NCERT Analog- 12.7.7

EE23BTECH11045 - Palavelli Srija*

Question 12.7.7: A charged $30\mu F$ capacitor is connected to a $27mH$ inductor. What is the angular frequency of free oscillations of the circuit?

Solution:

Symbol	Description	Value
C	Capacitance	$30\mu F$
L	Inductance	$27mH$
ω_0	Angular Frequency	??

TABLE 0
INPUT PARAMETERS

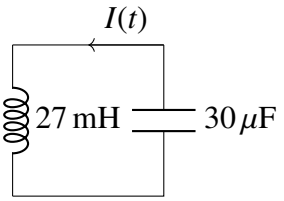

Fig. 0. LC Circuit Diagram

The differential equation is given by:

$$L\frac{dI}{dt} + \frac{1}{C}\int I\,dt = 0 \tag{1}$$

$$\frac{d}{dt}(L\frac{dI}{dt} + \frac{1}{C}\int I\,dt) = 0 \tag{2}$$

$$\frac{d^2I}{dt^2}L + \frac{I}{C} = 0 \tag{3}$$

$$I(t) = V_0\sqrt{\frac{C}{L}}\sin(\frac{1}{\sqrt{LC}}t) \tag{4}$$

V_0 is the starting voltage on the capacitor.

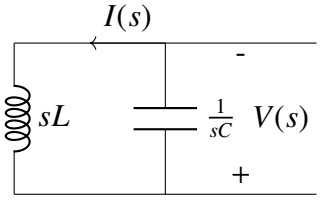

Fig. 0. LC Circuit Diagram

Net impedence of LC circuit

$$Z = R_L + R_C \tag{5}$$

$$= Ls + \frac{1}{sC} \tag{6}$$

At resonance, the resistance of capacitor and inductor cancel out as follows:

$$Ls + \frac{1}{sC} = 0 \tag{7}$$

$$\implies s = j\frac{1}{\sqrt{LC}} \tag{8}$$

s can be expressed in terms of angular resonance frequency as

$$s = j\omega_0 \tag{9}$$

on comparing (6) and (7)

$$\omega_0 = \frac{1}{\sqrt{LC}} \tag{10}$$

$$\omega_0 = \frac{1}{\sqrt{(30\times10^{-6})\times(27\times10^{-3})}} \tag{11}$$

$$= \frac{1}{\sqrt{8.1\times10^{-7}}} \tag{12}$$

$$\approx 1.11\times10^{3}\ \text{rad/s} \tag{13}$$