Question 39 Exercise(8.5)

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***Abstract*—A question based on intersection of equal chords.**

Download all python codes from

```
svn co https://github.com/Srihari123456/Summer
    -2020/tree/master/geometry/circle/codes
```

Download all LaTeX-Tikz codes from

```
svn co https://github.com/Srihari123456/Summer
    -2020/tree/master/geometry/circle/figs
```

1 Question

1) If two equal chords of a circle intersect within the circle, prove that the line joining the point of intersection to the center makes equal angles with the chords.

2 Construction

1) The figure for a circle obtained in the question looks like Fig. **??**, with radius **r**, center **O** and equal chords **AB** and **CD** whose point of intersection is **X**.

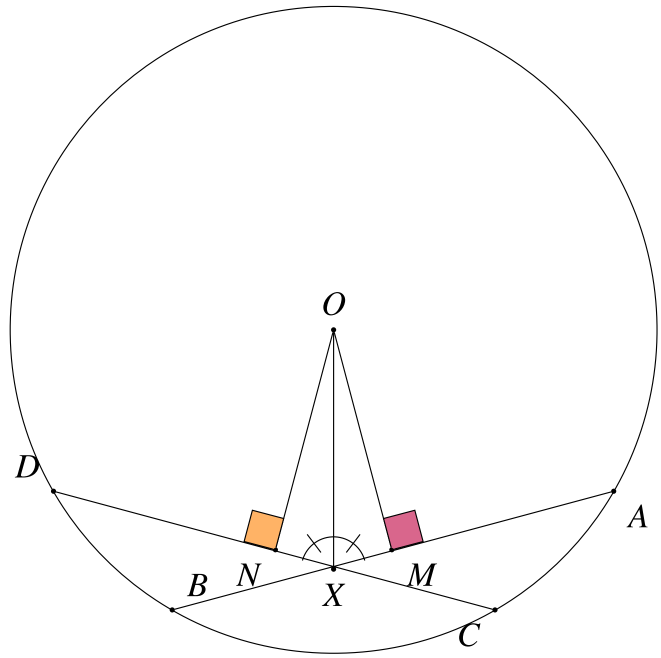

Fig. 1: Triangle by Latex-Tikz

3 Solution

a) First we need to show that a perpendicular from the center of a circle to a chord bisects the chord.

   Take a circle C of radius **r**=2cm, whose center is **O** such that **O**= $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$. **AB** is a chord such that **OX** $\perp$ **AB**. We need to show that **AX** = **BX**.

   i) $\triangle OAX \cong \triangle OBX$by RHS rule as:
      A) $\angle OXA = \angle OXB$ {90°}
      B) **OA** = **OB** {radius of the circle}
      C) **OX** = **OX** {Common}

   Therefore **AX** = **BX**
   Hence the perpendicular from the center of a circle to a chord bisects the chord.

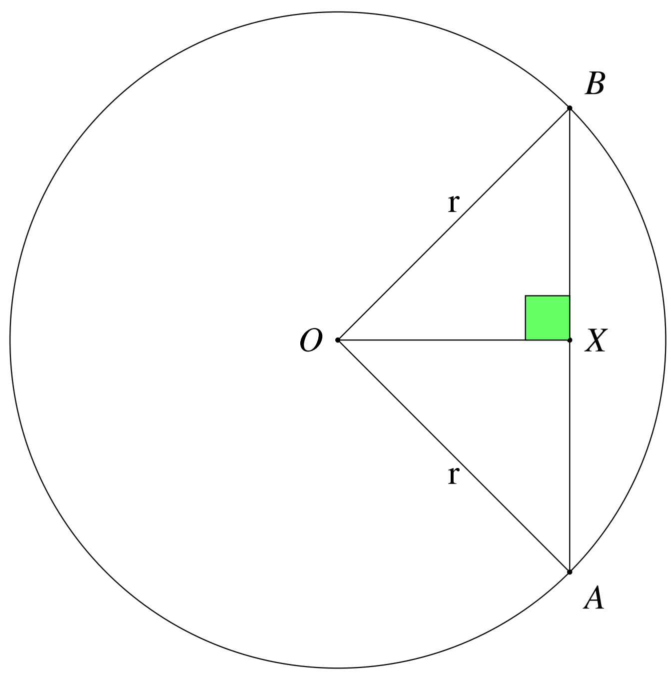

Fig. 2: Circle by Latex-Tikz

2) Next we need to show that equal chords of a

circle are equidistant from the center.

Take a circle C of radius **r**=2cm, whose center is **O** such that **O**= $\begin{pmatrix}0\\0\end{pmatrix}$. **AB** and **CD** are chords such that **OX** $\perp$ **AB** and **OY** $\perp$ **CD**. We need to show that **OX** = **OY**.

a) Since **OX** $\perp$ **AB**, **AX** = **BX** = $\frac{\mathbf{AB}}{2}$ {perpendicular from the center of a circle to a chord bisects the chord. }
Similarly **CY** = **DY** = $\frac{\mathbf{CY}}{2}$

b) As **AB** = **CD**
$\frac{\mathbf{AB}}{2} = \frac{\mathbf{CD}}{2}$
Hence **AX** = **CY**

c) In $\triangle AOX$ and $\triangle COY$

i) $\angle OXA = \angle OYC$ {90°}
ii) **OA** = **OC** {radius of the circle}
iii) **AX** = **CY** {Common}

Hence $\triangle AOX \cong \triangle COY$by RHS rule.
Therefore, **OX** = **OY**

Hence the equal chords of a circle are equidistant from the center.

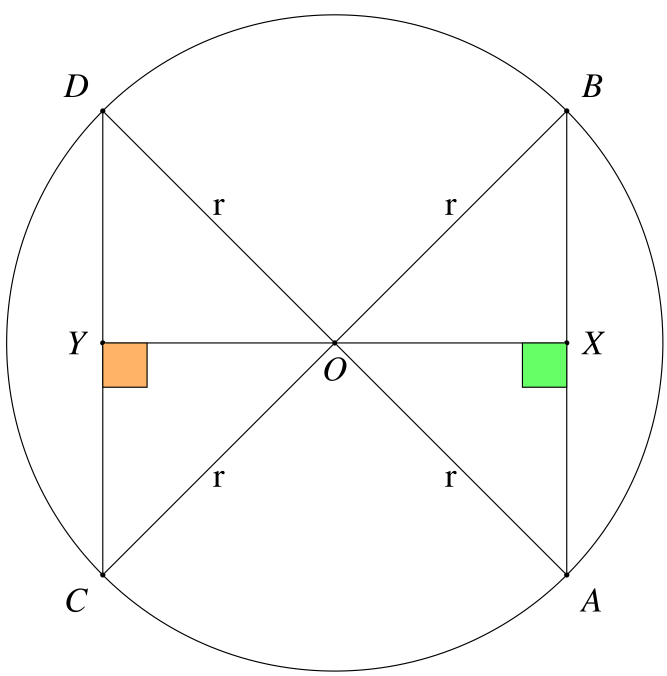

Fig. 3: Circle by Latex-Tikz