Propulsion of Space Vehicles

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Abstract

In this paper, we discuss about how mechanism of propulsion takes place for space vehicles, the governing equations for propulsion, the chemical propulsion system, study of the effectiveness of propellant in thermodynamic combustion process. Replacement of single staged by multi-staged system of rocket over time are analyzed studied. The analyzing of the effect of gravity and drag force on the propulsion system are complied in python for solving of complex differential equations. Finally, the importance of the control system with its basic principles are discussed in the later part of the paper.

1 Introduction

In today's world, everyone is looking forward towards the space science. Space vehicle Propulsion technologies are intended to provide efficient exploration of our solar system and help mission designers to plan their missions accordingly. Its major applications are launching satellites to geostationary orbits to connect the world and also on other planets like mars,moon,jupiter to get the crucial information about these planets. There are many big government and non-government organisations are working continuously towards these technologies like NASA,space-X, ISRO,etc.

In space vehicles propulsion system,a large number of technical sub-systems are required to operate in co-ordination with each other to carry out the whole process smoothly without failures. Some of its prominent sub-systems include mechanics of the propulsion, thermodynamics of the propulsion, rocket engine and nozzle design and the propulsion control system. In propulsion mechanics, basic idea of the change in velocity of rocket w.r.t. time and mass flow of the exhaust gas, the acceleration at different stages, and time for next stage of propulsion are considered. In thermodynamic process, the reaction of the the fuel and oxidizer which generates the exhaust gas which is the source of the thrust force is analyzed . The study of the efficient design of nozzles which helps the exhaust gases to expand and accelerate to high velocities are important for the efficiency of the propulsion mechanism. For the control of thrust propulsion direction and propulsion rate, sophisticated controllers are needed to be designed for the space vehicle to reach its desired destination.

2 Background

Even before 20th century, there were many hypothesis that were proposed related to propulsion of space vehicles. But no one of them were mathematically proven. But at the start of 20th century, a Russian school teacher named "Konstantin Tsiolkovsky" gave a realistic proposal to spaceflights. He gave a mathematical Rocket equation by applying Newton's second law of motion which relates rocket engine exhaust velocity to the change in velocity of vehicle itself. Then, In 1919 Robert H. Goddard's published a paper "A Method of Reaching Extreme Altitudes" where he had explained that by using de Laval nozzle and liquid fuel, rockets would get sufficient power for their interplanetary travel and he actually flew the world's first liquid propellant rocket. Both these scientists had given the rocket theory for a single staged rocket.But mathematically it was approximated that with a single staged rocket,it is nearly impossible to generate such power from earth surface to reach to the space.Then in 1935, Louis Damblanc gave the concept of multi-staged rockets when he proved mathematically that multi-staged rockets took less energy to generate more velocity.Then in later years,there were many practical attempts and advancements done in multi-staged rockets like clustering process(adding more than 1 rocket in 1st stage to make the takeoff possible easily), adjusting the nozzle shape and size, on improving the Efficiency of propulsion and in proper staging of rockets.
The propulsion system itself consists of different types which operates on different principles.

Types of propulsion technology

(1) Chemical propulsion:- It is a type of propulsion within which the thrust force is generated by the products of a chemical reaction, usually by the oxidation of fuel. These reactions produce hot gas which expands to generate the required thrust. In today's world, most of the rocket engines use this technology.

(2) Non-chemical propulsion:- Usually, electric propulsion are called as Non-Chemical Propulsion. They basically use the concept of Coulomb force or Lorentz force to produce thrust by accelerating the reaction mass electrostatically or electromagnetically. As of now, they are still not able to produce significant thrust to lift off a rocket but they can be used as in-space propulsion(propulsion systems used in the vacuum of space).

(3) Advanced propulsion technologies:- This technology is still under devel-

opment like nuclear thermal propulsion, electric propulsion, satellite tethers, and solar sails. The main purpose of this technology is to provide affordable, efficient and lighter rockets.

