

NCERT Discrete - 11.9.5.21

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Question 11.9.5.21:

1) Find the sum of the following series up to n terms:

a) $5 + 55 + 555 + \dots$

b) $.6 + .66 + .666 + \dots$

Solution:

Symbol	Value	Z-Transform
$x_1(n)$	$\{5, 55, 555, \dots\}$	$X_1(z)$
$x_2(n)$	$\{0.6, 0.66, 0.666, \dots\}$	$X_2(z)$
$s_1(n)$	$\{5 + 55 + 555 + \dots \text{ up to } n \text{ terms}\}$	$S_1(z)$
$s_2(n)$	$\{0.6 + 0.66 + 0.666 + \dots \text{ up to } n \text{ terms}\}$	$S_2(z)$

TABLE 1
INPUT PARAMETERS

1) For $x_1(n)$:

$$x_1(n) = 5 \left(\frac{10^{n+1} - 1}{10 - 1} \right) u(n) \quad (1)$$

$$x_1(n) \xleftrightarrow{Z} X_1(z) \quad (2)$$

$$X_1(z) = \frac{50}{9} \left(\frac{1}{1 - 10z^{-1}} \right) - \frac{5}{9} \left(\frac{1}{1 - z^{-1}} \right) \quad (3)$$

$$s_1(n) = 5 \sum_{i=0}^{n-1} \frac{(10^{i+1} - 1)}{10 - 1} \quad (4)$$

$$s_1(n) = \left(5 \frac{(10^{n+1} - 1)}{10 - 1} \right) * u(n) \quad (5)$$

$$s_1(n) \xleftrightarrow{Z} S_1(z) \quad (6)$$

$$S_1(z) = \left(\frac{50}{9} \frac{1}{(1 - 10z^{-1})} - \frac{5}{9} \frac{1}{(1 - z^{-1})} \right) \left(\frac{1}{1 - z^{-1}} \right) \quad (7)$$

$$= \frac{50}{81} \left(\frac{10}{1 - 10z^{-1}} - \frac{1}{1 - z^{-1}} \right) - \frac{5}{9} \left(\frac{1}{(1 - z^{-1})^2} \right) \quad (8)$$

from (??)

$$s_1(n) = \frac{50}{81} (10^{n+1} - 1) u(n) - \frac{5}{9} (n + 1) u(n) \quad (9)$$

$$= \frac{5}{81} (10^{n+2} - 9n - 19) u(n) \quad \{n \geq 0\} \quad (10)$$

2) For $x_2(n)$:

$$x_2(n) = 0.6 \left(\frac{1 - 10^{-(n+1)}}{1 - 0.1} \right) u(n) \quad (11)$$

$$x_2(n) \xleftrightarrow{Z} X_2(z) \quad (12)$$

$$X_2(z) = \frac{2}{3} \left(\frac{1}{1 - z^{-1}} \right) - \frac{1}{15} \left(\frac{1}{1 - (10z)^{-1}} \right) \quad (13)$$

$$s_2(n) = 0.6 \sum_{i=0}^{n-1} \frac{(1 - 10^{-(i+1)})}{1 - 10^{-1}} \quad (14)$$

$$s_2(n) = \left(0.6 \frac{(1 - 10^{-(n+1)})}{1 - 0.1} \right) * u(n) \quad (15)$$

$$s_2(n) \xleftrightarrow{Z} S_2(z) \quad (16)$$

$$S_2(z) = \left(\frac{2}{3} \frac{1}{(1 - z^{-1})} - \frac{1}{15} \frac{1}{(1 - (10z)^{-1})} \right) \left(\frac{1}{1 - z^{-1}} \right) \quad (17)$$

$$= \frac{2}{3} \left(\frac{1}{(1 - z^{-1})^2} \right) - \frac{2}{27} \left(\frac{1}{1 - z^{-1}} - \frac{10^{-1}}{1 - (10z)^{-1}} \right) \quad (18)$$

from (??)

$$s_2(n) = \frac{2}{3} (n + 1) u(n) - \frac{2}{27} (1 - 10^{-(n+1)}) u(n) \quad (19)$$

$$= \frac{2}{27} (10^{-(n+1)} + 9n + 8) u(n) \quad \{n \geq 0\} \quad (20)$$

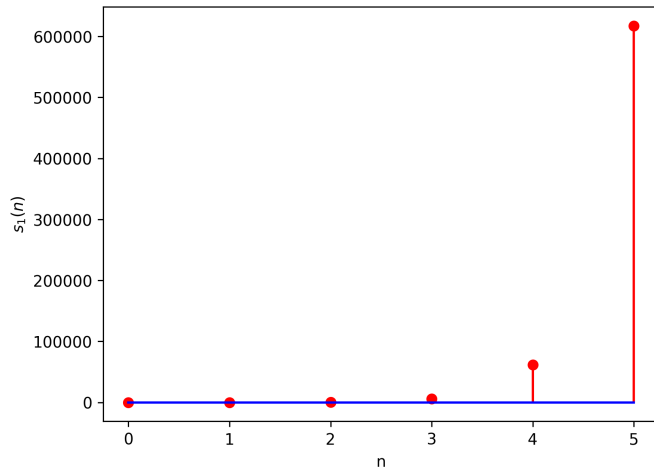


Fig. 2. Stem plot of $s_1(n)$

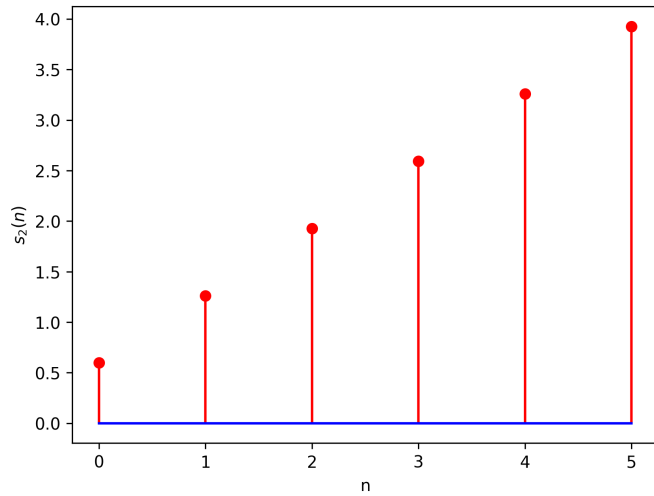


Fig. 2. Stem plot of $s_2(n)$