Binomial Probability distribution

Let there be n independent trials in an experiment. Let a random variable X denote the no of successes in 'n' trials. Let p be the probability of success and q that of a failure in a single trial so that $p+q=1$. Let the trials be independent and p be constant for every trial.

Let us find the probability of r successes in 'n' trials.

r successes can be obtained n trials in ${}^nc_r$ ways.

$$P(X=r) = {}^nc_r \, P(\underbrace{S\,S\,S \cdots S}_{r \text{ times}} \quad \underbrace{F\,F \cdots F}_{(n-r) \text{ times}})$$

$$= {}^nc_r \, P(S)\,P(S) \cdots P(S) \cdots P(F)\,P(F) \cdots P(F).$$

If n independent trials constitute one experiment and this experiment is repeated N times then the frequency of r successes is $N \cdot {}^nc_r p^r q^{n-r}$.

Mean & variance of binomial distribution:-

$$P(r) = {}^nc_r \, p^r q^{n-r}$$

$$\text{Mean } \mu = \sum_{r=0}^{n} r\,P(r) = \sum r \, {}^nc_r \, p^r q^{n-r}$$

$$= 0 + 1 \cdot {}^nc_1 \, q^{n-1} p + 2 \cdot {}^nc_2 \, q^{n-2} p^2 + 3 \cdot {}^nc_3 \, q^{n-3} p^3 + \cdots + n \cdot {}^nc_n \, p^n$$

$$= n q^{n-1} p + 2 \frac{n(n-1)}{2 \cdot 1} q^{n-2} p^2 + 3 \frac{n(n-1)(n-2)}{3 \cdot 2 \cdot 1} q^{n-3} p^3 + \cdots + n p^n$$

$$= np \left[{}^{n-1}c_0 \, q^{n-1} + {}^{n-1}c_1 \, q^{n-2} p + {}^{n-1}c_2 \, q^{n-3} p^2 + \cdots + {}^{n-1}c_{n-1} \, p^{n-1} \right]$$

$$= np \, (q+p)^{n-1} = np \qquad \because p+q=1$$

$$\boxed{\mu = np}$$

$$V(x) = E(X^2) - E(X)$$

Variance:
$$\sigma^2 = \sum_{r=0}^{n} r^2 P(r) - \mu^2$$

$$= \sum_{r=0}^{n} \{ r(r-1) + r \} P(r) - \mu^2$$

$$= \sum_{r=0}^{n} r P(r) + \sum_{r=0}^{n} r(r-1) P(r) - \mu^2$$

$$\sigma^2 = \mu + \sum r(r-1) \, {}^nc_r \, q^{n-r} p^r - \mu^2$$

$$= [\mu + 2\cdot 1\, n(n-1)\, q^{n-2} p^2 + 3\cdot 2\, {}^nC_3\, q^{n-3} p^3 + \cdots + n(n-1)\, {}^nC_n\, p^n] - \mu^2$$

$$= \mu + [2\cdot 1 \frac{n(n-1)}{2\cdot 1} q^{n-2} p^2 + 3\cdot 2 \cdot \frac{n(n-1)(n-2)}{3\cdot 2\cdot 1} q^{n-3} p^3 + \cdots + n(n-1) p^n] - \mu^2$$

$$= \mu + [n(n-1) q^{n-2} p^2 + n(n-1)(n-2) q^{n-3} p^3 + \cdots + n(n-1) p^n] - \mu^2$$

$$= \mu + n(n-1) p^2 [q^{n-2} + (n-2) q^{n-3} p + \cdots + p^{n-2}] - \mu^2$$

$$= \mu + n(n-1) p^2 [{}^{n-2}C_0\, q^{n-2} + {}^{n-2}C_1\, q^{n-3} p + \cdots + {}^{n-2}C_{n-2}\, p^{n-2}] - \mu^2$$

$$= \mu + n(n-1) p^2 (q+p)^{n-2} - \mu^2 = \mu + n(n-1) p^2 - \mu^2$$

$$= np + n(n-1) p^2 - n^2 p^2 = np(1-p) = npq.$$

$$\boxed{\sigma^2 = npq}$$

~~SCD~~ S.D of B.D is $\sqrt{npq}$.

