Capacitated K-center

Open atmost k facilities from set F.

Acign clients to these facilities

Each facility can serve at most U clients. (capacity constraint).

Minimize the maximum distance clients travel to facility.

Ex:- 9 customers 4 warehouses

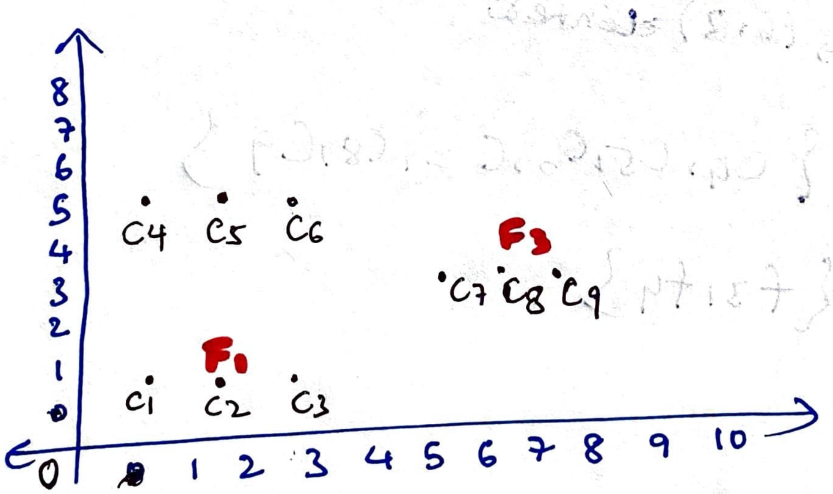
$C_1(1,1)$, $C_2(2,1)$ $C_3(3,1)$ $C_4(1,5)$, $C_5(2,5)$ $C_6(3,5)$ $C_7(5,3)$

$C_8(6,3)$ $C_9(7,3)$

Facilities: $F_1(2,1)$ $F_2(2,5)$ $F_3(6,3)$ $F_4(4,3)$

Constraint: $k=2$ (only 2 ware house)

$U=4$ (Warehouse serves 4 customers).



Vanilla Clustering - ignores constraint & minimize the maximize distance.

$C_1 \to$ Distance $((C_1,F_1)(C_3,F_1)) = 1$

$C_2 \to$ Distance $((C_4,F_3)) = 5.4$ Largest radius. F_3 serves 6 customers

(X)

Expanding the clusters

Try $r=6$ $\quad C_1 r = (r_1 + r') = (6+5.4) = 11.4$

~~$C_2(r) = (r_2+r') = (6+5.4) = 11.4$~~

Customers in Ball 1 :- Centered at f_1

$C_1 : D = 1.0$ ✓
$C_2 : D = 0$ ✓
$C_3 ; D = 1.0$ ✓
$C_4 D = 4.1$ ✓
$C_5 D = 4.0$ ✓
$C_6 D = 4.1$ ✓
$C_7 D = 4.5$ ✓
$C_8 D = 5.7$ ✓
$C_9 D = 6.7$ ✓ ≤ 11.4

Ball 1

Facility Ball (≤ 11.4)

$f_1 D = 0$ ✓
$f_2 D = 4$ ✓
$f_3 D = 2.8$ ✓
$f_4 D = 4.5$ ✓

All are fitting

~~Customer in Ball 2 | Centered f_2~~

~~$C_1 D = 5.4$ ✓~~
~~$C_2 D = 0$ ✓~~
~~$C_3 D = 1.0$ ✓~~
~~$C_4 D = 4.1$ ✓~~
~~$C_5 D = 4.0$ ✓~~
~~$C_6 D = 4.1$ ✓~~
~~$C_7 D = 4.5$ ✓~~
~~$C_8 D = 5.7$ ✓~~
~~$C_9 D = 6.7$ ✓~~

