COVID-19 in India : A Time Series Analysis

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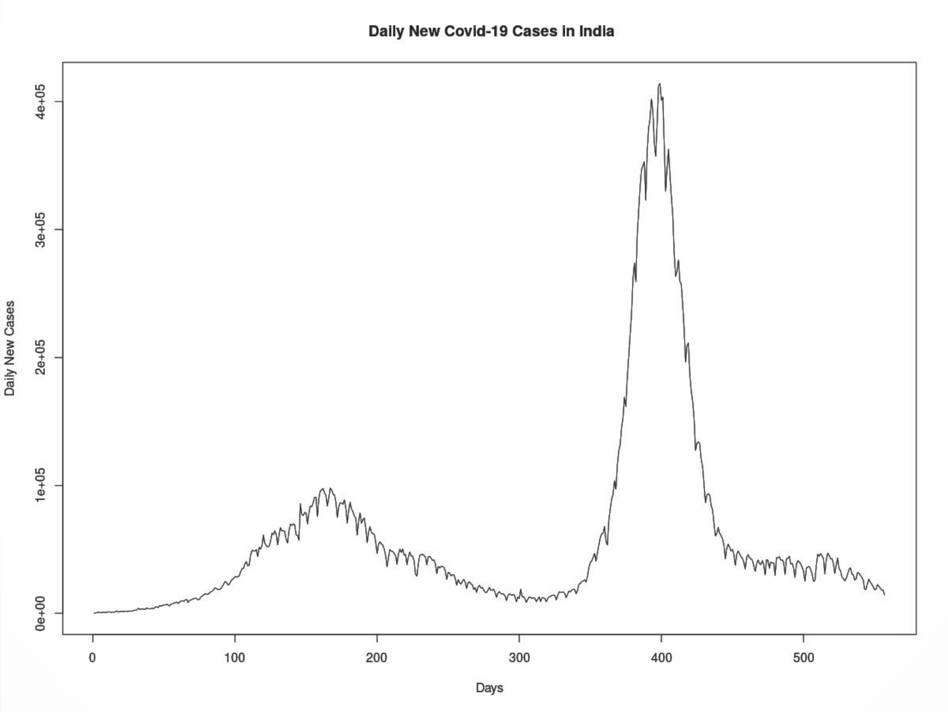
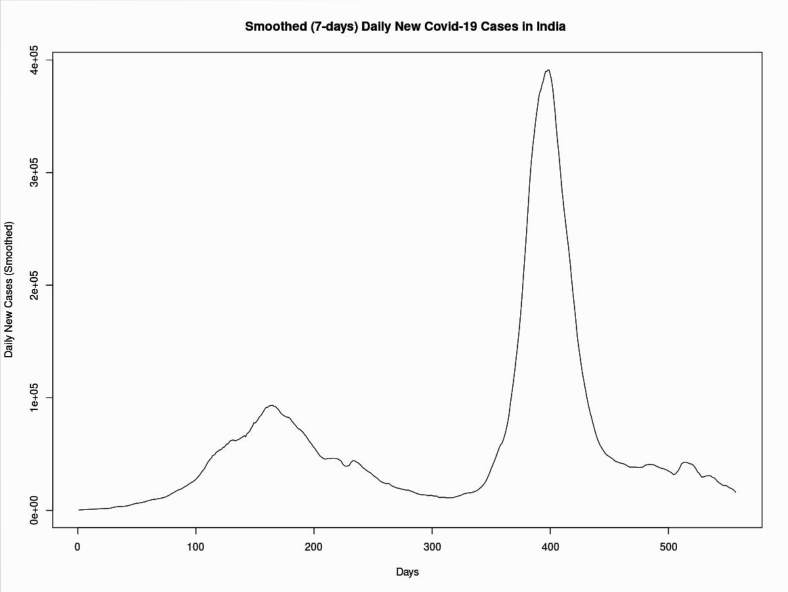
Project Objective:

The COVID-19 pandemic has led to a dramatic loss of human life worldwide and presents an unprecedented challenge to public health. India has been one of the most affected countries in the world, recording more than 3.43 crore cases and 4.5 lakh deaths till the end of October 2021. Here we have tried to study the daily cases and daily deaths due to COVID-19 using various time series modelling techniques.

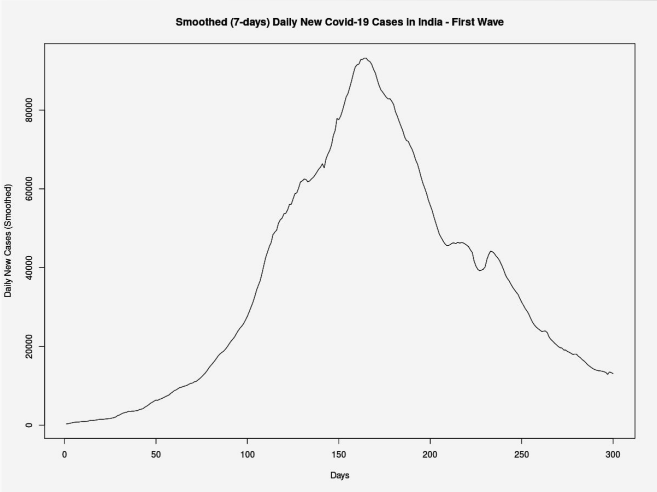
Daily Cases
(03-04-2020 - 11-10-2021)



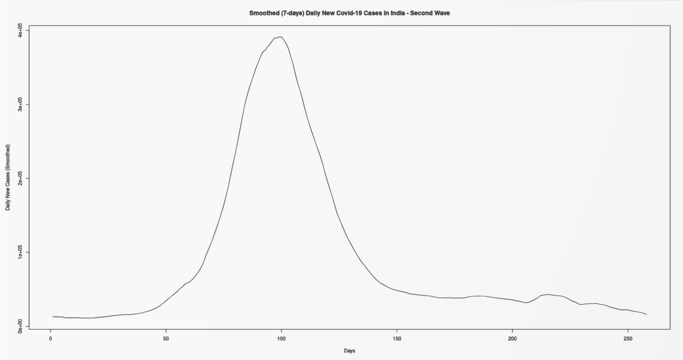
Smoothed Cases
(7-day moving average)



First Wave



Second Wave

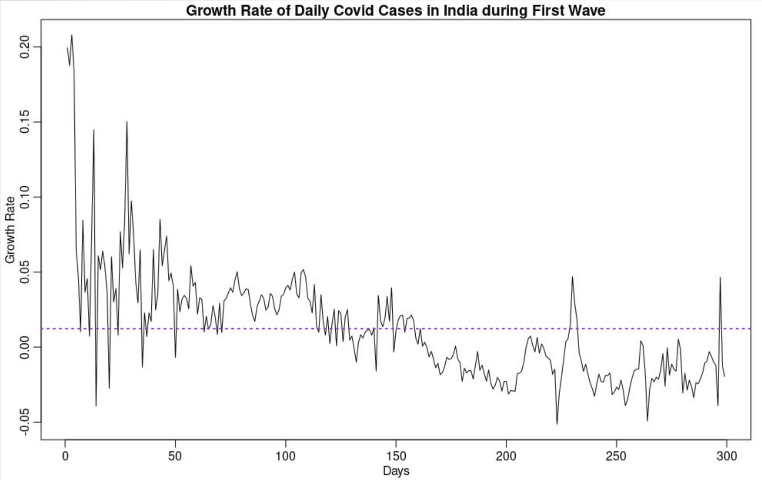


First Wave Analysis (Cases)

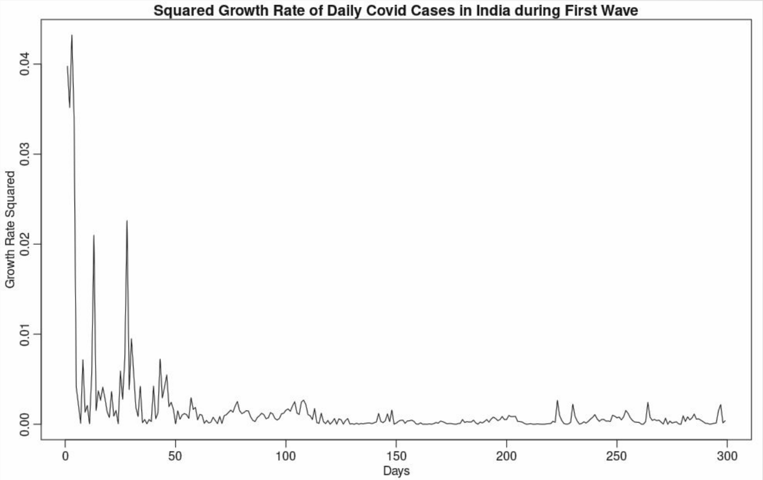
Transformation : Original Time Series X_t → $log(X_t/X_{t-1})$ **(Growth Rate)**

Growth rate of daily cases

(obtained by log differencing the data)

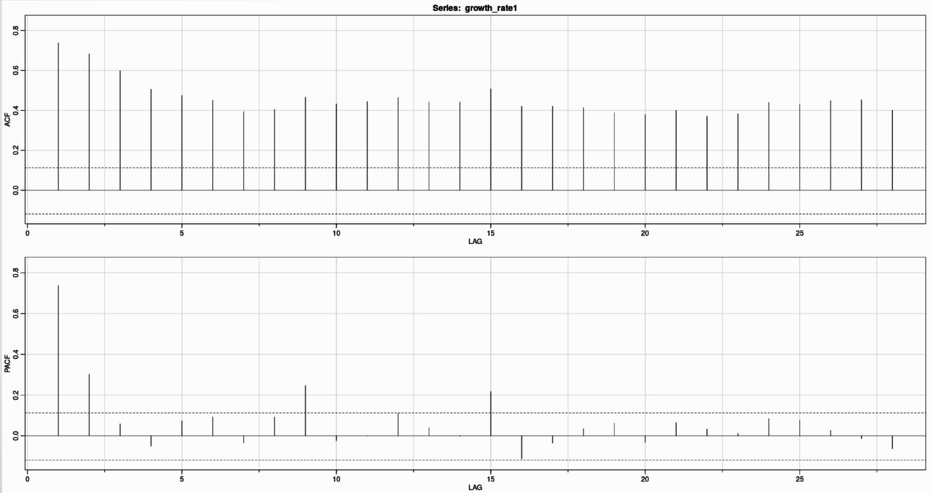


Squared growth rate



Observation : The mean appears to be non-constant and there are signs of volatility in the data.

