

NCERT Analog- 12.7.7

EE23BTECH11045 - Palavelli Srija*

Question 12.7.7: A charged $30\mu F$ capacitor is connected to a $27mH$ inductor. What is the angular frequency of free oscillations of the circuit?

Solution:

Symbol	Description	Value
C	Capacitance	$30\mu F$
L	Inductance	$27mH$
ω_0	Angular Frequency	??

TABLE 0
INPUT PARAMETERS

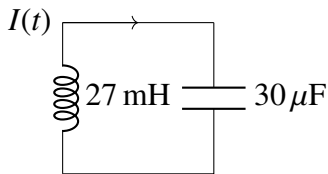


Fig. 0. LC Circuit Diagram

The differential equation is given by:

$$L \frac{dI}{dt} + \frac{1}{C} \int I dt = 0 \quad (1)$$

$$\frac{d}{dt} \left(L \frac{dI}{dt} + \frac{1}{C} \int I dt \right) = 0 \quad (2)$$

$$\frac{d^2 I}{dt^2} L + \frac{I}{C} = 0 \quad (3)$$

$$I = V_0 \sqrt{\frac{L}{C}} \sin\left(\frac{1}{\sqrt{LC}} t\right) \quad (4)$$

V_0 is the starting voltage on the capacitor.

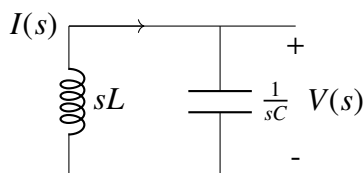


Fig. 0. LC Circuit Diagram

Net impedance of LC circuit

$$Z = R_L + R_C \quad (5)$$

$$= Ls + \frac{1}{sC} \quad (6)$$

At resonance, the resistance of capacitor and inductor cancel out as follows:

$$Ls + \frac{1}{sC} = 0 \quad (7)$$

$$\Rightarrow s = j \frac{1}{\sqrt{LC}} \quad (8)$$

s can be expressed in terms of angular resonance frequency as

$$s = j\omega_0 \quad (9)$$

on comparing (6) and (7)

$$\omega_0 = \frac{1}{\sqrt{LC}} \quad (10)$$

$$\omega_0 = \frac{1}{\sqrt{(30 \times 10^{-6}) \times (27 \times 10^{-3})}} \quad (11)$$

$$= \frac{1}{\sqrt{8.1 \times 10^{-7}}} \quad (12)$$

$$\approx 1.11 \times 10^3 \text{ rad/s} \quad (13)$$