Assignment 4: Word Embeddings

Welcome to the fourth (and last) programming assignment of Course 2!

In this assignment, you will practice how to compute word embeddings and use them for sentiment analysis.

- To implement sentiment analysis, you can go beyond counting the number of positive words and negative words.
- You can find a way to represent each word numerically, by a vector.
- The vector could then represent syntactic (i.e. parts of speech) and semantic (i.e. meaning) structures.

In this assignment, you will explore a classic way of generating word embeddings or representations.

- You will implement a famous model called the continuous bag of words (CBOW) model.

By completing this assignment you will:

- Train word vectors from scratch.
- Learn how to create batches of data.
- Understand how backpropagation works.
- Plot and visualize your learned word vectors.

Knowing how to train these models will give you a better understanding of word vectors, which are building blocks to many applications in natural language processing.

Important Note on Submission to the AutoGrader

Before submitting your assignment to the AutoGrader, please make sure you are not doing the following:

1. You have not added any *extra* `print` statement(s) in the assignment.
2. You have not added any *extra* code cell(s) in the assignment.
3. You have not changed any of the function parameters.

4. You are not using any global variables inside your graded exercises. Unless specifically instructed to do so, please refrain from it and use the local variables instead.
5. You are not changing the assignment code where it is not required, like creating *extra* variables.

If you do any of the following, you will get something like, `Grader Error: Grader feedback not found` (or similarly unexpected) error upon submitting your assignment. Before asking for help/debugging the errors in your assignment, check for these first. If this is the case, and you don't remember the changes you have made, you can get a fresh copy of

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1 - The Continuous Bag of Words Model

Let's take a look at the following sentence:

> **'I am happy because I am learning'**.

- In continuous bag of words (CBOW) modeling, we try to predict the center word given a few context words (the words around the center

word).

- For example, if you were to choose a context half-size of say $C = 2$, then you would try to predict the word **happy** given the context that includes 2 words before and 2 words after the center word:

> C words before: [I, am]

> C words after: [because, I]

- In other words:

$$context = [I, am, because, I]$$

$$target = happy$$

The structure of your model will look like this:

alternate text

Figure 1

Where $\overline{x}$ is the average of all the one hot vectors of the context words.

