MODULE-4
Statistical Technique –II

Applications in Engineering:

1.Probability and statistics in any many **engineering** fields are applicable to the testing and reliability assessment of engineered systems. There are many phenomena in **engineering** that cannot be accurately modeled computationally, and will require testing in order to predict its performance.

2. The **binomial distribution** allows us to calculate the probability of observing a certain number of successes in a given number of trials. ... Trials with two possible outcomes are often used as the building blocks of random experiments and can be useful to **engineers**.

3. The Poisson Distribution is a theoretical discrete probability distribution that is very useful in situations where the discrete events occur in a continuous manner. This has a huge application in many practical scenarios like determining the number of calls received per minute at a call centre or the number of unbaked cookies in a batch at a bakery, and much more

4. Normal or **Gaussian Distribution** deals with the analysis of items which exhibit failure due to wear, such as mechanical devices.

Module Syllabus:

Probability and Distribution: Introduction, Addition and multiplication law of probability, Conditional probability, Baye's theorem, Random variables (Discrete and Continuous Random variable) Probability mass function and Probability density function, Expectation and variance, Discrete and Continuous Probability distribution: Binomial, Poission and Normal distributions.

KIET GROUP OF INSTITUTIONS, GHAZIABAD

B. TECH IInd YEAR, 2020-21

MATHEMATICS –IV KAS 402

(PDE, Probability and Statistics)

Course Outcome

After completion of this course, students will be able to learn

CO-1	Identify the application of partial differential equations and apply for solving Linear and non-linear partial differential equation .	**BL-1,3**
CO-2	Understand the classification of second order partial differential equations and by using the method of separation of variables to evaluate the general solution of Heat, Wave, Laplace equations and Transmission lines.	**BL-1,3**
CO-3	Remember the concept of moments, skewness, kurtosis and moment generating function and analyze the linear and non linear regression.	**BL-1,4**
CO-4	To remember the concept of probability, random variable and apply for solving the problem related to discrete and continuous probability distributions.	**BL-1,3**
CO-5	Understand the statistical method of data samples , hypothesis testing and applying the study of control chart and their properties.	**BL-2,3**

CHAPTER -1
PROBABILITY THEORY

1.Random experiments, sample space and events
1.1 Random experiments

If we record the number of road traffic accidents in Ghana every day, we will find a large variation in the numbers we get each day. On some days, many accidents will be recorded while on other days, very few accidents will be recorded. That is, there is a large variation in the number of road traffic accidents which occur in Ghana every day. We cannot therefore tell in advance, the number of road traffic accidents in Ghana on a given day. The same sort of remark can be made about the number of babies who are born in Accra every week, the number of customers who enter a bank in a given time interval when the bank is open. Such experiments, because their outcomes are uncertain, are called ***random experiments***.

Example 1

Picking a ball from a box containing 20 numbered balls, is a random experiment, since the process can lead to one of the 20 possible outcomes.

Example 2

Consider the experiment of rolling a six-sided die and observing the number which appears on the uppermost face of the die. The result can be any of the numbers 1, 2, 3, …, 6. This is a random experiment since the outcome is uncertain.

Example 3

If we measure the distance between two points *A* and *B*, many times, under the same conditions, we expect to have the same result. This is therefore not a random experiment. It is a ***deterministic experiment***.
If a deterministic experiment is repeated many times under exactly the same conditions, we expect to have the same result.

Probability allows us to quantify the variability in the outcome of a random experiment. However, before we can introduce probability, it is necessary to specify the space of outcomes and the events on which it will be defined.

.1.2 Sample space

In statistics, the set of all possible outcomes of an experiment is called the ***sample space*** of the experiment, because it usually consists of the things that can happen when one takes a sample. Sample spaces are usually denoted by the letter *S*. spaces are usually denoted by the letter *S*.

Example 1

Consider the experiment of rolling a red die and a green die and observing the number which appears on the uppermost face of each die. The sample space of the experiment consists of the following array of 36 outcomes.

The first coordinate of each point is the number which appears on the red die, while the second coordinate is the number which appears on the green die.

The first coordinate of each point is the number which appears on the red die, while the second coordinate is the number which appears on the green die.

	Green die					
Red die	(1, 1)	(1, 2)	(1, 3)	(1, 4)	(1, 5)	(1, 6)
	(2, 1)	(2, 2)	(2, 3)	(2, 4)	(2, 5)	(2, 6)
	.	.	.	.	.	.
	(6, 1)	(6, 2)	(6, 3)	(6, 4)	(6, 5)	(6, 6)

Using the standard notation for sets, we can express this sample space as follows:

$S = \{(i, j): i = 1, 2, 3, 4, 5, 6; j = 1, 2, 3, 4, 5, 6\}$.

Example 2

Define a sample space for each of the following experiments.

(a) The heights, in centimetres, of five children are 60, 65, 70, 45, 48. Select a child from this group of children, then measure and record the child's height.

(b) Select a number at random from the interval [0, 2] of real numbers. Record the value of the number selected.

Solution

(a) $S = \{60, 65, 70, 45, 48\}$. (b) $S = \{x: 0 \leq x \leq 2$, where x is a real number$\}$.

1.3 Event

A subset of a sample space is called an ***event***.

1.3.1 Types of Events:

1.3.1.1 .Mutually exclusive (or disjoint) events

Any two events that cannot occur simultaneously, so that their intersection is the impossible event, are said to be ***mutually exclusive*** (or disjoint). Thus two events A and B are mutually exclusive if and only if A $A \cap B = \emptyset$.
Example : In tossing a coin head and tail are mutually exclusive

1.3.1.2. Exhaustive Events:

The total number of all possible outcomes in any trial is called the Exhaustive Event. For example : In tossing a coin, there are two exhaustive cases, head and tail.

1.3.1.3 Equally Likely Event :

Events are said to be equally likely if there is no reason to expect any one in the preference of the other event.Example: When a card is drawn from a pack of 52 cards, any card may be appear hence all 52 different cases are equally likely.

1.4 Other useful facts concerning operations on events

The following results can be verified by means of Venn diagrams.

1.Commutative law

If A, B are two events, then

$AUB = BUA$; $A \cap B = B \cap A$

2.Associative Law: If A.B,C are three event ($AUB) \cup C = A \cup (B \cup C)$;

$(A \cap B) \cap C = A \cap (B \cap C)$

3. Distributive Law

If A , B and C are three events, then $(AUB) \cap C = A \cup (B \cap C)$; $(A \cap B) \cup C = A \cap (B \cap C)$

4.De Morgan Law

If A, B are two events, the $(AUB)' = B' \cap A'$; $(A \cap B)' = (B' \cup A'$

1.5 Definition of Probability:

If a trial results in n exhaustive, mutually exclusive and equally likely cases and m of them are favourable to the happening of an event E , then the probability of happening of event E is given by

$$P(E) = \frac{Favourable\ number\ of\ cases}{Exhaustive\ number\ of\ cases} = \frac{m}{n}$$

In another way , if in n trials, an event E happens m times, then the probability of happening of E is given by

$$p = P(E) = \lim_{n\to\infty} \frac{m}{n}$$

Question 1 : A bag con*tai*ns 50 tickets numbered 1,2,3….,50 of which five are drawn at random and arranged in ascending order of magnitude ($x_1<x_2<x_3<x_4<x_5$).What is the probability that x_3=30?