~~facility~~

~~$f_1 = 0$~~
~~$f_2 =$~~

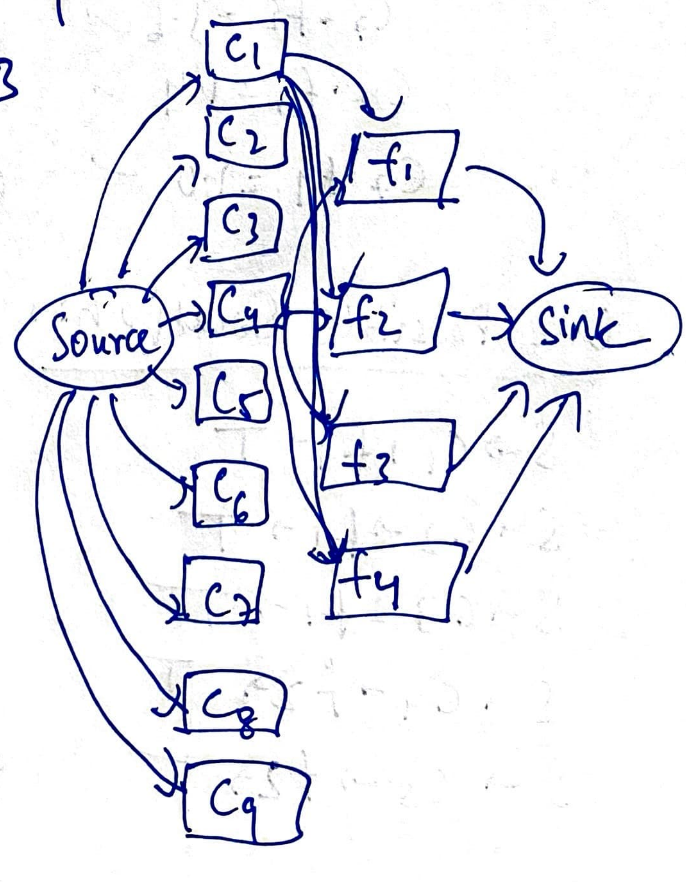
Customer in Ball 2 : centered at f_3
≤ 11.4

$C_1 D = 7.4$ ✓
$C_2 D = 4.5$ ✓
$C_3 D = 3.6$ ✓
$C_4 D = 1.4$ ✓
$C_5 D = 0.0$ ✓
$C_6 D = 1.4$ ✓
$C_7 D = 5.7$ ✓
$C_8 D = 5.0$ ✓
$C_9 D = 5.0$ ✓ ≤ 11.4

Facility Ball

$f_3 = 0.0$ ✓
$f_4 = 2.0$ ✓
$f_2 = 4.5$ ✓
$f_1 = 4.5$ ✓

All are fitting.



$S \to c_1\ [0/1]$ $c_1(1,1)$ $\boxed{\leq 6.}$ $c_3(3,1)$ $c_5(2,5)$

$S \to c_2\ [0/1]$ $c_1 \to f_1\ D = 1.0$ ✓ $c_3 \to f_1 = 1.0$ ✓ $c_5 \to f_1 = 4$ ✓

$S \to c_3\ [0/1]$ $c_1 \to f_2\ D = 4.1$ ✓ $c_3 \to f_2 = 4.1$ ✓ $c_5 \to f_2 = 0$ ✓

$S \to c_4\ [0/1]$ $c_1 \to f_3\ D = 5.7$ ✓ $c_3 \to f_3 = 5.0$ ✓ $c_5 \to f_3 = 3.6$ ✓

$S \to c_5\ [0/1]$ $c_1 \to f_4\ D = 3.6$ ✓ $c_3 \to f_4 = 2.2$ ✓ $c_5 \to f_4 = 2.2$ ✓

$S \to c_6\ [0/1]$ $c_2(2,1)$ $c_4(1,5)$ $c_6(3,5)$

$S \to c_7\ [0/1]$ $c_2 \to f_1\ D = 0$ ✓ $c_4 \to f_1 = 4.1$ ✓ $c_6 \to f_1 = 4.1$ ✓

$S \to c_8\ [0/1]$ $c_2 \to f_2\ D = 4$ ✓ $c_4 \to f_2 = 1$ ✓ $c_6 \to f_2 = 1.0$ ✓