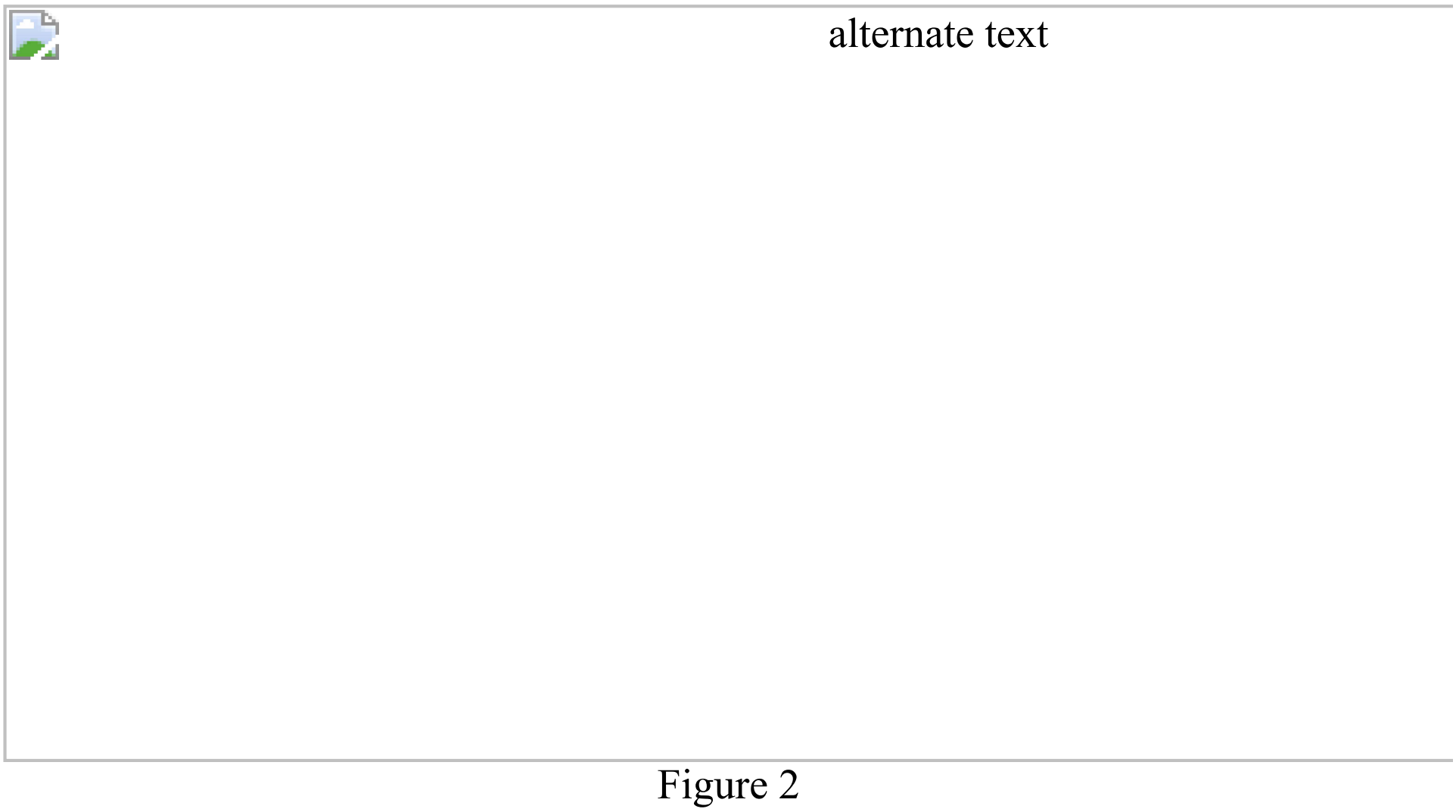


Figure 2

Once you have encoded all the context words, you can use $\overline{x}$ as the input to your model.

```
# Import Python libraries and helper functions (in utils2)
import nltk
from nltk.tokenize import word_tokenize
import numpy as np
from collections import Counter
from utils2 import sigmoid, get_batches, compute_pca, get_dict
import w4_unittest

nltk.download('punkt')
```

```
[nltk_data] Downloading package punkt to /home/jovya
n/nltk_data...
[nltk_data]   Unzipping tokenizers/punkt.zip.

True
```

```
# Download sentence tokenizer
nltk.data.path.append('.')
```

```
# Load, tokenize and process the data
import re
with open('./data/shakespeare.txt') as f:
    data = f.read()
data = re.sub(r'[,!?;-]', '.',data)
data = nltk.word_tokenize(data)
data = [ ch.lower() for ch in data if ch.isalpha() or ch == '.'
print("Number of tokens:",len(data),'\n', data[:15])
```

```
Number of tokens: 60996
 ['o', 'for', 'a', 'muse', 'of', 'fire', '.', 'that',
'would', 'ascend', 'the', 'brightest', 'heaven', 'o
f', 'invention']
```

```
# Compute the frequency distribution of the words in the dataset
fdist = nltk.FreqDist(word for word in data)
print("Size of vocabulary: ",len(fdist) )
print("Most frequent tokens: " fdist.most_common(20) ) # print
```

```
Size of vocabulary:  5778
Most frequent tokens:  [('.', 9630), ('the', 1521),
('and', 1394), ('i', 1257), ('to', 1159), ('of', 109
3), ('my', 857), ('that', 781), ('in', 770), ('a', 75
2), ('you', 748), ('is', 630), ('not', 559), ('for',
467), ('it', 460), ('with', 441), ('his', 434), ('bu
t', 417), ('me', 417), ('your', 397)]
```

Mapping words to indices and indices to words

We provide a helper function to create a dictionary that maps words to indices and indices to words.

```
# get_dict creates two dictionaries, converting words to indices
word2Ind, Ind2word = get_dict(data)
V = len(word2Ind)
print("Size of vocabulary: ", V)
```

```
Size of vocabulary:  5778
```

```
# example of word to index mapping
print("Index of the word 'king' :  ",word2Ind['king'] )
print("Word which has index 2743:  ",Ind2word[2743] )
```

```
Index of the word 'king' :   2745
Word which has index 2743:   kindness
```

2 - Training the Model

2.1 - Initializing the Model

You will now initialize two matrices and two vectors.