m.g.f of B.D:

$$M_x(t) = E(e^{tx}) = (q + pe^t)^n.$$

Moments about mean of B.D.

$$M_{x-np}(t) = (qe^{-pt} + pe^{qt})^n$$

$$= \left[q\left[1 - pt + \frac{p^2t^2}{2!} - \frac{p^3t^3}{3!} + \cdots\right] + p\left[1 + qt + \frac{q^2t^2}{2!} + \cdots\right]\right]^n$$

$$= \left[(q+p) + \frac{t^2}{2!} pq(p+q) + \frac{t^3}{3!} pq(p^2 - q^2) + \cdots\right]^n$$

$$= \left[1 + \left\{\frac{t^2}{2!} pq + \frac{t^3}{3!} pq(q-p) + \frac{t^4}{4!} qp(1-3pq) + \cdots\right\}\right]^n$$

$$= \left[1 + {}^nC_1 \left\{\frac{t^2}{2!} pq + \frac{t^3}{3!} pq(q-p) + \cdots\right\} + {}^nC_2 \left\{\quad\right\}^2 + \cdots\right]$$

μ_2 = Coefficient of $\frac{t^2}{2!} = npq$

μ_3 = " " $\frac{t^3}{3!} = npq(q-p)$

μ_4 = " " $\frac{t^4}{4!}$

Qn Comment on the following -

(i) For a B.D. mean is 6 & variance is 9.

(ii) A die is tossed thrice. A success is getting 1 or 6 on a toss. Find the mean & V. of no of success.

Soln (i) $np = 6$, $npq = 9$

$q = \frac{9}{np} = \frac{9}{6} = \frac{3}{2} > 1$

$\because 0 \le q \le 1$

False.

(ii) Prob. of getting success (1 or 6) on a toss $= \frac{2}{6} = \frac{1}{3}$

$q = 1 - \frac{1}{3} = \frac{2}{3}$

No. of tosses of a die, $n = 3$

Mean $np = 1$, $npq = \frac{2}{3}$ A.

Qn - Out of 800 families with 4 children each, how many families would be expected to have (i) 2 boys & 2 girls (ii) at least one boy (iii) at most two girls? Assume equal probabilities for boys & girls.

Soln $\because$ probabilities for boys & girls are equal,

p = probability of having a boy $= \frac{1}{2}$.

q = " " " " girl $= \frac{1}{2}$.

$n = 4$, $N = 800$.

(i) Expected no. of families of having 2 boys & 2 girls is -

$= 800 \cdot {}^4C_2 \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^2 = 300$

(ii) Expected no of families having at least one boy -

$800 \left[P(X=1) + P(X=2) + P(X=3) + P(X=4) \right]$

$= 800 \left[{}^4C_1 \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right) + {}^4C_2 \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^2 + {}^4C_3 \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)^3 + {}^4C_4 \left(\frac{1}{2}\right)^4 \right]$

$= 800 \times \frac{1}{16} \left[4 + 6 + 4 + 1 \right] = 750.$

(iii). $800 \times {}^4C_4 (\frac{1}{2})^4 = 50$

(iv) $800 \times (P(X=4) + P(X=3) + P(X=2))$

$= 800 \times \left[{}^4C_4 (\frac{1}{2})^4 + {}^4C_3 (\frac{1}{2})(\frac{1}{2})^3 + {}^4C_2 (\frac{1}{2})^2 (\frac{1}{2})^2 \right] = 550$ A

Q fit a binomial distⁿ to the following frm data -

x_i	0	1	2	3	4
f_i	30	62	46	10	2

x	f	fx
0	30	0
1	62	62
2	46	92
3	10	30
4	2	8

$\Sigma f = 150$

$\Sigma fx = 192$

Mean of observation $= \frac{192}{150} = 1.28$

$np = 1.28$

$4p = 1.28$

$p = 0.32$, $q = 1 - 0.32 = 0.68$

$N = 150$

Hence the binomial distⁿ $= N(p+q)^n$

$= 150(0.68 + 0.32)^4$

H.W A Student is given a true-false examination with 8 questions. If he corrects at least 7 questions, he passes the examination. find the probability that he will pass given that he guesses all questions.

