

# CIRCLES

EE1030 : MATRIX THEORY

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(EE24BTECH11026)

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## I. JEE ADVANCED/IIT-JEE

(2006-5M,-2)

### A. Comprehension Based Question

1) *Passage* : 1

$ABCD$  is a square of side length 2 units.  $C_1$  is the circle touching all the sides of the square  $ABCD$  and  $C_2$  is the *circumcircle* of square  $ABCD$ .  $L$  is a fixed line in same plane and  $R$  is a fixed point.

- 1) If  $P$  is any point of  $C_1$  and  $Q$  is another point on  $C_2$ , then  $\frac{PA^2+PB^2+PC^2+PD^2}{QA^2+QB^2+QC^2+QD^2}$   
(2006-5M,-2)

- a) 0.75
- b) 1.25
- c) 1
- d) 0.5

- 2) If a circle is such that it touches the line  $L$  and the circle  $C_1$  externally, such that both the circles are on the same side of the line, then locus of centre of the circle (2006-5M,-2)

- a) ellipse
- b) hyperbola
- c) parabola
- d) circle

- 3) A line  $L'$  through  $A$  is drawn parallel to  $BD$ . Point  $S$  moves such that its distances from the line  $BD$  and the vertex  $A$  are equal. If locus of  $S$  cuts  $L'$  at  $T_2$  and  $T_3$  and  $AC$  at  $T_1$ , then area of  $\Delta T_1 T_2 T_3$  is

- a)  $1/2$  sq.units
- b)  $2/3$  sq.units
- c) 1 sq.units
- d) 2 sq.units

2) *Passage* : 2

A circle  $C$  of radius 1 unit is inscribed in an equilateral triangle  $PQR$ . The points of contact of  $C$  with sides  $PQ, QR, RP$  are  $D, E, F$  respectively. The line  $PQ$  is given by the equation  $\sqrt{3}x + y - 6 = 0$  and the point  $D$  is  $(3\sqrt{3}/2, 3/2)$ . Further, it is given that the origin and the centre of  $C$  are on same side of line  $PQ$ .

- 4) The equation of circle  $C$  is (2008)

- a)  $(x - 2\sqrt{3})^2 + (y - 1)^2 = 1$
- b)  $(x - 2\sqrt{3})^2 + (y + 1/2)^2 = 1$
- c)  $(x - \sqrt{3})^2 + (y - 1)^2 = 1$
- d)  $(x - \sqrt{3})^2 + (y + 1)^2 = 1$

### B. Assertion & Reason Type Questions

- 1) Tangents are drawn from point  $(17, 7)$  to the circle  $x^2 + y^2 = 169$ .  
STATEMENT-1: The tangents are mutually perpendicular. because  
STATEMENT-2: The locus of all points from which mutually perpendicular tangents can

be drawn to a given circle is  $x^2 + y^2 = 338$ .  
(2007-3M)

- a) Statement-1 is True, statement-2 is True; Statement-2 is a correct explanation for Statement-1.
- b) Statement-1 is True, statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1.
- c) Statement-1 is True, Statement-2 is False
- d) Statement-1 is False, Statement-2 is True.

2) Consider  $L_1 : 2x + 3y + p - 3 = 0$   
 $L_2 : 2x + 3y + p + 3 = 0$   
 where  $p$  is a real number, and  $C : x^2 + y^2 + 6x - 10y + 30 = 0$   
 STATEMENT-1: If line  $L_1$  is a chord of circle  $C$ , then line  $L_2$  is not always a diameter of circle  $C$   
 and  
 STATEMENT-2: If line  $L_1$  is a diameter of circle  $C$ , then line  $L_2$  is not a chord of circle  $C$ .  
 (2008)

- a) Statement-1 is True, statement-2 is True; Statement-2 is a correct explanation for Statement-1.
- b) Statement-1 is True, statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1.
- c) Statement-1 is True, Statement-2 is False
- d) Statement-1 is False, Statement-2 is True.

### C. Integer Value Correct Type

- 1) The centres of two circles  $C_1$  and  $C_2$  each of unit radius are at a distance of 6 units from each other. Let  $P$  be the midpoint of the line segment joining the centres of  $C_1$  and  $C_2$  and  $C$  be a circle touching circles  $C_1$  and  $C_2$  externally. If a common tangent to  $C_1$  and  $C$  passing through  $P$  is also a common tangent to  $C_2$  and  $C$ , then the radius of circle  $C$  is

(2009)

- 2) The straight line  $2x - 3y = 1$  divides the circular region  $x^2 + y^2 \leq 6$  into two parts. If  $S$  is  $\{ (2, 3/4), (5/2, 3/4), (1/4, -1/4), (1/8, 1/4) \}$  then the number of point(s) in  $S$  lying inside the smaller part is (2011)
- 3) For how many values of  $p$ , the circle  $x^2 + y^2 + 2x + 4y - p = 0$  and the coordinate axes have exactly three common points? (JEE Adv. 2017)
- 4) Let the point  $B$  be the reflection of the point  $A(2, 3)$  with respect to the line  $8x - 6y - 23 = 0$ . Let  $T_A$  and  $T_B$  be circles of radii 2 and 1 with centres  $A$  and  $B$  respectively. Let  $T$  be a common tangent to the circles  $T_A$  and  $T_B$  such that both the circles are on the same side of  $T$ . If  $C$  is the point of intersection of  $T$  and the line passing through  $A$  and  $B$ , then the length of the line segment  $AC$  is (JEE Adv. 2019)

## II. JEE MAIN / AIEEE

- 1) If the chord  $y = mx + 1$  of the circle  $x^2 + y^2 = 1$  subtends an angle of measure  $45^\circ$  at the major segment of the circle then the value of  $m$  is (2002)
- a)  $2 \pm \sqrt{2}$
- b)  $-2 \pm \sqrt{2}$
- c)  $-1 \pm \sqrt{2}$
- d) none of this

- 2) The centres of a set of circles, each of radius 3, lie on the circle  $x^2 + y^2 = 25$ . The locus of any point in the set is (2002)

- a)  $4 \leq x^2 + y^2 \leq 64$
- b)  $x^2 + y^2 \leq 25$
- c)  $x^2 + y^2 \geq 25$

d)  $3 \leq x^2 + y^2 \leq 9$

- 3) The centre of the circle passing through  $(0,0)$  and  $(1,0)$  and touching the circle  $x^2 + y^2 = 9$  is (2002)

a)  $(1/2, 1/2)$

b)  $(1/2, -\sqrt{2})$

c)  $(3/2, 1/2)$

d)  $(1/2, 3/2)$

- 4) The equation of a circle with origin as a centre and passing through equilateral triangle whose median is of length  $3a$  is (2002)

a)  $x^2 + y^2 = 9a^2$

b)  $x^2 + y^2 = 16a^2$

c)  $x^2 + y^2 = 4a^2$

d)  $x^2 + y^2 = a^2$

- 5) If the two circles  $(x - 1)^2 + (y - 3)^2 = r^2$  and  $x^2 + y^2 - 8x + 2y + 8 = 0$  intersect in two distinct points, then (2003)

a)  $r > 2$

b)  $2 < r < 8$

c)  $r < 2$

d)  $r = 2$