3 Results

The propulsion of the space vehicles are affected by a large number of external factors that makes it important to consider their effects for their smooth and secure control. Some of the prominent external factors that affect the propulsion system are:

1. Gravitational field
2. Drag force due to wind fraction
3. Deviation of the thrust vector angle from set point due to small factors

The forces of equations governing the propulsion of the space vehicle(rocket) can be given by:

For rocket momentum is-

$$\begin{aligned} p &= m(t)v(t) \\ dp &= vdm + mdv \\ p &= m(t)v(t) \\ dp &= vdm + mdv \end{aligned}$$

for mass ejecting out-

$$dp = -dm(v - v_g)$$

So the total momentum is given by-

$$\begin{aligned} dp = vdm + mdv - dm(v - v_g) = mdv + v_g dm \\ \implies \frac{dp}{dt} = m\frac{dv}{dt} + v_g \frac{dm}{dt} ....i \end{aligned}$$

The external forces acting are-

$$\sum F = -m(t)g - bv^2 .....ii$$

Equating i and ii:

$$\frac{dv}{dt} = -g - \frac{b}{m(t)}v^2 - \frac{v_g}{m(t)}\frac{dm(t)}{dt} .....iii$$

where,

$m(t)$ is mass of payload + mass of structural load + mass of propellent
v_g is the speed the rocket expels fuel
b be the coefficient of friction
$v(t)$ is the speed of the rocket and
g is the gravity

There can be different conditions from which the space vehicles can be propelled where the effect of the external factors can vary accordingly. The cases of conditions are:

- **CASE I: Neglecting gravity and drag forces**

drag coefficient,
$b = 0$ and $g = 0$

$$\frac{dv}{dt} = -\frac{v_g}{m(t)}\frac{dm(t)}{dt} .....iv$$

For simplification we will be dealing with momentum domain only. Let us assume that u is that the rate of the rocket, v is that the rate of the exhaust from the rocket, A is that the space of the exhaust nozzle.

change in rocket momentum:-

$$M(u + du) - Mu = Mdu$$

change in exhaust momentum:-

$$dm(u - v) - dmu = -dmv$$

change in system momentum :-

$$Mdu - dmv$$

let say $\dot{m}$ is that the rate at that the mass exploit the nozzle. The sign of this term is negative as a result of the rocket is losing mass because the propellants are exhausted.

$$\dot{m}dt = -dM$$

Substituting into the momentum equation:

$$Mdu = -v_g dM$$
$$du = -V_g dM/M$$

We can now integrate this equation:

$$\vec{\Delta}u = -V_g ln(M)$$

where $\vec{\Delta}$u represents the change in velocity
And, the boundaries of integration are from the initial mass of the rocket to the final mass of the rocket. The fast mass of the rocket M, the mass is composed of 2 main components, the empty mass me and also the propellant mass mp. The empty mass doesn't modification with time, however the mass of propellants will change as the rocket will modification with time:

$$M(t) = me + mp(t)$$

Initially, the mass of the rocket mf contains the empty mass and the propellant at lift off. At the end of the burn, the mass of the rocket contains only the empty mass:

$$M_i = m_f = m_e + m_p$$
$$M_f = m_e$$

Substituting for these values we obtain:

$$\vec{\Delta}u = V_g ln(me/mf)$$

So, we got the rocket equation. But still there was a problem with this single staged rocket. At that time it was approximated that rockets required 10-12 Km/sec speed to enter into the orbit but it was not possible for the scientists to achieve that orbit velocity with a single rocket.It is proven that if the masses are exhausted in stages, it will produce more velocity in comparison to combined masses exhausted simultaneously.
Now,we will look at the multi-stage rocket equation. So, in multi-stage rocket equation,the net velocity change would be the sum of change in velocities in different stages. It will be like:-

$$\vec{\Delta}V = \vec{\Delta}V1 + \vec{\Delta}V2 + ...\vec{\Delta}Vn$$

where n is no. of stages

$$\vec{\Delta}V = \sum_{i=1}^{n} V_g ln(me_i/mf_i)$$

Now, as we know about the velocity change, we can find the acceleration,work done,velocities at different points and therefore the time taken during the process. Now, to achieve the orbit velocity, scientists put the number of stages and clustering in the Ist stage accordingly. We can relate it with a space vehicle(GSLV) used by ISRO which is a 4 staged rocket and payload is attached at the uppermost point. In the first stage, 4 rockets are used to give the sufficient upthrust force to uplift the rocket in its initial stage. Then,when the booster rocket gets detached, then the next rocket will automatically start working in the next stage and this this process will continue untill the last stage. And, then finally the last stage rocket will start and put the rocket into orbit with its orbit velocity.

