

a) If there is an optimal solution with $\delta = 0$ then
by statement (2)

$$y_i(\vec{w}^T \vec{x}_i + \theta) \geq 1 - \delta$$
$$\Rightarrow \delta = 0 \rightarrow y_i(\vec{w}^T \vec{x}_i + \theta) \geq 1$$

This means:

$$(\vec{w}^T \vec{x}_i + \theta) \geq 1 \geq 0 \text{ when } y_i = 1 \text{ \& } (x_i, y_i) \in D$$

and

$$(\vec{w}^T \vec{x}_i + \theta) \leq -1 < 0 \text{ when } y_i = -1 \text{ \& } (x_i, y_i) \in D$$

$$\therefore (\vec{w}^T \vec{x}_i + \theta) \geq 0 \text{ when } y_i = 1 \text{ \& } (x_i, y_i) \in D$$

and

$$(\vec{w}^T \vec{x}_i + \theta) < 0 \text{ when } y_i = -1 \text{ \& } (x_i, y_i) \in D.$$

The above statement is equivalent to statement (1)
and since it is true this implies statement (1)
is true.

\Rightarrow since the conditions satisfy statement (1)
D is linearly separable.

\therefore If there is an ~~optimal~~ optimal solution with $\delta = 0$
then D is linearly separable. ✓

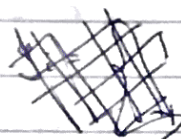
b) If there exists a hyperplane that satisfies condition (2) w/ $\delta > 0$

then:

if $\delta < 1$ then

$$y_i (\vec{w}^T \vec{x}_i + \theta) \geq 1 - \delta > 0$$

therefore by the same logic as part a:



$$\vec{w}^T \vec{x}_i + \theta \geq 0 \text{ if } y_i = 1$$

$$\vec{w}^T \vec{x}_i + \theta \leq 0 \text{ if } y_i = -1$$

which means

if $\delta < 1$ then D is linearly separable

However if ~~$\delta < 1$~~ we know of a $\delta \geq 1$ that satisfies condition 2 we ~~do~~ cannot tell if D is linearly separable.

If we know the optimal δ for the set is greater than or equal to 1. this means.

~~hyperplane~~ $y_i (\vec{w}^T \vec{x}_i + \theta) \geq 1 - \delta \leq 0$

$$y_i (\vec{w}^T \vec{x}_i + \theta) \geq 0 \text{ or negative } \#$$

~~if~~ $(\vec{w}^T \vec{x}_i + \theta) \geq 0 \text{ or negative } \# \text{ if } y_i = 1$

$$(\vec{w}^T \vec{x}_i + \theta) \leq 0 \text{ or positive } \# \text{ if } y_i = -1$$

Since there is overlap, D is not linearly separable.

if the optimal $\delta \geq 1$ then D is NOT linearly separable

c) The optimal solution occurs when $\delta = 0$.

an optimal solution occurs when

$$\begin{aligned} y_i(\vec{w}^T \vec{x}_i + \theta) &\geq -\delta \\ y_i(\vec{w}^T \vec{x}_i + \theta) &\geq 0. \end{aligned}$$

this is satisfied when

$$\vec{w} = \vec{0} \quad \& \quad \theta = 0$$

This an issue b/c this formula will output this solution
where $\vec{w} = \vec{0}$ & $\theta = 0$ & $\delta = 0$
which does not give a separating hyperplane.

d) $\vec{x}_1^T = [1 \ 1 \ 1] \quad y_1 = 1 \quad \vec{x}_2^T = [-1 \ -1 \ -1] \quad y_2 = -1$

Since there are only 2 points $\Rightarrow D$ is linearly separable.

$\delta = 0$ is optimal.

for (x_1, y_1) $y_i(\vec{w}^T \vec{x}_i + \theta) \geq 1$

$\Rightarrow 1 \left(\vec{w}^T \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} + \theta \right) \geq 1$

$(w_1 + w_2 + w_3 + \theta) \geq 1$

for (x_2, y_2)

$\Rightarrow -1 \left([w_1 \ w_2 \ w_3] \begin{bmatrix} -1 \\ -1 \\ -1 \end{bmatrix} + \theta \right) \geq 1$

$-1(-w_1 - w_2 - w_3 + \theta) \geq 1$

(x_1, y_1)

$w_1 + w_2 + w_3 + \theta \geq 1$

$w_1 + w_2 + w_3 \geq 1 - \theta$

(x_2, y_2)

$w_1 + w_2 + w_3 - \theta \geq 1$

$w_1 + w_2 + w_3 \geq 1 + \theta$

\therefore Any solution where both above eqs are satisfied works.

~~$w_1 + w_2 + w_3 \geq 1 + \theta$~~

One possible solution is:

$\vec{w}^T = [1 \ 1 \ 1]$ $\theta = 0.5$ $\delta = 0$

This is an optimal solution because it minimizes δ and satisfies statement (2).