- The first matrix (W_1) is of dimension $N \times V$, where V is the number of words in your vocabulary and N is the dimension of your word vector.
- The second matrix (W_2) is of dimension $V \times N$.
- Vector b_1 has dimensions $N \times 1$
- Vector b_2 has dimensions $V \times 1$.
- b_1 and b_2 are the bias vectors of the linear layers from matrices W_1 and W_2.

The overall structure of the model will look as in Figure 1, but at this stage we are just initializing the parameters.

Exercise 1 - initialize_model

Please use numpy.random.rand to generate matrices that are initialized with random values from a uniform distribution, ranging between 0 and 1.

Note: In the next cell you will encounter a random seed. Please **DO NOT** modify this seed so your solution can be tested correctly.

```python
import numpy as np

def initialize_model(N, V, random_seed=1):
    '''
    Inputs:
        N:  dimension of hidden vector
        V:  dimension of vocabulary
        random_seed: random seed for consistent results in the
    Outputs:
        W1, W2, b1, b2: initialized weights and biases
    '''

    np.random.seed(random_seed)

    # W1 has shape (N, V)
    W1 = np.random.rand(N, V)

    # W2 has shape (V, N)
    W2 = np.random.rand(V, N)

    # b1 has shape (N, 1)
    b1 = np.random.rand(N, 1)

    # b2 has shape (V, 1)
    b2 = np.random.rand(V, 1)

    return W1, W2, b1, b2
```

```python
# Test your function example.
tmp_N = 4
tmp_V = 10
tmp_W1, tmp_W2, tmp_b1, tmp_b2 = initialize_model(tmp_N,tmp_V)
assert tmp_W1.shape == ((tmp_N,tmp_V))
assert tmp_W2.shape == ((tmp_V,tmp_N))
print(f"tmp_W1.shape: {tmp_W1.shape}")
print(f"tmp_W2.shape: {tmp_W2.shape}")
print(f"tmp_b1.shape: {tmp_b1.shape}")
print(f"tmp_b2.shape: {tmp_b2.shape}")
```

```
tmp_W1.shape: (4, 10)
tmp_W2.shape: (10, 4)
tmp_b1.shape: (4, 1)
tmp_b2.shape: (10, 1)
```

Expected Output

```
tmp_W1.shape: (4, 10)
tmp_W2.shape: (10, 4)
tmp_b1.shape: (4, 1)
tmp_b2.shape: (10, 1)
```

```
# Test your function
w4_unittest.test_initialize_model(initialize_model)
```

```
All tests passed
```

2.2 - Softmax

Before we can start training the model, we need to implement the softmax function as defined in equation 5:

$$\text{softmax}(z_i) = \frac{e^{z_i}}{\sum_{i=0}^{V-1} e^{z_i}} \tag{5}$$

- Array indexing in code starts at 0.
- V is the number of words in the vocabulary (which is also the number of rows of z).
- i goes from 0 to |V| - 1.

Exercise 2 - softmax

Instructions: Implement the softmax function below.

- Assume that the input z to `softmax` is a 2D array
- Each training example is represented by a vector of shape (V, 1) in this 2D array.
- There may be more than one column, in the 2D array, because you can put in a batch of examples to increase efficiency. Let's call the batch size lowercase m, so the z array has shape (V, m)
- When taking the sum from $i = 1 \cdots V - 1$, take the sum for each column (each example) separately.

Please use

- numpy.exp
- numpy.sum (set the axis so that you take the sum of each column in z)

```
import numpy as np

def softmax(z):
    '''
    Inputs:
        z: output scores from the hidden layer, shape (V, m)
    Outputs:
        yhat: prediction (estimate of y), same shape as z
    '''
    # Subtract max for numerical stability
    z_stable = z - np.max(z, axis=0, keepdims=True)

    exp_z = np.exp(z_stable)
    sum_exp_z = np.sum(exp_z, axis=0, keepdims=True)

    yhat = exp_z / sum_exp_z

    return yhat
```

```
# Test the function
tmp = np.array([[1,2,3],
                [1,1,1]
               ])
tmp_sm = softmax(tmp)
display(tmp_sm)
```

```
array([[0.5       , 0.73105858, 0.88079708],
       [0.5       , 0.26894142, 0.11920292]])
```

