NCERT Discrete - 11.9.5.21

EE23BTECH11045 - Palavelli Srija*

Question 11.9.5.21:

- 1) Find the sum of the following series up to n terms:
 - a) $5 + 55 + 555 + \dots$
 - b) .6 + .66 + .666 + ...

Solution:

Symbol	Value	Z-Transform
$x_1(n)$	{5,55,555,}	$X_1(z)$
$x_2(n)$	$\{0.6, 0.66, 0.666, \ldots\}$	$X_2(z)$
$s_1(n)$	$\{5 + 55 + 555 + \dots \text{ up to } n \text{ terms}\}$	$S_1(z)$
$s_2(n)$	$\{0.6 + 0.66 + 0.666 + \dots \text{ up to } n \text{ terms}\}\$	$S_2(z)$

TABLE 1 INPUT PARAMETERS

1) For $x_1(n)$:

$$x_1(n) = 5\left(\frac{10^{n+1} - 1}{10 - 1}\right)u(n) \tag{1}$$

$$x_1(n) \stackrel{\mathcal{Z}}{\longleftrightarrow} X_1(z)$$
 (2)

$$X_1(z) = \frac{50}{9} \left(\frac{1}{1 - 10z^{-1}} \right) - \frac{5}{9} \left(\frac{1}{1 - z^{-1}} \right), |z| > 10$$
(3)

$$s_1(n) = 5 \sum_{i=0}^{n-1} \frac{(10^{i+1} - 1)}{10 - 1}$$
 (4)

$$s_1(n) = \left(5\frac{(10^{n+1} - 1)}{10 - 1}\right) * u(n)$$
 (5)

$$s_1(n) \stackrel{\mathcal{Z}}{\longleftrightarrow} S_1(z)$$
 (6)

$$S_1(z) = \left(\frac{50}{9} \frac{1}{(1 - 10z^{-1})} - \frac{5}{9} \frac{1}{(1 - z^{-1})}\right) \left(\frac{1}{1 - z^{-1}}\right), |z| > 10$$
(7)

$$= \frac{50}{81} \left(\frac{10}{1 - 10z^{-1}} - \frac{1}{1 - z^{-1}} \right) - \frac{5}{9} \left(\frac{1}{(1 - z^{-1})^2} \right), |z| > 10$$
(8)

from (??)

$$s_1(n) = \frac{50}{81} (10^{n+1} - 1)u(n) - \frac{5}{9} (n+1)u(n)$$
 (9)
= $\frac{5}{81} (10^{n+2} - 9n - 19)u(n)$ $\{n \ge 0\}$
(10)

2) For $x_2(n)$:

$$x_2(n) = 0.6 \left(\frac{1 - 10^{-(n+1)}}{1 - 0.1} \right) u(n)$$
 (11)

$$x_2(n) \stackrel{\mathcal{Z}}{\longleftrightarrow} X_2(z)$$
 (12)

$$X_2(z) = \frac{2}{3} \left(\frac{1}{1 - z^{-1}} \right) - \frac{1}{15} \left(\frac{1}{1 - (10z)^{-1}} \right), |z| > 1$$
(13)

$$s_2(n) = 0.6 \sum_{i=0}^{n-1} \frac{(1 - 10^{-(i+1)})}{1 - 10^{-1}}$$
 (14)

$$s_2(n) = \left(0.6 \frac{(1 - 10^{-(n+1)})}{1 - 0.1}\right) * u(n)$$
 (15)

$$s_2(n) \stackrel{\mathcal{Z}}{\longleftrightarrow} S_2(z)$$
 (16)

$$S_2(z) = \left(\frac{2}{3} \frac{1}{(1 - z^{-1})} - \frac{1}{15} \frac{1}{(1 - (10z)^{-1})}\right) \left(\frac{1}{1 - z^{-1}}\right), |z| > 1$$
(17)

$$= \frac{2}{3} \left(\frac{1}{(1-z^{-1})^2} \right)$$

$$- \frac{2}{27} \left(\frac{1}{1-z^{-1}} - \frac{10^{-1}}{1-(10z)^{-1}} \right), |z| > 1$$
(18)

from (??)

$$s_2(n) = \frac{2}{3}(n+1)u(n) - \frac{2}{27}(1 - 10^{-(n+1)})u(n)$$
(19)

$$= \frac{2}{27}(10^{-(n+1)} + 9n + 8)u(n) \qquad \{n \ge 0\}$$

$$\frac{2}{27}(10^{-(n+1)} + 9n + 8)u(n) \qquad \{n \ge 0\}$$
(20)

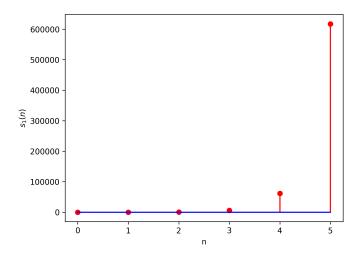


Fig. 2. Stem plot of $s_1(n)$

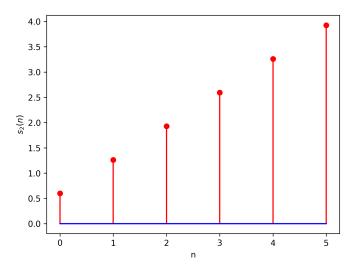


Fig. 2. Stem plot of $s_2(n)$