Similarity of Triangles

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Question

Exercise 8.1(Q no.51)

O is a point in the interior of $\triangle ABC$. D is a point on OA. If DE $\parallel$ OB and DF $\parallel$ OC. Show that EF $\parallel$ BC.

Codes and Figures

The python code for the figure is

```
./codes/similartriangle.py
```

The latex- tikz code is

```
./figs/constructionpic.tex
```

The above latex code can be compiled as standalone document

```
./figs/constructionpic_standalone.tex
```

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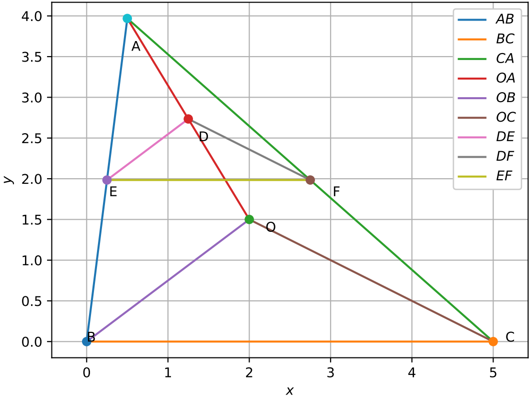
4.0
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
y
0
1
2
3
4
5
x
A
B
C
D
E
F
O
AB
BC
CA
OA
OB
OC
DE
DF
EF

(a) By Python

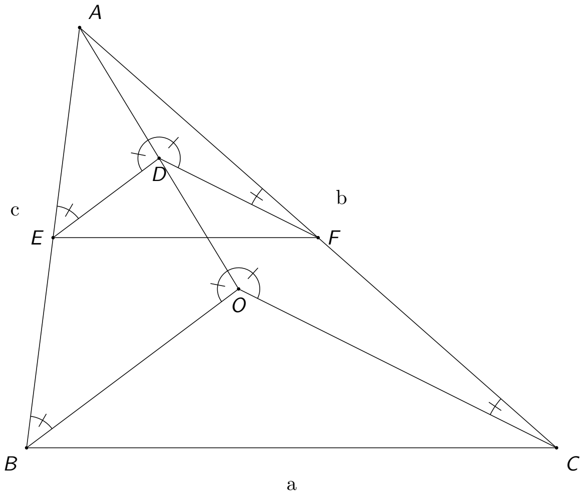

Figure: By Latex-tikz

Construction method

The values used for constructing the triangles in both Python and LaTeX-Tikz is given below:

Initial Input Values	
Parameter	**Value**
a	**5**
b	**6**
c	**4**

Table: To construct $\triangle ABC$

Finding the coordinates of various points of $\triangle ABC$:
From the information provided, let

$$B = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad C = \begin{pmatrix} a \\ 0 \end{pmatrix} \quad A = \begin{pmatrix} p \\ q \end{pmatrix}$$

Given a point O, we need to determine whether it lies inside $\triangle ABC$. Consider 3 vectors v_1, v_2 and v_3 which are orthogonal to vectors AB,BC and CA which are ordered counterclock-wise.

Let $A = \begin{pmatrix} x_1 \\ y_1 \end{pmatrix} \quad B = \begin{pmatrix} x_2 \\ y_2 \end{pmatrix} \quad C = \begin{pmatrix} x_3 \\ y_3 \end{pmatrix} \quad O = \begin{pmatrix} x \\ y \end{pmatrix}$

$$AB = B - A = \begin{pmatrix} x_2 - x_1 \\ y_2 - y_1 \end{pmatrix}$$

$$BC = C - B = \begin{pmatrix} x_3 - x_2 \\ y_3 - y_2 \end{pmatrix}$$

$$CA = A - C = \begin{pmatrix} x_1 - x_3 \\ y_1 - y_3 \end{pmatrix}$$

Construction method

Equation of AB is
$(y_2-y_1)x+(-x_2+x_1)y=x_1(y_2-y_1)-y_1(x_2-x_1)$
As v_1 is orthogonal to AB, equation of v_1 is

$$(-x_2+x_1)x-(y_2-y_1)y=d \quad (1)$$

where d is some constant.Hence v_1 is represented as $\begin{pmatrix} y_2-y_1 \\ -x_2+x_1 \end{pmatrix}$
Similarly v_2 is represented as $\begin{pmatrix} y_3-y_2 \\ -x_3+x_2 \end{pmatrix}$

