NCERT Discrete - 11.9.5.21

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Question 11.9.5.21:

1) Find the sum of the following series up to n terms:
   a) $5 + 55 + 555 + \ldots$
   b) $.6 + .66 + .666 + \ldots$

Solution:

Parameter	Description	Value
$x_1(0)$	first term of series 1	5
$x_2(0)$	first term of series 2	0.6
$x_1(n)$	(n+1)th term of series 1	$5(\frac{10^{n+1}-1}{10-1})u(n)$
$x_2(n)$	(n+1)th term of series 2	$0.6(\frac{1-10^{-(n+1)}}{1-0.1})u(n)$
$s_1(n)$	sum of n terms of series 1	??
$s_2(n)$	sum of n terms of series 2	??

TABLE 1
INPUT PARAMETERS

$$x_1(n) = 5\left(\frac{10^{n+1}-1}{10-1}\right)u(n) \tag{1}$$

$$x_1(n) \overset{Z}{\leftrightarrow} X_1(Z) \tag{2}$$

$$X_1(Z) = \frac{50}{9}\left(\frac{1}{1-10z^{-1}}\right) - \frac{5}{9}\left(\frac{1}{1-z^{-1}}\right) \tag{3}$$

$$s_1(n) = 5\sum_{i=0}^{n-1}\frac{(10^{i+1}-1)}{10-1} \tag{4}$$

$$s_1(n) = 5\frac{(10^{n+1}-1)}{10-1} * u(n) \tag{5}$$

$$s_1(n) \overset{Z}{\leftrightarrow} S_1(Z) \tag{6}$$

$$S_1(Z) = \left(\frac{50}{9}\frac{1}{(1-10z^{-1})} - \frac{5}{9}\frac{1}{(1-z^{-1})}\right)\left(\frac{1}{1-z^{-1}}\right) \tag{7}$$

$$= \frac{50}{81}\left(\frac{10}{1-10z^{-1}} - \frac{1}{1-z^{-1}}\right) - \frac{5}{9}\left(\frac{1}{(1-z^{-1})^2}\right) \tag{8}$$

(9)

from (**??**)

$$s_1(n) = \frac{50}{81}(10^{n+1}-1)u(n) - \frac{5}{9}(n+1)u(n) \tag{10}$$

$$= \frac{5}{81}(10^{n+2}-9n-19)u(n) \tag{11}$$

$$x_2(n) = 0.6\left(\frac{1-10^{-(n+1)}}{1-0.1}\right)u(n) \tag{12}$$

$$x_2(n) \overset{Z}{\leftrightarrow} X_2(Z) \tag{13}$$

$$X_2(Z) = \frac{2}{3}\left(\frac{1}{1-z^{-1}}\right) - \frac{1}{15}\left(\frac{1}{1-(10z)^{-1}}\right) \tag{14}$$

$$s_2(n) = 0.6\sum_{i=0}^{n-1}\frac{(1-10^{-(i+1)})}{1-10^{-1}} \tag{15}$$

$$s_2(n) = 0.6\frac{1-10^{-(n+1)}}{1-0.1} * u(n) \tag{16}$$

$$s_2(n) \overset{Z}{\leftrightarrow} S_2(Z) \tag{17}$$

$$S_2(Z) = \left(\frac{2}{3}\frac{1}{(1-z^{-1})} - \frac{1}{15}\frac{1}{(1-(10z)^{-1})}\right)\left(\frac{1}{1-z^{-1}}\right) \tag{18}$$

$$= \frac{2}{3}\left(\frac{1}{(1-z^{-1})^2}\right) - \frac{2}{27}\left(\frac{1}{1-z^{-1}} - \frac{10^{-1}}{1-(10z)^{-1}}\right) \tag{19}$$

(20)

from (**??**)

$$s_2(n) = \frac{2}{27}(1-10^{-(n+1)})u(n) - \frac{2}{3}(n+1)u(n) \tag{21}$$

$$= \frac{2}{27}(10^{-(n+1)}+9n+8)u(n) \tag{22}$$

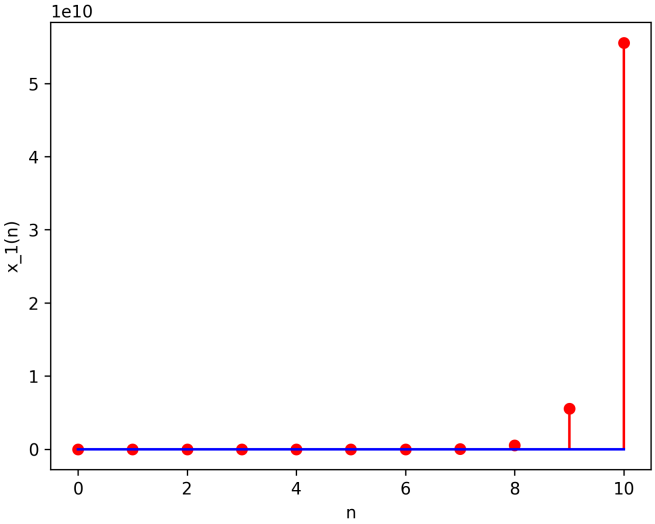

Fig. 1. Stem plot of $x(n)$

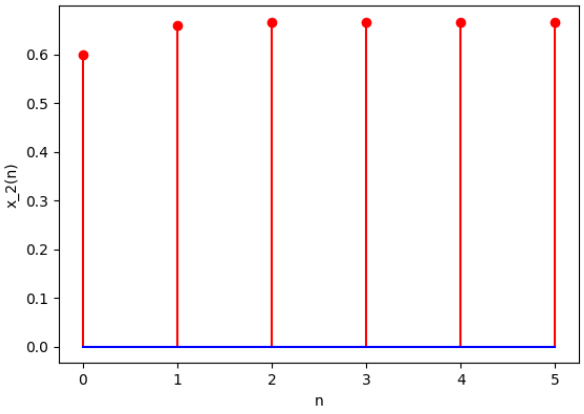

Fig. 1. Stem plot of $x_2(n)$