Expected Ouput

```
array([[0.5       , 0.73105858, 0.88079708],
       [0.5       , 0.26894142, 0.11920292]])
```

```
# Test your function
w4_unittest.test_softmax(softmax)
```

```
All tests passed
```

2.3 - Forward Propagation

Exercise 3 - forward_prop

Implement the forward propagation z according to equations (1) to (3).

$$h = W_1 X + b_1 \tag{1}$$
$$h = ReLU(h) \tag{2}$$
$$z = W_2 h + b_2 \tag{3}$$

For that, you will use as activation the Rectified Linear Unit (ReLU) given by:

$$f(h) = \max(0, h) \tag{6}$$

Hints

```python
import numpy as np

def forward_prop(x, W1, W2, b1, b2):
    '''
    Inputs:
        x:  average one-hot vector for the context, shape (V, m
        W1, W2, b1, b2:  matrices and biases to be learned
    Outputs:
        z:  output score vector, shape (V, m)
        h:  hidden layer activation, shape (N, m)
    '''

    # Calculate hidden layer pre-activation
    h = np.dot(W1, x) + b1   # shape (N, m)

    # Apply ReLU activation
    h = np.maximum(0, h)     # shape (N, m)

    # Calculate output layer scores
    z = np.dot(W2, h) + b2   # shape (V, m)

    return z, h
```

```
# Test the function

# Create some inputs
tmp_N = 2
tmp_V = 3
tmp_x = np.array([[0,1,0]]).T
#print(tmp_x)
tmp_W1, tmp_W2, tmp_b1, tmp_b2 = initialize_model(N=tmp_N,V=tmp

print(f"x has shape {tmp_x.shape}")
print(f"N is {tmp_N} and vocabulary size V is {tmp_V}")

# call function
tmp_z, tmp_h = forward_prop(tmp_x, tmp_W1, tmp_W2, tmp_b1, tmp_
print("call forward_prop")
print()
# Look at output
print(f"z has shape {tmp_z.shape}")
print("z has values:")
print(tmp_z)

print()

print(f"h has shape {tmp_h.shape}")
print("h has values:")
print(tmp_h)
```

```
x has shape (3, 1)
N is 2 and vocabulary size V is 3
call forward_prop

z has shape (3, 1)
z has values:
[[0.55379268]
 [1.58960774]
 [1.50722933]]

h has shape (2, 1)
h has values:
[[0.92477674]
 [1.02487333]]
```

Expected output

```
x has shape (3, 1)
N is 2 and vocabulary size V is 3
call forward_prop

z has shape (3, 1)
z has values:
[[0.55379268]
 [1.58960774]
 [1.50722933]]

h has shape (2, 1)
h has values:
[[0.92477674]
 [1.02487333]]
```

```
# Test your function
w4_unittest.test_forward_prop(forward_prop)
```

```
All tests passed
```

2.4 - Cost Function

- We have implemented the *cross-entropy* cost function for you.

```
# compute_cost: cross-entropy cost function
def compute_cost(y, yhat, batch_size):

    # cost function
    logprobs = np.multiply(np.log(yhat),y)
    cost = - 1/batch_size * np.sum(logprobs)
    cost = np.squeeze(cost)
    return cost
```

```
# Test the function
tmp_C = 2
tmp_N = 50
tmp_batch_size = 4
tmp_word2Ind, tmp_Ind2word = get_dict(data)
tmp_V = len(word2Ind)

tmp_x, tmp_y = next(get_batches(data, tmp_word2Ind, tmp_V,tmp_C

print(f"tmp_x.shape {tmp_x.shape}")
print(f"tmp_y.shape {tmp_y.shape}")

tmp_W1, tmp_W2, tmp_b1, tmp_b2 = initialize_model(tmp_N,tmp_V)

print(f"tmp_W1.shape {tmp_W1.shape}")
print(f"tmp_W2.shape {tmp_W2.shape}")
print(f"tmp_b1.shape {tmp_b1.shape}")
print(f"tmp_b2.shape {tmp_b2.shape}")

tmp_z, tmp_h = forward_prop(tmp_x, tmp_W1, tmp_W2, tmp_b1, tmp_
print(f"tmp_z.shape: {tmp_z.shape}")
print(f"tmp_h.shape: {tmp_h.shape}")

tmp_yhat = softmax(tmp_z)
print(f"tmp_yhat.shape: {tmp_yhat.shape}")

tmp_cost = compute_cost(tmp_y, tmp_yhat, tmp_batch_size)
print("call compute_cost")
print(f"tmp_cost {tmp_cost:.4f}")
```

```
tmp_x.shape (5778, 4)
tmp_y.shape (5778, 4)
tmp_W1.shape (50, 5778)
tmp_W2.shape (5778, 50)
tmp_b1.shape (50, 1)
tmp_b2.shape (5778, 1)
tmp_z.shape: (5778, 4)
tmp_h.shape: (50, 4)
tmp_yhat.shape: (5778, 4)
call compute_cost
tmp_cost 10.5788
```

