number of ways to select four men from the pool of 10 is C(10,4). The total number of possible committees is C(22,7), so the required probability is:

    P(3 women and 4 men) = C(12,3) * C(10,4) / C(22,7) = 0.191 or 19.1%.

    (II) To determine the probability that the committee will consist of all women, we need to find the number of ways in which seven women can be selected, and divide that by the total number of possible committees. The number of ways to select seven women from the pool of 12 is C(12,7), and the total number of possible committees is C(22,7). So, the required probability is:

    P(all women) = C(12,7) / C(22,7) = 0.026 or 2.6%.

    (b). Find the number of permutations of the word "COMBINATION"

    The number of permutations of the word "COMBINATION" is 11!. The number of permutations of a set of n distinct elements is given by n! (n factorial), which is the product of all positive integers less than or equal to n. In this case, n = 11, so the number of permutations is 11!.

11! = 11 x 10 x 9 x 8 x 7 x 6 x 5 x 4 x 3 x 2 x 1 = 39916800.

Therefore, the number of permutations of the word "COMBINATION" is 39916800.

5. (b). Find the constant c such that the function f(x) =c*x*x when 0<x<3 and 0, otherwise, is a density function and compute P(1< X <2)