NCERT Analog- 12.7.7

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Question 12.7.7: Let $x(t) = 10\cos(10.5\omega t)$ be passed through an LTI system with impulse response $h(t) = \pi\left(\frac{\sin(\omega t)}{\pi t}\right)^2 \cos(10\omega t)$. The output of the system is:
(GATE EC 2023)

Solution

Given $h(t)$ is Real and Even. When a sinusoidal input is applied to an LTI system with an even impulse response, the output will also be sinusoidal.

$x(t) \longrightarrow \boxed{h(t)} \longrightarrow y(t)$

$$y(t) = H(\omega)\Big|_{\omega=10.5\omega} \cdot 10\cos(10.5\omega t) \tag{1}$$

$$h(t) = f(t)\cos(10\omega t) \tag{2}$$

$$f(t) = \pi\left(\frac{\sin(\omega t)}{\pi t}\right)^2 \tag{3}$$

$$\tag{4}$$

The Fourier transform of $f(t)$:

$$f(t) \overset{\mathcal{F}}{\longleftrightarrow} F(\omega) \tag{5}$$

$$F(\omega) = \pi\int_{-\infty}^{\infty}\left(\frac{\sin(\omega t)}{\pi t}\right)^2 e^{-j\omega t}\,dt \tag{6}$$

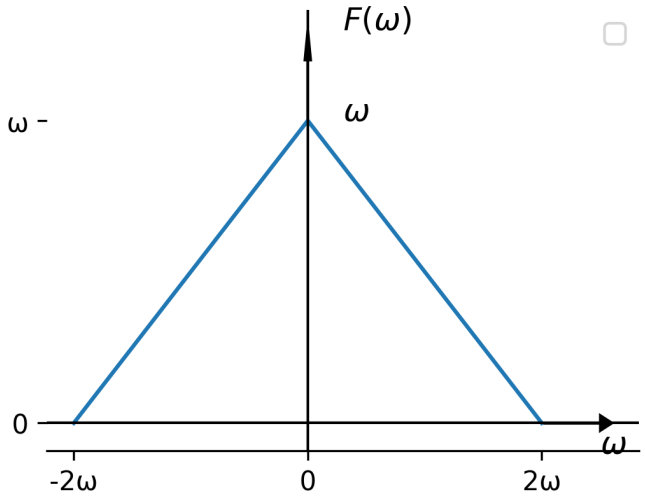

Fig. 0

The frequency response $H(\omega)$:

$$H(\omega) = \frac{1}{2}\left[F(\omega + 10\omega) + F(\omega - 10\omega)\right] \tag{7}$$

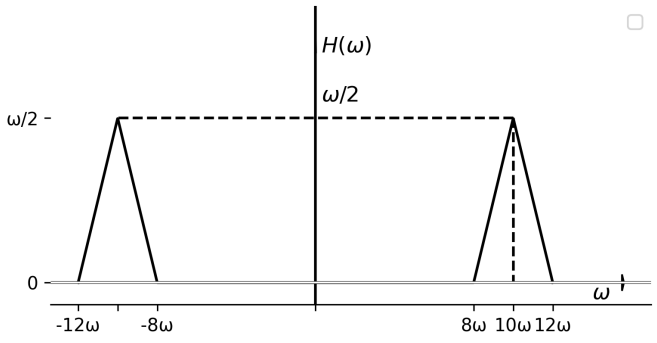

Fig. 0

$$H(\omega)\Big|_{\omega=10.5\omega} = \frac{3}{8}\omega \tag{8}$$

The output $y(t)$:

$$y(t) = \frac{3}{8}\omega \cdot 10\cos(10.5\omega t) \tag{9}$$

$$= \frac{15}{4}\omega\cos(10.5\omega t) \tag{10}$$