Solution. Exhaustive number of cases If x_3=30,then the two tickets with number x_1 and x_2 must come out of 29 tickets numbered 1 to 29 and this can be done in $29C_2$ ways. The other two tickets with number x_3 and x_5 must come out of the 20 tickets number 31 to 50 and this can be draw in $20C_2$ ways

Favorable number of cases =$29C_2 \times 20C_2$

Required Probability =$29C_2 \times 20C_2/50C_5$ = 551/15134

Question 2:A has 2 shares ina lottery in which there are 3 prizes and 5blanks;Bhasshares in a lottery in which there are 4 prize and 6 blanks . Show that A's chance of success to B's as 27:35.

Solution: A can draw two tickets (out of 3+5=8)in $8C_3$ =28 ways

A will get the blanks in $5C_2$ =10 ways

A can win prize in 28-10=18 ways

Hence A's chance of success =18/28=9/14

B can draw 3 tickets in 10C 3=120 ways ; B will get all blanks in $6C_3$=20 ways

B can win a prize in 120-20 =100 ways.

Hence B's chance of success=100/120

Question 3 **:** A bag contain 7 white, 6 red and 5 black balls.Two balls are drawn at random . find the probability that they both will be white.

Sol. Total number of balls= 7+6+5=18

Out of 18 balls,2 can be drawn in $18C_2$ ways.

Exhaustive number of cases=$18C_2 = \frac{18\times17}{2\times1} = 153$

Out of 7 white balls,2 can be drawn in $7C_2 = \frac{7\times6}{2\times1} = 21\ ways$

Favorable number of cases=21

Probability = $\frac{21}{153} = \frac{7}{51}$

1.6 Addition Theorem on Probability:

Statement: If A and B are any two events, then
$P(A \cup B) = P(A) + P(B) - P(A \cap B)$
P(A or B)=P(A) +P(B)-P(A and B)
IF A, B AND C ARE ANY THREE EVENTS,THEN
$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) + P(A \cap B \cap C)$

1.7 Conditional Probability:

The probability of the happening of an event A when another event is known to have already happen is called conditional probability and is denoted by P(A/B). If the events A and B are independent , then P(A/ B)=P(A)

1.8 MULTIPLICATIVE LAW OF PROBABILITY

The probability of simultaneous occurrence of two events is equal to the probability of one of the events multiplied by the conditional probability of the other, i.e for two two events A and B,

$$P(A \cap B) = P(A) \times P(\frac{B}{A})$$

Where $P(\frac{B}{A})$ represents the conditional probability of occurrence of B when the event A has already happened.

Question 5 . A can hit a target 4 times in 5 shots ; B 3 times in 4 Shots; C twice in 3 shots. They fire a volley. What is the probability that at least two shots hit?

Sol. Probability of A's hitting the target=$\frac{4}{5}$
Probability of B's hitting the target=$\frac{3}{4}$
Probability of C's hitting the target=$\frac{2}{3}$.
For at least two hits, we may
(i) A,B,C all hit the target, the probability for which is

- $$\frac{4}{5} \times \frac{3}{4} \times \frac{2}{3} = \frac{24}{60}$$

(ii) A, B hit the target and C misses it , the probability for which is

$$\frac{4}{5} \times \frac{3}{4} \times \frac{1}{3} = \frac{12}{60}$$

(iii) B, C hit the target and A misses it , the probability for which is

$$\frac{1}{5} \times \frac{3}{4} \times \frac{2}{3} = \frac{6}{60}$$

-

(iv) A, C hit the target and B misses it , the probability for which is

$$\frac{4}{5} \times \frac{1}{4} \times \frac{2}{3} = \frac{8}{60}$$

Since these are mutually exclusive Probability=$\frac{24}{60}+\frac{12}{60}+\frac{6}{60}+\frac{8}{60}=\frac{50}{60}=\frac{5}{6}$

Question 6: A has 2 shares in a lottery in which there are 3 prizes and 5 blanks; B has 3 shares in a lottery in which there are 4 prizes and 6 blanks . Show that A's chance of success is to B's as 27:35

Solution : A can draw 2 tickets in $8C_3 = 28$ ways.
A will get the blanks in $5C_2$=10 ways , A can win a prize in 28-10=18 ways
Hence A's chance of success=$\frac{18}{28} = \frac{9}{14}$
B will get the 3tickets in $10C_3$=120 ways , B will get all blanks in $5C_3$=20 ways

B can win a prize in 120-20 =100ways

Hence B's chance of success=$\frac{100}{120}=\frac{5}{6}$

$\therefore A'schance : B'schance = \frac{9}{14}:\frac{5}{6} = 27:35$

1.9 Baye's theorem:

If $E_1, E_2, \ldots\ldots\ldots\ldots\ldots, E_n$, are mutually exclusive and exhaustive events with $P(E_i)\neq 0$, (i=1,2,….n) Of a random experiment then for any arbitrary event A of the sample space of the above experiment With P(A)>0 , we have

$$P\left({}^{E_i}/_{A}\right) = \frac{P(E_i)P\left({}^{A}/_{E_i}\right)}{\sum_{i=1}^{n} P(E_i)P\left({}^{A}/_{E_i}\right)}$$

Question 7 . The contents of urns I ,II, III are as follows : 1 white, 2 black and 3 red ball 2 white, 1 black and red balls, 4 white, 5 black and 3 red balls .One urn is chosen at randomly and two balls drawn. They happen to be white and red ,find the probability that they come from urns I ,II or III.

Solution:

Let E_1: urn I is chosen; E_2: urn II is chosen; E_3: urn III is chosen

A: the two balls are white and red

We have to find $P\left({}^{E_1}/_{A}\right), P\left({}^{E_2}/_{A}\right)$ and $P\left({}^{E_3}/_{A}\right)$

$P(E_1) = P(E_2) = P(E_3) = \frac{1}{3}$

$P\left({}^{A}/_{E_1}\right) = \frac{1_{C_1} \times 3_{C_1}}{6_{C_2}} = \frac{1}{5}$; $P\left({}^{A}/_{E_1}\right) = \frac{2_{C_1} \times 1_{C_1}}{4_{C_2}} = \frac{1}{3}$ and $P\left({}^{A}/_{E_3}\right) = \frac{4_{C_1} \times 3_{C_1}}{12_{C_2}} = \frac{2}{11}$

By Baye's theorem

$$P\left({}^{E_1}/_{A}\right) = \frac{P(E_1)P\left({}^{A}/_{E_1}\right)}{P(E_1)P\left({}^{A}/_{E_1}\right) + P(E_2)P\left({}^{A}/_{E_2}\right) + P(E_3)P\left({}^{A}/_{E_3}\right)} = \frac{\frac{1}{3}\times\frac{1}{5}}{\frac{1}{3}\times\frac{1}{5}+\frac{1}{3}\times\frac{1}{3}+\frac{1}{3}\times\frac{2}{11}} = \frac{35}{118}$$

Similarly $P\left({}^{E_2}/_{A}\right) = \frac{55}{118}$, $P\left({}^{E_3}/_{A}\right) = \frac{11}{59}$

Question 8: In a bolt factory, machines A,B and C manufacture respectively 25% ,35% and 40% of the total,of their output 5,4 and 2per cent are defective bolts .A bolt is drawn at random from the product and is found to be defective. What is the probability that it was manufactured by machine B?

