

TUTORIAL: 4

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Ques

Solution: → $T(n) = aT(n/b) + f(n^2)$
 $a > 1, b > 1$

On comparing

$$a = 3, b = 2, f(n) = n^2$$

$$\text{Now, } C = \log_b a = \log_2 3 = 1.584$$

$$n^c = n^{1.584} < n^2$$

$$\therefore f(n) > n^c$$

$$T(n) = \Theta(n^2)$$

Soln:-

Soln:- $a > 1, b > 1$

$$a = 4, b = 2, f(n) = n^2$$

$$C = \log_2 4 = 2$$

$$n^c = n^2 = f(n) = n^2$$

$$\therefore T(n) = \Theta(n^2 \log_2 n)$$

Soln:-

Soln:- $a = 1$

$$b = 2$$

$$f(n) = 2^n$$

$$C = \log_b a = \log_2 1 = 0$$

$$n^c = n^0 = 1$$

$$f(n) > n^c$$

$$T(n) = \Theta$$

Ques

Soln $a = 2^n$

$$b = 2, f(n) = n^2$$

$$C = \log_b a = \log_2 2^n = n$$

$$n^c \Rightarrow n^n$$

$$f(n) = n^c$$

$$f(n) = \Theta(n^2 \log_2 n)$$

Soln:-

Soln:- $a = 16, b = 4$

$$f(n) = n$$

$$C = \log_4 16 = \log_4 (4)^2 = 2 \log_4 4 = 2$$

$$n^c \Rightarrow n^2$$

$$f(n) < n^c$$

$$\therefore T(n) = \Theta(n^2)$$

Soln

Soln:- $a = 2, b = 2$

$$f(n) = n \log n$$

$$C = \log_2 2 = 1$$

$$n \log n > n$$

$$f(n) > n^c$$

$$T(n) = \Theta(n \log n)$$

Ques 7
Solution $a=2, b=2, f(n) = n/\log n$

$$c = \log_2 2 = 1$$

$$n^c = n^1 = n$$

$$\therefore \frac{n}{\log n} < n$$

$$f(n) < n^c$$

$$T(n) = \Theta(n)$$

Ques 8
Solution $\rightarrow a=2, b=4, f(n) = n^{0.5}$

$$c = \log_4 2 = \log_4 2^1 = 0.5$$

$$n^c = n^{0.5}$$

$$n^{0.5} < n^{0.5}$$

$$\therefore T(n) = \Theta(n^{0.5})$$

Ques 9

Solution $\rightarrow a=0.5, b=2$

$a \geq 1$ but here $a = 0.5$ so we cannot apply Master's theorem

Ques 10

Solution $\rightarrow a=16, b=4, f(n) = n!$

$$\therefore c = \log_4 16 = \log_4 4^2 = 2$$

$$n^c = n^2$$

$$\text{As } n! > n^2$$

$$\therefore T(n) = \Theta(n!)$$

Ques 11 \rightarrow

Solution $\rightarrow a=4, b=2, f(n) = \log n$

$$c = \log_2 4 = \log_2 2^2 = 2$$

$$n^c = n^2$$

$$f(n) = \log n$$

$$\therefore \log n < n^2$$

$$f(n) < n^c$$

$$T(n) = \Theta(n^2)$$

$$= \Theta(n^2)$$

Ques 12 \rightarrow

Solution $\rightarrow a=\sqrt{n}, b=2$

$$c = \log_2 a = \log_2 \sqrt{n} = \frac{1}{2} \log_2 n$$

$$\therefore \frac{1}{2} \log_2 n < \log(n)$$

$$\therefore f(n) > n^c$$

$$T(n) = \Theta(f(n))$$

$$\Theta(\log(n))$$

Ques 13 \rightarrow

Solution $\rightarrow a=3, b=3$

$$c = \log_3 3 = \log_3 3^1 = 1$$

$$n^c = n^1 = n$$

$$\text{As } \text{sqrt}(n) < n$$

$$f(n) < n^c$$

$$T(n) = \Theta(n)$$

Ques 14

Solution $\rightarrow a=3, b=2, f(n) = n$

$$c = \log_2 3 = \log_2 3^1 = 1.5849$$

$$n^c = n^{1.5849} \Rightarrow n < n^{1.5849}$$

$$f(n) < n^c$$

$$T(n) = \Theta(n^{1.5849})$$

2.1 →
Solution! →

$$a=4, b=2$$

$$c = \log_b a = \log_2 4 = 2$$

$$n^c = n^2$$

$$n < n^2 \text{ (for any constant)}$$

$$\theta(n) < n^c$$

$$f(n) = \theta(n^2)$$

Ques 16:-

Solution! → $a=3, b=4, \theta(n) = n \log n$

$$c = \log_b a = \log_4 3 = 0.792$$

$$n^c = n^{0.792}$$

$$n^{0.792} < n \log n$$

$$T(n) = \theta(n \log n)$$

Ques 17! →

Solution! → $a=3, b=3$

$$c = \log_b a = \log_3 3 = 1$$

$$\theta(n) = n/2$$

$$\therefore n/2 < n$$

$$\theta(n) < n^c$$

$$\therefore T(n) = \theta(n)$$

Ques 18! →

Solution! → $a=6, b=3$

$$c = \log_b a = \log_3 6 = 1.6309$$

$$n^c = n^{1.6309}$$

$$n^{1.6309} < n^2 \log n$$

$$\therefore T(n) = \theta(n^2 \log n)$$

Ques 19

Solution $a=4, b=2, f(n) = \frac{n}{\log n}$
 $c = \log_b a = \log_2 4 = 2$

$$n^c = n^2$$

$$\frac{n}{\log n} < n^2$$

$$T(n) = \Theta(n^2)$$

Ques 20 \rightarrow

Solution $\rightarrow a=64, b=8$

$$c = \log_b a = \log_8 64 = \log_8 (8)^2 = 2$$

$$c = 2$$

$$n^c = n^2$$

$$n^2 \log n > n^2$$

$$T(n) = \Theta(n^2 \log n)$$

Ques 21 \rightarrow

Solution $a=4, b=3, f(n) = n^2$

$$c = \log_b a = \log_3 4 = 1.712$$

$$n^c = n^{1.712}$$

$$n^{1.712} < n^2$$

$$\therefore T(n) = \Theta(n^2)$$

Ques 22 \rightarrow

Solution $\rightarrow a=1, b=2$

$$c = \log_b a = \log_2 1 = 0$$

$$n^c = n^0 = 1$$

$$n(2 - \cos x) > n^c$$

$$T(n) = \Theta(n(2 - \cos x))$$