NCERT Analog- 12.7.7

EE23BTECH11045 - Palavelli Srija*

Question 12.7.7: A charged $30\mu F$ capacitor is connected to a $27mH$ inductor. What is the angular frequency of free oscillations of the circuit?

Solution:

Symbol	Description	Value
C	Capacitance	$30\mu F$
L	Inductance	$27mH$
ω_0	Angular Frequency	??

TABLE 0
INPUT PARAMETERS

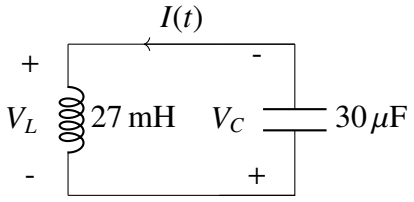

Fig. 0. LC Circuit Diagram

at $t = 0^-$ $\quad V_C = -V_0, \quad I(0) = 0, \quad V_L = V_0$

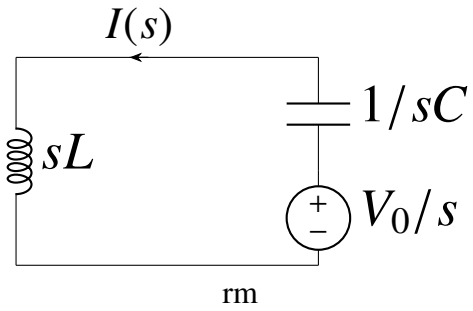

Fig. 0. LC Circuit Diagram in S domain

$$sLI(s) + \frac{1}{sC}I(s) = \frac{V_0}{s} \tag{1}$$

$$I(s) = \frac{V_0 C}{s^2 LC + 1} \tag{2}$$

$$\mathcal{L}^{-1}\{I(s)\} = I(t) \tag{3}$$

$$I(t) = \frac{V_0 C}{\sqrt{LC}} \sin\left(t\frac{1}{\sqrt{LC}}\right) \tag{4}$$

$$I(t) = V_0\sqrt{\frac{C}{L}} \sin\left(t\frac{1}{\sqrt{LC}}\right) \tag{5}$$

Net impedence of LC circuit

$$Z = R_L + R_C \tag{6}$$

$$= Ls + \frac{1}{sC} \tag{7}$$

At resonance, the resistance of capacitor and inductor cancel out as follows:

$$Ls + \frac{1}{sC} = 0 \tag{8}$$

$$\implies s = j\frac{1}{\sqrt{LC}} \tag{9}$$

s can be expressed in terms of angular resonance frequency as

$$s = j\omega_0 \tag{10}$$

on comparing (12) and (13)

$$\omega_0 = \frac{1}{\sqrt{LC}} \tag{11}$$

$$\omega_0 = \frac{1}{\sqrt{(30\times10^{-6})\times(27\times10^{-3})}} \tag{12}$$

$$= \frac{1}{\sqrt{8.1\times10^{-7}}} \tag{13}$$

$$\approx 1.11\times10^{3}\ \text{rad/s} \tag{14}$$