Solution. Let E_1,E_2,and E_3 denote the events that a bolt selected at random is manufacture is the machine A, Band C respectively and let H denote the event of its being defective. Then

$P(E_1)=0.25, P(E_2)=0.35, P(E_3)=0.40$

The probability of drawing a defective bolt manufactured by machine A is

$P(H/E_1)=0.05$

Similarly, $P(H/E_2)=0.04$ and $P(H/E_3)=0.02$

By Baye's Theorem, we have

$$P\left({}^{E_2}/_{H}\right) = \frac{P(E_2)P\left({}^{H}/_{E_2}\right)}{P(E_1)P\left({}^{H}/_{E_1}\right)+P(E_2)P\left({}^{H}/_{E_2}\right)+P(E_3)P\left({}^{H}/_{E_3}\right)} = \frac{0.35\times0.04}{0.25\times0.05+0.35\times0.04+0.40\times0.02} = 0.41$$

Exercise Problems:

1. Three groups of children contains respectively 3 girls and 1 boy; 2 girls and 2 boys ; 3 boys and 1 girl. One child is selected at random from each group . Show that the chance that the three selected consist of 1 girl and 2 boys is$\frac{13}{32}$
2. A box A contain 2 white and 4 black balls. Another box B contains 5 white and 7 black balls . A ball is transferred from the box A to the box B, then a ball is drawn from the box B . Find the probability that it is white.

   Ans $\frac{16}{39}$.
3. A husband and wife appear in an interview for two vacancies in the same post . The probability of husband's selection is $\frac{1}{7}$.and that of wife selection is $\frac{1}{5}$ .What is the probability that (i) both of them will be selected (ii) Only one of them will be selected (iii) none will be selected .

   Ans(i) $\frac{1}{35}$.(ii) $\frac{2}{7}$.(iii) $\frac{24}{39}$.
4. Two dice are tossed once . Find the probability of getting an even number on the first die or a total of 8.
5. A,B and C, in order, toss a coin . The first one to throw a head wins. If A starts , find their respective chance of winning

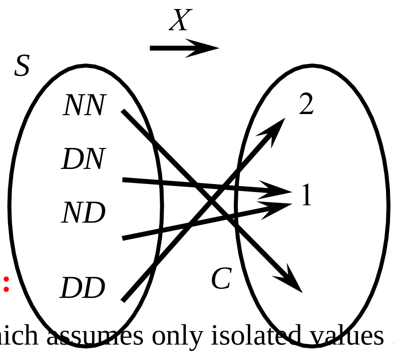
   Answer:$\frac{4}{7}$.(ii$\frac{2}{7}$.(iii)$\frac{1}{7}$

Chapter 2

Random Variable

1.The concept of a random variable

Results of random experiments are often summarized in terms of numerical values. Consider, for example, the experiment of testing two electronic components. When an electronic component is tested, it is either defective or non-defective. The sample space of the experiment may therefore be written as $S = \{NN, DN, ND, DD\}$, where N denotes non-defective and D denotes defective.Let X denote the number of electronic components which are defective. One is naturally interested in the possible values of X. Can X take the value 3? What about the value 1.5? The values X can take are 0, 1 and 2. Notice that X takes the value 0 at the sample point NN and the value 1 at the sample points DN and ND. What value does X take at the sample poin It can be seen that X assigns a unique real number $X(s)$ to each sample point s of S (see Fig. 2.1). X is therefore a function with domain S and co-domain $C = \{0, 1, 2\}$. Such a function is called a **random variable**.



1.1Types of Random Variable:

1.1.1Discrete Random Variable :

A discrete random variable is one which assumes only isolated values . For Example:The number of heads in 4 tosses of a coin is a discrete random variable as it can not be assume values more than 0,1,2,34.

1..1.2Continuous Random Variable :

A continuous random variable is one which assume any value within the interval , .Example: height of a group of individuals.

1.2 PROBABILITY FUNCTION

Let x_1, x_2, x_3 …… be the values of a discrete random variable X and let p_1, p_2, p_3 …. B e the corresponding probabilities.

Then $P(X=x)=p(x)= \begin{cases} p(x_i) & \text{where } x=x_i,\ i=1,2\ldots\ldots \\ 0 & \text{otherwise} \end{cases}$

is called the probability function of the discrete random variable X.

1.2.1 PROBABILITY DISTRIBUTIONS OF A DISCRETE RANDOM VARIABLE

The set of ordered pairs [x_i, $p(x_i)$] is called the probability distribution of a discrete random variable X provided

$p(x_i) \geq 0$ and $\sum p(x_i) = 1$

1.3 MEAN AND VARIANCE OF RANDOM VARIABLE

X	x_1	x_2	x_3
P(X)	p_1	p_2	p_3

We denote mean by $= \frac{\sum p_i x_i}{\sum p_i}$, $Standard\ Deviation = +\sqrt{Variance}$

Question 1. A die is tossed thrice. A success is getting 1 or 6 on a toss. Find the mean of the number of successes.

Sol . Let X denote the number of success. Clearly X can take the values 0,1,2,3.

Probability of success= $\frac{2}{6}=\frac{1}{3}$; Probability of failure = 1-$\frac{1}{3}=\frac{2}{3}$

P(X=0) =P(no success)=P(all 3failure)= $\frac{2}{3} X \frac{2}{3} X \frac{2}{3} = \frac{8}{27}$

P(x=1)=P (one success)=P(one success and 2 failure)= $3C_1\ X \frac{1}{3} X \frac{2}{3} X \frac{2}{3} = \frac{12}{27}$

P(x=2)=P (two success)=P(two success and 1 failure)= $3C_2\ X \frac{1}{3} X \frac{1}{3} X \frac{2}{3} = \frac{6}{27}$

P(x=3)=P (all three success)= $3C_3\ X \frac{1}{3} X \frac{1}{3} X \frac{1}{3} = \frac{1}{27}$

X	0	1	2	3
P(X)	$\frac{8}{27}$	$\frac{12}{27}$	$\frac{6}{27}$	$\frac{1}{27}$

Mean $= \frac{\sum p_i x_i}{\sum p_i}$=1

Question 2 . A random Variable X has the following probability function

X	0	1	2	3	4	5	6	7

P(X)	0	k	$2k$	3k	3k	k^2	$2k^2$	$7k^2+k$

(i) Find k (ii) Evaluate P(X<6) , P(X≥6), $P(3 < X \le 6)$

Sol. (i) $\sum_{x=0}^{7} p(x) = 1$

0+k+2k+3k+3k+$k^2 + 2k^2 + 7k^2 + k = 1$

$k = \frac{1}{10}$

(ii) P(X<6)=P(X=0)+P(X=1)+………P(X=5)