Expected output

```
tmp_x.shape (5778, 4)
tmp_y.shape (5778, 4)
tmp_W1.shape (50, 5778)
tmp_W2.shape (5778, 50)
tmp_b1.shape (50, 1)
tmp_b2.shape (5778, 1)
tmp_z.shape: (5778, 4)
tmp_h.shape: (50, 4)
tmp_yhat.shape: (5778, 4)
call compute_cost
tmp_cost 10.5788
```

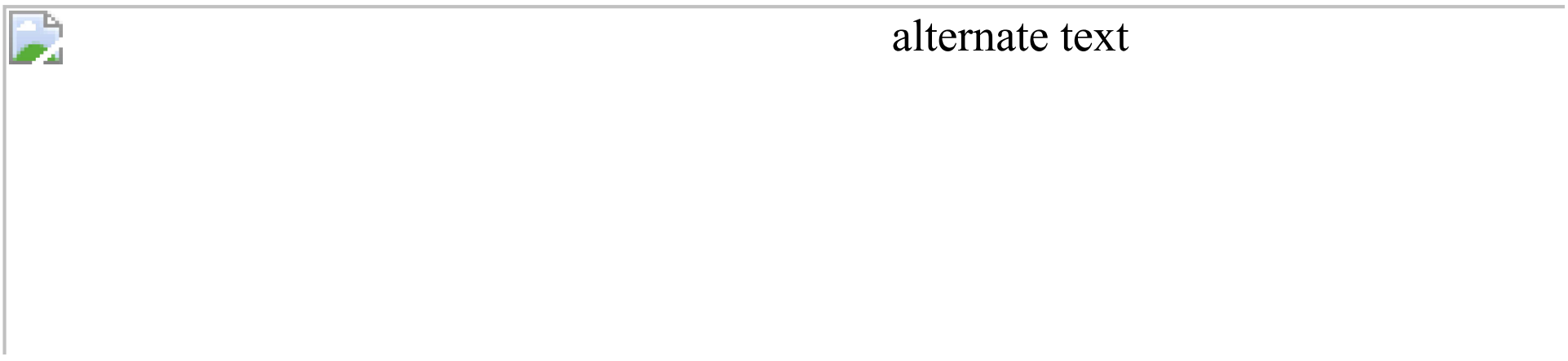
2.5 - Training the Model - Backpropagation

Exercise 4 - back_prop

Now that you have understood how the CBOW model works, you will train it.
You created a function for the forward propagation. Now you will implement a function that computes the gradients to backpropagate the errors.

Note: `z1` is calculated as `W1·x + b1` in this function. In practice, you would save it already when making forward propagation and just re-use here, but for simplicity, it is calculated again in `back_prop`.

As reference, below are the equations of backpropagation as taught in the lecture:


alternate text

```
import numpy as np

def back_prop(x, yhat, y, h, W1, W2, b1, b2, batch_size):
    '''
    Inputs:
        x:  average one-hot vector for the context, shape (V, m
        yhat: prediction (estimate of y), shape (V, m)
        y: target vector, shape (V, m)
        h: hidden layer activations, shape (N, m)
        W1, W2, b1, b2: matrices and biases
        batch_size: batch size, m
    Outputs:
        grad_W1, grad_W2, grad_b1, grad_b2: gradients
    '''

    # Compute z1 as W1·x + b1
    z1 = np.dot(W1, x) + b1  # shape (N, m)

    # Compute l1 = W2^T * (yhat - y)
    l1 = np.dot(W2.T, (yhat - y))  # shape (N, m)

    # Apply ReLU derivative: if z1 < 0, set l1 = 0
    l1[z1 < 0] = 0

    # Compute gradient for W1
    grad_W1 = np.dot(l1, x.T) / batch_size  # shape (N, V)

    # Compute gradient for W2
    grad_W2 = np.dot((yhat - y), h.T) / batch_size  # shape (V,

    # Compute gradient for b1
    grad_b1 = np.sum(l1, axis=1, keepdims=True) / batch_size  #

    # Compute gradient for b2
    grad_b2 = np.sum((yhat - y), axis=1, keepdims=True) / batch

    return grad_W1, grad_W2, grad_b1, grad_b2
```

```
# Test the function
tmp_C = 2
tmp_N = 50
tmp_batch_size = 4
tmp_word2Ind, tmp_Ind2word = get_dict(data)
tmp_V = len(word2Ind)



# get a batch of data
tmp_x, tmp_y = next(get_batches(data, tmp_word2Ind, tmp_V,tmp_C

print("get a batch of data")
print(f"tmp_x.shape {tmp_x.shape}")
print(f"tmp_y.shape {tmp_y.shape}")

print()
print("Initialize weights and biases")
tmp_W1, tmp_W2, tmp_b1, tmp_b2 = initialize_model(tmp_N,tmp_V)

print(f"tmp_W1.shape {tmp_W1.shape}")
print(f"tmp_W2.shape {tmp_W2.shape}")
print(f"tmp_b1.shape {tmp_b1.shape}")
print(f"tmp_b2.shape {tmp_b2.shape}")

print()
print("Forwad prop to get z and h")
tmp_z, tmp_h = forward_prop(tmp_x, tmp_W1, tmp_W2, tmp_b1, tmp_
print(f"tmp_z.shape: {tmp_z.shape}")
print(f"tmp_h.shape: {tmp_h.shape}")

print()
print("Get yhat by calling softmax")
tmp_yhat = softmax(tmp_z)
print(f"tmp_yhat.shape: {tmp_yhat.shape}")

tmp_m = (2*tmp_C)
tmp_grad_W1, tmp_grad_W2, tmp_grad_b1, tmp_grad_b2 = back_prop(

print()
print("call back_prop")
print(f"tmp_grad_W1.shape {tmp_grad_W1.shape}")
print(f"tmp_grad_W2.shape {tmp_grad_W2.shape}")
print(f"tmp_grad_b1.shape {tmp_grad_b1.shape}")
```

```
get a batch of data
tmp_x.shape (5778, 4)
tmp_y.shape (5778, 4)

Initialize weights and biases
tmp_W1.shape (50, 5778)
tmp_W2.shape (5778, 50)
tmp_b1.shape (50, 1)
tmp_b2.shape (5778, 1)

Forwad prop to get z and h
tmp_z.shape: (5778, 4)
tmp_h.shape: (50, 4)

Get yhat by calling softmax
tmp_yhat.shape: (5778, 4)

call back_prop
tmp_grad_W1.shape (50, 5778)
tmp_grad_W2.shape (5778, 50)
tmp_grad_b1.shape (50, 1)
tmp_grad_b2.shape (5778, 1)
```

Expected output

```
    get a batch of data
    tmp_x.shape (5778, 4)
    tmp_y.shape (5778, 4)

    Initialize weights and biases
    tmp_W1.shape (50, 5778)
    tmp_W2.shape (5778, 50)
    tmp_b1.shape (50, 1)
    tmp_b2.shape (5778, 1)
```

```
# Test your function
w4_unittest.test_back_prop(back_prop)
```

```
All tests passed
```

2.6 - Gradient Descent

Exercise 5 - gradient_descent

Now that you have implemented a function to compute the gradients, you will implement batch gradient descent over your training set.