Q - A die is thrown five times. If getting an odd no is a success. find the probability of getting at least four success.

Solⁿ - $n = 5$, $p =$ probability of getting a success in a single trial $= \frac{3}{6} = \frac{1}{2}$

$q = \frac{1}{2}$.

$P(X \geq 4) = P(X=4) + P(X=5)$

$= {}^5C_4 p^4 q^{5-4} + {}^5C_5 p^5 q^{5-5}$

$= 5(\frac{1}{2})^4(\frac{1}{2}) + (\frac{1}{2})^5 = \frac{5+1}{32} = \frac{6}{32} = \frac{3}{16}$ A

Poisson distribution As a limiting case of binomial distribution:-

When n is very large & p is very small, application of binomial dist^n is very laborious. However, if we assume that as $n \to \infty$ and $p \to 0$ such that np always remains finite, say λ, we get the poisson approximation to the binomial dist^n.

For a binomial dist^n -

$$P(X=r) = {}^nC_r \, p^r q^{n-r} \qquad \boxed{p+q=1}$$

$$= \frac{n(n-1)(n-2)\cdots(n-r+1)}{\lfloor r \; \cancel{\lfloor n-r}} \times (1-p)^{n-r} p^r$$

$$= \frac{n(n-1)(n-2)\cdots(n-r+1)}{\lfloor r} \left(1-\frac{\lambda}{n}\right)^{n-r} \times \left(\frac{\lambda}{n}\right)^r$$

$$= \frac{\lambda^r}{\lfloor r} \; \frac{n(n-1)(n-2)\cdots(n-r+1)}{n^r} \; \frac{\left(1-\frac{\lambda}{n}\right)^n}{\left(1-\frac{\lambda}{n}\right)^r} \quad \because np=\lambda$$

$$= \frac{\lambda^r}{\lfloor r} \left(\frac{n}{n}\right)\left(\frac{n-1}{n}\right)\left(\frac{n-2}{n}\right)\cdots\left(\frac{n-r+1}{n}\right) \frac{\left(1-\frac{\lambda}{n}\right)^n}{\left(1-\frac{\lambda}{n}\right)^r}$$

$$= \frac{\lambda^r}{\lfloor r}\left(1-\frac{1}{n}\right)\left(1-\frac{2}{n}\right)\cdots\left(1-\frac{r-1}{n}\right) \times \frac{\left(\left(1-\frac{\lambda}{n}\right)^{-\frac{n}{\lambda}}\right)^{-\lambda}}{\left(1-\frac{\lambda}{n}\right)^r}$$

as $n \to \infty$ then each $(r-1)$ factor

$$\left(1-\frac{1}{n}\right)\left(1-\frac{2}{n}\right)\cdots\left(1-\frac{r-1}{n}\right) \text{ tends to } 1.$$

also $\left(1-\frac{\lambda}{n}\right)^r \to 1$

We know that $\lim_{n\to\infty}\left(1 \pm \frac{1}{n}\right)^n \to e$

$$\therefore \left(\left(1-\frac{\lambda}{n}\right)^{-\frac{n}{\lambda}}\right)^{-\lambda} \to e^{-\lambda} \text{ as } n \to \infty$$

Hence in the limiting case when $n \to \infty$

$$P(X=r) = \frac{e^{-\lambda}\lambda^r}{\lfloor r}, \quad r = 0, 1, 2, \ldots$$

... out razor blad...

Mean & variance of the poisson dist^n.