ACF AND PACF plots of Growth rate

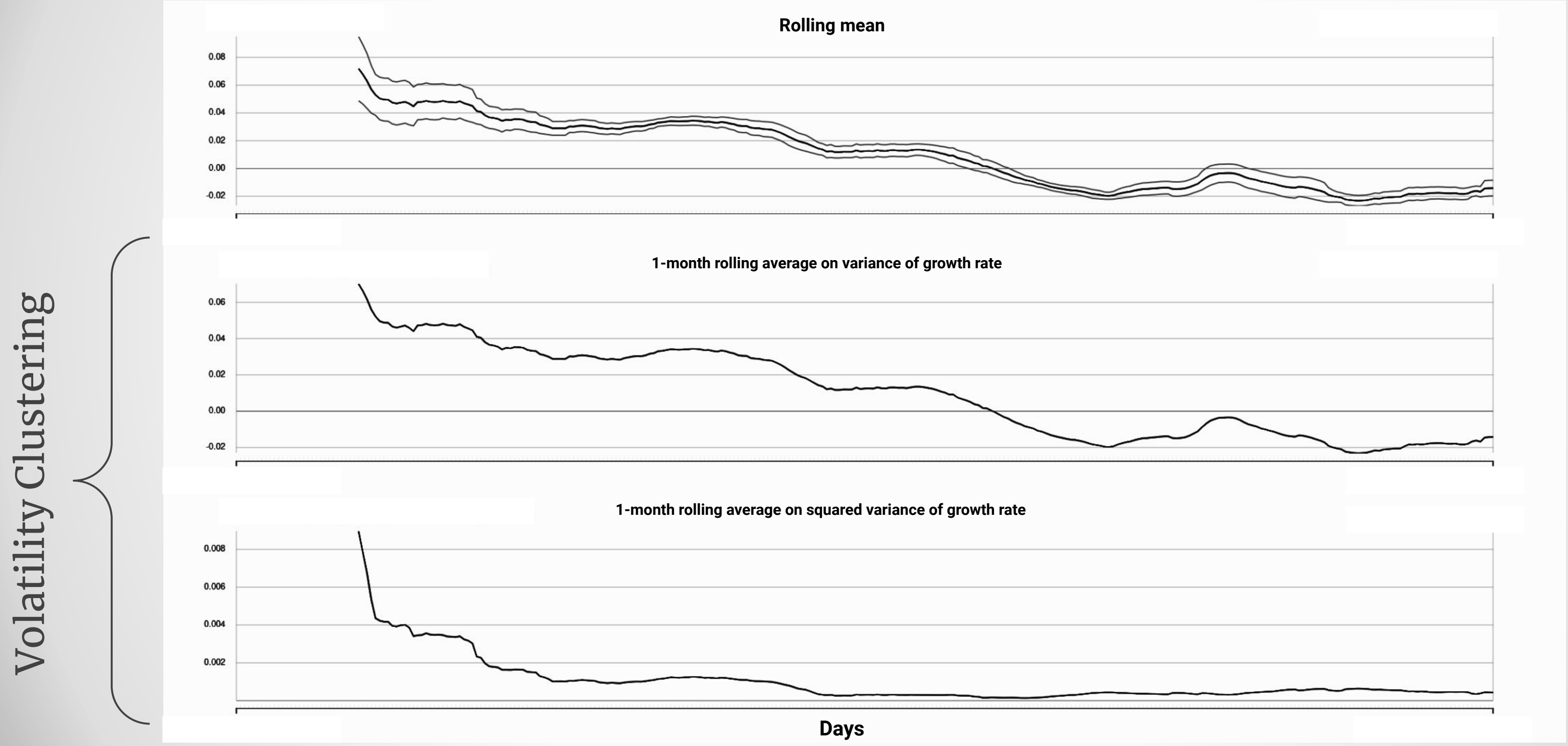


Unit Root Test

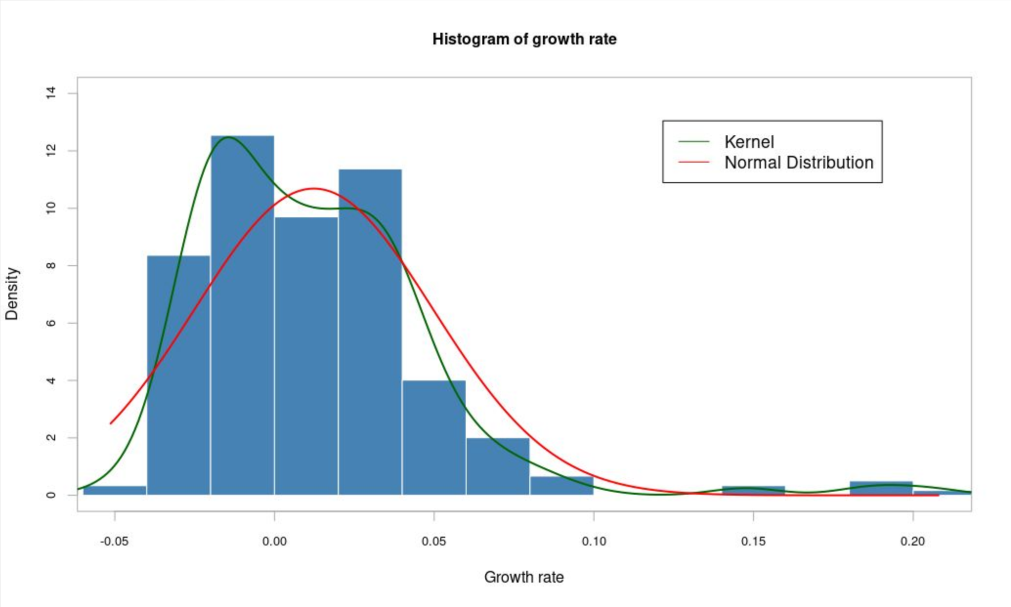
```
Value of test-statistic is: -5.8257 17.3265

Critical values for test statistics:
      1pct  5pct 10pct
tau2 -3.44 -2.87 -2.57
phi1  6.47  4.61  3.79
```

Unit root not present,
but we must check for volatility before making any remark on the stationarity of the time series

Rolling mean and variance of growth rate



Histogram of growth rate (first wave)



Positively skewed

Skewness = 1.948

Modelling first wave using ARMA-GARCH:

ARMA GARCH (m,n,p,q) MODEL:

$$\begin{aligned} x_t &= \mu + \sum_{i=1}^{m} a_i x_{t-i} + \sum_{j=1}^{n} b_j \epsilon_{t-j} + \epsilon_t \\ \epsilon_t &= z_t \sigma_t \\ z_t &= N(0,1) \\ \sigma_t^2 &= w + \sum_{i=1}^{p} \alpha_i \epsilon_{t-i}^2 + \sum_{i=1}^{q} \beta_j \sigma_{t-j}^2 \end{aligned}$$

Model using minimum AiCc

ORDER	AIC	BIC	AICc
1 0 4 2	-5.62992079931565	-5.48675474340154	-5.24504107422973
1 0 4 3	-5.62297726797339	-5.46679611606709	-5.12642554383545
1 0 4 4	-5.6158289880832	-5.44663274018472	-4.99299161784099
$\boxed{1\ 1\ 1\ 1}$	$\boxed{-5.72123634288985}$	$\boxed{-5.61711557495232}$	$\boxed{-5.58518192112114}$
1 1 1 2	-5.69723477831194	-5.58009891438222	-5.49245662131536
1 1 1 3	-5.67430627940743	-5.54415531948552	-5.38663504653072
1 1 1 4	-5.66688465247231	-5.5237185965582	-5.28200492738639

Model using minimum BIC

3 2 4 2	-5.58855625065093	-5.39332981076806	-4.66869562347323
3 2 4 3	-5.59188706685718	-5.38364553098212	-4.50097797594808
3 2 4 4	-5.62245477549124	-5.40119814362398	-4.3452617930351
$\boxed{3\ 3\ 1\ 1}$	$\boxed{-5.77680907247234}$	$\boxed{-5.62062792056604}$	$\boxed{-5.2802573483344}$
3 3 1 2	-5.77542442656479	-5.6062281786663	-5.15258705632258
3 3 1 3	-5.77849284571589	-5.59628150182521	-5.014603956827
3 3 1 4	-5.58312196558509	-5.38789552570222	-4.66326133840739

Suggested Models

(minimum AICc)

- Mean model: ARMA(1,1)
- Variance model: GARCH(1,1)

$Y_{t+1} = -0.00297 + 0.9635 * Y_t + Z_{t+1} - 0.3662 * Z_t$

$\sigma^2_{t+1} = 0.000007 + 0.1332 * Z^2_t + 0.8303 * \sigma^2_t$

(minimum BIC)

