9.3.12.A

EE24BTECH11026 - G.Srihaas

QUESTION

A part of monthly hostel charges is fixed and the remaining depends on the number of days one has taken food in the mess. When a student A takes food for 20 days she has to pay ₹1000 as hostel charges whereas a student B, who takes food for 26 days, pays ₹1180 as hostel charges. Find the fixed charges and the cost of food per day.

SOLUTION

Lets assume x is the fixed charge and y as the extra per day charge.
From given we can say,

$$x + 20y = 1000 \tag{0.1}$$

$$x + 26y = 1080 \tag{0.2}$$

The above equations can be written in the form $A\mathbf{x} = \mathbf{b}$
Where,

$$A = \begin{pmatrix} 1 & 20 \\ 1 & 26 \end{pmatrix} \tag{0.3}$$

$$b = \begin{pmatrix} 1000 \\ 1080 \end{pmatrix} \tag{0.4}$$

The matrix A can be decomposed into:

$$A = L \cdot U, \tag{0.5}$$

where:

$$L = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}, \tag{0.6}$$

$$U = \begin{pmatrix} 1 & 20 \\ 0 & 6 \end{pmatrix}. \tag{0.7}$$

Factorization of LU:
Given a matrix $\mathbf{A}$ of size $n \times n$, LU decomposition is performed row by row and column by column. The update equations are as follows:
1. Start by initializing $\mathbf{L}$ as the identity matrix $\mathbf{L} = \mathbf{I}$ and $\mathbf{U}$ as a copy of $\mathbf{A}$.
2. For each column $j \geq k$, the entries of U in the k-th row are updated as:

$$U_{k,j} = A_{k,j} - \sum_{m=1}^{k-1} L_{k,m} \cdot U_{m,j} \quad \forall \quad j \geq k \tag{0.8}$$

3. For each row $i > k$, the entries of L in the k-th column are updated as:

$$L_{i,k} = \frac{1}{U_{k,k}} \left(A_{i,k} - \sum_{m=1}^{k-1} L_{i,m} \cdot U_{m,k} \right) \quad \forall \quad i > k \tag{0.9}$$

The system $A\mathbf{x} = \mathbf{b}$ is transformed into $L \cdot U \cdot \mathbf{x} = \mathbf{b}$. Let $\mathbf{y}$ satisfy $L\mathbf{y} = \mathbf{b}$:

$$\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} 1000 \\ 1080 \end{pmatrix}. \tag{0.10}$$

Using forward substitution:

$$y_1 = 1000 \tag{0.11}$$

$$y_1 + y_2 = 1080 \tag{0.12}$$

$$y_2 = 80 \tag{0.13}$$

Thus:

$$\mathbf{y} = \begin{pmatrix} 1000 \\ 80 \end{pmatrix}. \tag{0.14}$$

Next, solve $U\mathbf{x} = \mathbf{y}$:

$$\begin{pmatrix} 1 & 20 \\ 0 & 6 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1000 \\ 80 \end{pmatrix}. \tag{0.15}$$

Using backward substitution:

$$6y = 80 \tag{0.16}$$

$$y = \frac{40}{3} = 13.33 \tag{0.17}$$

$$x + 20y = 1000 \tag{0.18}$$

$$x = \frac{2200}{3} = 733.33 \tag{0.19}$$

Hence the fixed charge is ₹733.33 and extra per day cost is ₹13.33.

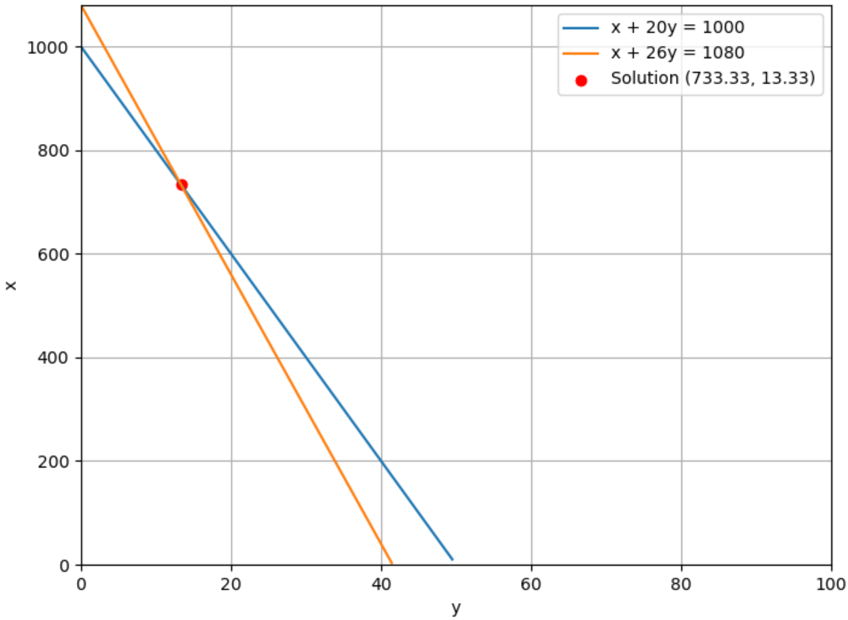

Fig. 0.1: Solution to set of linear equations