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NCERT Analog- 12.7.7

EE23BTECH11045 - Palavelli Srija*

Question 12.7.7: A charged $30\mu F$ capacitor is connected to a 27mH inductor. What is the angular frequency of free oscillations of the circuit? **Solution:**

Symbol	Description	Value
C	Capacitance	$30\mu F$
L	Inductance	27 <i>mH</i>
ω_0	Angular Frequency	??
TABLE 0		

INPUT PARAMETERS

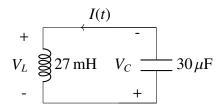


Fig. 0. LC Circuit Diagram

at
$$t = 0^ V_C = -V_0$$
, $I(0) = 0$, $V_L = V_0$

$$I(t) = A_1 \cos\left(\frac{1}{\sqrt{LC}}t\right) + A_2 \sin\left(\frac{1}{\sqrt{LC}}t\right)$$
 (1)

at
$$t = 0$$
, $A_1 = 0$ (2)

$$I(t) = A_2 \sin\left(\frac{1}{\sqrt{IC}}t\right) \tag{3}$$

$$\frac{dI}{dt}(0^+) = \frac{V_0}{I} \tag{4}$$

$$\frac{dI}{dt} = A_2 \frac{1}{\sqrt{IC}} \cos\left(\frac{1}{\sqrt{IC}}t\right) \tag{5}$$

$$\frac{V_0}{L} = A_2 \frac{1}{\sqrt{LC}} \tag{6}$$

$$A_2 = V_0 \sqrt{\frac{C}{L}} \tag{7}$$

$$I(t) = V_0 \sqrt{\frac{C}{L}} \sin\left(\frac{1}{\sqrt{LC}}t\right) \tag{8}$$

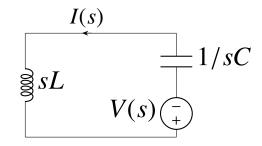


Fig. 0. LC Circuit Diagram in S domain

Net impedence of LC circuit

$$Z = R_L + R_C \tag{9}$$

$$= Ls + \frac{1}{sC} \tag{10}$$

At resonance, the resistance of capacitor and inductor cancel out as follows:

$$Ls + \frac{1}{sC} = 0 \tag{11}$$

$$\implies s = j \frac{1}{\sqrt{LC}} \tag{12}$$

s can be expressed in terms of angular resonance frequency as

$$s = j\omega_0 \tag{13}$$

on comparing (12) and (13)

$$\omega_0 = \frac{1}{\sqrt{LC}} \tag{14}$$

$$\omega_0 = \frac{1}{\sqrt{(30 \times 10^{-6}) \times (27 \times 10^{-3})}}$$
 (15)

$$=\frac{1}{\sqrt{8.1\times10^{-7}}}\tag{16}$$

$$\approx 1.11 \times 10^3 \,\text{rad/s} \tag{17}$$