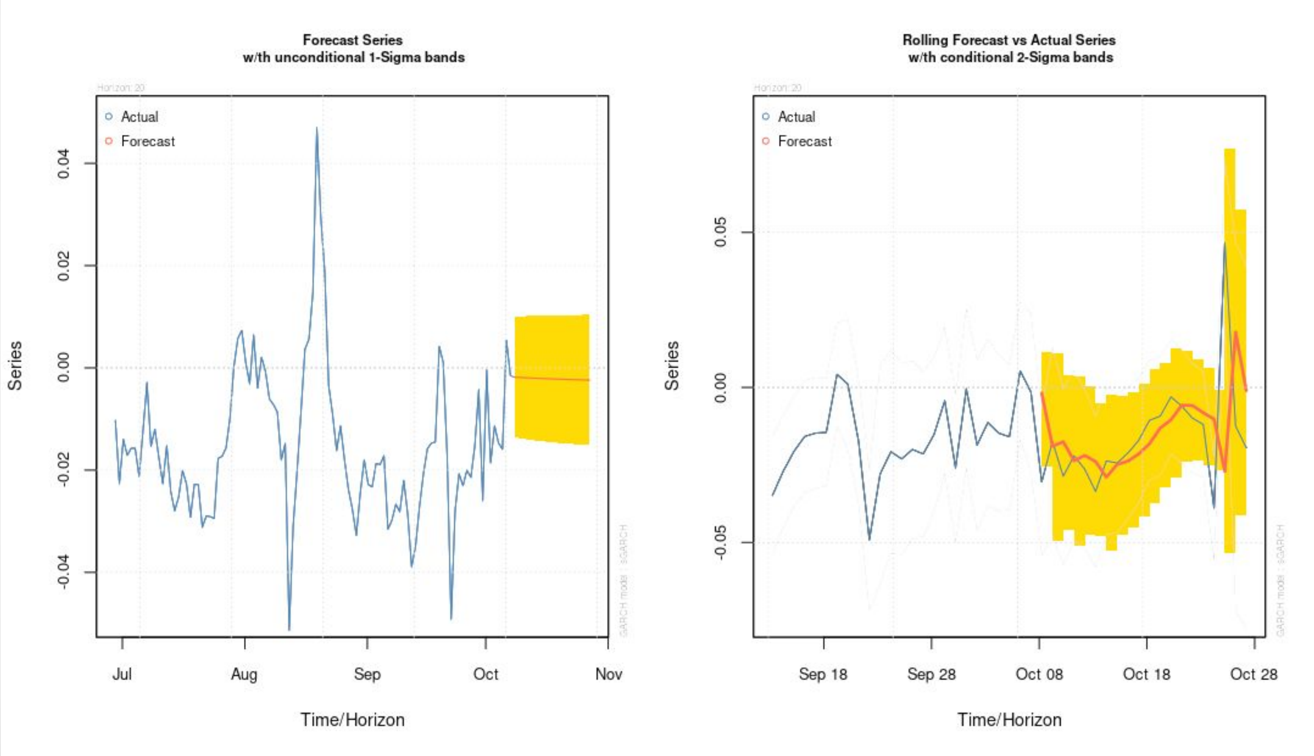
- Mean model: ARMA(3,3)
- Variance model: GARCH(1,1)

$Y_{t+1} = 0.188022 + 2.578620 * Y_t - 2.327144 * Y_{t-1} + 0.748875 * Y_{t-2} + Z_{t+1} - 2.090002 * Z_t + 1.626706 * Z_{t-1} - 0.472380 * Z_{t-2}$

$\sigma^2_{t+1} = 0.000007 + 0.157867 * Z^2_t + 0.807702 * \sigma^2_t$

Forecasting First wave growth rate using ARMA-GARCH:

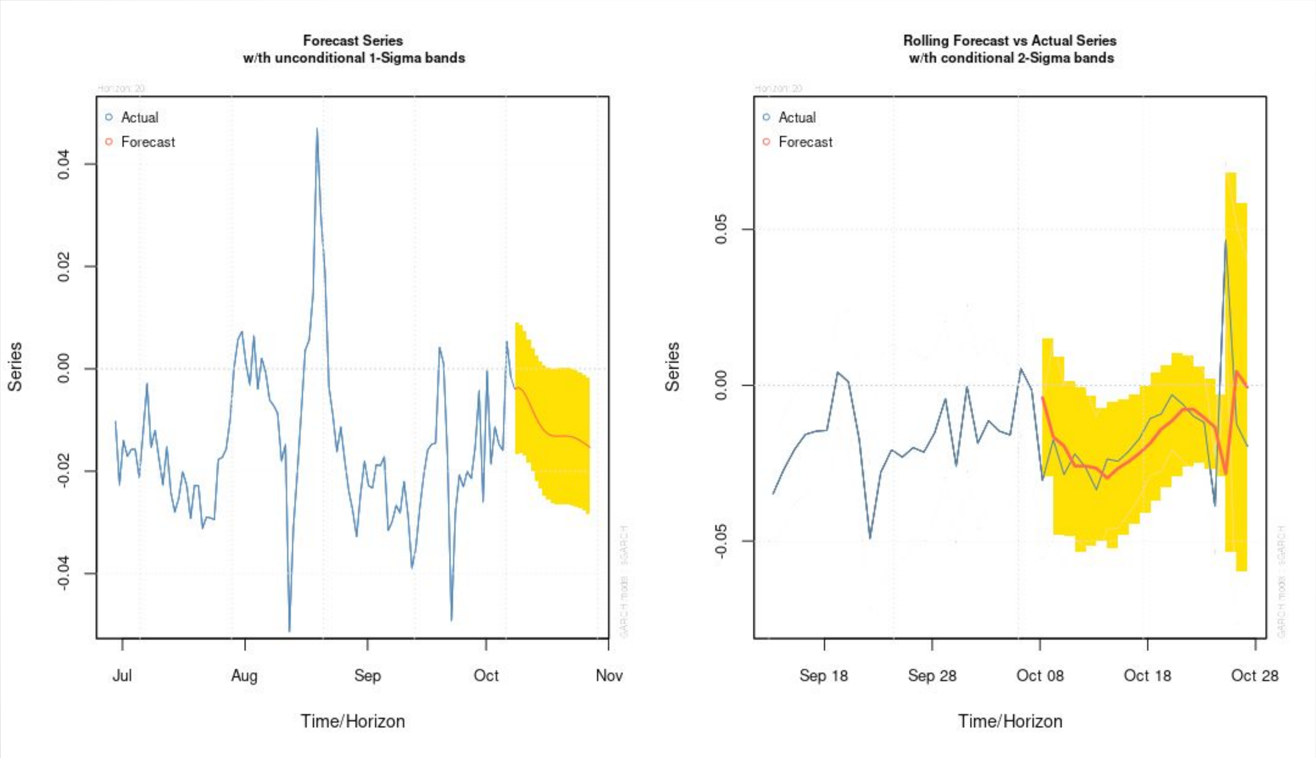
(minimum AICc model)



MSE : 0.0004870874

Forecasting First wave growth rate using ARMA-GARCH:

(minimum BIC model)

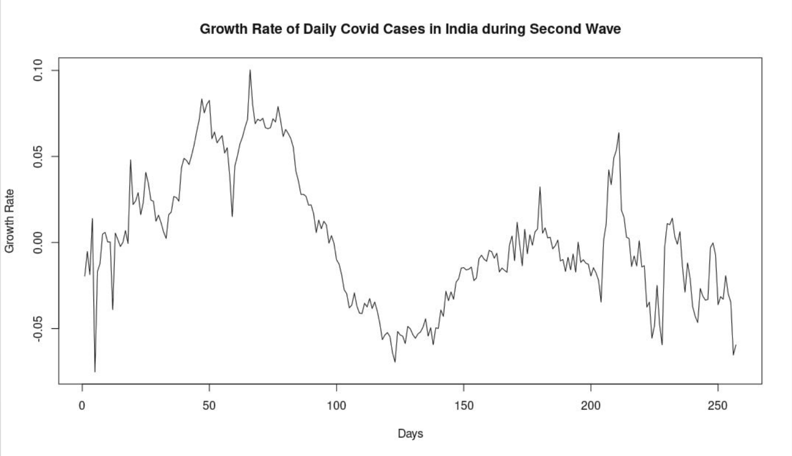


MSE : 0.0003856014

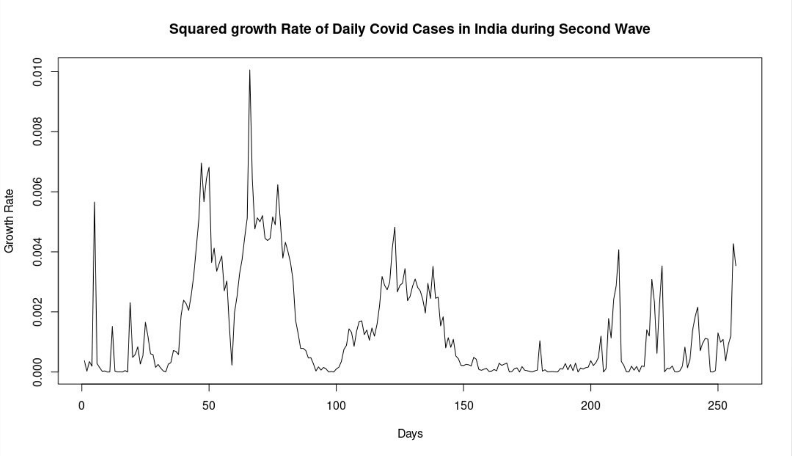
Second Wave Analysis (Cases)

Transformation : Original Time Series X_t → $log(X_t/X_{t-1})$ **(Growth Rate)**

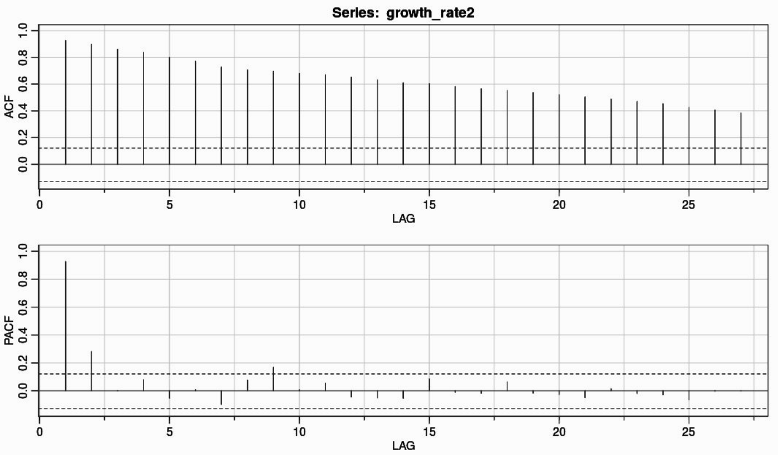
Growth rate of daily cases
(obtained by log differencing the data)



Squared growth rate

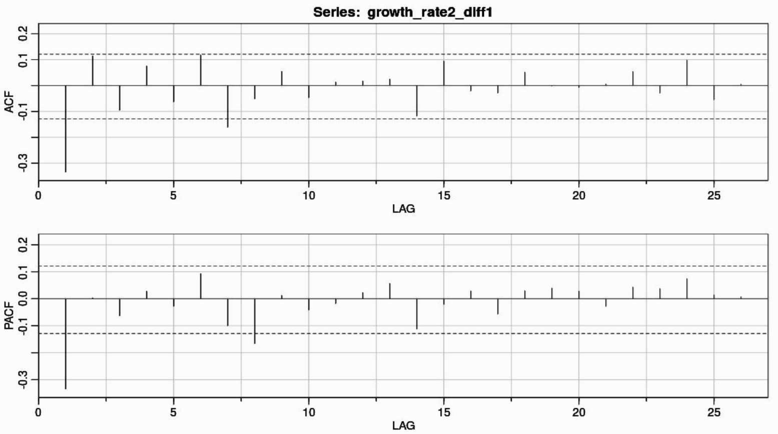


ACF and PACF plots of Growth rate during 2nd wave




```
Value of test-statistic is: -1.8078 1.6883

Critical values for test statistics:
      1pct  5pct 10pct
tau2 -3.44 -2.87 -2.57
phi1  6.47  4.61  3.79
```

