HW #2 ~ PROB #3

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a) If there is an optimal solution with $\delta = 0$ then by statement (2)

$$y_i(\vec{\omega}^T \vec{x}_i + \theta) \geq 1 - \delta$$

$$\Rightarrow \delta = 0 \rightarrow \quad y_i(\vec{\omega}^T \vec{x}_i + \theta) \geq 1$$

This means:

$(\vec{\omega}^T \vec{x}_i + \theta) \geq 1 \geq 0$ when $y_i = 1$ & $(x_i, y_i) \in D$

and $(\vec{\omega}^T \vec{x}_i + \theta) \leq -1 < 0$ when $y_i = -1$ & $(x_i, y_i) \in D$

$\therefore$ $(\vec{\omega}^T \vec{x}_i + \theta) \geq 0$ when $y_i = 1$ & $(x_i, y_i) \in D$

and $(\vec{\omega}^T \vec{x}_i + \theta) < 0$ when $y_i = -1$ & $(x_i, y_i) \in D$.

The above statement is equivalent to statement (1) and since it is true this implies statement (1) is true.

$\Rightarrow$ since the conditions satisfy statement (1) D is linearly separable.

$\therefore$ If there is an ~~solution~~ optimal solution with $\delta = 0$ then D is linearly separable. ✓

b) If there exists a hyperplane that satisfies condition (2) w/ $\delta > 0$

then:

if $\delta < 1$ then

$$y_i(\vec{w}^T \vec{x}_i + \theta) \ge 1 - \delta > 0$$

therefore by the same logic as part a:

$$\vec{w}^T \vec{x}_i + \theta \ge 0 \quad \text{if } y_i = 1$$

$$\Rightarrow \; \& \quad \vec{w}^T \vec{x}_i + \theta \le 0 \quad \text{if } y_i = -1$$

which means

$\boxed{\text{if } \delta < 1 \text{ then } D \text{ is linearly separable}}$

However if we know of a $\delta \ge 1$ that satisfies condition 2 we cannot tell if D is linearly separable.

If we know the optimal δ for the set is greater than or equal to 1. this means.

$$y_i(\vec{w}^T \vec{x}_i + \theta) \ge 1 - \delta \le 0$$

$$y_i(\vec{w}^T \vec{x}_i + \theta) \ge 0 \text{ or negative \#}$$

$$(\vec{w}^T \vec{x}_i + \theta) \ge 0 \text{ or negative \#} \quad \text{if } y_i = 1$$

$$(w^T x_i + \theta) \le 0 \text{ or positive \#} \quad \text{if } y_i = -1$$

Since there is overlap, D is not linearly separable.

$\boxed{\text{if the optimal } \delta \ge 1 \text{ then } D \text{ is NOT linearly separable}}$

c) The optimal solution occurs when $\delta = 0$

an optimal solution occurs when

$$y_i(\vec{w}^T \vec{x}_i + \theta) \geq -\delta$$

$$y_i(\vec{w}^T \vec{x}_i + \theta) \geq 0$$

this is satisfied when

$$\vec{w}^T = \vec{0} \quad \& \quad \theta = 0$$

This an issue b/c this formula will output this solution where $\vec{w} = \vec{0}$ & $\theta = 0$ & $\delta = 0$ which does not give a separating hyperplane.

d) $\vec{x}_1^T = [1, 1 \quad 1]$ $y_1 = 1$ $\vec{x}_2^T = [-1 \ -1 \ -1]$ $y_2 = -1$

since there are only 2 points $\Rightarrow$ D is linearly separable

$\delta = 0$ is optimal

for (x_1, y_1)

$$y_i(\vec{w}^T \vec{x}_i + \theta) \geq 1$$

$$\Rightarrow \quad 1\left(\vec{w}^T \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} + \theta\right) \geq 1$$

$$(w_1 + w_2 + w_3 + \theta) \geq 1$$

for (x_2, y_2)

$$\Rightarrow \quad -1\left([w_1 \ w_2 \ w_3] \begin{bmatrix} -1 \\ -1 \\ -1 \end{bmatrix} + \theta\right) \geq 1$$

$$-1(-w_1 - w_2 - w_3 + \theta) \geq 1$$

(x_1, y_1)

$$w_1 + w_2 + w_3 + \theta \geq 1$$

$$w_1 + w_2 + w_3 \geq 1 - \theta$$

(x_2, y_2)

$$w_1 + w_2 + w_3 - \theta \geq 1$$

$$w_1 + w_2 + w_3 \geq 1 + \theta$$

$\therefore$ Any solution where both above eqs are satisfied works.

One possible solution is:

$$\boxed{\begin{array}{l} \vec{w}^T = [1 \quad 1 \quad 1] \\ \theta = 0.5 \\ \delta = 0 \end{array}}$$

This is an optimal solution because it minimizes δ and satisfies statement (2).