

NCERT Discrete - 11.9.1.8

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Question 11.9.1.8:

- 1) Find the seventh term of the sequence where the nth term is given by $a_n = \frac{n^2}{2^n}$

Solution:

$$x(n) = \frac{(n+1)^2}{2^{(n+1)}} u(n) \quad (1)$$

Parameter	Value
$x(n)$	$\frac{(n+1)^2}{2^{(n+1)}} u(n)$
$x(6)$?

TABLE 1
INPUT PARAMETERS

$$\begin{aligned} x(6) &= \frac{(6+1)^2}{2^{(6+1)}} \\ &= \frac{49}{128} \end{aligned} \quad (2)$$

- 1) Scaling property:

$$a^n u(n) \xleftrightarrow{Z} \frac{1}{(1 - az^{-1})}, \quad |z| > |a| \quad (3)$$

- 2) Differentiation property:

$$\begin{aligned} nu(n) &\xleftrightarrow{Z} (-z) \frac{dY(z)}{dz} \\ \Rightarrow nu(n) &\xleftrightarrow{Z} \frac{z^{-1}}{(1 - z^{-1})^2}, \quad |z| > 1 \quad (4) \\ \Rightarrow n^2 u(n) &\xleftrightarrow{Z} \frac{z^{-1}(1 + z^{-1})}{(1 - z^{-1})^3}, \quad |z| > 1 \quad (5) \end{aligned}$$

- 3) Time shifting property:

$$y(n - k) \xleftrightarrow{Z} z^{-k} Y(z) \quad (6)$$

The Z transform of $x(n)$ is given by:
from(4)

$$\frac{u(n)}{2^n} \xleftrightarrow{Z} \frac{1}{(1 - (2z)^{-1})}, \quad |z| > \frac{1}{2} \quad (7)$$

from(5)

$$\frac{n}{2^n} u(n) \xleftrightarrow{Z} \frac{(2z)^{-1}}{(1 - (2z)^{-1})^2}, \quad |z| > \frac{1}{2} \quad (8)$$

$$\frac{n^2}{2^n} u(n) \xleftrightarrow{Z} \frac{(2z)^{-1}(1 + (2z)^{-1})}{(1 - (2z)^{-1})^3}, \quad |z| > \frac{1}{2} \quad (9)$$

from(8)

$$\frac{(n+1)^2}{2^{(n+1)}} u(n) \xleftrightarrow{Z} (z) \frac{(2z)^{-1}(1 + (2z)^{-1})}{(1 - (2z)^{-1})^3}, \quad |z| > \frac{1}{2} \quad (10)$$

$$X(z) = \frac{1 + (2z)^{-1}}{2(1 - (2z)^{-1})^3}, \quad |z| > \frac{1}{2} \quad (11)$$

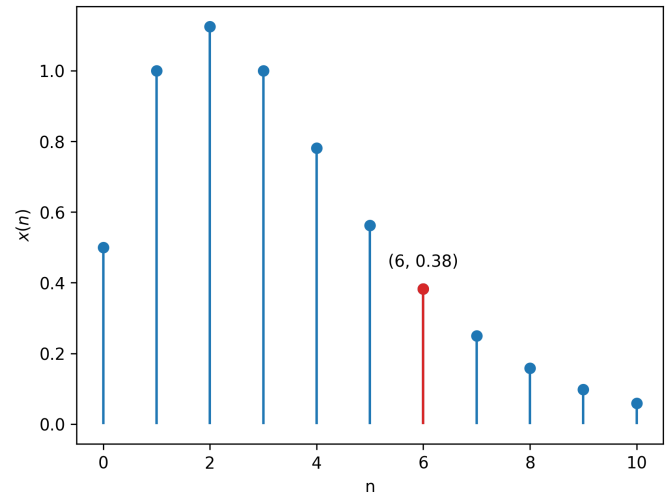


Fig. 3. Stem plot of $x(n)$