v_3 is represented as $\begin{pmatrix} y_1-y_3 \\ -x_1+x_3 \end{pmatrix}$
Position vector of O w.r.t A is $v_1^{'} = \begin{pmatrix} x-x_1 \\ y-y_1 \end{pmatrix}$
Position vector of O w.r.t B is $v_2^{'} = \begin{pmatrix} x-x_2 \\ y-y_2 \end{pmatrix}$
Position vector of O w.r.t C is $v_3^{'} = \begin{pmatrix} x-x_3 \\ y-y_3 \end{pmatrix}$
Now we compute the dot products:
O lies inside $\triangle ABC$ only if dot_1 ,dot_2 and dot_3 are all $\geqslant 0$, where $dot_1 = v_1 \cdot v_1^{'}$ $\quad dot_2 = v_2 \cdot v_2^{'}$ $\quad dot_3 = v_3 \cdot v_3^{'}$.

Construction method

Let the arbitrary interior point O be represented as $\begin{pmatrix} 2 \\ 1.5 \end{pmatrix}$.

D is a point on line AO such that DE $\parallel$ OB and DF $\parallel$ OC.

Determination of points D,E and F:

As DE $\parallel$ OB, by basic proportionality theorem the points E and D, divide the lines AB and AO respectively in the same ratio.

Hence we choose points E and D such that

$$\frac{AE}{EB} = \frac{AD}{DO} \tag{2}$$

Similarly point F is chosen such that the points F and D, divide the lines AC and AO respectively in the same ratio such that

$$\frac{AF}{FC} = \frac{AD}{DO} \tag{3}$$

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Derived Values	
Parameter	**Value**
p	**0.5**
q	**3.96**

Table: To construct $\triangle ABC$

Construction method

If the point D divides the line AO in the ratio x:y, the coordinates of D is given by section formula as:

$$D = \frac{yA + xO}{x + y} \tag{4}$$

Similarly the coordinates of points E and F is given by

$$E = \frac{yA + xB}{x + y} \tag{5}$$

$$F = \frac{yA + xC}{x + y} \tag{6}$$

Let us assume the points divide the respective lines in the ratio 1:1. Then the coordinates of points D, E and F is

$$D = \begin{pmatrix} 1.25 \\ 2.73 \end{pmatrix} \qquad E = \begin{pmatrix} 0.25 \\ 1.98 \end{pmatrix}$$

$$F = \begin{pmatrix} 2.75 \\ 1.98 \end{pmatrix}$$

To check whether D lies on line AO, substituting the values of the x and y co-ordinate of D must satisfy the equation of line AO. Equation of line joining two points (x_1, y_1) and (x_2, y_2) is given by

$$\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1} \tag{7}$$

Solution

$\triangle EAD \sim \triangle BAO$ by AAA Similarity:
Since DE $\parallel$ OB,

1. $\angle DEA = \angle OBA \quad \{Alternate\ Interior\ Angles\}$
2. $\angle ADE = \angle AOB \quad \{Alternate\ Interior\ Angles\}$
3. $\angle EAD = \angle BAO \quad \{Common\ angle\}$

Therefore

$$\frac{AE}{AB} = \frac{AD}{AO} \tag{8}$$

Solution

Similarly $\triangle FDA \sim \triangle COA$ by AAA Similarity:
Since DF $\parallel$ OC,

1. $\angle DFA = \angle OCA \quad \{Alternate\ Interior\ Angles\}$
2. $\angle ADF = \angle AOC \quad \{Alternate\ Interior\ Angles\}$
3. $\angle FAD = \angle CAO \quad \{Common\ angle\}$

Therefore

$$\frac{AF}{AC} = \frac{AD}{AO} \tag{9}$$

Hence from the above we conclude,

$$\frac{AF}{AC} = \frac{AE}{AB} = \frac{AD}{AO} \tag{10}$$

As the ratio of the sides is the same, $\triangle$ ABC $\sim$ $\triangle$ AEF, which means $\angle AFE = \angle ACB$ and $\angle AEF = \angle ABC$ as similar triangles have same angles. i.e.

$$EF \parallel QR \tag{11}$$

Hence Proved.