Propulsion efficiency

$$\eta_{\text{p}} = \frac{\text{Work done by rocket per unit time}}{\text{Work done by rocket per unit time} + \text{Work wasted by rocket per unit time}}$$

Since, Work done by rocket:-

$$F * distance = \dot{m}VjL$$

Work done by rocket/time= $\dot{m}V_gV$
where V is the average velocity of rocket

And, work wasted by rocket $= \frac{1}{2}\dot{m}(V - Vj)^2$
Now,

$$\eta_{\text{p}} = \frac{\dot{m}VjV}{\dot{m}V_gV + \frac{1}{2}\dot{m}(V - V_g)^2}$$

After solving, we'll get;

$$\eta_{\text{p}} = \frac{2\frac{V}{Vj}}{1 + (\frac{V}{Vj})^2}$$

And, if we will differentiate the right term and put it to zero, we will get the maximum value of η_{p} at $V = V_g$.
So, the maximum Propulsion efficiency would be 1 at $V = V_g$.

- **CASE II: Neglecting drag forces**

drag coefficient, b=0

$$\frac{dv}{dt} = -g - \frac{v_g}{m(t)} \frac{dm(t)}{dt} .....v$$

- **CASE III: Considering gravity and drag forces**

$$\frac{dv}{dt} = -g - \frac{b}{m(t)} v^2 - \frac{v_g}{m(t)} \frac{dm(t)}{dt} .....iii$$

- **Algorithm**

```
# importing libraries
import numpy as np
import sympy as smp
import scipy as sp
from scipy.integrate import solve_ivp
import matplotlib.pyplot as plt
import math

# considering one stage rocket
Mu=3 # typical mass of payload(in %)
Ms=17 #typical mass of structure(in %)
Mp=80 #typical mass of propellent(in %)
Mi=Mu+Ms+Mp # initial mass of rocket
g = 9.81 # gravity assumed to be constant
vg = 500 # fuel emitted at 800m/s
b = [0,0.2] # friction coefficient b[0] implies zero, b[1]=1
t, n = smp.symbols('t n')
f =Mp* pow(math.exp(1),-1*n*t) # assume 'f' (fuel) as a exponential function of
dfdt = smp.diff(f, t).simplify() #fuel loss rate
f = smp.lambdify([t, n], f)
dfdt = smp.lambdify([t, n], dfdt)


#CASE_I: constructing differential equations for position and # velocity as func
def dS3dt(t,S,vg,Mu,Ms,n):
    x, v = S[0], S[1]
    dxdt= v
    dvdt = - vg/(Mu+Ms+f(t,n))*dfdt(t,n)
    return [dxdt,dvdt]
```

```
#case_II:    drag force tends to zero
def dS2dt(t,S,vg,Mu,Ms,n):
    x, v = S[0], S[1]
    dxdt= v
    dvdt = -g- vg/(Mu+Ms+f(t,n))*dfdt(t,n)
    return [dxdt,dvdt]
#CASE_III:
def dSdt(t,S,vg,Mu,Ms,n):
    x, v = S[0], S[1]
    dxdt= v
    dvdt = -g- b[1]/(Mu+Ms+f(t,n))*v**2  - vg/(Mu+Ms+f(t,n))*dfdt(t,n)
    if (dvdt<0)*(dxdt<0)*(x<=0):
        dxdt=0
        dvdt=0
    return [dxdt,dvdt]
# plotting of rest of graphs of all position vs time and velocity vs time is att
```

- **Control of thrust propulsion vector and propulsion rate**

As the velocity of the rocket is very large and the space is of infinite dimension it is very crucial for the rocket to follow a proper defined path and so that it reaches its exact destination in the space with controlled propulsion rate. The small deviation over time increases rapidly and can cause the space vehicle to change its entire route. This level of precise and accurate control and navigation is carried out by computer controller at every steps so that the rocket follow a fixed trajectory and travel with velocity in allowable range.