Hint: For that, you will use `initialize_model` and the `back_prop` functions which you just created (and the `compute_cost` function). You can also use the provided `get_batches` helper function:

```
for x, y in get_batches(data, word2Ind, V, C, batch_size):

...
```

Also: print the cost after each batch is processed (use batch size = 128)

```
def gradient_descent(data, word2Ind, N, V, num_iters, alpha=0.0
                     random_seed=282, initialize_model=initiali
                     get_batches=get_batches, forward_prop=forw
                     softmax=softmax, compute_cost=compute_cost
                     back_prop=back_prop):

    '''
    Batch gradient descent for CBOW

    Inputs:
        data:       text data
        word2Ind:   mapping from words to indices
        N:          hidden layer dimension
        V:          vocabulary size
        num_iters: number of iterations
        alpha:      learning rate
        random_seed: random seed
        initialize_model, get_batches, forward_prop, softmax, c
    Outputs:
        W1, W2, b1, b2: updated model parameters
    '''

    # Initialize model parameters
    W1, W2, b1, b2 = initialize_model(N, V, random_seed=random_

    batch_size = 128
    iters = 0
    C = 2  # context window

    for x, y in get_batches(data, word2Ind, V, C, batch_size):
        # Forward propagation
        z, h = forward_prop(x, W1, W2, b1, b2)

        # Softmax predictions
        yhat = softmax(z)

        # Compute cost (pass batch_size)
        cost = compute_cost(yhat, y, batch_size)
        if (iters + 1) % 10 == 0:
            print(f"iters: {iters + 1} cost: {cost:.6f}")

        # Backpropagation to compute gradients
        grad_W1, grad_W2, grad_b1, grad_b2 = back_prop(x, yhat,
```

```
        # Update parameters
        W1 -= alpha * grad_W1
        W2 -= alpha * grad_W2
        b1 -= alpha * grad_b1
        b2 -= alpha * grad_b2

        iters += 1
        if iters == num_iters:
            break
        if iters % 100 == 0:
            alpha *= 0.66  # decay learning rate every 100 iter

    return W1, W2, b1, b2
```

```
# test your function

C = 2
N = 50
word2Ind, Ind2word = get_dict(data)
V = len(word2Ind)
num_iters = 150
print("Call gradient_descent")
W1, W2, b1, b2 = gradient_descent(data, word2Ind, N, V, num_ite
```

```
Call gradient_descent
iters: 10 cost: inf
iters: 20 cost: inf
iters: 30 cost: inf
iters: 40 cost: inf
iters: 50 cost: inf
iters: 60 cost: inf
iters: 70 cost: inf
iters: 80 cost: inf
iters: 90 cost: inf
iters: 100 cost: inf
iters: 110 cost: inf
iters: 120 cost: inf
iters: 130 cost: inf
iters: 140 cost: inf
iters: 150 cost: inf
```

Expected Output

```
iters: 10 cost: 9.686791
iters: 20 cost: 10.297529
iters: 30 cost: 10.051127
iters: 40 cost: 9.685962
iters: 50 cost: 9.369307
iters: 60 cost: 9.400293
iters: 70 cost: 9.060542
iters: 80 cost: 9.054266
iters: 90 cost: 8.765818
iters: 100 cost: 8.516531
iters: 110 cost: 8.708745
iters: 120 cost: 8.660616
iters: 130 cost: 8.544338
iters: 140 cost: 8.454268
iters: 150 cost: 8.475693
```

Your numbers may differ a bit depending on which version of Python you're using.

```
# Test your function
```

```
name default_check
iters: 10 cost: inf
name small_check
iters: 10 cost: inf
 All tests passed
```

3 - Visualizing the Word Vectors

In this part you will visualize the word vectors trained using the function you just coded above.

