Assignment - IV

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40.) A multiple choice test consists of 8 Questions and 3 Answers to each question (of which only one is correct). If a student answers each question by rolling a balanced die and checking the first answer if he get 1 or 2, the second answer if he gets 3 or 4, the third answer if he get 5 or 6. What is the probability that he will get exactly four correct answers?

soln: Let X be the event that the student gets a correct answer.

X is independent and 'n' repeated trials

$\therefore X \sim b(x; n, \theta) = \binom{n}{x} \theta^{x} (1-\theta)^{n-x}$ for $x = 0, 1, 2 \ldots n$

$n = 8$ as. $\theta = 1/3$

$\therefore P(X=4): b(4; 8, 1/3) = \binom{8}{4} \left(\frac{1}{3}\right)^{4} \left(1 - \frac{1}{3}\right)^{8-4}$

$= \frac{8 \times 7 \times 6 \times 5 \times 4!}{1 \times 2 \times 3 \times 4 \times 4!} \times \frac{1}{81} \times (0.66)^{4}$

$= \frac{70 \times 0.189}{81}$

$= 0.163$

41) An automobile safety engineer claims that 1 in 10 automobile accidents is due to driver fatigue. Using the formula for binomial dist'n round to 4 decimal places, 3/5 accidents are due to driver fatigue

solu Let X be event that automobile accidents occur due to driver fatigue

X - is independent, 'n' repeated trials,

$n = 5, \theta = 1/10$

$\therefore X \sim$ Binomial $b(x; n, \theta) = \binom{n}{x} \theta^{x} (1-\theta)^{n-x}$

$$P(X \geq 3) = \binom{5}{3}\left(\frac{1}{10}\right)^{3}\left(1-\frac{1}{10}\right)^{5-3} + \binom{5}{4}\left(\frac{1}{10}\right)^{4}\left(1-\frac{1}{10}\right)^{5-4} + \binom{5}{5}\left(\frac{1}{10}\right)^{5}\left(1-\frac{1}{10}\right)^{5-5}$$

$$= \frac{107}{12500}$$

$$= 0.0086$$

43) If 40 percent of the mice used in an experiment will become very aggressive within 1 minute after having administrative drug. Find the probability that exactly 6 of 15 mice will get very aggressive within 1 minute using

a) $b(6; 15, 0.4) = \binom{15}{6} (0.4)^{6} (1-0.4)^{15-6}$

$= 0.2066$

b.) Table value $n = 15$, $x = 6$, $\theta = 0.4$ is 0.2066

44.) Y - high school seniors capable of doing college work actually going to college

$n = 18$, $\theta = 0.5$

a) $P(X = 10)$: from the Table I

0.1669

b.) $P(X \geq 10) = P(X = 10) + P(X = 11) + \cdots + P(X = 18)$

$= 0.1669 + 0.1214 + \cdots + 0.0000$

$= 0.4073$

c) $P(X \leq 8) = P(X = 0) + P(X = 1) + \cdots + P(X = 8)$

$= 0.0000 + 0.0001 + \cdots + 0.1669$

$= 0.4073.$

72.) $\lambda = 150 \times 0.014 = 2.1$ Poisson distribution.

$P(X = 2) = \frac{(2.1)^{2} e^{-2.1}}{2!}$, ~~for x = 0,1,2,~~

$= 0.27002$

77.) $\lambda = 3.3$, $x = 2$

$P(X = 2) = \frac{(3.3)^{2} e^{-3.3}}{2!}$

$= 0.2008$

80.) $\lambda = 0.25$ for 1 yard.

$\lambda = 2(0.25) = 0.50$ for 2 yards

a) $P(X \le 1)$. from Table

$x=0, x=1$ at $\lambda = 0.5$.

$P(X=0) + P(X=1)$

$0.6065 + 0.3033 \Rightarrow 0.9098$

b) using computer printout.

$P(X \le 1) = 0.9098$.

