# 10.3.5.4.1

EE24BTECH11026 - G.Srihaas

# Problem Statement

**QUESTION**

A part of monthly hostel charges is fixed and the remaining depends on the number of days one has taken food in the mess. When a student $A$ takes food for 20 days she has to pay 1000 as hostel charges whereas a student $B$, who takes food for 26 days, pays 1180 as hostel charges. Find the fixed charges and the cost of food per day.

# Solution

**SOLUTION**

Let's assume $x$ is the fixed charge and $y$ as the extra per day charge.
From given we can say,

$$x + 20y = 1000 \tag{1}$$
$$x + 26y = 1080 \tag{2}$$

The above equations can be written in the form $A\mathbf{x} = \mathbf{b}$
Where,

$$A = \begin{pmatrix} 1 & 20 \\ 1 & 26 \end{pmatrix} \tag{3}$$
$$b = \begin{pmatrix} 1000 \\ 1080 \end{pmatrix} \tag{4}$$

# LU Decomposition

The matrix $A$ can be decomposed into:

$$A = L \cdot U, \tag{5}$$

where:

$$L = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}, \tag{6}$$

$$U = \begin{pmatrix} 1 & 20 \\ 0 & 6 \end{pmatrix}. \tag{7}$$

# Factorization of LU:DOLITTLE'S ALOGORITHM

Given a matrix **A** of size $n \times n$, LU decomposition is performed row by row and column by column. The update equations are as follows:

1. Start by initializing **L** as the identity matrix $\mathbf{L} = \mathbf{I}$ and **U** as a copy of **A**.

2. For each column $j \geq k$, the entries of $U$ in the $k$-th row are updated as:

$$U_{k,j} = A_{k,j} - \sum_{m=1}^{k-1} L_{k,m} \cdot U_{m,j} \quad \forall \quad j \geq k \tag{8}$$

3. For each row $i > k$, the entries of $L$ in the $k$-th column are updated as:

$$L_{i,k} = \frac{1}{U_{k,k}} \left( A_{i,k} - \sum_{m=1}^{k-1} L_{i,m} \cdot U_{m,k} \right) \quad \forall \quad i > k \tag{9}$$

# Solving the System

The system $A\mathbf{x} = \mathbf{b}$ is transformed into $L \cdot U \cdot \mathbf{x} = \mathbf{b}$. Let $\mathbf{y}$ satisfy $L\mathbf{y} = \mathbf{b}$:

$$\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} 1000 \\ 1080 \end{pmatrix}. \tag{10}$$

Using forward substitution:

$$\begin{aligned} y_1 &= 1000 \qquad (11) \\ y_1 + y_2 &= 1080 \qquad (12) \\ y_2 &= 80 \qquad (13) \end{aligned}$$

Thus:

$$\mathbf{y} = \begin{pmatrix} 1000 \\ 80 \end{pmatrix}. \tag{14}$$

# Backward Substitution

Next, solve $U\mathbf{x} = \mathbf{y}$:

$$\begin{pmatrix} 1 & 20 \\ 0 & 6 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1000 \\ 80 \end{pmatrix}. \tag{15}$$

Using backward substitution:

$$6y = 80 \tag{16}$$

$$y = \frac{40}{3} = 13.33 \tag{17}$$

$$x + 20y = 1000 \tag{18}$$

$$x = \frac{2200}{3} = 733.33 \tag{19}$$

Hence the fixed charge is 733.33 and extra per day cost is 13.33.

# Graphical Representation

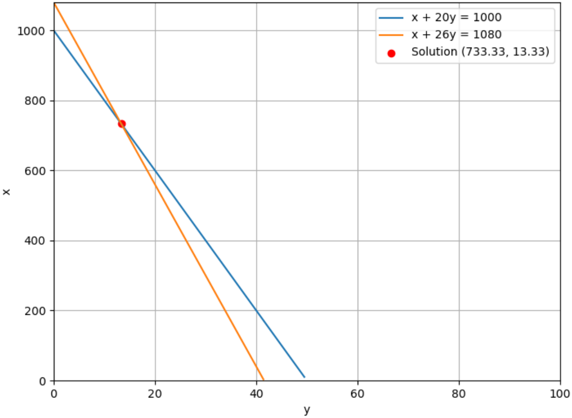


Figure: Solution to set of linear equations

# C-code

https://github.com/Srihaas15/EE1003/10.3.5.4.1/codes/LU.c

# Python-code

https://github.com/Srihaas15/EE1003/10.3.5.4.1/codes/plot.py