$=k + 2k + 3k + k^2 = 8k + k^2 = \frac{8}{10} + \frac{1}{100} = \frac{81}{100}$

(ii) P(X≥6)=P(X=6)+P(X=7)= $9k^2 + k = \frac{9}{100} + \frac{1}{10} = \frac{19}{100}$

(iii) $P(3 < X \le 6) = P(X = 4) + P(X = 5) + P(X = 6) = 3k + k^2 + 2k^2 = \frac{3}{10} +$

2 Continuous Distributions:

The probability mass function of the random variable. Corresponding to every continuous random variable *X*, there is a function *f*, called the **probability density function** (p.d.f.) of *X* such that

(i) $f(x) \ge 0$ (ii) $\int_{-\infty}^{\infty} f(x)\,dx = 1$ (iii) $P(a \le X \le b) = \int_a^b f(x)dx$

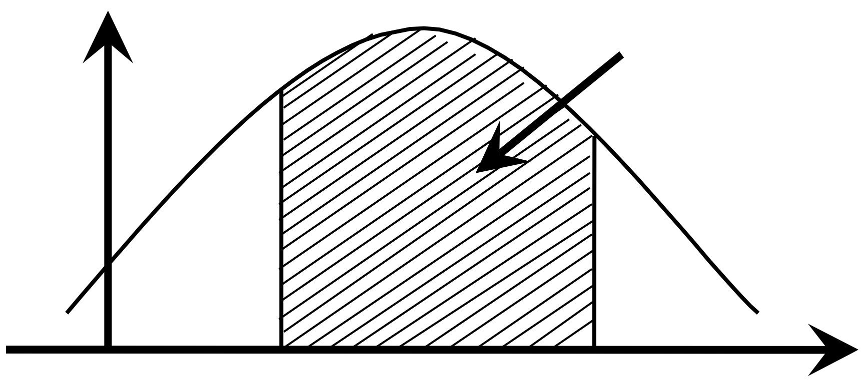


Fig.2

For a complete characterization of a continuous random variable, it is necessary and sufficient to know the p.d.f. of the random variable Geometrically, relation means the following The probability that a continuous random variable *X* takes values in the interval (*a*, *b*) is equal to the area of the region defined by the p.d.f. of *X*, the straight lines *x* = *a* and *x* = *b*, and the *x*-axis (see Fig. 2.).

A consequence of *X* being a continuous random variable is that for any he range of *X*, say *x*,

$$P(X = x) = \int_a^x f(x)dx \text{ value in t}$$

Question3 . If f(x) has a probability density $cx^2, 0 < x < 1$, determine c and find the probability that $\frac{1}{3}$<x<$\frac{1}{2}$.

Soluion. f(x) will have a probability density , if $\int_0^1 c\,x^2\,dx = 1$

$\left[\left(\frac{1}{3}\right) cx^3\right]_0^1 = 1\ i.e\ ; c = 3$

P($\frac{1}{3}$<x<$\frac{1}{2}$.)=$\int_{\frac{1}{3}}^{\frac{1}{2}} 3\,x^2\,dx = \frac{19}{216}$

Question 4 A continuous random variable X has a pdf $f(x) = 3x^2, 0 < x < 1$, find and b such that (i)$P(X \leq a) = P(X > a)$, and (ii) $P(X > b) = 0.05$

Solution : Since $P(X \leq a) = P(X > a)$, each must be equal to $\frac{1}{2}$,because the total probability is always unity

$$P(X \leq a) = \frac{1}{2}$$

$\int_0^a 3\, x^2\, dx = \frac{1}{2} \rightarrow a = \left(\frac{1}{2}\right)^{\frac{1}{3}}$

(ii) $P(X > b) = 0.05$

$$\int_b^1 f(x)\, dx = 0.05 \rightarrow 1 - b^3$$

Question5 : What is the expected value of the number of points that will be obtained in a single throw with an ordinary die Find variance also?

Solution It assumes the value 1 ,2,,3,4,5,6 with probability $\frac{1}{6}$ in each case.

$E(X) = p_1\, x_1 + p_2\, x_2 + \ldots\ldots\ldots p_6\, x_6 = \frac{1}{6}.1 + \frac{1}{6}.2 + \cdots \ldots \ldots \ldots .. + \frac{1}{6}.6 = 3.5$

$var(x) = E(x^2) - [E(x)]^2 = \frac{35}{12}$

Exercise Questions:

(1) Two bad eggs are mixed accidently with 10 good ones . Find the probability distribution of the number of bad eggs in 3 , drawn at random , without replacement from this lot .

Answer :

X	0	1	3
P(X)	1/7	2/12	5/12

(2). A die is tossed twice. Getting a number greater than 4 is considered as a success. Find the variance of the probability distribution of the number of successes. Ans

(3). Show that the symmetrical distribution $f(x) = \frac{2a}{\pi}\left(\frac{1}{a^2+x^2}\right)$, -a<x<a represents a probability density function.

Chapter -3
Binomial Distribution:

This distribution was discovered by James Bernoulli. This is a discrete distribution. It occurs in cases of repeated trials such as students writing an examination, births in a hospital etc. Here all the trials are assumed to be independent and each trial has only two outcomes namely success and failure.

Let an experiment consist of "n" independent trials. Let it succeed "x" times. Let "p" be the probability of success and "q" be the probability of failure in each trial. p + q = 1

The probability of getting x successe= p.p.p............p(x times) $= p^x$

The probability of getting (n – x) failures = q.q.q.q[(n – x) times] = $q^{(n-x)}$

From multiplication theorem, the probability of getting x successes and (n – x) failures is $p^x q^{(n-x)}$.

This is the probability of getting x successes in one combination. There are such ${}^nC_x$ mutually exclusive combinations each with probability $p^x q^{(n-x)}$.

From addition theorem the probability of getting x success in ${}^nC_x\, p^x q^{(n-x)}$.

Notation: b(x; n, p) denotes a binomial distribution with x successes, n trials and with p as the probability of success.

$b(x; n, p) = {}^nC_x\, p^x q^{(n-x)}$, x = 0, 1, 2, 3, ... , n.

Parameters of Binomial distribution:

In b(x; n, p) there are 3 constants viz., n, p and q. Since q = 1- p, hence there are only 2 independent constants namely n and p. These are called the parameters of binomial distribution.