81.) $\lambda = 5.2$

a) $P(X=3)$ ⇓ 0.1293

b.) $P(X \ge 10)$ ⇓ $P(X=10) + P(X=11) + \dots + P(X=17)$ ⇓ 0.0397

c.) $P(4 \le X \le 6)$ ⇓ $P(X=4) + P(X=5) + P(X=6)$ ⇓ 0.4944

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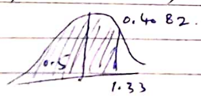
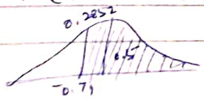
62.) a) $P(Z < 1.33) = 0.5 + 0.4082 = 0.9082$.



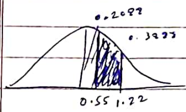
b.) $P(Z \ge -0.79) = 0.5 + 0.2852 = 0.7852$



c) $P(0.55 < Z < 1.22) \Rightarrow P(Z=1.22) - P(Z=0.55)$

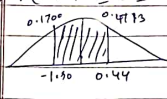
$= 0.3888 - 0.2088$

$= 0.18$



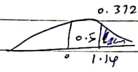
d.) $P(-1.90 \le Z \le 0.44) = 0.1700 + 0.4713$

$= 0.6413$



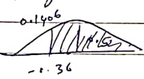
63.) a.) greater than 1.14 : $P(Z > 1.14) = 0.5 - 0.3729$

$= 0.1271$



b.) greater than -0.36 : $P(Z > -0.36) = 0.5 + 0.1406$

$= 0.6406$

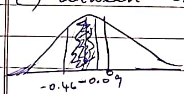


c.) between -0.46 and -0.09 $P(-0.46 < Z < -0.09)$

$\phi(0.46) - \phi(0.09)$

$= 0.1772 - 0.0359$

$= 0.1413$

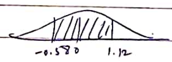


d.) between -0.58 and 1.12

$\phi(0.58) + \phi(1.12)$

$= 0.2190 + 0.3686$

$= 0.5876$



64.) a.) $P(0 < Z < Z_1) = 0.4306$

$P(Z < Z_1) - 0.5 = 0.4306$

$P(Z < Z_1) = 0.9306$

$Z_1 = 1.48$.

b) $P(Z \geq Z_2) = 0.7704$

$1 - P(Z < Z_2) = 0.7704$

$P(Z < Z_2) = 1 - 0.7704 = 0.2296$

$Z_2 = -0$

c) $P(Z > Z_3) = 0.2912$

$1 - P(Z \leq Z_3)$

$= 1 - 0.2912$

$= 0.7088$

$Z_3 = 0.55$

d) $P(-Z_4 \leq Z < Z_4)$

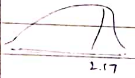
$\Rightarrow P(Z < Z_4) - P(Z < -Z_4) = 0.9700$

$P(Z < Z_4) - [1 - P(Z < -Z_4)] = 0.9700$

$P(Z < Z_4) + P(Z < Z_4) = 0.9700$

$2P(Z < Z_4) = 0.9700 + 1$

$P(Z < Z_4) = 1.700/2 \Rightarrow P(Z < Z_4) = 0.9850$



$Z_4 = 2.17$

65.) a.) between 0 and z : 0.4726

$P(0 < Z < z) = 0.4726$

$P(Z < z) = 0.9726.$

$\therefore Z = 1.92$

b.) $P(Z < z) = 0.9868$

$Z = 2.22$

c) $P(Z > z) = 0.1314$

$1 - P(Z \leq z) = 0.1314$

$P(Z \leq z) = 1 - 0.1314$

$P(Z < z) = 0.8686$

$z = 1.12$

d.) $P(-z < Z < z) = 0.8502$

$2P(Z < z) = 0.8502$

$P(Z < z) = 0.4251$

$z = 1.44$

66.) a.) one standard deviation of mean.

$P(|X - \mu| < \sigma) = P(\mu - \sigma < X < \mu + \sigma)$

$Z = \dfrac{X - \mu}{\sigma}$

$= P(-1 < Z < 1)$

$= 2\phi(1)$

$= 0.6826$

b) Two standard deviations of mean.

