11.16.4.9.2

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QUESTION

If 4-digit numbers greater than 5,000 are randomly formed from the digits 0, 1, 3, 5, and 7, what is the probability of forming a number divisible by 5 when the repetition of digits is not allowed?

SOLUTION

To solve this problem, we need to determine the probability of forming a 4-digit number greater than 5,000 using the digits 0, 1, 3, 5, 7 without repetition, such that the number is divisible by 5.

Total Valid 4-Digit Numbers A 4-digit number greater than 5,000 must start with either 5 or 7. The remaining digits can be any of the remaining digits without repetition.

- First Digit: 2 choices (5 or 7).
- Second Digit: 4 choices (excluding the first digit).
- Third Digit: 3 choices (excluding the first two digits).
- Fourth Digit: 2 choices (excluding the first three digits).

Total valid numbers:

$$2 \times 4 \times 3 \times 2 = 48 \tag{0.1}$$

Numbers Divisible by 5 A number is divisible by 5 if its last digit is either 0 or 5. We consider two cases:

1) - Last Digit is 0
   - First Digit: 2 choices (5 or 7).
   - Second Digit: 3 choices (excluding the first digit and 0).
   - Third Digit: 2 choices (excluding the first two digits and 0).
   - Fourth Digit: 1 choice (0).

   Total numbers:

$$2 \times 3 \times 2 \times 1 = 12 \tag{1.1}$$

2) - Last Digit is 5
   - First Digit: 1 choice (7, since 5 is already used as the last digit).
   - Second Digit: 3 choices (excluding the first digit and 5).
   - Third Digit: 2 choices (excluding the first two digits and 5).
   - Fourth Digit: 1 choice (5).

   Total numbers:

$$1 \times 3 \times 2 \times 1 = 6 \tag{2.1}$$

Total numbers divisible by 5:

$$12 + 6 = 18 \tag{2.2}$$

Probability Calculation The probability P is the ratio of numbers divisible by 5 to the total valid numbers:

$$P = \frac{18}{48} = \frac{3}{8} = 0.375 \tag{2.3}$$

The above steps are summarized as follows.

1) Generate All Valid 4-Digit Numbers:
   - Generate all combinations of 4-digit numbers starting with 5 or 7.
   - Ensure no repetition of digits.
2) Check Divisibility by 5:
   - For each valid number, check if the last digit is 0 or 5.
3) Count and Calculate Probability:
   - Count the total valid numbers and the numbers divisible by 5.
   - Compute the probability as the ratio of the two counts.