Note: since b(x; n, p) is same as the $(x + 1)^{th}$ term in the binomial expansion of $(q + p)^n$, hence this distribution is called the "Binomial Distribution"

Mean of the Binomial Distribution:

For the binomial distribution $P(r) = nC_r\, p^r q^{n-r}$

$$Mean\ \mu = \sum_{r=0}^{n} r.nC_r\, p^r q^{n-r}$$

$$=0 + 1.nC_1\, p^1 q^{n-1} + 2.\ nC_2\, p^2 q^{n-2} + 3.nC_3\, p^3 q^{n-3} + \cdots n.nC_n\, p^n q^{n-n}$$

$$=nq^{n-1}p + 2\,\frac{n\,(n-1)}{2}\,p^2 q^{n-2} + \cdots \ldots\ldots\ldots\ldots. + np^n$$

$$=nq^{n-1}p + n(n-1)\,p^2 q^{n-2} + \cdots \ldots\ldots\ldots\ldots. + np^n$$

$$=np[(q+p)^n]=\text{np}$$

Variance of the binomial distribution=

The arithmetic mean

$$\frac{\sum_{x=0}^{n} xp(x)}{\sum_{x=0}^{n} p(x)} = \sum_{0}^{n} xp(x)$$

$$= \sum_{0}^{n} x\ n_{c_x} p^x q^{n-x} = \sum_{0}^{n} \frac{xn!}{x!\,(n-x)!} p^x q^{n-x} = np \sum_{0}^{n} \frac{(n-1)!}{(x-1)!\,(n-x)!} p^{x-1} q^{n-x}$$

$$= np \sum_{0}^{n} n - 1_{c_{x-1}}\, p^{x-1} q^{n-x} = np(q+p)^{n-1} = np$$

Mean of the binomial distribution$=\bar{x} = np$

The variance σ^2 of binomial distribution is given by

$$\sigma^2 = \mu_2 = \frac{\sum_{x=0}^{n}(x-\bar{x})^2 p(x)}{\sum_{x=0}^{n} p(x)} = \sum_{0}^{n} x^2\, p(x) - \left(\sum_{0}^{n} xp(x)\right)^2 \ as \ \sum_{0}^{n} p(x) = 1$$

$$= \sum_{0}^{n} x^2\ n_{c_x} p^x q^{n-x} - (np)^2 = \sum_{0}^{n} \frac{x^2 n!}{x!\,(n-x)!} p^x q^{n-x} - (np)^2 \ as\ \bar{x} = np$$

$$= np\sum_{1}^{n} \frac{x(n-1)!\,p^{x-1}q^{n-x}}{(x-1)!\,(n-x)!} - = np\sum_{1}^{n} \frac{(x-1+1)(n-1)!\,p^{x-1}q^{n-x}}{(x-1)!\,(n-x)!} - (np)^2$$

$$np\sum_{2}^{n} \frac{(n-1)!\,p^{x-1}q^{n-x}}{(x-2)!\,(n-x)!} + np\sum_{1}^{n} \frac{(n-1)!\,p^{x-1}q^{n-x}}{(x-1)!\,(n-x)!} - (np)^2$$

$= np^2(n-1)\sum_0^n\ n-2_{c_{x-2}} p^{x-2}q^{n-x} + np\sum_1^{n-1}\ n-1_{c_{x-1}} p^{x-1}q^{n-x} - (np)^2$

$= np^2(n-1)(q+p)^{n-2} + np(q+p)^{n-1} - (np)^2$

$= n(n-1)p^2.1 + np.1 - (np)^2 = np(1-p) = npq$

So $\sigma^2 = npq$

Also we can easily show that

$\mu_3 = npq(q-p),\ \ \mu_4 = npq[1+3(n-2)pq]$

$\beta_1 = \frac{{\mu_3}^2}{{\mu_2}^3} = \frac{n^2p^2q^2(q-p)^2}{n^3p^3q^3} = \frac{(q-p)^2}{npq}$

Question 1: It has been claimed that in 60% of all solar heat installations the utility bills is reduced by at least one third. Accordingly what ae the probabilities that the utility bill will be reduced by at least one third in (i) four or five installations (ii) at least four of five installations?

Solution: n = 5, p = 0.6, q = 1 – p = 0.4

(i) b(4; 5, 0.6) = ${}^5C_4\ (0.6)^4\ (0.4)^1 = 5(0.6)^4(0.4) = 0.2592$

(ii) at least 4 means 4 or 5

b(5; 5, 0.6) = ${}^5C_5\ (0.6)^5\ (0.4)^0 = 0.0778$

Probability in at least four installations = b(4; 5, 0.6) + b(5; 5, 0.6)

= 0.2592 + 0.0778=0.337

Question2 :If X is binomially distributed with 6 trials and probability of success equal to $\frac{1}{4}$ at each attempt, what is probability of (a) exactly 4 successes (b) at least one successes.

Solution: (a) Exactly 4 successes

$P(X=4) = 6_{C_4}\left(\frac{1}{4}\right)^4\left(\frac{3}{4}\right)^2 = 15 \times \frac{1}{256} \times \frac{9}{16} = \frac{135}{4096}$

$= .033$

(a) $P(X \geq 4) = 1 - P(X=0) = 1 - \left(\frac{3}{4}\right)^6$

$= 1 - \frac{729}{4026} = \frac{3367}{4096} = .82$

Example 3 when an unbiased coin is tossed 8 times what Probability obtaining (a) Less than 4 heads (b) More than 5 heads

Solution:

Let H –no of heads

(a) $P(H \leq 3) = P(H=0) + P(H=1) + P(H=2) + P(H=3)$

$= \left(\frac{1}{2}\right)^8 + 8_{C_1}\left(\frac{1}{2}\right)^1\left(\frac{1}{2}\right)^7 + 8_{C_2}\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^6 + 8_{C_3}\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^5$

$$= \left(\frac{1}{2}\right)^8 + 8\left(\frac{1}{2}\right)^8 + 28\left(\frac{1}{2}\right)^8 + 56\left(\frac{1}{2}\right)^8$$
$$= 93\left(\frac{1}{2}\right)^8$$
$$= \frac{93}{256}$$
$$= .822$$

(b) $P(H > 5) = P(H = 6) + P(H = 7) + P(H = 8)$

$$= 8_{C_6}\left(\frac{1}{2}\right)^6\left(\frac{1}{2}\right)^2 + 8_{C_7}\left(\frac{1}{2}\right)^7\left(\frac{1}{2}\right) + \left(\frac{1}{2}\right)^8$$
$$= 28\left(\frac{1}{2}\right)^8 + 8\left(\frac{1}{2}\right)^8 + \left(\frac{1}{2}\right)^8$$
$$= 37\left(\frac{1}{2}\right)^8 = \frac{37}{256} = .1445$$

Question 4:Out of 800 families with 4 children each, how many families would be expected to have (i) 2 boys and 2 girls (ii) at least one boy(iii) no girl? Assume equal probabilities for boys and girls?

Solution: since probabilities of boys and girls are equal .p= probability of having a boy= $\frac{1}{2}$; q =probability of having a girl= $\frac{1}{2}$; n=4, N=800

(i) The expected no of families having 2 boys and 2 girls= $800 \times 4_{C_2}\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^2 = 800 \times 6 \times \frac{1}{16} = 300$

(ii) The expected no of families having at least one boy

$$= 800\,[4_{C_1}\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^1 + 4_{C_2}\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^2 + 4_{C_3}\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^1 + 4_{C_4}\left(\frac{1}{2}\right)^4\left(\frac{1}{2}\right)^0]$$
$$=800 \times \frac{1}{16} \times [4 + 6 + 4 + 1] = 750$$

(iii) The expected no of families having no girl= $800 \times 4_{C_4}\left(\frac{1}{2}\right)^4\left(\frac{1}{2}\right)^0$=5

Question5 : Assuming that half the population are consumers of chocolate, so that the chance of an individual being a consumer is ½ andassuming that each of the 100 investigators takes 10 individuals to see whether they are consumers. How many investigators would you expect to report that three people or less were consumers.

