GATE 2023 EC 49

EE23BTECH11045 - Palavelli Srija*

Question 12.7.7: Let $x(t) = 10\cos(10.5\omega t)$ be passed through an LTI system with impulse response $h(t) = \pi\left(\frac{\sin(\omega t)}{\pi t}\right)^2 \cos(10\omega t)$. The output of the system is:

Solution:

Symbol	Description	Value
$x(t)$	input	$10\cos(10.5\omega t)$
$h(t)$	impulse	$\pi\left(\frac{\sin(\omega t)}{\pi t}\right)^2 \cos(10\omega t)$
$y(t)$	output	??

TABLE 0: Input Parameters

Given $h(t)$ is real and even. When a sinusoidal input is applied to an LTI system with an even impulse response, the output will also be sinusoidal. Hence, $y(t) = A \cdot 10\cos(10.5\omega t + \theta)$.

$$x(t) \rightarrow \boxed{\text{h(t)}} \rightarrow y(t)$$

$$\text{Let } f(t) = \pi\left(\frac{\sin(\omega t)}{\pi t}\right)^2 \tag{1}$$

$$h(t) = f(t)\cos(10\omega t) \tag{2}$$

Using

$$x_1(t) \cdot x_2(t) \overset{\mathcal{F}}{\longleftrightarrow} X_1(\omega) * X_2(\omega) \tag{3}$$

$$\left(\frac{\sin(\omega t)}{\pi t}\right) \cdot \left(\frac{\sin(\omega t)}{\pi t}\right) \overset{\mathcal{F}}{\longleftrightarrow} X_1(\omega) * X_2(\omega) \tag{4}$$

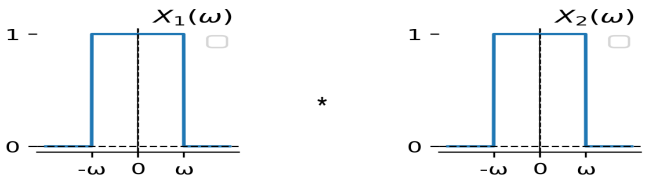

Fig. 0

$$\left(\frac{\sin(\omega t)}{\pi t}\right)^2 \overset{\mathcal{F}}{\longleftrightarrow} X_3(\omega) \tag{5}$$

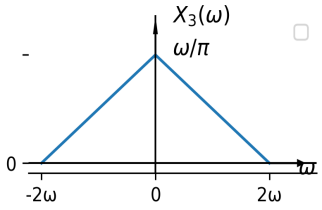

Fig. 0

$$\pi\left(\frac{\sin(\omega t)}{\pi t}\right)^2 \overset{\mathcal{F}}{\longleftrightarrow} X_4(\omega) \tag{6}$$

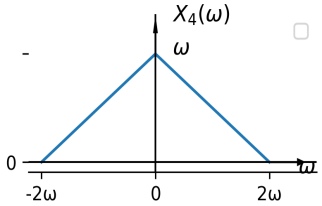

Fig. 0

From modulating property:

$$f(t)\cos(\omega_0 t) \overset{\mathcal{F}}{\longleftrightarrow} \frac{1}{2}\left[F(\omega+\omega_0) + F(\omega-\omega_0)\right] \tag{7}$$

$$H(\omega) = \frac{1}{2}\left[F(\omega+10\omega) + F(\omega-10\omega)\right] \tag{8}$$

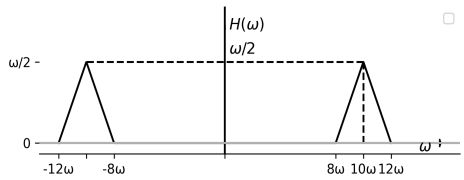

Fig. 0

$$\frac{\frac{\omega}{2} - 0}{10\omega - 12\omega} = \frac{|H(10.5\omega)| - 0}{10.5\omega - 12\omega} \tag{9}$$

$$A = |H(10.5\omega)| = \frac{3}{8}\omega \quad \text{and} \quad \theta = \angle H(10.5\omega) = 0^\circ \tag{10}$$

The output $y(t)$:

$$y(t) = \frac{3}{8}\omega \cdot 10\cos(10.5\omega t) \tag{11}$$

$$= \frac{15}{4}\omega\cos(10.5\omega t) \tag{12}$$

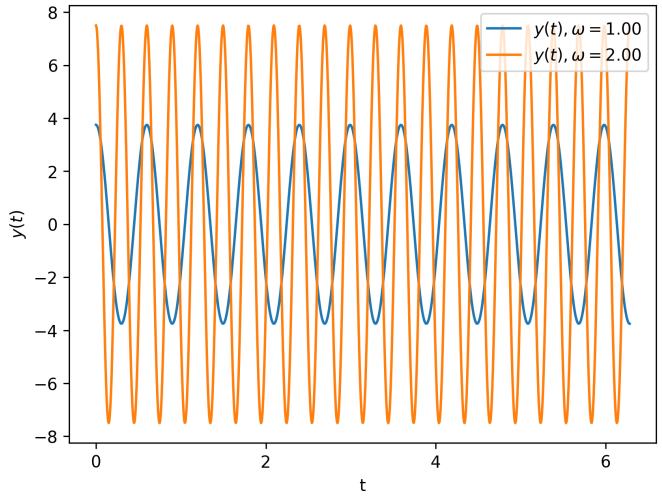

Fig. 0