

NCERT Analog- 12.7.7

EE23BTECH11045 - Palavelli Srija*

Question 12.7.7: A charged $30\mu F$ capacitor is connected to a $27mH$ inductor. What is the angular frequency of free oscillations of the circuit?

Solution:

Symbol	Description	Value
C	Capacitance	$30\mu F$
L	Inductance	$27mH$
ω_0	Angular Frequency	??

TABLE 0
INPUT PARAMETERS

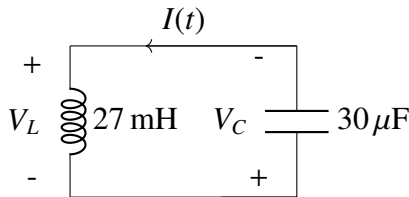


Fig. 0. LC Circuit Diagram

$$\text{at } t = 0^- \quad V_C = -V_0, \quad I(0) = 0, \quad V_L = V_0$$

$$I(t) = A_1 \cos\left(\frac{1}{\sqrt{LC}}t\right) + A_2 \sin\left(\frac{1}{\sqrt{LC}}t\right) \quad (1)$$

$$\text{at } t = 0, \quad A_1 = 0 \quad (2)$$

$$I(t) = A_2 \sin\left(\frac{1}{\sqrt{LC}}t\right) \quad (3)$$

$$\frac{dI}{dt}(0^+) = \frac{V_0}{L} \quad (4)$$

$$\frac{dI}{dt} = A_2 \frac{1}{\sqrt{LC}} \cos\left(\frac{1}{\sqrt{LC}}t\right) \quad (5)$$

$$\frac{V_0}{L} = A_2 \frac{1}{\sqrt{LC}} \quad (6)$$

$$A_2 = V_0 \sqrt{\frac{C}{L}} \quad (7)$$

$$I(t) = V_0 \sqrt{\frac{C}{L}} \sin\left(\frac{1}{\sqrt{LC}}t\right) \quad (8)$$

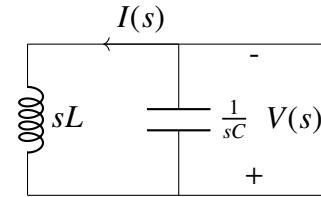


Fig. 0. LC Circuit Diagram in S domain

Net impedance of LC circuit

$$Z = R_L + R_C \quad (9)$$

$$= Ls + \frac{1}{sC} \quad (10)$$

At resonance, the resistance of capacitor and inductor cancel out as follows:

$$Ls + \frac{1}{sC} = 0 \quad (11)$$

$$\Rightarrow s = j \frac{1}{\sqrt{LC}} \quad (12)$$

s can be expressed in terms of angular resonance frequency as

$$s = j\omega_0 \quad (13)$$

on comparing (12) and (13)

$$\omega_0 = \frac{1}{\sqrt{LC}} \quad (14)$$

$$\omega_0 = \frac{1}{\sqrt{(30 \times 10^{-6}) \times (27 \times 10^{-3})}} \quad (15)$$

$$= \frac{1}{\sqrt{8.1 \times 10^{-7}}} \quad (16)$$

$$\approx 1.11 \times 10^3 \text{ rad/s} \quad (17)$$