9.3.12.A

EE24BTECH11026 - G.Srihaas

QUESTION

Which of the following differential equations has $y = x$ as one of its particular solution?

$$(A)\frac{d^2y}{dx^2} - x^2\frac{dy}{dx} + xy = x \tag{0.1}$$

Solution: NUMERICAL METHOD

Assume $y = x$ is a solution to $\frac{d^2y}{dx^2} - x^2\frac{dy}{dx} + xy = x$
Therefore it would satisfy the equation.

Consider the following equations

$$y = x \tag{0.2}$$
$$\frac{dy}{dx} = 1 \tag{0.3}$$
$$\frac{d^2y}{dx^2} = 0 \tag{0.4}$$

Substituting them in L.H.S of the given equation is as follows

$$L.H.S = 0 - x^2 * 1 + x^2 \tag{0.5}$$
$$L.H.S = 0 \tag{0.6}$$
$$R.H.S = x \tag{0.7}$$
$$L.H.S \neq R.H.S \tag{0.8}$$

Hence, we can say thatbour assumption was wrong and therefore $y = x$ is not a solution to the given equation.

COMPUTATIONAL METHOD

First,Plot the graph of $y = x$
Consider,

$$\frac{d^2y}{dx^2} - x^2\frac{dy}{dx} + xy = x \tag{0.9}$$

Now using the equations,

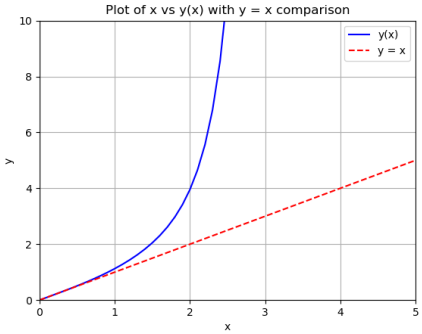
$$y(x+h) = y(x) + hy'(x) \tag{0.10}$$

$$y'(x+h) = y'(x) + hy''(x) \tag{0.11}$$

from first principle and assuming the initial conditions $y(0) = 0$, $y'(0) = 1$ and $h = 0.1$ we can find multiple values for $y(x)$ by varying x

Now we can plot those various points on the graph and join them to give a curve which is the solution of the given differential equation.

If the curve coincides with $y = x$, then $y = x$ would be a solution.



Clearly, they dont coincide hence $y = x$ is not a solution to the given differential equation.