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Question 51 Exercise(8.1)

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Abstract—A question based on similarity of triangles.

Download all python codes from

svn co https://github.com/Srihari123456/Summer -2020/tree/master/geometry/triangle/codes

Download all LATEX-Tikz codes from

svn co https://github.com/Srihari123456/Summer -2020/tree/master/geometry/triangle/figs

1 Question

1) **O** is a point in the interior of △**ABC**. **D** is a point on **OA**. If **DE** || **OB** and **DF** || **OC**. Show that **EF** || **BC**.

2 Construction

The figure for a triangle obtained in the question looks like Fig. 1, with sides a,b,c, an arbitrary interior point O and a point D on line AO.

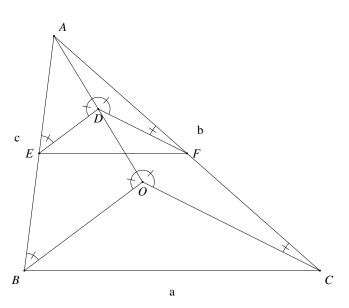


Fig. 1: Triangle by Latex-Tikz

The values used for constructing the triangles in both Python and LATEX-Tikz is given in Table

Initial Input Values	
Parameter	Value
a	5
b	6
С	4

TABLE I: To construct $\triangle ABC$

I:

2) Finding the coordinates of various points of Fig. 1:

From the information provided in the Table I: let

$$\mathbf{B} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad \mathbf{C} = \begin{pmatrix} a \\ 0 \end{pmatrix} \quad \mathbf{A} = \begin{pmatrix} p \\ q \end{pmatrix}$$

The derived value of \mathbf{p} and \mathbf{q} is available in Table II.

- 3) Given a point \mathbf{O} , we need to determine whether it lies inside $\triangle ABC$. A point \mathbf{O} is said to lie inside $\triangle ABC$ if and only if all of the cross products $\mathbf{AB} \times \mathbf{AO}$, $\mathbf{BC} \times \mathbf{BP}$ and $\mathbf{CA} \times \mathbf{CP}$ are ≥ 0
- 4) Let the arbitrary interior point **O** be represented as $\binom{2}{1.5}$.

D is a point on line **AO** such that **DE** \parallel **OB** and **DF** \parallel **OC**.

5) Determination of points D,E and F: As DE || OB, by basic proportionality theorem the points E and D, divide the lines AB and AO respectively in the same ratio.

Hence we choose points **E** and **D** such that

$$\frac{AE}{EB} = \frac{AD}{DO} \tag{2.0.1}$$

Similarly point **F** is chosen such that the points **F** and **D**, divide the lines **AC** and **AO** respectively in the same ratio such that

$$\frac{AF}{FC} = \frac{AD}{DO} \tag{2.0.2}$$

Derived Values	
Parameter	Value
p	0.5
q	3.96

TABLE II: To construct $\triangle ABC$

6) If the point **D** divides the line **AO** in the ratio x:y, the coordinates of **D** is given by section formula as:

$$\mathbf{D} = \frac{y\mathbf{A} + x\mathbf{O}}{x + y} \tag{2.0.3}$$

Similarly the coordinates of points **E** and **F** is given by

$$\mathbf{E} = \frac{y\mathbf{A} + x\mathbf{B}}{x + y} \tag{2.0.4}$$

$$\mathbf{F} = \frac{y\mathbf{A} + x\mathbf{C}}{x + y} \tag{2.0.5}$$

Let us assume the points divide the respective lines in the ratio 1:1. Then the coordinates of points **D**, **E** and **F** is

$$\mathbf{D} = \begin{pmatrix} 1.25 \\ 2.73 \end{pmatrix} \quad \mathbf{E} = \begin{pmatrix} 0.25 \\ 1.98 \end{pmatrix} \quad \mathbf{F} = \begin{pmatrix} 2.75 \\ 1.98 \end{pmatrix}$$

The following Python code generates Fig. 2

./codes/similartriangle.py

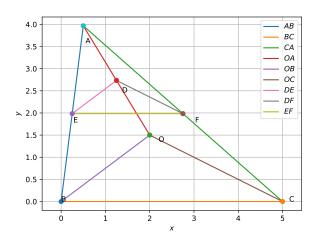


Fig. 2: Triangle generated using python

The equivalent LATEX- tikz code generating Fig. 1 is

./figs/constructionpic.tex

The above LATEX code can be compiled as a standalone document as

To Show:: We need to prove that EF || BC.

3 Solution

- 1) $\triangle EAD \sim \triangle BAO$ by AAA Similarity: Since **DE** \parallel **OB**,
 - a) $\angle DEA = \angle OBA$ {Alternate Interior Angles}
 - b) $\angle ADE = \angle AOB$ {Alternate Interior Angles}
 - c) $\angle EAD = \angle BAO$ {Common angle}

Therefore

$$\frac{\mathbf{AE}}{\mathbf{AB}} = \frac{\mathbf{AD}}{\mathbf{AO}} \tag{3.0.1}$$

- 2) Similarly $\triangle FDA \sim \triangle COA$ by AAA Similarity: Since **DF** \parallel **OC**,
 - a) $\angle DFA = \angle OCA$ {Alternate Interior Angles}
 - b) $\angle ADF = \angle AOC$ {Alternate Interior Angles}
 - c) $\angle FAD = \angle CAO$ {Common angle}

Therefore

$$\frac{\mathbf{AF}}{\mathbf{AC}} = \frac{\mathbf{AD}}{\mathbf{AO}} \tag{3.0.2}$$

3) Hence from the above we conclude,

$$\frac{\mathbf{AF}}{\mathbf{AC}} = \frac{\mathbf{AE}}{\mathbf{AB}} = \frac{\mathbf{AD}}{\mathbf{AO}} \tag{3.0.3}$$

As the ratio of the sides is the same, \triangle ABC \sim \triangle AEF, which means $\angle AFE = \angle ACB$ and $\angle AEF = \angle ABC$ as similar triangles have same angles. i.e.

$$\mathbf{EF} \parallel \mathbf{OR} \tag{3.0.4}$$

Hence Proved.