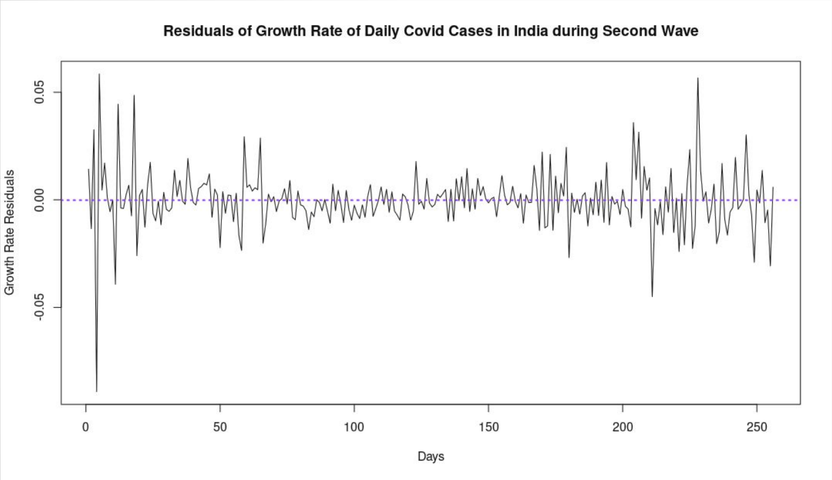
(lag-1 differencing)

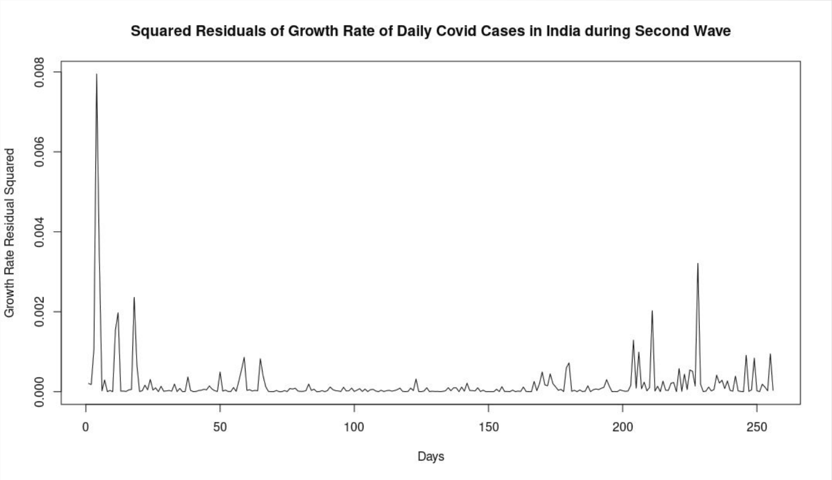



```
Value of test-statistic is: -12.8141 82.1023

Critical values for test statistics:
      1pct  5pct 10pct
tau2 -3.44 -2.87 -2.57
phi1  6.47  4.61  3.79
```

Differenced growth rate of daily cases

Squared differenced growth rate of daily cases

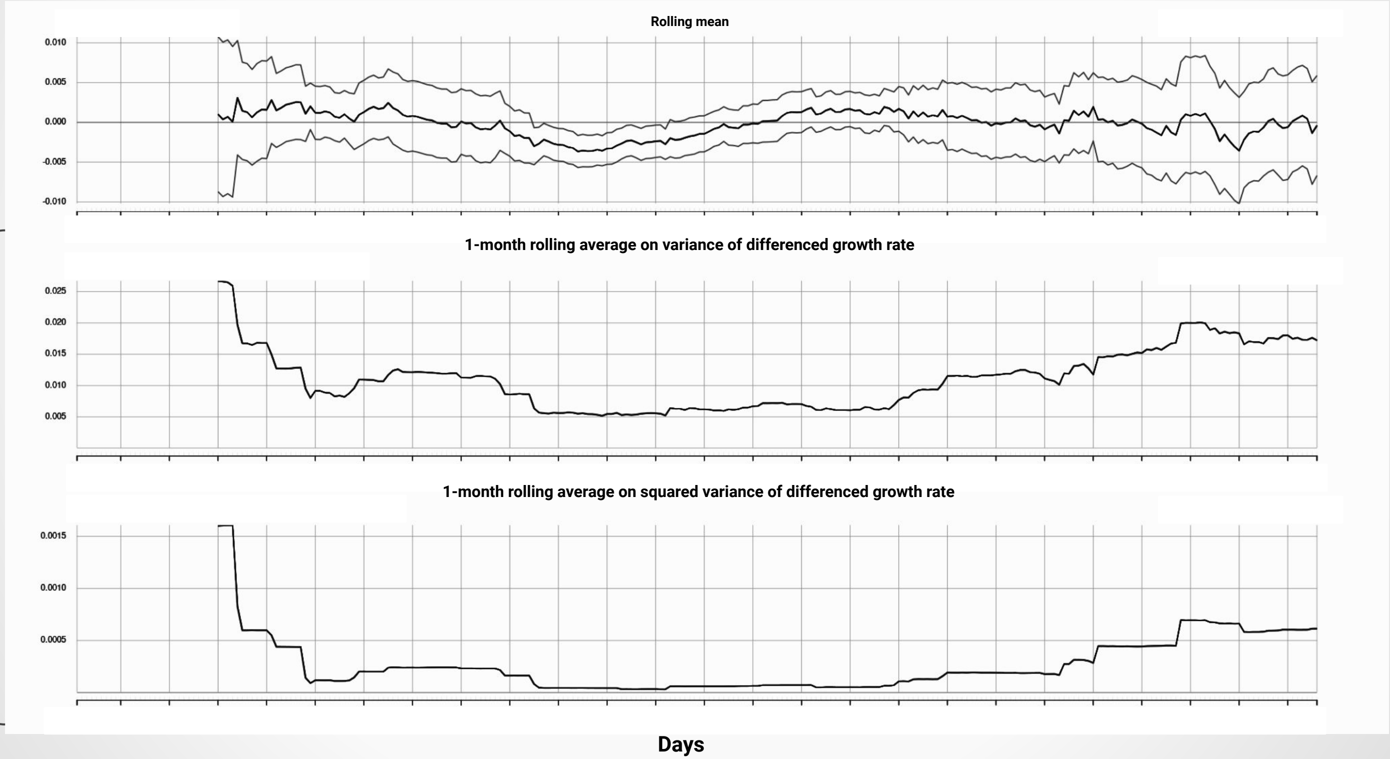




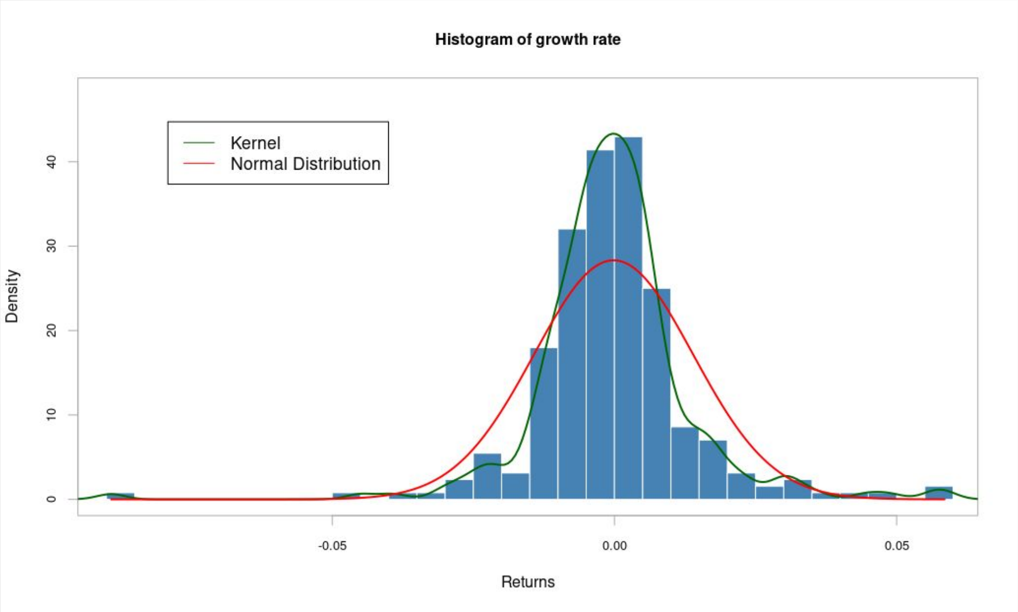
Observation : The mean appears to be somewhat constant and there are signs of volatility in the data.

