GATE 2023 EC 49

EE23BTECH11045 - Palavelli Srija*

Question 12.7.7: Let $x(t) = 10\cos(10.5\omega t)$ be passed through an LTI system with impulse response $h(t) = \pi\left(\frac{\sin(\omega t)}{\pi t}\right)^2 \cos(10\omega t)$. The output of the system is:

Solution:

Symbol	Description	Value
$x(t)$	input	$10\cos(10.5\omega t)$
$h(t)$	impulse	$\pi\left(\frac{\sin(\omega t)}{\pi t}\right)^2 \cos(10\omega t)$
$y(t)$	output	??

TABLE 0: Input Parameters

Given $h(t)$ is Real and Even.

$$x(t) \rightarrow \boxed{h(t)} \rightarrow y(t)$$

$$y(t) = H(\omega)\Big|_{\omega=10.5\omega} \cdot 10\cos(10.5\omega t) \quad (1)$$

$$h(t) = f(t)\cos(10\omega t) \quad (2)$$

$$f(t) = \pi\left(\frac{\sin(\omega t)}{\pi t}\right)^2 \quad (3)$$

The Fourier transform of $f(t)$:

$$f(t) \overset{\mathcal{F}}{\longleftrightarrow} F(\omega) \quad (4)$$

$$F(\omega) = \pi \int_{-\infty}^{\infty} \left(\frac{\sin(\omega t)}{\pi t}\right)^2 e^{-j\omega t}\, dt \quad (5)$$

Fig. 0
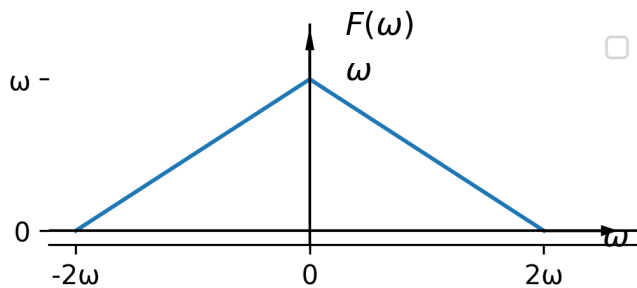
The frequency response $H(\omega)$:

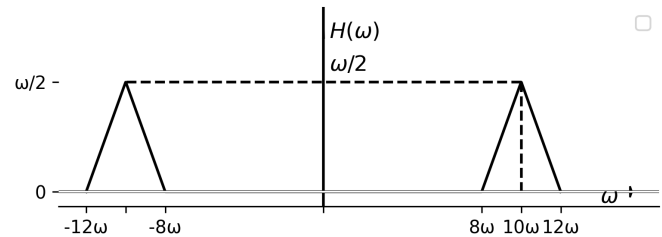$$H(\omega) = \frac{1}{2}\left[F(\omega + 10\omega) + F(\omega - 10\omega)\right] \quad (6)$$

Fig. 0

$$H(\omega)\Big|_{\omega=10.5\omega} = \frac{3}{8}\omega \quad (7)$$

The output $y(t)$:

$$y(t) = \frac{3}{8}\omega \cdot 10\cos(10.5\omega t) \quad (8)$$

$$= \frac{15}{4}\omega\cos(10.5\omega t) \quad (9)$$