For the Poisson dist^n - $P(r) = \dfrac{e^{-\lambda}\lambda^{r}}{\lfloor r}$

Mean $\mu = \sum_{r=0}^{\infty} r P(r) = \sum r \dfrac{e^{-\lambda}\lambda^{r}}{\lfloor r}$

$$= e^{-\lambda}\sum_{r=1}^{\infty}\frac{\lambda^{r}}{\lfloor r-1} = e^{-\lambda}\left[\lambda + \frac{\lambda^2}{\lfloor 1} + \frac{\lambda^3}{\lfloor 2} + \cdots\right]$$

$$= e^{-\lambda}\lambda\left[1 + \frac{\lambda}{\lfloor 1} + \frac{\lambda^2}{\lfloor 2} + \cdots\right]$$

$$= e^{-\lambda}\lambda e^{\lambda} = \lambda$$

$$\boxed{\mu = \lambda}$$

Variance $\sigma^2 = \sum r^2 P(r) - \left(\sum r P(r)\right)^2$

$$= \sum r^2 \frac{e^{-\lambda}\lambda^{r}}{\lfloor r} - \lambda^2$$

$$= e^{-\lambda}\sum \frac{r^2\lambda^{r}}{\lfloor r} - \lambda^2$$

$$= e^{-\lambda}\left[\frac{1^2\lambda}{\lfloor 1} + \frac{2^2\lambda^2}{\lfloor 2} + \frac{3^2\lambda^3}{\lfloor 3} + \cdots\right] - \lambda^2$$

$$= e^{-\lambda}\lambda\left[1 + \frac{2\lambda}{\lfloor 1} + \frac{3\lambda^2}{\lfloor 2} + \frac{4\lambda^3}{\lfloor 3} + \cdots\right] - \lambda^2$$

$$= e^{-\lambda}\lambda\left[1 + \frac{(1+1)\lambda}{\lfloor 1} + \frac{(1+2)\lambda^2}{\lfloor 2} + \frac{(1+3)\lambda^3}{\lfloor 3} + \cdots\right] - \lambda^2$$

$$= e^{-\lambda}\lambda\left[\left(1 + \frac{\lambda}{\lfloor 1} + \frac{\lambda^2}{\lfloor 2} + \frac{\lambda^3}{\lfloor 3} + \cdots\right) + \left(\frac{\lambda}{\lfloor 1} + \frac{2\lambda^2}{\lfloor 2} + \frac{3\lambda^3}{\lfloor 3} + \cdots\right)\right]$$

$$= e^{-\lambda}\lambda\left[e^{\lambda} + \lambda\left(1 + \frac{\lambda}{\lfloor 1} + \frac{\lambda^2}{\lfloor 2} + \cdots\right)\right] - \lambda^2 \quad -\lambda^2$$

$$= e^{-\lambda}\lambda\left[e^{\lambda} + \lambda e^{\lambda}\right) - \lambda^2$$

$$= \lambda + \lambda^2 - \lambda^2 = \lambda$$

Thus the variance of Poisson dist^n is also λ.

Q. In a certain factory turning out razor blades, there is a small chance of 0.002 of any blade to be defective. The blades are supplied in packets of 10. Calculate the app. no of packets containing no defective, one defective and two defective blades in a consignment of 10,000 packets.

Given $e^{-0.02} = 0.9802$.

Sol^n

$p(\text{defective}) = 0.002$

$n = 10$ no of blades in a packet.

$\lambda = np = 0.02$.

$N = 10,000$

(i) ~~pr~~ prob of having no defective $P(0) = \frac{e^{-0.02}(0.02)^0}{\lfloor 0}$

$= 0.9802$.

App. no of packets having zero defective in consignment.

$= .9802 \times 10000 = 9802$

ii App " " " " one " "

$= 10000 \times P(1)$

$= 10000 \times \frac{e^{-0.02}(0.02)^1}{\lfloor 1} = 0.19604$

$= ~~.019604~~ ~~0.019601~~ \simeq 194$

iii " " " " two " "

$= 10000 \times P(2)$ $= .000196$.