Rolling mean and variance of differenced growth rate

Volatility Clustering



Histogram of diff growth rate (second wave)



Negatively skewed

Skewness = -0.2387

Modelling second wave using ARMA-GARCH:

ARMA GARCH(m,n,p,q) MODEL:

$$x_t = \mu + \sum_{i=1}^{m} a_i x_{t-i} + \sum_{j=1}^{n} b_j \epsilon_{t-j} + \epsilon_t$$
$$\epsilon_t = z_t \sigma_t$$
$$z_t = N(0,1)$$
$$\sigma_t^2 = w + \sum_{i=1}^{p} \alpha_i \epsilon_{t-i}^2 + \sum_{i=1}^{q} \beta_j \sigma_{t-j}^2$$

(Same) Model using minimum AICc/BIC

ORDER	AIC	BIC	AICc
0 3 3 3	-6.18394309001847	-5.99313879567384	-5.4522357729453
0 3 3 4	-6.16445330465548	-5.95897175689973	-5.26649412098201
0 3 4 1	-6.14225764069676	-5.96613059976326	-5.55926168927976
0 3 4 2	-6.13878098227539	-5.94797668793076	-5.40707366520222
0 3 4 3	-6.16346574672102	-5.95798419896527	-5.26550656304755
0 3 4 4	-6.15597871610209	-5.9358199149352	-5.07401150298733
1 0 1 1	-6.18432729092107	-6.08158651704319	-6.08908919568298
1 0 1 2	-6.18049873563322	-6.06308070834422	-6.02113618583242
1 0 1 3	-6.20750010708086	-6.07540482638073	-5.96750010708086
1 0 1 4	-6.18570476963518	-6.03893223552393	-5.84835537204482
1 0 2 1	-6.17532764420936	-6.05790961692036	-6.01596509440856
1 0 2 2	-6.17202415785167	-6.03992887715154	-5.93202415785167
1 0 2 3	-6.19953980652047	-6.05276727240922	-5.86219040893011

Suggested Model (minimum AICc/BIC)

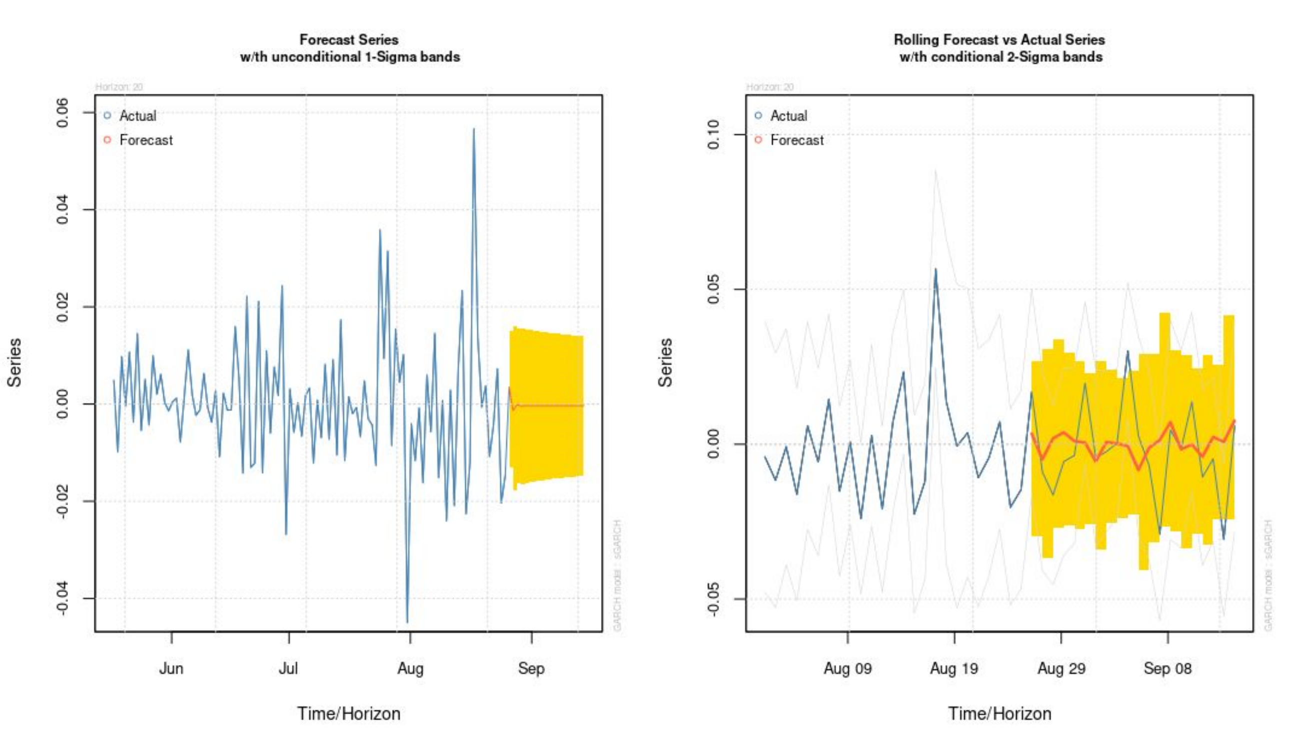
- Mean model: ARMA(1,0)
- Variance model: GARCH(1,1)

$$Y_{t+1} = -0.000345 - 0.261105 * Y_t + Z_{t+1}$$

$$\sigma^2_{t+1} = 0.00001 + 0.163929 * Z^2_t + 0.777992 * \sigma^2_t$$

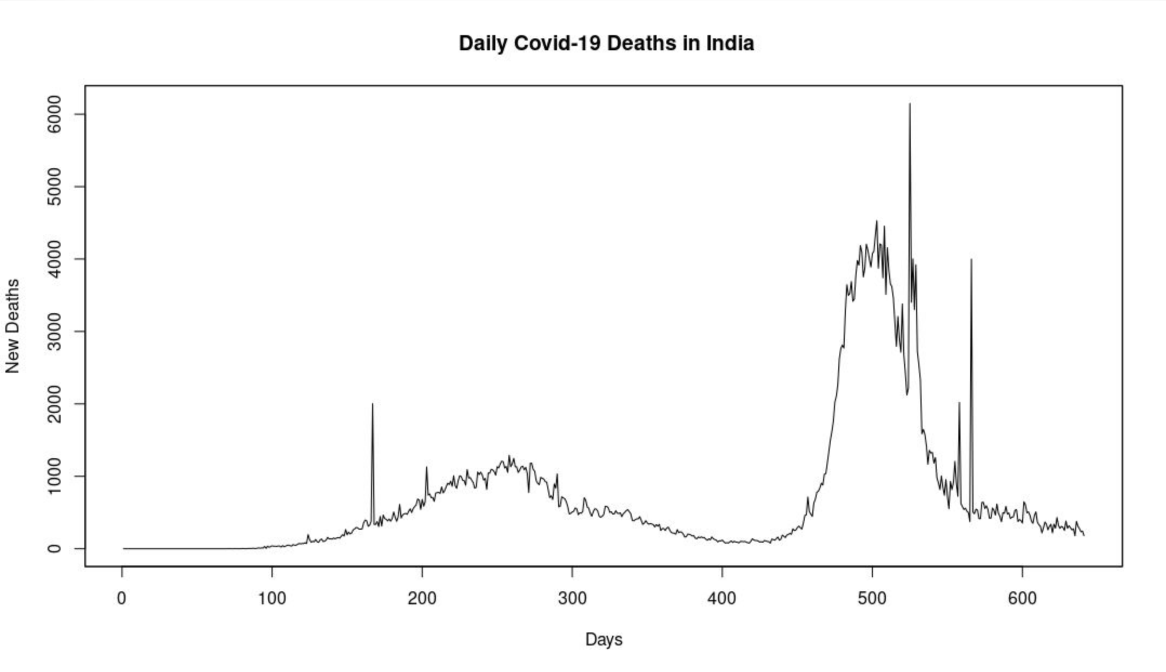
Forecasting Second wave growth rate using ARMA-GARCH:

(minimum AICc/BIC model)

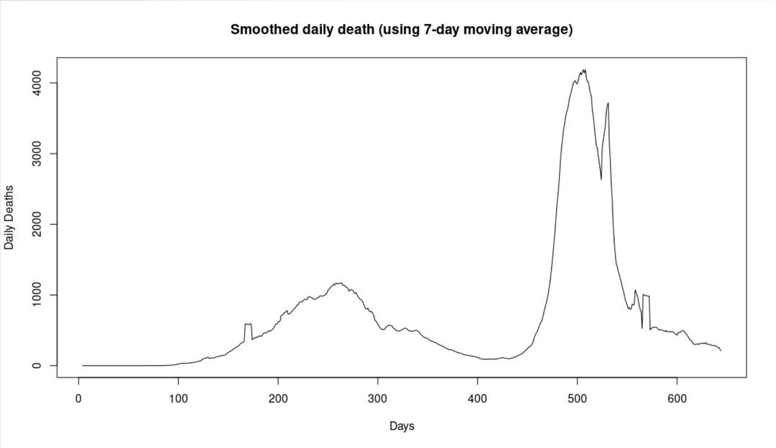


MSE :
0.0002039771

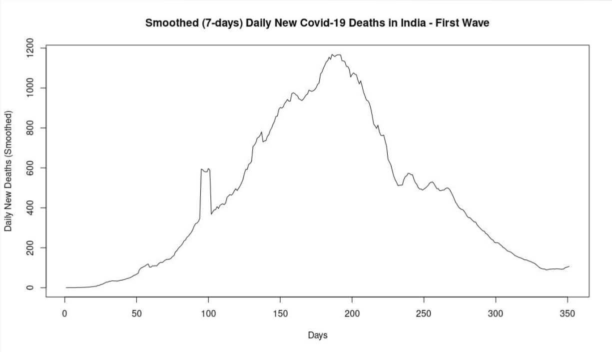
Daily Deaths
(03-04-2020 - 11-10-2021)



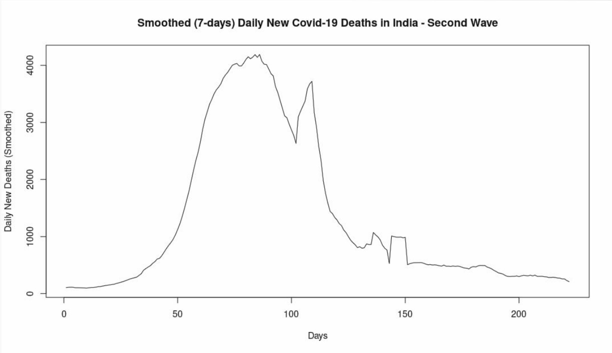
Smoothed Deaths
(7-day moving average)



