CIRCLES

EE1030 : MATRIX THEORY

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(EE24BTECH11026)

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1 JEE Advanced/IIT-JEE

1.1 Comprehension Based Question

1.1.1 Passage : 1

$ABCD$ is a square of side length 2 units.C_1 is the circle touching all the sides of the square $ABCD$ and C_2 is the *circumcircle* of square $ABCD$.L is a fixed line in same plane and R is a fixed point.

1) If P is any point of C_1 and Q is another point on C_2,then $\frac{PA^2+PB^2+PC^2+PD^2}{QA^2+QB^2+QC^2+QD^2}$ (2006-5M,-2)

   a) 0.75
   b) 1.25
   c) 1
   d) 0.5

2) If a circle is such that it touches the line L and the circle C_1 externally,such that both the circles are on the same side of the line,then locus of centre of the circle (2006-5M,-2)

   a) ellipse
   b) hyperbola
   c) parabola
   d) circle

3) A line L'through A is drawn parallel to BD.Point S moves such that its distances from the line BD and the vertex A are equal.If locus of S cuts L' at T_2 and T_3 and AC at T_1,then area of $\Delta T_1T_2T_3$ is (2006-5M,-2)

   a) 1/2 sq.units
   b) 2/3 sq.units
   c) 1 sq.units
   d) 2 sq.units

1.1.2 Passage : 2

A circle C of radius 1 unit is inscribed in an equilateral triangle PQR.The points of contact of C with sides PQ,QR,RP are D,E,F respectively.The line PQ is given by the equation $\sqrt{3}x + y - 6 = 0$ and the point D is $\left(3\sqrt{3}/2, 3/2\right)$.Further,it is given that the origin and the centre of C are on same side of line PQ.

4) The equation of circle C is (2008)
   a) $(x - 2\sqrt{3})^2 + (y - 1)^2 = 1$
   b) $(x - 2\sqrt{3})^2 + (y + 1/2)^2 = 1$
   c) $(x - \sqrt{3})^2 + (y - 1)^2 = 1$
   d) $(x - \sqrt{3})^2 + (y + 1)^2 = 1$

1.2 Assertion & Reason Type Questions

1) Tangents are drawn from point $(17, 7)$ to the circle $x^2 + y^2 = 169$.
   STATEMENT-1:The tangents are mutually perpendicular.because
   STATEMENT-2:The locus of all points from which mutually perpendicular tangents can be drawn to a given circle is $x^2 + y^2 = 338$. (2007-3M)

   a) Statement-1 is True,statement-2 is True;Statement-2 is a correct explantion for Statement-1.
   b) Statement-1 is True,statement-2 is True;Statement-2 is NOT a correct explantion for Statement-1.
   c) Statement-1 is True,Statement-2 is False
   d) Statement-1 is False,Statement-2 is True.

2) Consider $L_1 : 2x + 3y + p - 3 = 0$
   $L_2 : 2x + 3y + p + 3 = 0$
   where p is a real number,and C: $x^2 + y^2 + 6x - 10y + 30 = 0$
   STATEMENT-1:If line L_1 is a chord of circle C ,then line L_2 is not always a diameter of circle C
   and
   STATEMENT-2:If line L_1 is a diameter of circle C ,then line L_2 is not a chord of circle C. (2008)

   a) Statement-1 is True,statement-2 is True;Statement-2 is a correct explantion for Statement-1.
   b) Statement-1 is True,statement-2 is True;Statement-2 is NOT a correct explantion for Statement-1.
   c) Statement-1 is True,Statement-2 is False
   d) Statement-1 is False,Statement-2 is True.

1.3 Integer Value Correct Type

1) The centres of two circles C_1 and C_2 each of unit radius are at a distance of 6 units from each other. Let P be the midpoint of the line segment joining the centres of C_1 and C_2 and C be a circle touching circles C_1 and C_2 externally.If a common tangent to C_1 and C passing through P is also a common tangent to C_2 and C, then the radius of circle C is (2009)

2) The straight line $2x - 3y = 1$ divides the circular region $x^2 + y^2 \leq 6$ into two parts. If S is $\{ (2, 3/4), (5/2, 3/4), (1/4, -1/4), (1/8, 1/4) \}$ then the number of point(s) in S lying inside the smaller part is (2011)

3) For how many values of p, the circle $x^2 + y^2 + 2x + 4y - p = 0$ and the coordinate axes have exactly three common points? (JEE Adv. 2017)

4) Let the point B be the reflection of the point A$(2, 3)$ with respect to the line $8x - 6y - 23 = 0$.Let T_A and T_B be circles of radii 2 and 1 with centres A and B respectively.Let T be a common tangent to the circles T_A and T_B such that both the circles are on the same side of T.If C is the point of intersection of T and the line passing through A and B,then the length of the line segment AC is (JEE Adv. 2019)

2 JEE Main / AIEEE

1) If the chord $y = mx + 1$ of the circle $x^2 + y^2 = 1$ subtends an angle of measure $45°$ at the major segment of the circle then the value of m is (2002)
   a) $2 \pm \sqrt{2}$
   b) $-2 \pm \sqrt{2}$
   c) $-1 \pm \sqrt{2}$
   d) none of this

2) The centres of a set of circles,each of radius 3, lie on the circle $x^2 + y^2 = 25$.The locus of any point in the set is (2002)
   a) $4 \leq x^2 + y^2 \leq 64$
   b) $x^2 + y^2 \leq 25$
   c) $x^2 + y^2 \geq 25$
   d) $3 \leq x^2 + y^2 \leq 9$

3) The centre of the circle passing through $(0, 0)$ *and* $(1, 0)$ and touching the circle $x^2 + y^2 = 9$ is (2002)
   a) $(1/2, 1/2)$
   b) $\left(1/2, -\sqrt{2}\right)$
   c) $(3/2, 1/2)$
   d) $(1/2, 3/2)$

4) The equation of a circle with origin as a centre and passing through equilateral triangle whose median is of length $3a$ is (2002)
   a) $x^2 + y^2 = 9a^2$
   b) $x^2 + y^2 = 16a^2$
   c) $x^2 + y^2 = 4a^2$
   d) $x^2 + y^2 = a^2$

5) If the two circles $(x - 1)^2 + (y - 3)^2 = r^2$ and $x^2 + y^2 - 8x + 2y + 8 = 0$ intersect in two distinct points, then (2003)
   a) $r > 2$
   b) $2 < r < 8$

c) $r < 2$
d) $r = 2$