$= ~~100~~ 1.96 \simeq 2$

Q. (i) Fit a poisson dist^n to the following data and calculate theoritical frequencies –

Deaths ;	0	1	2	3	4
freq ;	122	60	15	2	1

(ii) The frequency of accidents per shift in a factory is

Sol^n(i) Mean of given dist^n $= \frac{\Sigma fx}{\Sigma f} = 0.5 \Rightarrow \lambda$

Required Poisson dist^n $= N \times \frac{e^{-\lambda} \lambda^r}{\lfloor r} = 200 \times \frac{e^{-0.5} (.5)^r}{\lfloor r}$

$= 121.306 \frac{(.5)^r}{\lfloor r}$

r	$N \cdot P(r)$	theoritical
0	$121.306 \times \frac{(0.5)^0}{\lfloor 0} = 121.306$	121
1	$= 60.653$	61
2	$= 15.163$	15
3	$= 2.527$	3
4	$= 0.3159.$	0
		200

Qn A manufacturer knows that the condensors he makes contain on an average 1% of defectives. He packs them in boxes of 100. What is the probability that a box picked at random will contain 4 or more faulty condensors.

Sol^n $p = .01$, $n = 100$

$np = 1$

$P(r) = \frac{e^{-\lambda} \lambda^r}{\lfloor r} = \frac{e^{-1}}{\lfloor r}$

$P(4 \text{ or more faulty condensors}) = P(4) + P(5) + \cdots + P(100)$

$= 1 - (P(0) + P(1) + P(2) + P(3))$

$= 1 - \left(\frac{e^{-1}}{\lfloor 0} + \frac{e^{-1}}{\lfloor 1} + \frac{e^{-1}}{\lfloor 2} + \frac{e^{-1}}{\lfloor 3} \right)$

$= 1 - \frac{8}{3e} = 1 - 0.981$

$= 0.019$

Normal dist^n

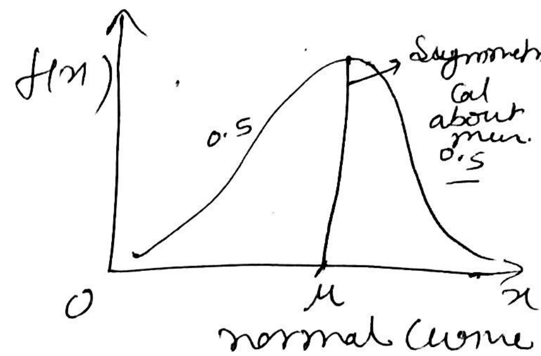
Def^n: $f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$

Properties –
$f(x) \geq 0$.
$\int_{-\infty}^{\infty} f(x)\,dx = 1$
area under the normal curve

The probability density fn for the normal dist^n in standard form is –

$$f(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z^2}, \quad \boxed{z = \frac{x-\mu}{\sigma}}$$

Mean And variance of Normal dist^n –



$$\bar{x} = \frac{\int_{-\infty}^{\infty} x f(x)\,dx}{\int_{-\infty}^{\infty} f(x)\,dx}$$

$$\bar{x} = \int_{-\infty}^{\infty} x \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} dx.$$

Put $\frac{x-\mu}{\sigma} = z$, $x = \mu + \sigma z$, & $dx = \sigma dz$.

$$\bar{x} = \int_{-\infty}^{\infty} (\mu + \sigma z) \frac{1}{\cancel{\sigma}\sqrt{2\pi}} e^{-\frac{1}{2}z^2} (\cancel{\sigma} dz)$$

$$\bar{x} = \mu \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z^2} dz + \frac{\sigma}{\sqrt{2\pi}} \int_{-\infty}^{\infty} z e^{-\frac{z^2}{2}} dz.$$

$$= \mu + \frac{\sigma}{\sqrt{2\pi}} \underbrace{\int_{-\infty}^{\infty} z e^{-\frac{z^2}{2}} d\left(\frac{z^2}{2}\right)}_{=0}$$