First Wave

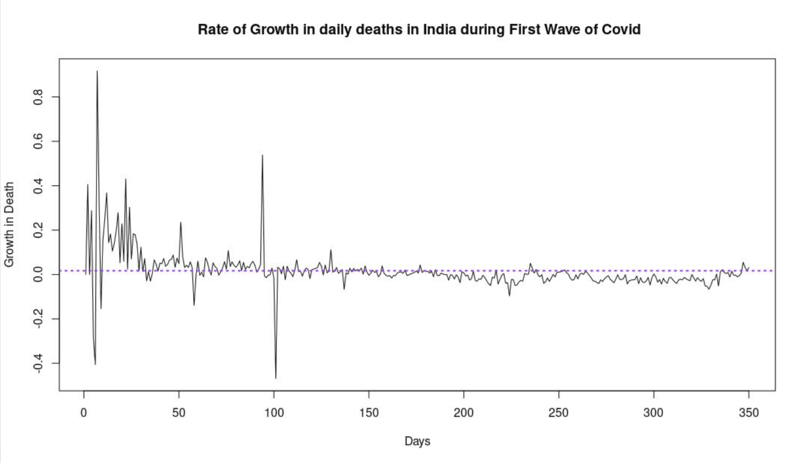


Second Wave

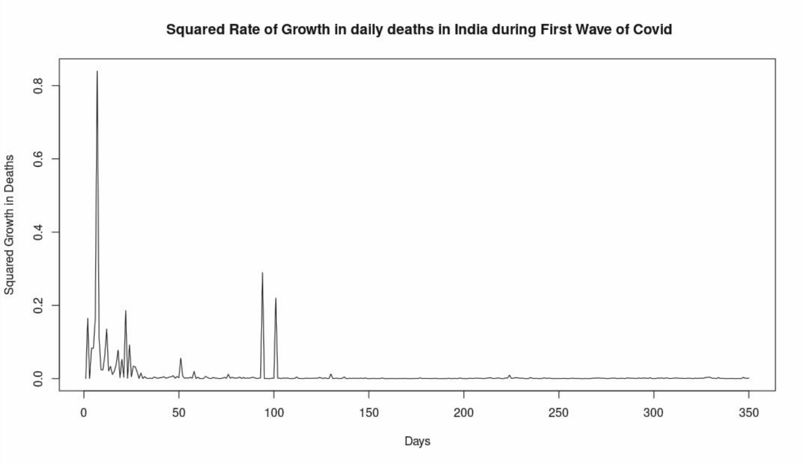


Transformation : Original Time Series X_t → $log(X_t/X_{t-1})$ **(Growth Rate)**

Growth rate of daily deaths
(obtained by log differencing the data)

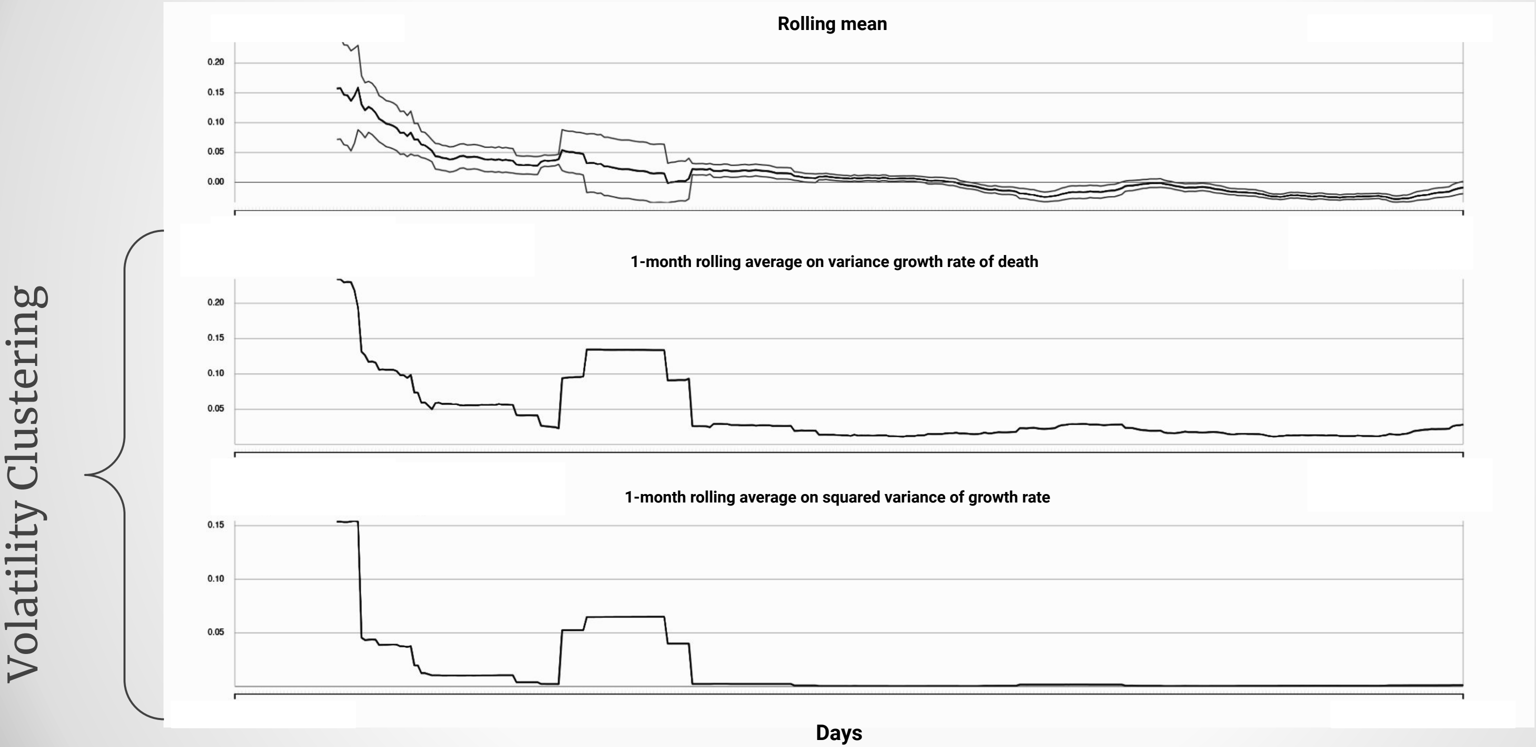


Squared growth rate of daily deaths

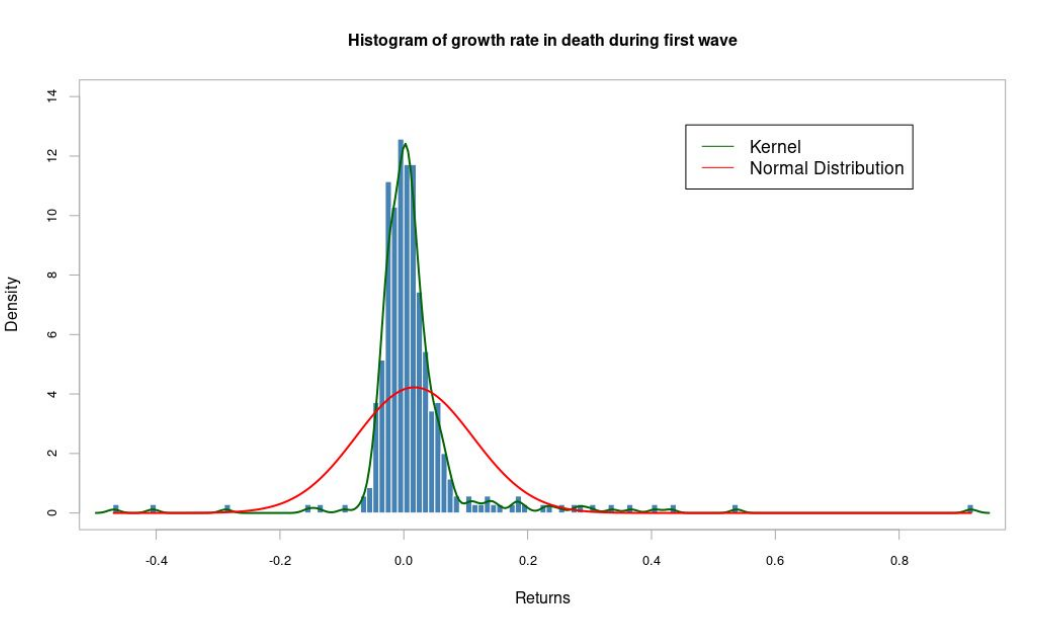


Observation : The mean appears to be constant in the long run and there are signs of volatility in the data.

Rolling mean and variance of growth rate of death (first wave)



Histogram of growth rate of deaths (first wave)



Positively skewed

Skewness = 3.2735

Modelling first wave deaths using ARMA-GARCH:

ARMA GARCH(m,n,p,q) MODEL:

$$
\begin{aligned}
x_t &= \mu + \textstyle\sum_{i=1}^{m} a_i x_{t-i} + \sum_{j=1}^{n} b_j \epsilon_{t-j} + \epsilon_t \\
\epsilon_t &= z_t \sigma_t \\
z_t &= N(0,1) \\
\sigma_t^2 &= w + \textstyle\sum_{i=1}^{p} \alpha_i \epsilon_{t-i}^2 + \sum_{i=1}^{q} \beta_j \sigma_{t-j}^2
\end{aligned}
$$

Model using minimum AiCc

ORDER	AIC	BIC	AICc
1 0 3 1	-3.89739740799553	-3.7937857901466	-3.51222585927427
1 0 3 2	-3.89799631828047	-3.78287229844833	-3.43179624872299
1 0 3 3	-3.91241302121452	-3.78577659939917	-3.31533728790966
1 1 1 1	-4.13797635440513	-4.04587713853942	-3.76175833386495
1 1 1 2	-4.13404375325452	-4.0304321354056	-3.69130012619648
1 1 1 3	-4.12053899374946	-4.00541497391732	-3.61608156387625
1 1 2 1	-4.13384471994539	-4.03023310209647	-3.69398939481553

Model using minimum BIC

2 1 3 1	-4.14895072446738	-4.02231430265203	-3.52875449476532
2 1 3 2	-4.14524579527963	-4.00709697148107	-3.42400223337438
2 1 3 3	-4.14616250727461	-3.99650128149283	-3.30692759932491
2 2 1 1	-4.18755610302275	-4.07243208319062	-3.61560089254615
2 2 1 2	-4.18152539758308	-4.05488897576773	-3.52306321790904
2 2 1 3	-4.16643146158867	-4.02828263779011	-3.4448574861314
2 2 2 1	-4.18149549649751	-4.05485907468216	-3.52957126867425

Suggested Models

(minimum AICc)

- Mean model: ARMA(1,1)
- Variance model: GARCH(1,1)

$Y_{t+1} = -0.016215 + 0.9765 * Y_t + Z_{t+1} - 0.8204 * Z_t$

$\sigma^2_{t+1} = 0.00007 + 0.1079 * Z^2_t + 0.8363 * \sigma^2_t$

(minimum BIC)