Solution: Chance of an individuals to be a consumers $= p = \frac{1}{2}$ hence

$q = \frac{1}{2}$ Probability that less than or equal to three people, are consumers

$(P(r \leq 3) = P(r = 0) + P(r = 1) + P(r = 2) + P(r = 3)$

$P(r) = nC_r\, p^r q^{n-r} = {}^{10}C_r \left(\frac{1}{2}\right)^r \left(\frac{1}{2}\right)^{10-r} = {}^{10}C_r\left(\frac{1}{2}\right)^{10}$

Required Probability$=\left(\frac{1}{2}\right)^{10} [{}^{10}C_0+{}^{10}C_1+{}^{10}C_2+{}^{10}C_3]$

$=\left(\frac{1}{2}\right)^{10} [1 + 10 + \frac{10.9}{1.2}+\frac{10.9.8}{1.2.3}] = \frac{176}{32\times 32} = \frac{11}{64} = 0.172$

The required number of investigators reporting that out of 10, at the most three are consumers $=100 \times 0.172 = 17.2 = 17$ investigators (approx.)

Exercise

1. Four persons are chosen at random from a group containing 3 men, 2 women and 4 children . Show that the chance that exactly two of them will be children is 5/21.
2. Six dice are thrown 729 times. How many times do you expect at least three dice to show a five or six? Ans:
3. A and B throw alternately with a pair of ordinary dice. A wins if he throws 6 before B throws 7 and B wins if he throws 7 before A throws 6 . If A begins , find their respective chance of winning.
   Ans : 30/61,31/61
4. A speaks truth in 60% cases and B in 70% cases . In what percentage of cases are they likely to contradict each other in stating the same fact? Ans.
5. Out of 320 families with 5 children each, what percentage would be expected to have (i) 2 boys and 3 girls (ii) at least one boy? Assume equal probability for boys and girls.

Poisson Distribution

1.Poisson distribution is a limiting form of the binomial distribution.

Proof :In the binomial distribution let us consider n as very large and p as very small such that np=constant =m

$$P(r) = \mathrm{n}C_r\, p^r q^{n-r} = \frac{n(n-1)(n-2)\ldots\ldots(n-r+1)}{r!}\, p^r\, q^{n-r}$$

$$= \frac{np\,(np-p)(np-2p)\ldots\ldots(np-pr+p)}{r!}\,(1-p)^{n-r}$$

$$= \frac{m\,(m-p)(m-2p)\ldots\ldots(m-pr+p)}{r!}\,\left(1-\frac{m}{n}\right)^{n-r}$$

$$= \frac{m\,(m-p)(m-2p)\ldots\ldots(m-pr+p)\left(1-\frac{m}{n}\right)^{n}}{r!\left(1-\frac{m}{n}\right)^{r}}$$

$$\lim_{n\to\infty} P(r) = \frac{m\,(m-0)(m-0)\ldots\ldots(m-0)\ \ \lim_{n\to\infty}\left(1-\frac{m}{n}\right)^{n}}{r!(1-0)^{r}} = \frac{m^r}{r!}\,e^{-m}$$

Therefore in Poisson distribution the probabilities of 0,1,2……r,……successes are

$$e^{-m},\ m\,e^{-m},\ \frac{m^2}{2!}\,e^{-m},\ldots\ldots\ldots\frac{m^r}{r!}\,e^{-m},\ldots\ldots$$

The sum of all the probabilities= $e^{-m}\left(1+m+\frac{m^2}{2!}+\frac{m^3}{3!}+\cdots\ldots\right) = e^{-m}e^{m} = 1$

Thus, Poisson's distribution is a probability distribution. There is only one constant of this distribution which is m=np . Here $P(r) = \frac{m^r}{r!}\,e^{-m}$

1.1Mean of the Poisson's Distribution:

Mean=$\sum rP(r)$ $\quad p(r) = 0p(0) + 1p(1) + 2p(2) + \cdots\ldots\ldots\infty$

$$=\sum r\,\frac{m^r}{r!}\,e^{-m} = m\sum_{r=1}^{\infty}\frac{e^{-m}m^{r-1}}{r-1!} = m\,e^{-m}.e^{m} = m$$

Mean of the Poisson's Distribution=m

Also the variance = $\boldsymbol{\sigma^2 = \mu_2 = npq = m \quad since\ q \to 1\ as\ p \to 0}$

1.2Applications of Poisson's Distribution

This distribution is applied to problems

(i) Arrival pattern of defective vehicle in a workshop
(ii) Patients in a hospitals
(iii) Telephone calls
(iv) Demand pattern for certain spare parts
(v) Emission of radioactive (α) particles.

.**Question1:** Suppose that a book of 600 pages contain 40 printing mistakes . Assume that these errors are randomly distributed throughout the book and r, the number of error per page has a poisson distribution. What is the probability that 10 pages selected at random will be free from errors?

Sol : $p = \frac{40}{600} = \frac{1}{15}, \qquad n = 10$

$$m = np = 10\left(\frac{1}{15}\right) = \frac{2}{3}$$

$$P(r) = \frac{m^r}{r!}\,e^{-m} = \frac{\left(\frac{2}{3}\right)^r}{r!}\,e^{-\frac{2}{3}}$$

$P(0) = \frac{m^r}{r!}\, e^{-m} = \frac{\left(\frac{2}{3}\right)^0}{0!}\, e^{-\frac{2}{3}} = 0.51$

Question2: In a certain factory turning out razor blades, there is a small chance of 0.002 for any blade to be defective . The blades are supplied in a packet of 10 . Calculate the approximate number of packets containing no defective, one defective and two defective blades in a consignment of 10,000 packets.

Solution : p(defective)=0.002

n=10

$m = np = 10 \times 0.002 = 0.02$

No. of packets in the consignment, N=10,000

Probability of having no defective = $P(0) = \frac{0.02^0}{0!}\, e^{-0.02} = 0.9802$

Approximate no. of packets having one defective in the consignment=$0.9802 \times 10{,}000 = 9802$

(ii) Probability of having one defective = $P(1) = \frac{0.02^1}{1!}\, e^{-0.02} = 0.019604$

Approximate no. of packets having one defective in the consignment=$0.019604 \times 10{,}000 = 196$

(iii) Probability of having two defective = $P(2) = \frac{(0.02)^2}{2!}\, e^{-0.02} = 0.000196$

Approximate no. of packets having two defective in the consignment=$0.000196 \times 10{,}000 = 1.96 \sim 2$

Question 3: Fit a Poisson Distribution to the following data and calculate theoretical frequencies

Deaths	0	1	2	3	4
Frequencies	122	60	15	2	1

Sol. (i) Mean of the given distribution =$\frac{\sum fx}{\sum f}$

$$m = \frac{60 + 30 + 6 + 4}{200} = 0.5$$

Poisson Distribution = $P(r) = N.\frac{m^r}{r!}\, e^{-m} = 200.\frac{0.5^r}{r!}\, e^{-0.5} = 121.306\,\frac{0.5^r}{r!}$

r	N .P(r)	Theoretical Frequency
0	$121.306\,\frac{(0.5)^0}{0!} = 121.306$	121
1	$121.306\,\frac{(0.5)^1}{1!} = 60.653$	61
2;	$121.306\,\frac{(0.5)^2}{2!} = 15.163$	15