$P(|X - \mu| < 2\sigma) = P(\mu - 2\sigma < X \ \mu + 2\sigma)$

$Z = \dfrac{X - \mu}{\sigma} \Rightarrow P(-2 < Z < 2)$

$= 2\phi(2)$

$= 0.9544$

c.) three standard deviations of mean

$P(|X - \mu| < 3\sigma) = P(\mu - 3\sigma < X \ \mu + 3\sigma)$

$= P(-3 < Z < 3) = 2\phi(3)$

$= 0.9974$

d) four standard deviation of mean

$P(|X-\mu| < 4\sigma) = P(\mu - 4\sigma < X < \mu + 4\sigma)$

$= P(-4 < Z < 4)$

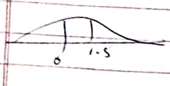
$= 2\phi(4) =$

$= 0.99994$

70) a) $P(X \geq 44.5)$: $\mu = 37.6$ $\sigma = 4.6$

$\cancel{Z} = \frac{44.5 - 37.6}{4.6} = 1.5$

$P(Z \geq 1.5) = 0.0668$



b) $P(X \leq 35.0)$ $Z = \frac{35.0 - 37.6}{4.6} = -0.57$

$P(Z \leq -0.57) = 0.2843$

c) $P(30 \leq X \leq 40.0) \cdot P(-1.65 \leq Z \leq 0.52)$

$0.4505 + 0.1985$

$= 0.649$

71) $\mu = 15.40$ $\sigma = 0.48$

a) $P(X \geq 16)$: $P(Z \geq 1.25)$

$0.5 - 0.3944$

0.1056

b) $P(X \leq 14.20)$ $P(Z \leq -2.5)$

$= 0.5 - 0.4938$

$= 0.0062$

c) $P(15.00 \leq X \leq 15.80) = P(-0.83 \leq Z \; 0.83)$

$= 2\phi(0.83)$

$= 2 \times 0.2967$

0.5934

72) $\sigma = 10$

$P(X < 82.5) = 0.8212$

$Z = 82.5$

72,73 => cumulative normal distribution topic is not taught to us.

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20.) In the test of a certain hypothesis, the P-value corresponding to the test statistic is 0.0316.

a.) $\alpha = 0.01$ b.) $\alpha = 0.05$ c.) $\alpha = 0.10$

a) p-value - 0.0316

$\therefore$ pvalue $> \alpha$

$0.0316 > 0.01$

Null hypothesis Accepted.

b) $0.0316 < 0.05$

$\therefore$ p value $< \alpha$

$0.0316 < 0.05$

Null hypotesis Rejected

c) 0.0316

$\therefore$ p valu

0.0316 <

Null hy

Reject.

21.) wrt. Example 1. verify that p-value corresponding to the observed value of the test statistic is 0.0046

$\bar{x} = 8.071$

p-value $= 2P(\bar{X} \geq \bar{x})$

$= 2(1 - P(\bar{X} < \bar{x}))$

$= 2(0.0023)$ $2(0.5 - 0.4977)$

$= 0.0046$

22.) p-value - 0.0808. verify.

$\bar{x} = 21,819$ 0.08226

$H_1 : \mu < 22,000$

$P(\bar{X} \leq \bar{x})$

$P(Z \leq 1.40)$

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24.) Test at 0.05 level of significance whether the mean of a random sample of size $n=16$ "significantly less than 10". If distribution from which the sample was taken is normal, $\bar{x} = 8.4$, $\sigma = 3.2$. What are null and Alternative hypothesis?

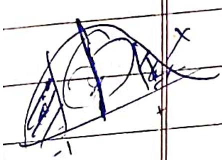
Soln: $n = 16$ $\quad \bar{x} = 8.4$ $\quad \sigma = 3.2$

$\alpha = 0.05$ $\qquad n < 30$

$\therefore$ Z-test.

$H_0 : \mu = 10$

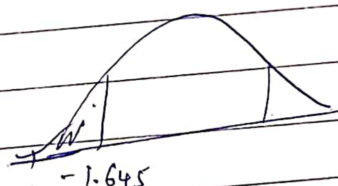
$H_1 : \mu < 10$



$$z = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \Rightarrow \frac{8.4 - 10}{3.2/\sqrt{16}} = \frac{-1.6}{0.8} = -2$$

Critical region: $\quad Z < -Z_\alpha$

$Z \leq -1.645$



$-2 \leq -1.645$

The value -2 test statistic lies in the critical region, therefore reject null hypothesis.

$\therefore$ Mean is significantly less than 10.

