

NCERT Analog- 12.7.7

EE23BTECH11045 - Palavelli Srija*

Question 12.7.7: A charged $30\mu F$ capacitor is connected to a $27mH$ inductor. What is the angular frequency of free oscillations of the circuit?

Solution:

Symbol	Description	Value
C	Capacitance	$30\mu F$
L	Inductance	$27mH$
ω_0	Angular Frequency	??

TABLE 0
INPUT PARAMETERS

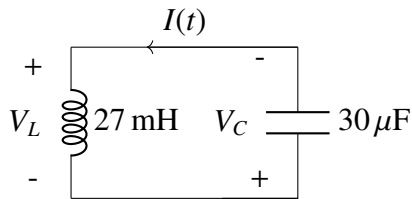


Fig. 0. LC Circuit Diagram

at $t = 0^-$ $V_C = -V_0$, $I(0) = 0$, $V_L = V_0$

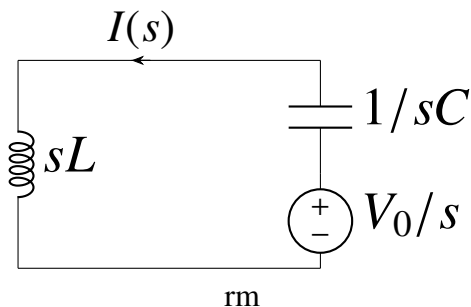


Fig. 0. LC Circuit Diagram in S domain

$$sLI(s) + \frac{1}{sC}I(s) = \frac{V_0}{s} \quad (1)$$

$$I(s) = \frac{V_0 C}{s^2 LC + 1} \quad (2)$$

$$\mathcal{L}^{-1}\{I(s)\} = I(t) \quad (3)$$

$$I(t) = \frac{V_0 C}{\sqrt{LC}} \sin\left(t \frac{1}{\sqrt{LC}}\right) \quad (4)$$

$$I(t) = V_0 \sqrt{\frac{C}{L}} \sin\left(t \frac{1}{\sqrt{LC}}\right) \quad (5)$$

Net impedance of LC circuit

$$Z = R_L + R_C \quad (6)$$

$$= Ls + \frac{1}{sC} \quad (7)$$

At resonance, the resistance of capacitor and inductor cancel out as follows:

$$Ls + \frac{1}{sC} = 0 \quad (8)$$

$$\Rightarrow s = j \frac{1}{\sqrt{LC}} \quad (9)$$

s can be expressed in terms of angular resonance frequency as

$$s = j\omega_0 \quad (10)$$

on comparing (9) and (10)

$$\omega_0 = \frac{1}{\sqrt{LC}} \quad (11)$$

$$\omega_0 = \frac{1}{\sqrt{(30 \times 10^{-6}) \times (27 \times 10^{-3})}} \quad (12)$$

$$= \frac{1}{\sqrt{8.1 \times 10^{-7}}} \quad (13)$$

$$\approx 1.11 \times 10^3 \text{ rad/s} \quad (14)$$