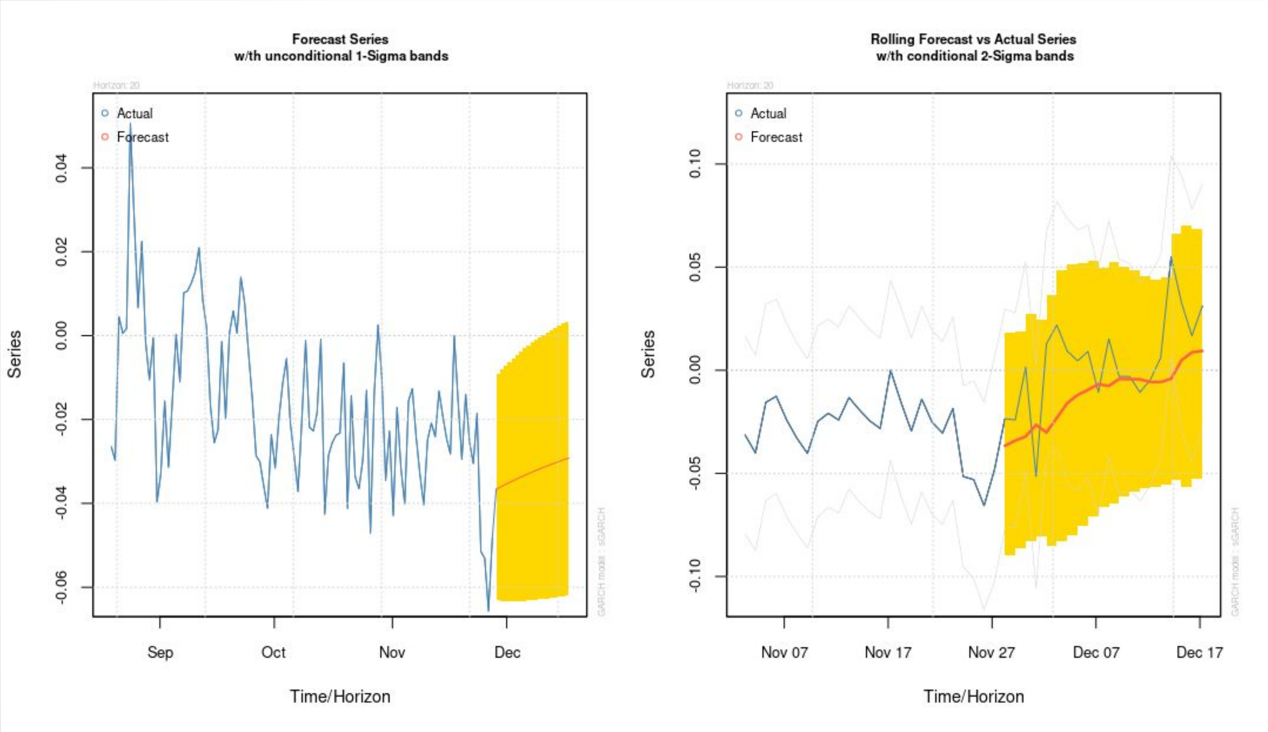
- Mean model: ARMA(2,2)
- Variance model: GARCH(1,1)

$Y_{t+1} = -0.029884 + 1.692425 * Y_t - 0.694092 * Y_{t-1} + Z_{t+1} - 1.478185 * Z_t + 0.494570 * Z_{t-1}$

$\sigma^2_{t+1} = 0.000087 + 0.130202 * Z^2_t + 0.794233 * \sigma^2_t$

Forecasting First wave death rate using ARMA-GARCH:

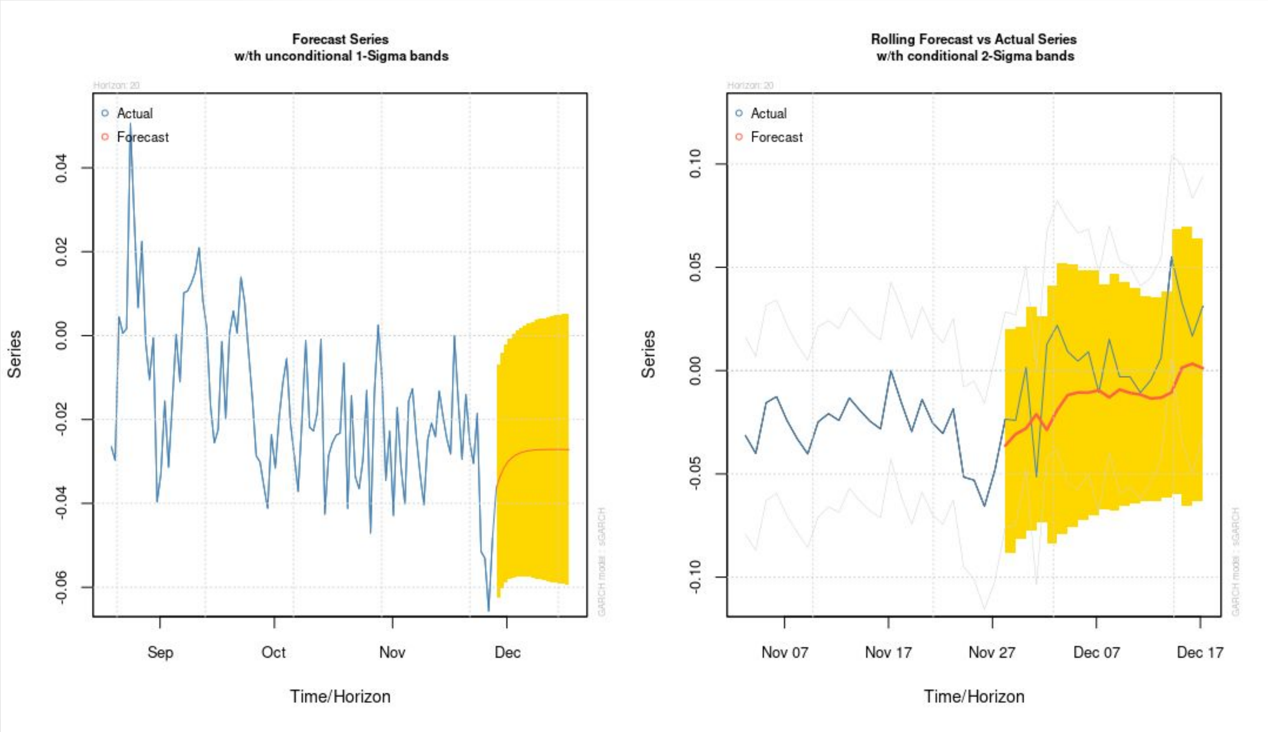
(minimum AICc model)



MSE : 0.001814932

Forecasting First wave death rate using ARMA-GARCH:

(minimum BIC model)



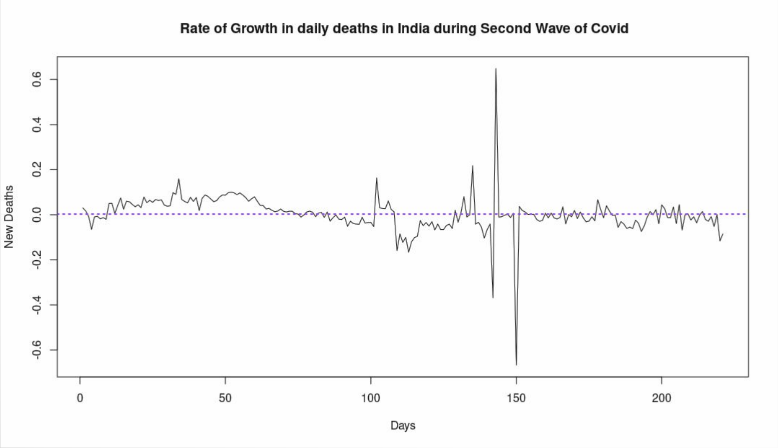
MSE :
0.001534467

Second Wave Analysis (Deaths)

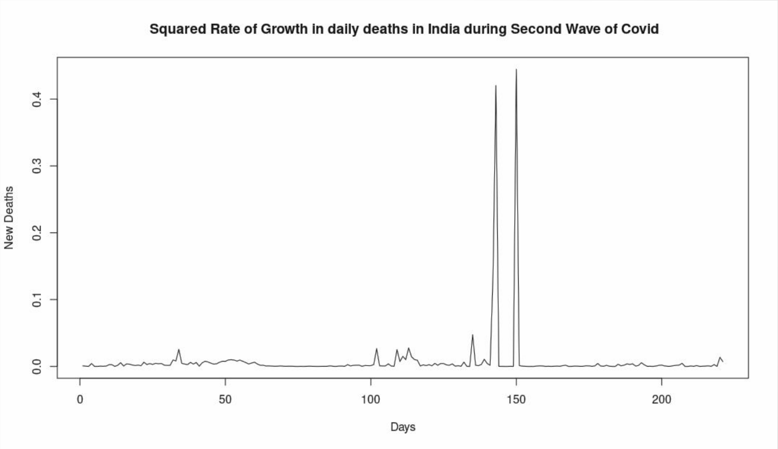
Transformation : Original Time Series X_t → $log(X_t/X_{t-1})$ **(Growth Rate)**

Growth rate of daily deaths

(obtained by log differencing the data)