$\because \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}} dz = 1$

$$\bar{x} = \mu$$

Variance $= \int_{-\infty}^{\infty} (x-\bar{x})^2 f(x)\, dx$

$$= \int_{-\infty}^{\infty} x^2 f(x)\, dx + \bar{x}^2 \int_{-\infty}^{\infty} f(x)\, dx - 2\bar{x} \int_{-\infty}^{\infty} x f(x)\, dx.$$

$$= \int_{-\infty}^{\infty} x^2 f(x)\, dx + \bar{x}^2 - 2\bar{x}^2$$

Solve for -

$$I = \int_{-\infty}^{\infty} x^2 f(x)\, dx$$

$$= \int_{-\infty}^{\infty} x^2 \cdot \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} dx.$$

Put $\frac{x-\mu}{\sigma} = z$, $x = \bar{x} + \sigma z$

$dx = \sigma dz$

$$I = \int_{-\infty}^{\infty} (\bar{x}+\sigma z)^2 \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{z^2}{2}} \sigma dz$$

$$= \frac{1}{\sqrt{2\pi}} \left[\sigma^2 \int_{-\infty}^{\infty} z^2 e^{-\frac{z^2}{2}} dz + \bar{x}^2 \int_{-\infty}^{\infty} e^{-\frac{z^2}{2}} dz + 2\sigma\bar{x} \int_{-\infty}^{\infty} z e^{-\frac{z^2}{2}} dz \right]$$

$$= \frac{1}{\sqrt{2\pi}} \left[\sigma^2 \int_{-\infty}^{\infty} z^2 e^{-\frac{z^2}{2}} dz + \bar{x}^2 + 2\sigma\bar{x} \times 0 \right]$$

$$= \frac{1}{\sqrt{2\pi}} \left[\sigma^2 \int_{-\infty}^{\infty} z^2 e^{-\frac{z^2}{2}} dz \right] + \bar{x}^2$$

$= 2\int_0^{\infty} z^2 e^{-\frac{z^2}{2}} dz$

$$= \frac{1}{\sqrt{2\pi}} \sigma^2 \times \sqrt{2\pi} + \bar{x}^2$$

$$= \sigma^2 + \bar{x}^2 \text{ a.}$$

Variance $= \sigma^2 + \bar{x}^2 - \bar{x}^2$

$= \sigma^2$

$\frac{z^2}{2} = t$

$z^2 = 2t$

$2z\,dz = 2dt$

$dz = \frac{dt}{\sqrt{2t}}$

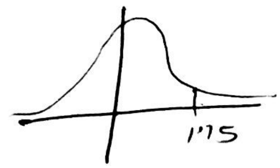
$2\int_0^{\infty} \frac{2t}{\sqrt{2}} e^{-t} dt$

$2\sqrt{2} \times \Gamma\frac{3}{2}$

Q. Assume mean height of soldiers to be 68.22 inches with a variance of 10.8 inches sq". How many soldiers in a regiment of 1000 would you expect to be over 6 feet tall, given that the area under the standard normal curve b/w $z=0$ to $z=0.35$ is 0.1368 and between $z=0$ and $z=1.15$ is 0.3746.

Sol^n $x = 6$ feet $= 72$ inches

$$z = \frac{x-\mu}{\sigma} = \frac{72-68.22}{\sqrt{10.8}}$$



$$z = 1.15$$

$$P(x>72) = P(z>1.15)$$
$$= 0.5 - P(0 \le z \le 1.15)$$
$$= 0.5 - 0.3746 = 0.1254.$$

Expected no of soldiers $= 1000 \times 0.1254$
$= 125.4 \simeq 125$ app.

