NCERT Analog- 12.7.7

EE23BTECH11045 - Palavelli Srija*

Question 12.7.7: A charged $30\mu F$ capacitor is connected to a $27mH$ inductor. What is the angular frequency of free oscillations of the circuit?

Solution:

Symbol	Description	Value
C	Capacitance	$30\mu F$
L	Inductance	$27mH$
ω_0	Angular Frequency	??

TABLE 0
INPUT PARAMETERS

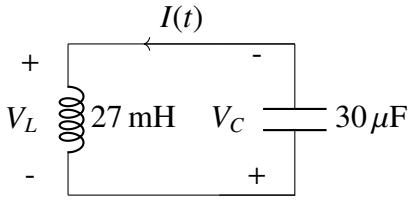

Fig. 0. LC Circuit Diagram

$$\text{at } t = 0^- \quad V_C = -V_0, \quad I(0) = 0, \quad V_L = V_0$$

$$I(t) = A_1 \cos\left(\frac{1}{\sqrt{LC}}t\right) + A_2 \sin\left(\frac{1}{\sqrt{LC}}t\right) \tag{1}$$

$$\text{at } t = 0, \quad A_1 = 0 \tag{2}$$

$$I(t) = A_2 \sin\left(\frac{1}{\sqrt{LC}}t\right) \tag{3}$$

$$\frac{dI}{dt}(0^+) = \frac{V_0}{L} \tag{4}$$

$$\frac{dI}{dt} = A_2 \frac{1}{\sqrt{LC}} \cos\left(\frac{1}{\sqrt{LC}}t\right) \tag{5}$$

$$\frac{V_0}{L} = A_2 \frac{1}{\sqrt{LC}} \tag{6}$$

$$A_2 = V_0 \sqrt{\frac{C}{L}} \tag{7}$$

$$I(t) = V_0 \sqrt{\frac{C}{L}} \sin\left(\frac{1}{\sqrt{LC}}t\right) \tag{8}$$

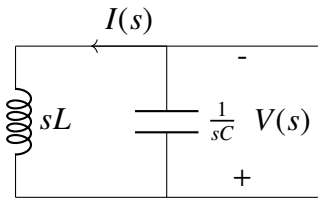

Fig. 0. LC Circuit Diagram in S domain

Net impedence of LC circuit

$$Z = R_L + R_C \tag{9}$$

$$= Ls + \frac{1}{sC} \tag{10}$$

At resonance, the resistance of capacitor and inductor cancel out as follows:

$$Ls + \frac{1}{sC} = 0 \tag{11}$$

$$\implies s = j\frac{1}{\sqrt{LC}} \tag{12}$$

s can be expressed in terms of angular resonance frequency as

$$s = j\omega_0 \tag{13}$$

on comparing (6) and (7)

$$\omega_0 = \frac{1}{\sqrt{LC}} \tag{14}$$

$$\omega_0 = \frac{1}{\sqrt{(30 \times 10^{-6}) \times (27 \times 10^{-3})}} \tag{15}$$

$$= \frac{1}{\sqrt{8.1 \times 10^{-7}}} \tag{16}$$

$$\approx 1.11 \times 10^3 \text{ rad/s} \tag{17}$$