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# NCERT Discrete - 11.9.5.21

## EE23BTECH11045 - Palavelli Srija\*

### **Question 11.9.5.21:**

- 1) Find the sum of the following series up to *n* terms:
  - a)  $5 + 55 + 555 + \dots$
  - b) .6 + .66 + .666 + ...

#### **Solution:**

Symbol	Value	Z-Transform
$x_1(n)$	{5,55,555,}	$X_1(z)$
$x_2(n)$	$\{0.6, 0.66, 0.666, \ldots\}$	$X_2(z)$
$s_1(n)$	$\{5 + 55 + 555 + \dots \text{ up to } n \text{ terms}\}$	$S_1(z)$
$s_2(n)$	$\{0.6 + 0.66 + 0.666 + \dots \text{ up to } n \text{ terms}\}\$	$S_2(z)$

# TABLE 1 INPUT PARAMETERS

## 1) For $x_1(n)$ :

$$x_1(n) = 5\left(\frac{10^{n+1} - 1}{10 - 1}\right)u(n) \tag{1}$$

$$x_1(n) \stackrel{\mathcal{Z}}{\longleftrightarrow} X_1(z)$$
 (2)

$$X_1(z) = \frac{50}{9} \left( \frac{1}{1 - 10z^{-1}} \right) - \frac{5}{9} \left( \frac{1}{1 - z^{-1}} \right), \quad |z| > 10$$

(3)

$$s_1(n) = 5 \sum_{i=0}^{n-1} \frac{(10^{i+1} - 1)}{10 - 1}$$
 (4)

$$s_1(n) = \left(5\frac{(10^{n+1} - 1)}{10 - 1}\right) * u(n)$$
 (5)

$$s_1(n) \stackrel{\mathcal{Z}}{\longleftrightarrow} S_1(z)$$
 (6)

# $S_1(z) = \left(\frac{50}{9} \frac{1}{(1 - 10z^{-1})} - \frac{5}{9} \frac{1}{(1 - z^{-1})}\right) \left(\frac{1}{1 - z^{-1}}\right), \quad |z| > 10$

$$= \frac{50}{81} \left( \frac{10}{1 - 10z^{-1}} - \frac{1}{1 - z^{-1}} \right) - \frac{5}{9} \left( \frac{1}{(1 - z^{-1})^2} \right), \quad |z| > 10$$

#### from (??)

$$s_1(n) = \frac{50}{81} (10^{n+1} - 1)u(n) - \frac{5}{9} (n+1)u(n)$$
 (9)  
=  $\frac{5}{81} (10^{n+2} - 9n - 19)u(n)$   $\{n \ge 0\}$   
(10)

2) For  $x_2(n)$ :

$$x_2(n) = 0.6 \left( \frac{1 - 10^{-(n+1)}}{1 - 0.1} \right) u(n)$$
 (11)

$$x_2(n) \stackrel{\mathcal{Z}}{\longleftrightarrow} X_2(z)$$
 (12)

$$X_2(z) = \frac{2}{3} \left( \frac{1}{1 - z^{-1}} \right) - \frac{1}{15} \left( \frac{1}{1 - (10z)^{-1}} \right), \quad |z| > 1$$
(13)

$$s_2(n) = 0.6 \sum_{i=0}^{n-1} \frac{(1 - 10^{-(i+1)})}{1 - 10^{-1}}$$
 (14)

$$s_2(n) = \left(0.6 \frac{(1 - 10^{-(n+1)})}{1 - 0.1}\right) * u(n)$$
 (15)

$$s_2(n) \stackrel{\mathcal{Z}}{\longleftrightarrow} S_2(z)$$
 (16)

$$S_2(z) = \left(\frac{2}{3} \frac{1}{(1 - z^{-1})} - \frac{1}{15} \frac{1}{(1 - (10z)^{-1})}\right) \left(\frac{1}{1 - z^{-1}}\right), \quad |z|$$
(17)

$$=\frac{2}{3}\left(\frac{1}{(1-z^{-1})^2}\right)-\frac{2}{27}\left(\frac{1}{1-z^{-1}}-\frac{10^{-1}}{1-(10z)^{-1}}\right),\tag{18}$$

from (??)

$$s_2(n) = \frac{2}{3}(n+1)u(n) - \frac{2}{27}(1 - 10^{-(n+1)})u(n)$$
(19)

$$= \frac{2}{27} (10^{-(n+1)} + 9n + 8)u(n) \qquad \{n \ge 0\}$$

$$(20)$$

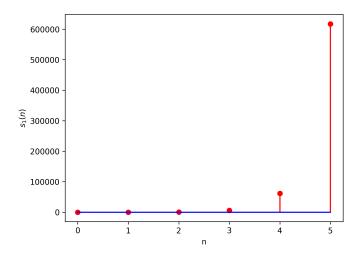


Fig. 2. Stem plot of  $s_1(n)$ 

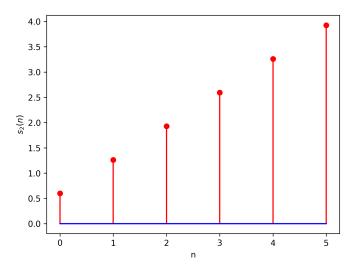


Fig. 2. Stem plot of  $s_2(n)$