

GATE 2021 EE 47

EE23BTECH11045 - Palavelli Srija*

Question: In the given figure, plant $G_p(s) = \frac{2.2}{(1+0.1s)(1+0.4s)(1+1.2s)}$ and compensator $G_c(s) = K \left(\frac{1+T_1s}{1+T_2s} \right)$. The external disturbance input is $D(s)$. It is desired that when the disturbance is a unit step, the steady-state error should not exceed 0.1 unit. The minimum value of K is

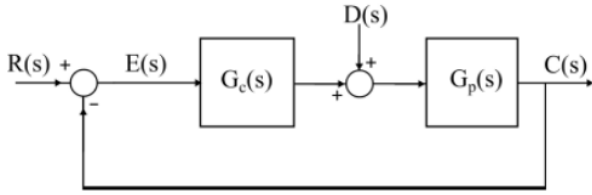


Fig. 0.

Solution:

Symbol	Value
$G_p(s)$	$\frac{2.2}{(1+0.1s)(1+0.4s)(1+1.2s)}$
$G_c(s)$	$K \left(\frac{1+T_1s}{1+T_2s} \right)$
e_{ss}	≤ 0.1
K_{min}	??

TABLE 0

INPUT PARAMETERS

$$D(s) = \mathcal{L}\{u(t)\}$$

$$= \frac{1}{s} \quad (10)$$

$$e_{ss} = \lim_{s \rightarrow 0} \left(\frac{-s \frac{1}{s} G_p(s)}{1 + G_c(s) G_p(s)} \right) \quad (11)$$

$$= \lim_{s \rightarrow 0} \frac{\frac{2.2}{(1+0.1s)(1+0.4s)(1+1.2s)}}{1 + K \left(\frac{1+T_1s}{1+T_2s} \right) \frac{2.2}{(1+0.1s)(1+0.4s)(1+1.2s)}} \quad (12)$$

$$= \lim_{s \rightarrow 0} \frac{2.2(1+T_2s)}{(1+0.1s)(1+0.4s)(1+1.2s)(1+T_2s) + 2.2K(1+T_1s)} \quad (13)$$

$$|e_{ss}| = \frac{2.2}{1 + 2.2K} \quad (14)$$

given

$$e_{ss} \leq 0.1 \quad (15)$$

$$\frac{2.2}{1 + 2.2K} \leq 0.1 \quad (16)$$

$$K \geq 9.54 \quad (17)$$

$$K_{min} = 9.54 \quad (18)$$

From Fig. 0

$$E(s) = R(s) - C(s) \quad (1)$$

Assume $R(s)=0$

$$E(s) = -C(s) \quad (2)$$

$$C(s) = (E(s)G_c(s) + D(s)) G_p(s) \quad (3)$$

$$-E(s) = (E(s)G_c(s) + D(s)) G_p(s) \quad (4)$$

$$E(s) = \frac{-D(s)G_p(s)}{1 + G_c(s)G_p(s)} \quad (5)$$

Using final value theorem

$$e_{ss} = \lim_{t \rightarrow \infty} e(t) = \lim_{s \rightarrow 0} sE(s) \quad (6)$$

Where $\mathcal{L}\{e(t)\} = E(s)$

$$e_{ss} = \lim_{s \rightarrow 0} sE(s) \quad (7)$$

$$= \lim_{s \rightarrow 0} \left(\frac{-sD(s)G_p(s)}{1 + G_c(s)G_p(s)} \right) \quad (8)$$

$$(9)$$