25.) According to the norms established for a reading comprehension test, eighth graders should average 84.3 with a standard deviation of 8.6. If 45 randomly selected eighth graders from a certain school district averaged 87.8, use the 4 steps in the initial part of section, to test the null hypothesis $\mu = 84.3$ against alternative hypo $\mu > 84.3$ at 0.01 level of significance.

$n = 45$ be the size of given sample

$\bar{x} = 87.8$ $\quad \sigma = 8.6$

$\alpha = 0.01$ $\qquad n > 30$

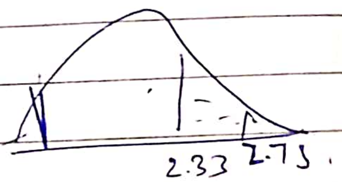
$H_0 : \mu = 84.3$

$H_1 : \mu > 84.3$

$$z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} \Rightarrow \frac{87.8 - 84.3}{8.6/\sqrt{45}}$$

$$= \frac{3.5}{1.28} = 2.73$$

$z > Z_{0.01} \Leftrightarrow Z > 2.33$ 2/2



$2.73 > 2.33$

Null hypothesis is rejected.

i.e the mean for this sample is significantly greater than 84.3. 29.)

27.) The security department of a factory wants to known whether the true average time required by the night gaurd to walk his round is 30 minutes. If, in a random sample of 32 rounds, the night guard averaged 30.8 minutes with a standard deviation 1.5 minutes, determine whether this is sufficient evidence to reject the null hypothesis $\mu = 30$ mins and alternative $\mu \neq 30$ mins, use 4 steps in the initial part and 0.01 significance.

31.

Soln: $H_0 : \mu = 30$ mins

$H_1 : \mu \neq 30$ mins

$\because n > 30$ & s is known

$n = 32$. $\quad \bar{x} = 30.8 \quad s = 1.5$ $\qquad z$-test.

$$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$$

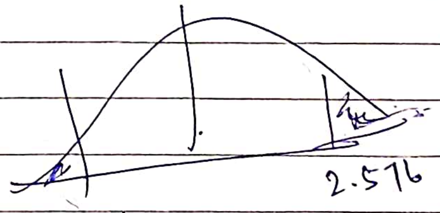
$$= \frac{30.8 - 30}{1.5/\sqrt{32}}$$

$$= \frac{0.8}{0.26} = 3.07$$

$\alpha = 0.005$

$\nu = n-1 = 31$

2.576



$\alpha/2$

$3.07 > 2.576.$

So, Reject the Null hypothesis.

29.) $n = 12$

$\bar{X} = 33.6$ $\alpha = 0.05$

$s = 2.3$ $H_0 : \mu = 35$

$H_1 : \mu < 35$

left Tail :

$\because$ sample sd is known,

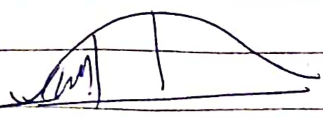
t-test:

$$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{33.6 - 35}{2.3/\sqrt{12}}$$

$$= -2.10$$

find critical value $t_{0.05,11} = -1.796$

$t \leq -t_{\alpha, n-1}$



$-2.10 \leq -1.796.$

$\therefore$ Rejecting Null hypothesis.

30.) $X_i = 14.5, 14.2, 14.4, 14.3, 14.6$ $\quad n = 5$

$\alpha = 0.05$

$H_0 : \mu = 14.0$

$H_1 : \mu \neq 14.0$

$$\bar{X} = \frac{14.5 + 14.2 + 14.4 + 14.3 + 14.6}{5} = 14.4$$

$$s = \sqrt{\frac{\Sigma (X_i - \bar{X})^2}{n-1}}$$

$$= \sqrt{\frac{0.01 + 0.04 + 0 + 0.01 + 0.04}{4}}$$

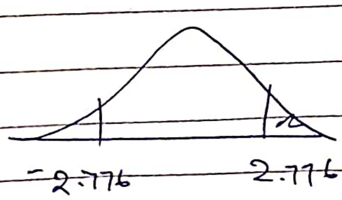
$$= \sqrt{\frac{0.1}{4}} = \sqrt{0.025} = 0.15811$$

$$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{14.4 - 14.0}{0.1581/\sqrt{5}} = \frac{0.4}{0.1581/2.236}$$

$$t = \frac{0.4}{0.07} = 5.65$$

Two tail Test:

$$t_{\alpha/2, n-1} = t_{0.025, 4} = 2.776$$



$$t > t_{\alpha/2, n-1}$$

$$5.65 > 2.776$$

$\therefore$ Rejecting the Null hypothesis.