Q. A sample of 100 dry battery cells tested to find the length of life produced the following results.

$\bar{x} = 12$ hours, ~~F~~ $\sigma = 3$ hours,

assuming the data to be normally distributed, what percentage battery cells are expected to have life -

(i) more than 15 hours (ii) less than 6 hours.

(iii) between 10 & 14 hours

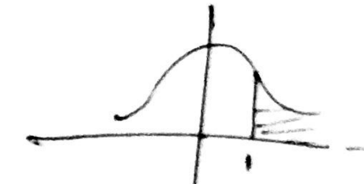
Sol^n Here x denotes the length of life of dry battery cells.

$$z = \frac{x-\bar{x}}{\sigma} = \frac{x-12}{3},$$

for the value of this see the table of "normal dist^n" at last of book.

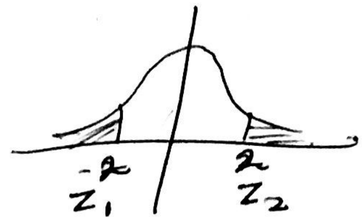
(i) ~~Z@~~ $x=15$, $z=1$.

$$P(x>15) = P(z>1)$$
$$= .5 - P(0 \le z \le 1)$$
$$= .5 - 0.3413$$
$$= 15.87\ \%$$



(ii) when $x=6$, $z=-2$

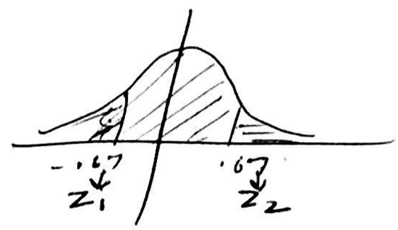
$$P(x<6) = P(z<-2)$$
$$= P(z>2)$$
$$= \cancel{\cdot 5}\; P(0<z<\infty) - P(0<z<2)$$
$$= .5 - .4772$$
$$= .0228 = 2.28\%$$



(iii) $x=10$, $z = -\frac{2}{3} = -.67$

$x=14$, $z = \frac{2}{3} = .67$

$$P(10<x<14)$$
$$= P(-0.67<z<.67)$$
$$= 2P(0<z<.67)$$
$$= 2\times .2485$$
$$= .4970 = 49.70\%$$



Q-8(c)

$\mu = 1.9$, $\sigma^2 = .01$

$x=2$ $\quad z = \frac{2.0-1.9}{\sqrt{.01}} = \frac{0.1}{\sqrt{.01}} = 1$

$$P(x\geq 2) = P(z\geq 1) = 0.5 - P(0<z<1)$$
$$= .5 - .3413$$
$$= .1587\times 1000 \approx 159$$

(ii)

$$z = \frac{2.1-1.9}{\sqrt{.01}} = \frac{.2}{\sqrt{.01}} = 2$$

$$P(x\geq 2.1) = P(z\geq 2)$$
$$= .5 - .4772$$
$$= .0228$$

No. of envelopes $.0228\times 1000$

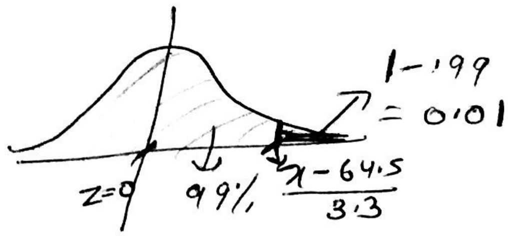
$$= 22.8 \approx 23$$

Q - If the heights of 300 students are normally distributed with mean 64.5 inches and standard deviation 3.3 inches, find the height below which 99% of the student lie.

Sol^n Given $\mu = 64.5$ & $\sigma = 3.3$ inches.

Suppose full area is 100%.

We need to find that x, under which 99% of student lie.



Now

Area between 0 & $\frac{x-64.5}{3.3} = 0.5 - 0.01 = 0.49$

from the table, for area .49, $z = 2.327$

the Corresponding value of x is -

$$\frac{x-64.5}{3.3} = 2.327$$

$\Rightarrow x = 72.18$ inches. Ans