The embedded system in the rocket contains sensors that senses the amount of the deviation of the thrust vector form its set point and calculate the error function. This error function is continuously calculated and provided as feedback to the controller (eg. PID, Phase-Lag Controller) at each and every control output. The controller in turn calculates the restoring torque required which helps the rocket to return to its set point range.

Similarly, the rate of propellant required to be burnt at each interval is controlled by the control system so that the propulsion velocity is confined at a fixed range required for the rocket to reach its destination.

- **Thermodynamics of propulsion**

In space vehicles the chemical energy of the fuel is directly converted to the kinetic energy. The combustible mixture of fuel is compressed and the heated. Then, the heated mixture is ejected through a nozzle with controlled direction to provide thrust reaction that jet pushes the rocket forward. The specific impulse is the impulse per unit mass of propellant burned which varies from fuel to fuel should be higher enough to increase the efficiency of the propulsion system.

The first law of thermodynamics states that-

$$
\begin{aligned}
dq = dE + w = dE + pdV \\
dq = dH - V_m dp \\
dq = dE + Fdp + pdV - V_m dp
\end{aligned}
$$

Now,

$$\rho = M/V_m$$

From hydrodynamics, the Euler equation neglecting effect of viscosity and gravitational

$$dp = \rho v dV$$

Now after substitution,

$$dq = dH + V_m + Mvd_v = d(H + l/2Mv^2) = d(H + K.E.)$$

where, M is molecular weight of gas,H is enthalpy,V_m is molar volume,$K.E$ represents kinetic energy of the flow of the gas out of the nozzle,ρ is density of gas

In the adiabatic process where there is no exchange of heat

$$
\begin{aligned}
dq = 0 \\
H + K.E. = constant \\
deltaH + \delta K.E. = 0 \\
deltaH + 0 - l/2Mv_e^2 = 0
\end{aligned}
$$

where, V_e is the exhaust velocity

$$
\begin{aligned}
v_e = \sqrt{2\delta H/M} \\
\delta H = C_p(T_c - T_e)
\end{aligned}
$$

where, T_c is the chamber temperature and T_e is the exhaust temperature
Also,

$$C_p = \gamma/(1-\gamma)R$$
$$T_c/T_e = p_e/p_c(1-\gamma)/\gamma$$

So, finally

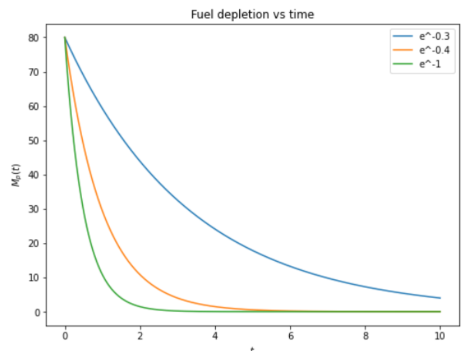
$$v_e = \sqrt{\frac{2\gamma R T_c}{(\gamma-1)M}(1-(\frac{p_e}{p_c})\frac{\gamma-1}{\gamma})}$$

- **Rocket Engine Nozzle**

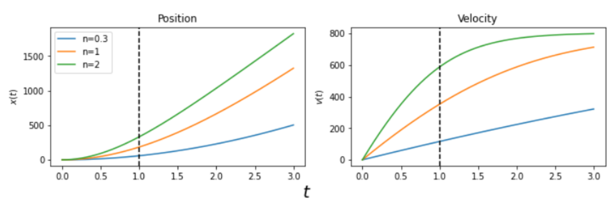
Rocket Engine Nozzle is one of the part of space vehicles engines which is used to expand and accelerate exhaust gases to high supersonic velocities. So, what actually happens in this nozzle is that we have high pressure exhaust gases being forced through a small throat which is narrow enough that the gases end up getting choked up at very high speeds. Beyond this, the nozzle expands outwards,therefore the pressure drops and eventually gases accelerates drastically(from high pressure to low pressure region) which end up going at supersonic speeds.