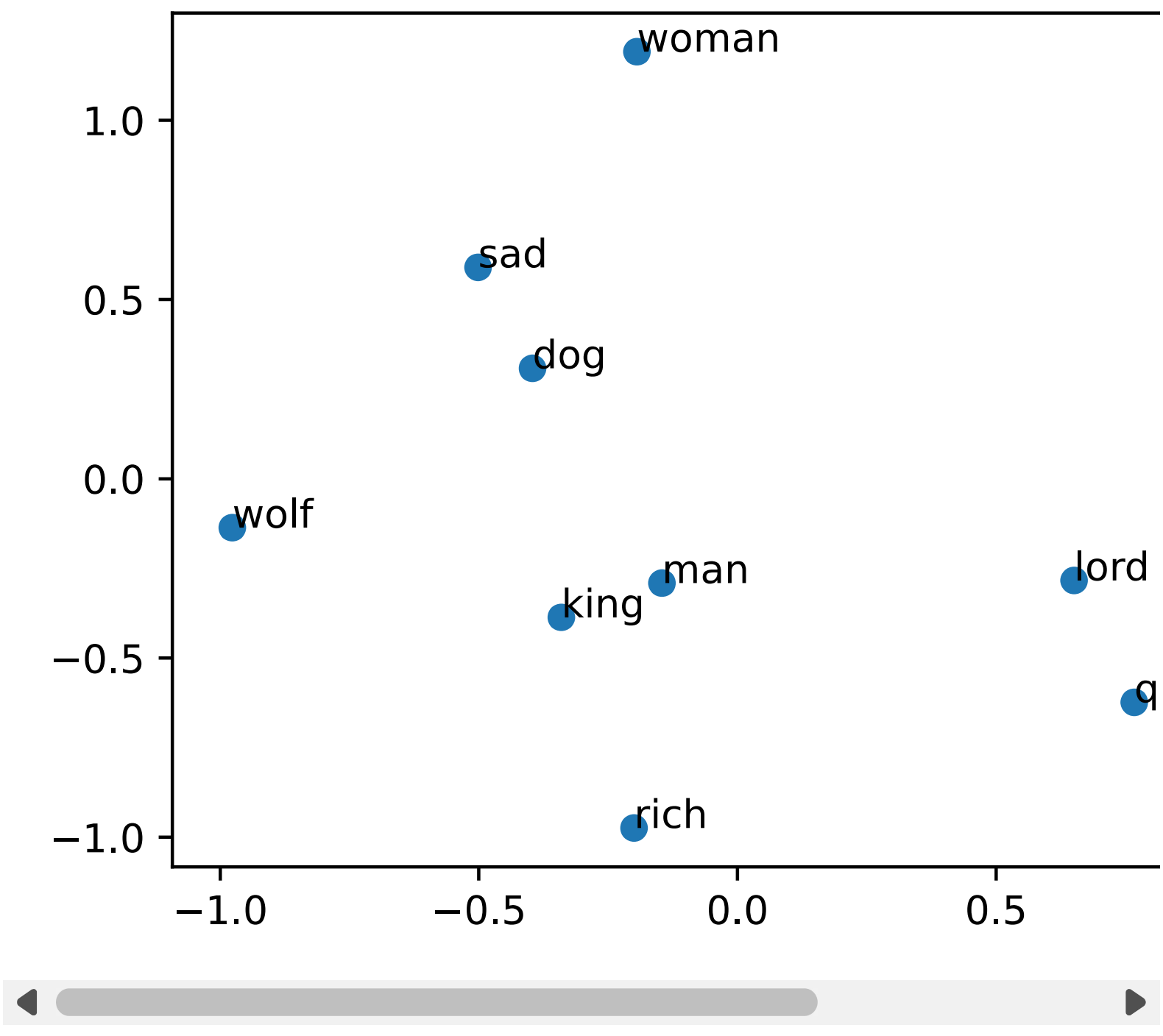
```
# visualizing the word vectors here
from matplotlib import pyplot
%config InlineBackend.figure_format = 'svg'
words = ['king', 'queen','lord','man', 'woman','dog','wolf',
         'rich','happy','sad']

embs = (W1.T + W2)/2.0

# given a list of words and the embeddings, it returns a matrix
idx = [word2Ind[word] for word in words]
X = embs[idx, :]
print(X.shape, idx)  # X.shape:  Number of words of dimension N
```

```
(10, 50) [2745, 3951, 2961, 3023, 5675, 1452, 5674, 4
191, 2316, 4278]
```

```
result= compute_pca(X, 2)
pyplot.scatter(result[:, 0], result[:, 1])
for i, word in enumerate(words):
    pyplot.annotate(word, xy=(result[i, 0], result[i, 1]))
pyplot.show()
```



You can see that man and king are next to each other. However, we have to be careful with the interpretation of this projected word vectors, since the PCA depends on the projection -- as shown in the following illustration.

```
result= compute_pca(X, 4)
pyplot.scatter(result[:, 3], result[:, 1])
for i, word in enumerate(words):
    pyplot.annotate(word, xy=(result[i, 3], result[i, 1]))
pyplot.show()
```