Therefore, the true mean is significantly different than 14.

34) $n = 10{,}000$

idk

35) $\mu_1 - \mu_2 = 0.2$

$\sigma_1 = 0.12 \qquad \sigma_2 = 0.14$

$n_1 = 50 \qquad n_2 = 40. \qquad \alpha = 0.05$

$$z = \frac{\bar{x}_1 - \bar{x}_2 - \delta}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} \qquad Z_{0.025} = 1.96.$$

$$-Z_{\alpha/2} \le \frac{\bar{x}_1 - \bar{x}_2 - 0.20}{\sqrt{\frac{(0.12)^2}{50} + \frac{(0.14)^2}{40}}} \le 1.96$$

$$-1.96 \times \sqrt{\frac{(0.12)^2}{50} + \frac{(0.14)^2}{40}} \le \bar{x}_1 - \bar{x}_2 - 0.20 \le 1.96 \times \sqrt{\frac{(0.12)^2}{50} + \frac{(0.14)^2}{40}}$$

$$-0.0546 \le \bar{x}_1 - \bar{x}_2 - 0.20 \le 0.0546$$

$$0.14533 \le \bar{x}_1 - \bar{x}_2 \le 0.25466$$

if

$\bar{x}_1 - \bar{x}_2 < 0.1453 \qquad \bar{x}_1 - \bar{x}_2 > 0.254$

Reject the null.

it's not there for assignment, just for exam purpose

38.)

38.) $n_1 = 400$ $\quad n_2 = 500$ $\quad \alpha = 0.05$

$\bar{x}_1 = 53.8$ $\quad x_2 = 54.5$

$s_1 = 2.4$ inches $\quad s_2 = 2.5$

$H_0 : \mu_1 - \mu_2 = -0.5$

$H_1 : \mu_1 - \mu_2 < -0.5$

$$t = \frac{\bar{x}_1 - \bar{x}_2 - \delta}{s_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

$$s_p = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}$$

$$= \frac{(400-1)(2.4)^2 + (500-1)(2.5)^2}{400+500-2}$$

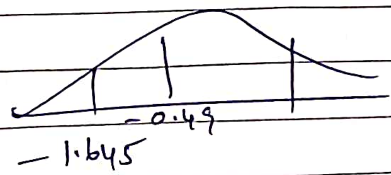
$$= 6.0322$$

$$t = \frac{53.8 - 54.5 + 0.5}{6.0322\sqrt{\frac{1}{400} + \frac{1}{500}}}$$

$$= -0.49426.$$

$$-t_{0.05,898} = -1.645$$

$$-1.645 < -0.49426.$$



Not inside critical region.

$\therefore$ Cannot Reject Null Hypothesis.

42)

$\alpha = 0.01$

$H_0: \mu_1 - \mu_2 = 0$

$n_1 = n_2 = 6$

$H_1: \mu_1 - \mu_2 \neq 0$

$$\bar{x}_1 = \frac{127 + 168 + 143 + 165 + 122 + 13\ldots}{6}$$

$$= 144$$

$$\bar{x}_2 = \frac{154 + 135 + 132 + 171 + 153 + \ldots}{6}$$

$$= 149$$

$$s_p^2 = \frac{1}{2}\left(s_1^2 + s_2^2\right) \qquad s_1 = \sqrt{\frac{\Sigma (x_i - \bar{x}_1)^2}{n-1}} \qquad s_2 = \sqrt{\frac{\Sigma (x_i - \ldots}{n-1}}$$

$$= 282.6001 \qquad s_1 = 19.05 \qquad s_2 = 14.21$$

$$s_p = \sqrt{s_p^2}$$

$$= 16.8107$$

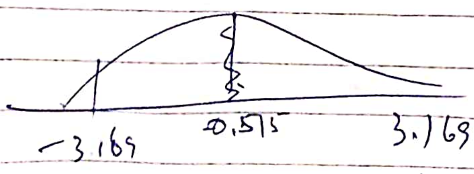
$$\therefore t = \frac{\bar{x}_1 - \bar{x}_2 - \delta}{s_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