3	$121.306\ \frac{(0.5)^3}{3!} = 2.527$	3
4	$121.306\ \frac{(0.5)^4}{4!} = 0.3159$	0
		Total =200

Question4: Suppose the number of telephone calls on an operator received from 9:00 to 9:05 follow a Poisson distribution with a mean 3. Find the probability that, The operator will receive no calls in that time interval tomorrow. (ii) In the next three days, the operator will receive a total of 1 call in that interval Given ($e^{-3} = 0.04978$)

Solution: $m = 3\ \ P(0) = \frac{m^0}{0!}\ e^{-m} = e^{-3} = 0.04978$

(ii) Required Probability=$P(0)P(0)P(1) + P(0)P(1)P(0) + P(1)P(0)P(0) =$

$$=\left(\frac{e^{-m}m^0}{0!}\right)^2 \frac{e^{-m}m}{1!} = 9\ (e^{-3})^3 = 0.00111$$

Question 5: Six coins are tossed 6400 times. Using the poisson distribution, determine the approximate probability of getting 6 heads x times.

Ans: Probability of getting one head with one coin=$\frac{1}{2}$

The probability of getting six heads with six coins =$\left(\frac{1}{2}\right)^6$=$\frac{1}{64}$

Average number of six heads with six coins in 6400 throws =$np =$ $6400 \times \frac{1}{64} = 100$

Approximate probability of getting 6 heads x times when the distribution is poisson

$$=\frac{m^x}{x!}\ e^{-m} = \frac{(100)^x}{x!}\ e^{-100}$$

Exercise:

1. It is given that 2% of the electric bulbs manufactured by accompany are defective. Using Poisson, distribution, find the probability that a sample of 200 bulbs will contain (i) no defective bulb (ii) two defective bulbs (iii) at the most three defective bulbs

   Ans: 00.018315 (ii) 0.146525 (iii) 0.43347
2. The incidence of occupational disease in an industry is such that the workmen have a 10% chance of suffering from it. What is the probability
   That in a group of 7, five or more will suffer from it.

   Ans: 0.0008 (ii)0.23816
3. A manufacturer of cotter pins knows that 5% of his product is defective.
4. If he sells cotter pins in boxes of 100 and guarantee that not more than 10 pins will be defective, what is the approximate prob. that a box will fail to meet the guaranteed quality? Ans: 0.007926
5. A certain screw making machine produces on average 2 defective screws out of 100, and packs them in boxes of 500. Find the probability that a box contains 15 defective scores. Ans: 0.0

CHAPTER-5
NORMAL DISTRIBUTION

Definition: The normal distribution is a continuous distribution. It can be derived from the binomial distribution in the limiting case when n , number of trials is very large, p the probability of success is close to $\frac{1}{2}$,

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}}\, e^{\frac{-1}{2}(\frac{x-\mu}{\sigma})^2}$$

If a variable x has the normal distribution with mean μ and standard deviation σ N (μ, σ^2). The graph of the normal distribution is called the normal curve. It is bell shaped and symmetrical about mean μ.
The graph of the normal distribution is called the normal curve . it is bell shaped and symmetrical about mean μ.
The total area under the normal curve above the x axis is 1.

Basic Properties of the Normal Distribution:

The probability density function of the normal distribution is given by

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}}\, e^{\frac{-1}{2}(\frac{x-\mu}{\sigma})^2}$$

(i) $f(x) \geq 0.$
(ii) $\int_{-\infty}^{\infty} f(x)dx = 1$

The total area under the normal curve above the x axis is 1.
(iii) The normal distribution is symmetrical about its mean.

(1) The A.M of the normal distribution is given by

$$\bar{x} = \frac{\int_{-\infty}^{\infty} x\, f(x)dx}{\int_{-\infty}^{\infty} f(x)dx}$$

$$\bar{x} = \int_{-\infty}^{\infty} x\, \frac{1}{\sigma\sqrt{2\pi}}\, e^{\frac{-1}{2}(\frac{x-\mu}{\sigma})^2} dx$$

Let $\frac{x-\mu}{\sigma} = z \rightarrow x = \mu + \sigma z \rightarrow dx = \sigma dz$

$$\bar{x} = \int_{-\infty}^{\infty} (\mu + \sigma z)\, \frac{1}{\sigma\sqrt{2\pi}}\, e^{\frac{-1}{2}z^2}(\sigma dz)$$

$$\bar{x} = \int_{-\infty}^{\infty} (\mu\, \frac{1}{\sigma\sqrt{2\pi}}\, e^{\frac{-1}{2}z^2}(\sigma dz + \sigma z \int_{-\infty}^{\infty} \frac{1}{\sigma\sqrt{2\pi}}\, e^{\frac{-1}{2}z^2}(\sigma dz)$$

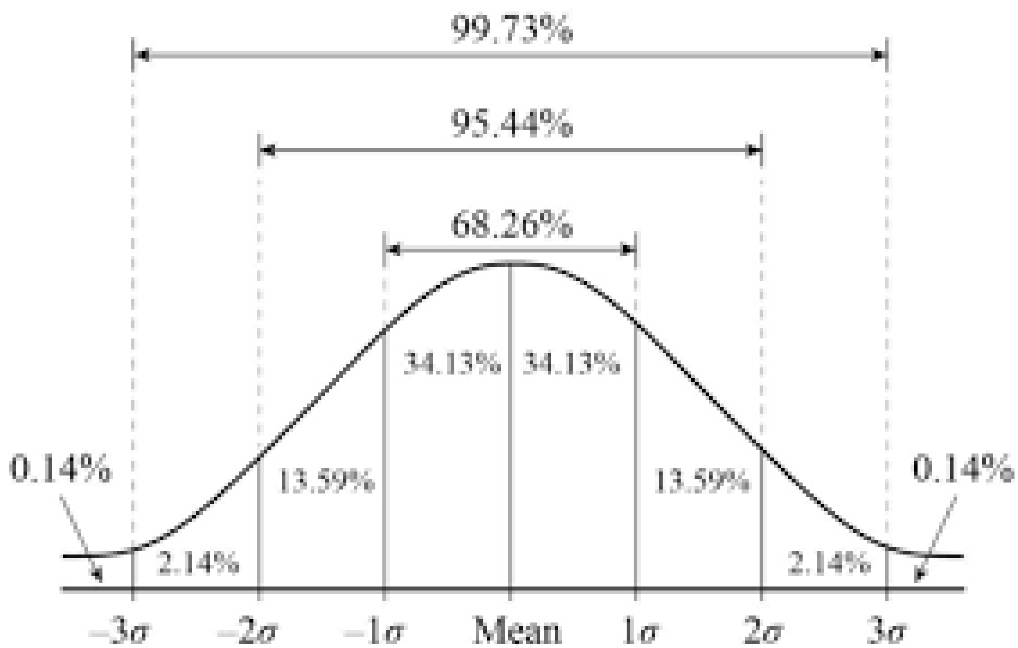
$$\bar{x} = \mu + \frac{\sigma}{\sqrt{2\pi}}(\frac{e^{\frac{-z^2}{2}}}{-1})_{-\infty}^{\infty};\ \bar{x} = \mu.$$

Variance of the Normal Distribution=σ^2

Area under Normal Curve:

By considering $z = \frac{x-\mu}{\sigma}$, standard normal curve is formed . The total area under this curve is 1.