$S \to c_9\ [0/1]$ $c_2 \to f_3\ D = 5$ ✓ $c_4 \to f_3 = 5.4$ ✓ $c_6 \to f_3 = 3.1$ ✓

$c_2 \to f_4\ D = 2.8$ ✓ $c_4 \to f_4 = 3.6$ ✓ $c_6 \to f_4 = 2.2$ ✓

$c_7(5,3)$ $c_8(6,3)$ $c_9(7,3)$

$c_7 \to f_1 = 3.6$ ✓ $c_8 \to f_1 = 4.5$ ✓ $c_9 \to f_1 = 5.4$ ✓

$c_7 \to f_2 = 3.6$ ✓ $c_8 \to f_2 = 4.5$ ✓ $c_9 \to f_2 = 5.4$ ✓

$c_7 \to f_3 = 1.4$ ✓ $c_8 \to f_3 = 0.0$ ✓ $c_9 \to f_3 = 1.4$ ✓

$c_7 \to f_4 = 1.0$ ✓ $c_8 \to f_4 = 2.0$ ✓ $c_9 \to f_4 = 3.6$ ✓

<u>One Possible Solution:-</u>

S = source
T = sink.

$S \to c_1 \to f_1 \to T$

$S \to c_2 \to f_1 \to T$

$S \to c_3 \to f_1 \to T$

$S \to c_4 \to f_2 \to T$

$S \to c_5 \to f_2 \to T$

$S \to c_6 \to f_2 \to T$

$S \to c_7 \to f_3 \to T$

$S \to c_8 \to f_3 \to T$

$S \to c_9 \to f_4 \to T$

All 4 are used but our $\boxed{k \leq 2}$

Only 2 facilities

One possible solution

$S \to c_1 \to f_2 \to T$ [1/1]

$S \to c_2 \to f_2 \to T$ [1/1]

$S \to c_3 \to f_3 \to T$ [1/1]

$S \to c_4 \to f_2 \to T$ [1/1]

$S \to c_5 \to f_2 \to T$ [1/1]

$S \to c_6 \to f_3 \to T$ [1/1]

$S \to c_7 \to f_3 \to T$ [1/1]

$S \to c_8 \to f_3 \to T$ [1/1]

c_9 is unreserved

$f_1 \to T$ [0/4]

$f_2 \to T$ [4/4]

$f_3 \to T$ (4/4)

$f_4 \to T$ [0/4]

f_2 serves (2, 5) = 4 customers

f_3 (6, 3) 8 = 4 customers.

∴ Serves the best 8 customers

& Infeasible

In short:- 1) Find k good facility locations (vanilla k-center)

2) Expand clusters with increase in radius.

3) Assign incidents to facilities respecting capacity 4 per facility.

Algo intuition:

→ Vanille K-centers:-

9 customers 4 warehouses

$C_1(1,1)$ $C_2(2,1)$ $C_3(3,1)$ $C_4(1,5)$ $C_5(2,5)$ $C_6(3,5)$ $C_7(5,3)$

$C_8(6,3)$ $C_9(7,3)$

$F_1(2,1)$ $(2,5)$ $F_3(6,3)$ $F_4(4,3)$

$k=2$, Constriant for facilities.

Suppose $F_1(2,1)$

Distance

$C_1 - F_1 = 1.0$

$C_2 - F_1 = 0.0$

$C_3 - F_1 = 1.0$

$C_4 - F_1 = 4.12$

$C_5 - F_1 = 4.0$

$C_6 - F_1 = 4.12$

$C_7 - F_1 = 3.61$

$C_8 - F_1 = 4.4$

$C_9 - F_1 = 5.39$

$F_1 + F_2 = 5.0$

$F_1 + F_3 = 4.12$

$F_1 + F_4 = 3.61$ (Worst distance)

$F_4 \rightarrow C_4, C_5, C_6, C_7, C_8, C_9$

$F_1 \rightarrow C_1, C_2, C_3$.