$$= \frac{144 - 149 + 0}{16.8\cancel{1} \cdot \sqrt{\frac{1}{6} + \frac{1}{6}}}$$

$$t = -0.515$$

$$-t_{0.005, 10}\ ,\ t_{0.005, 10}$$

$$-3.169\ ,\ 3.169$$



Not in critical region

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68. A random sample of size $n = 225$ is to be taken from an exponential population with $\theta = 4$. Based on the central limit theorem, what is the probability that the mean of the sample will exceed 4.5?

soln:

Central-limit theorem: $Z = \dfrac{\bar{X} - \mu}{\sigma/\sqrt{n}}$

$X_i \sim Exp(\theta = 4)$ for $i = 1, 2, 3, \ldots n$.

In exponential distribution:

$\mu = \theta = 4$ $\qquad n = 225$

$\sigma^2 = \theta^2 = 16$

$P(\bar{X} > 4.5) = ?$ $\Rightarrow 1 - P(\bar{X} \leq 4.5)$

$$1 - P\left(Z \leq \frac{4.5 - 4}{\frac{4}{\sqrt{225}}}\right)$$

$$1 - P\left(Z \leq \frac{0.5}{0.26}\right)$$

$$1 - P(Z \leq 1.88)$$

0.4699

0.5 $\qquad 1 - (0.5 + 0.4699)$

$1 - 0.9699$

0 $\qquad .88 \qquad 0.03005$

69. A random sample of size $n = 200$ is to be taken from a uniform population with $\alpha = 24, \beta = 48$. Based on Central limit theorem, what is the probability of the mean of the sample will be less than 35?

omit the problem..

70. A random sample of size 64 is taken from a normal population with $\mu = 51.4$ and $\sigma = 6.8$. What is the probability that the mean of the sample will

Let $X_i \sim N(\underset{\mu}{51.4}, \underset{\sigma}{6.8})$ for $i = 1, 2, .. n$

$n = 64$

a) exceed 52.9

$P(\bar{x} > 52.9)$

$= 1 - P(\bar{X} \leq 52.9) \rightarrow Z = \dfrac{\bar{X} - \mu}{\sigma/\sqrt{n}} = \dfrac{52.9 - 51.4}{6.8/\sqrt{64}}$

$1 - P(\cancel{\bar{X}} \leq 52.9)$

$= \dfrac{1.5}{6.8/8}$

$1 - P(Z \leq 1.76)$

$= \dfrac{1.5}{0.85}$

$1 - (0.5 + 0.4608)$

1.76

0.4608

$1 - 0.9608$

0.0392

$P(50.5 \leq \bar{X} \leq 52.3)$:

$P\left(\dfrac{50.5 - 51.4}{0.85} \leq Z \leq \dfrac{52.3 - 51.4}{0.85}\right)$

$-1.06 \qquad\qquad 1.06$

$P(-1.06 \leq Z \leq 1.06)$

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$= 2(0.3554)$

$0.7108.$

c) $P(\bar{X} < 50.6) = P\left(Z < \frac{50.6 - 51.4}{0.85}\right)$

$= P\left(Z < -\left(\frac{0.8}{0.85}\right)\right)$

$= P(Z < (-0.94)$

$= 0.5 - 0.3264$

$= 0.17361$

71.) A random sample of size 100 is taken from a norm population with $\sigma = 25$. What is the probability that the mean of the sample will differ from the mean of the population by 3 or more either way?

soln: $P(|\bar{X} - \mu| > 3) \Rightarrow P\left(\left|\frac{\bar{X} - \mu}{\sigma/\sqrt{n}}\right| > \frac{3}{25/\sqrt{100}}\right)$

$= P(|Z| > 1.2) \Rightarrow 0.5 + \phi(1.2)$

$= \cancel{2 \times \phi(1.2)}$

$\cancel{0.3849}$

$= [1 - P(Z \leq 1.2)]\, 2$

$[1 - (0.5 + 0.3849)]\, 2$

$(0.1146) \times 2$

0.23614