In the ideal condition, the exhaust pressure should be equal to external atmospheric pressure($P_e = P_a$) i.e. if this condition will match, we can get the optimum nozzle design which will generate the maximum thrust force when gases will pass through it. So, generally nozzles are designed for two cases.The first one is for the lower altitudes and second one when rocket is in higher altitudes(i.e. low pressure).So,for booster(Ist stage) rockets,the engine bells should be smaller to maintain high pressure. But as the rocket will go up at higher altitudes, we know that the atmospheric pressure decreases and hence to maintain the ideal condition, the exhaust pressure ought to be low. That's why the nozzles at third onward stages are much larger and wider in shape. However if the exhaust pressure is approximately less than 40 percent than atmospheric pressure,then flow separation can occur and can cause exhaust instabilities, damage to the nozzle, control difficulties of the engine. So, while designing nozzle,one needs to take care of these things.

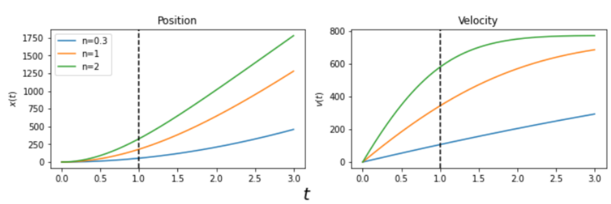
4 Conclusion



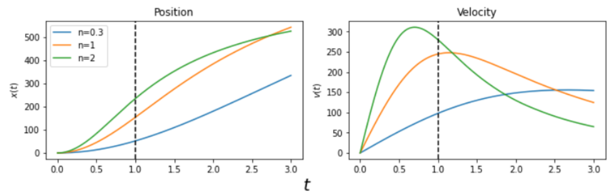
The above figure is the approximate exponential function of the depletion of fuel as function of time in which mass of the propellant M_p decreases exponentially in different orders taken as input to visualise the corresponding position and velocity of the space vehicle.



The above figure represents the position and velocity vectors in the time domain in the external conditions where gravity and drag force is negligible as compared to the thrust force. The vertical dotted line in the graph shows $1/10th$ of the trajectory. As the order of the exponential decrease rate increases velocity also changes from linear to non-linear profile.



The above figure represents the position and velocity vectors in the time domain in the external conditions where constant gravity is acting and drag force is negligible as compared to the thrust force. The results are much similar to the first case that shows that gravity of the earth can be neglected as the thrust force by propulsion is much higher as compared to it.



The above figure represents the position and velocity vectors in the time domain in the external conditions where both gravity and drag force(that is proportional to the square of the velocity) is considered. The results shows that the drag force significantly affects the velocity and the trajectory. As the vehicle follows its path, first the velocity increases to a maximum limit and then decreases further until the vehicle is in the atmosphere of the earth.

From the derived results of the exhaust velocity v_e from the thermodynamic principles,

It can be said higher the velocity of the exhaust velocity, higher is the effectiveness of the propellant fuel of the rocket.
And it depends upon-

a. higher temperature of the inside chamber, T_c
b. suitable γ
c. Lower Molar mass of the gas, M
d. Lower ratio of exhaust pressure and chamber pressure, $\frac{p_e}{p}{}_c$

5 Refrences

https://en.wikipedia.org/wiki/History_of_spaceflight
https://www.youtube.com/playlist?list=PL99EA5ECCC34949DB
https://www.grc.nasa.gov/WWW/K-12/rocket/rktpow.html
https://en.m.wikipedia.org/wiki/Rocket_engine_nozzle
https://web.mit.edu/16.unified/www/FALL/thermodynamics/notes/node81.
html https://colab.research.google.com/drive/1EuqkmLVy497hxwLyJf2MZUI7AoJkcfRf?
usp=sharing