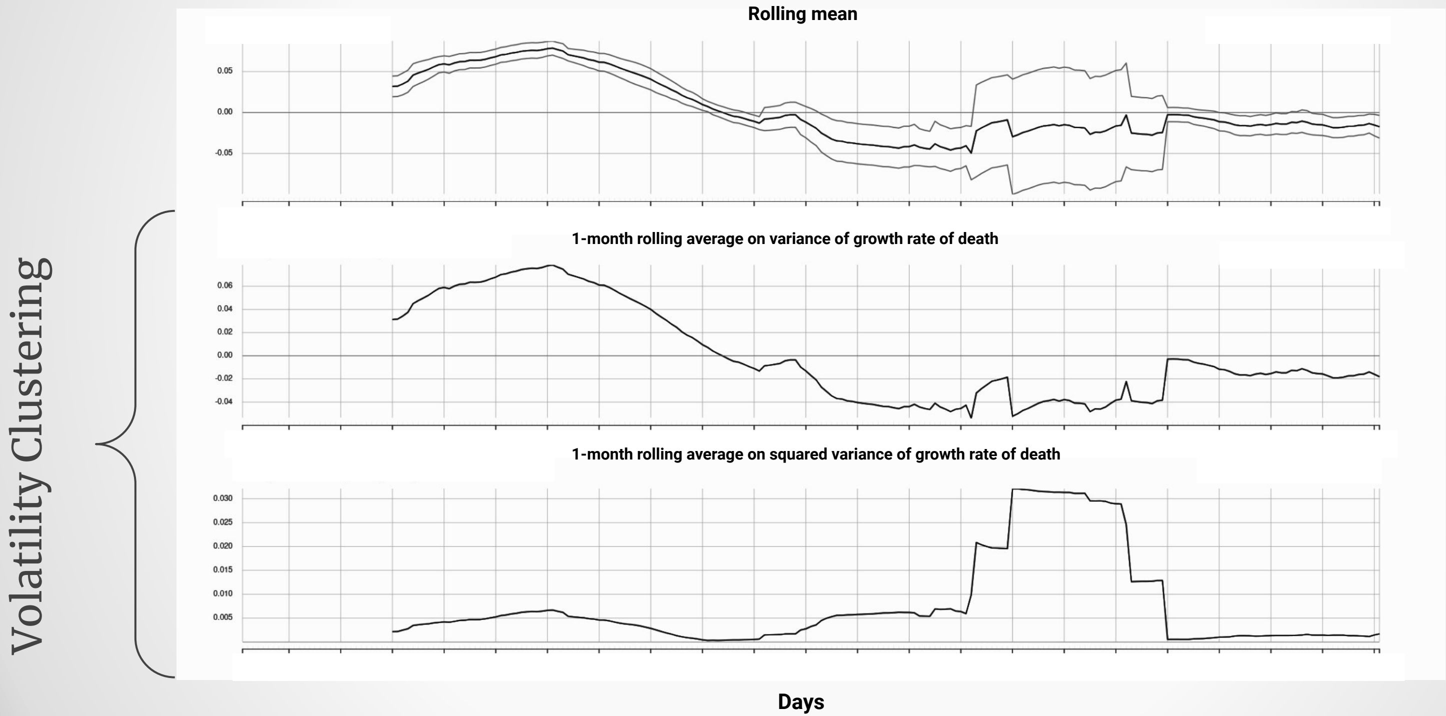


Squared growth rate of daily deaths

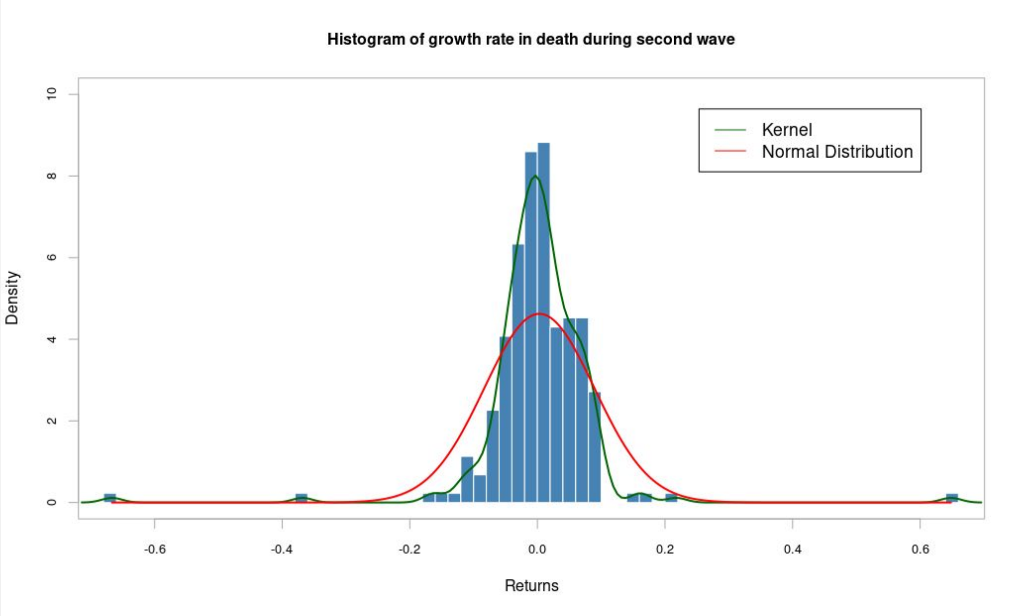


Observation : There are signs of volatility in the data.

Rolling mean and variance of growth rate of death (second wave)



Histogram of growth rate of deaths (second wave)



Negatively skewed

Skewness = −0.5321

Modelling second wave deaths using ARMA-GARCH:

ARMA GARCH(m,n,p,q) MODEL:

$$x_t = \mu + \sum_{i=1}^{m} a_i x_{t-i} + \sum_{j=1}^{n} b_j \epsilon_{t-j} + \epsilon_t$$
$$\epsilon_t = z_t \sigma_t$$
$$z_t = N(0,1)$$
$$\sigma_t^2 = w + \sum_{i=1}^{p} \alpha_i \epsilon_{t-i}^2 + \sum_{i=1}^{q} \beta_j \sigma_{t-j}^2$$

(Same) Model using minimum AICc/BIC

ORDER	AIC	BIC	AICc
1 0 2 1	-3.69806616498918	-3.56659134277788	-3.512880979804
1 0 2 2	-3.68811582409438	-3.54020664910666	-3.40904605665252
1 0 2 3	-3.65277292314993	-3.48842939538579	-3.26024955866395
1 0 3 1	-3.68664446392543	-3.53873528893771	-3.40757469648357
1 0 3 2	-3.67669420790666	-3.51235068014253	-3.28417084342068
1 0 3 3	-3.64282266802662	-3.46204478748607	-3.11700107178249
1 1 1 1	-3.87770036285046	-3.74622554063915	-3.69251517766527
1 1 1 2	-3.86571873084764	-3.71780955585992	-3.58664896340578
1 1 1 3	-3.86097952305993	-3.6966359952958	-3.46845615857395
1 1 2 1	-3.86840799946669	-3.72049882447897	-3.58933823202483
1 1 2 2	-3.85845775544015	-3.69411422767601	-3.46593439095417
1 1 2 3	-3.85371699398897	-3.67293911344843	-3.32789539774484
1 1 3 1	-3.86366724373657	-3.69932371597244	-3.47114387925059

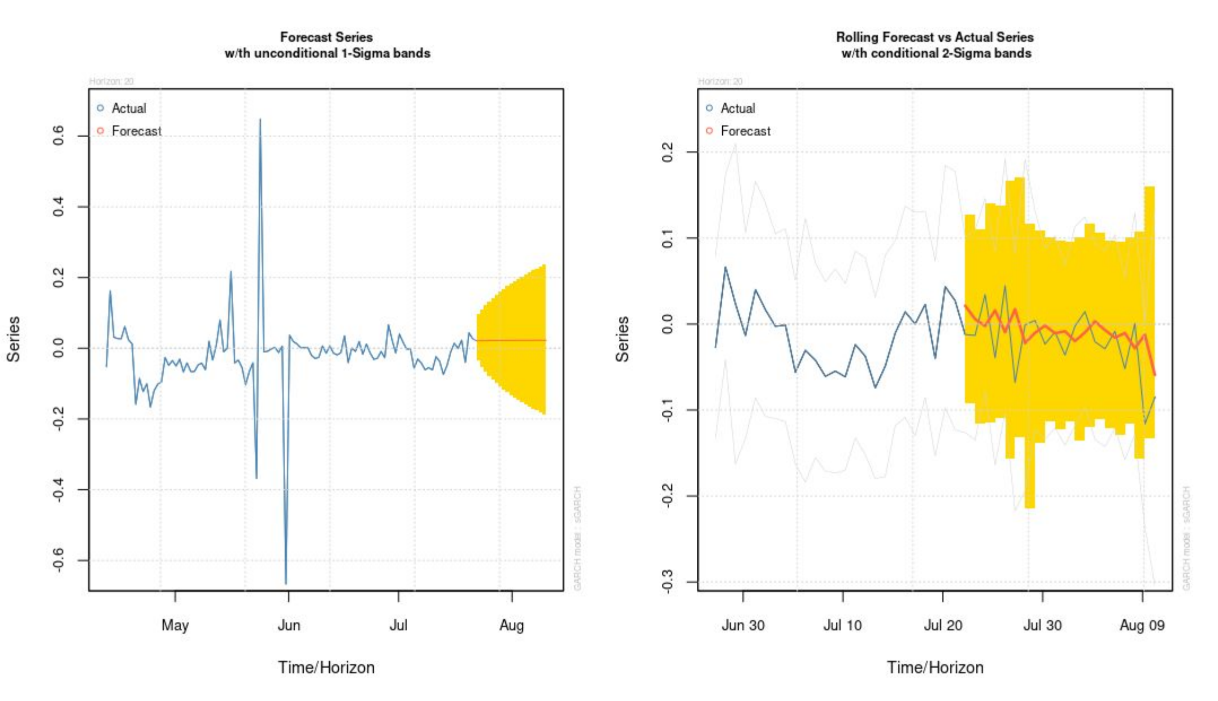
Suggested Model (minimum AICc/BIC)

- Mean model: ARMA(1,1)
- Variance model: GARCH(1,1)

$$Y_{t+1} = 0.023062 + 0.9527 * Y_t + Z_{t+1} - 0.4859 * Z_t$$

$$\sigma^2_{t+1} = 0.002274 + 0.8478 * Z^2_t + 0.1512 * \sigma^2_t$$

Modelling Second wave death rate using ARMA-GARCH: Forecasting



MSE :
0.003233904

Summary

DATA	MINIMUM AICc MODEL	MINIMUM BIC MODEL
Cases in Wave 1	ARMA-(1,1)-GARCH(1,1)	ARMA-(3,3)-GARCH(1,1)
Cases in Wave 2	ARMA-(1,0)-GARCH(1,1)	ARMA-(1,0)-GARCH(1,1)
Deaths in Wave 1	ARMA-(1,1)-GARCH(1,1)	ARMA-(2,2)-GARCH(1,1)
Deaths in Wave 2	ARMA-(1,1)-GARCH(1,1)	ARMA-(1,1)-GARCH(1,1)

Thank you!