The area under the curve is divided into two equal parts by z=0. The area between the ordinate z=0 any other ordinate can be noted from the table.



De Moivre made the discovery of this distribution in 1733. This distribution has an important application in the theory of errors made by chance in experimental measurements.

Question1: A sample of 100 dry battery cells tested to find the length of life produced the following results: $\bar{x} = 12\ hours, \sigma = 3\ hours$. Assuming the data to be normally distributed , what percentage of battery cells are expected to have life (i) More than 15 hours (ii) less than 6 hour (iii) Between 10 and 14 hour.

Solution : Here x denotes the length of life of dry battery cells. Also, $z = \frac{x-\mu}{\sigma} = \frac{x-12}{3}$

(i) When x=15, z=1

P(x>15)=P(z>1)

$=P(0 < z < \infty) - P(0 < z < 1)$

=0.5-0.3413=0.1587=15.87%

(ii) When $x = 6$, $z = -2$

$$P(x < 6) = P(z < -2)$$

$=P(z > 2)$

$= P(0 < z < \infty) - P(0 < z < 2)$

=0.5-0.4772=0.0228=2.28%

(iii) When $x = 10$, $z = -\frac{2}{3} = -0.67$

$x = 14$, $z = \frac{2}{3} = 0.6$

$$P(10 < x < 14) = P(-0.67 < z < 0.67)$$

$$=2P(0 < z < 0.67)$$
$$=2 \times 0.2485 = 0.497 = 49.7\%$$

Question2: Assume mean heights of soldier to be 68.22 inches with a variance of 10.8 inches square .How many soldier in a regiment of 1,000 would you expect to be over 6 feet tall , given that the area under the standard normal curve between z=0 and z=0.35 is 0.1368 and z=0 and z=1.15 is 0.3746.

solution:

$$x = 6\,feets = 72\,inches$$

$$z = \frac{x-\mu}{\sigma} = \frac{72-68.22}{3} = 1.15$$

$$P(x > 72) = P(z > 1.15) = 0.5 - P(0 \le z \le 1.15)$$
$$=0.5 - 0.3746 = 0.1254$$

Expected no. of soldier$=1000 \times 0.1254 = 125.4 \sim 125(app.)$

Question3: The life of army shoes is normally distributed with mean 8 months and standard deviation 2 months. If 5000 pairs are insured, how many pairs would be expected to need replacement after 12 month?

Solution: $Mean\ \mu = 8,$ Standard deviation $\sigma = 12$

Number of pair of shoes=5000, Total months(x)=12

When $x = 12, z = \frac{x-\mu}{\sigma} = \frac{12-8}{2} = 2$

Area $(z \ge 2) = 0.0228$

Number of pairs of shoes whose life is more than 12 months$=5000 \times 0.0228 = 114$

Pairs of Shoes needing replacement after 12 months=5000-114=4886

Question4: If the heights of 300 students are normally distributed with mean 64. 5 inches and standard deviation 3.3 inches , find the height below which 99% student lie.

Solution: $Mean\ \mu = 64.5\ inches,$ Standard deviation $\sigma = 3.3\ inches$

$$Area\ between\ 0\ and\ \frac{x-64.5}{3.3} = 0.99 - 0.5 = 0.49$$

From the table, for the area 0.49 z=2.327

The corresponding value of x is given by

$$\frac{x-64.5}{3.3} = 2.327$$

$$x - 64.5 = 7.68$$

$$x = 7.68 + 64.5 = 72.18\ inches$$

Hence 99% students are of heights less 6 ft 0.18 inches.

Question5: Assuming that the diameters of 1000 brass plugs taken consecutively from a machine, form a normal distribution with mean 0.7515 cm and standard deviation 0.002 cm , how many of the plugs are likely to be rejected If the approved diameter is $0.752 \pm 0.004\ cm$.

Solution. Tolerance limits of the diameter of non defective plugs are $0.752 - 0.004 = 0.748$ cm and $.752 + 0.004 = 0.756\ cm$

Standard normal variable $z = \frac{x-\mu}{\sigma}$

If $x_1 = 0.748, z_1 = \frac{0.748-0.7515}{0.002} = -1.75$

If $x_2 = 0.756, z_2 = \frac{0.756-0.7515}{0.02} = 2.25$

Area from $(z_1 = -1.75)$ to $(z_2 = 2.25)$$=P(-1.75 \le z \le 0) + P(0 \le z \le 2.25)$

$$= P(0 \le z \le 1.75) + P(0 \le z \le 2.25)$$
$$=0.4599 + 0.4878 = 0.9477$$

Number of plugs which are likely to be rejected== $1000 \times (1 - 0.9477) = 1000 \times 0.0523 = 52.3$
Hence approximately 52 plugs are likely to be rejected.

Exercise

Question1: 2000 students appeared in an examination. Distribution of marks is assumed to be normal with $Mean\ \mu = 30$ and $\sigma = 6.25$. How many students are expected to get marks? (i)Between $20\ and\ 40$ (ii) less than 35 and (iii) above 50.

Ans(i) 1781 (ii) 1576 (iii)1

Question2. Suppose the weight of 600 male students are normally distributed with mean $Mean\ \mu = 70\ kg$ and $\sigma = 5\ kg$. Find the number of students with weight

(i) Between 69 and 74 kg (ii) more than 76 kg.

Answer: (i) 220 (ii)69

Question3 : In an intelligence test administered to 1000 students, the average score was 42 and standard deviation 24. Find

(i) The expected number of students scoring more than 50
(ii) The number of students scoring between 30 and 54.
(iii) The value of score exceeded by top 100 students.

Answer(i) 371 (ii) 383, (iii) 72.72

Question**4**:In a normal distribution, 31% of the items are under 45 and 8% are over 64. Find the mean and standard deviation of the distribution.

Answer: $\sigma = 10, \mu = 50$.

Question5: The daily wages of 1000 workers are distributed around a mean of Rs. 140 and with a standard deviation of RS 10. Estimate the number of workers whose daily wages will be

(i) Between Rs. 140 and Rs. 144 (ii) less than Rs. 126 (iii) more than Rs. 160.

Answer: (i) 155(ii) 81(iii) 23

Important Links:

1. https://nptel.ac.in/courses/111/105/111105041/
2. https://www.youtube.com/watch?v=6x1pL9Yov1k
3. https://www.youtube.com/watch?v=8MpgZJHcB8w&app=desktop
4. https://www.youtube.com/watch?v=qvUT68tG_bo

References of Test Books :

1. Peter V. O'Neil, Advance Engineering Mathematics Thomson (Cengage) Learning, 2007.
2. N. P Bali; Dr. Manish Goyal, A Textbook of Engineering Mathematics, 10th Edition
3. JN Kapur, Mathematical Statistics, S. Chand & company Ltd.
4. BS Grewal, Higher Engineering Mathematics, Khanna Publishers.