MINIPROJECT FINAL REPORT

COVID-19 MODELLING

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ABSTRACT

Humanity deteriorated because of the global spread of the Corona virus. Mathematical models are well-known for assisting in the development of various tactics by providing an approximate time for infectious illnesses. Various mathematical models are built to explain the behaviour of the transmissible illness when it enters a community and preventative actions to be considered to forecast the path of the epidemic. We used numerous models in this project, including the Exponential Model, Logistic Model, and SIR Model, as well as the SIR Model with the inclusion of social psychologies, to analyse and explain the behaviour of SARS-CoV-2, commonly known as Covid-19 illness.

KEY WORDS

Mathematical Study, Covid19, Exponential Model, Logistic Growth Model, SIR Model, Crowd Effect, Argentina, Australia, Belgium

INTRODUCTION

The outbreak of novel coronavirus started in the late December 2019 in central China's Wuhan province which is regarded as a major manufacturing hub in China. "By 11th March 2020 the WHO had already recognised novel corona virus as a pandemic, formally naming severe acute respiratory syndrome SARS-COV-21 as Covid-19"[1]. Initially the outbreak was thought to be limited to a certain region or the people in contact with those people in the region. Later the neighbouring countries started experiencing the cases, but the magnitude of those cases outside the outbreak place was less. The initial cases reported outside China are small and reported in very few countries such as Thailand and Japan in the first 50 days. Later, the rest of world started experiencing increasing number of cases and it was then the WHO had considered it to be a pandemic. As a result of the outbreak several restrictions were enforced in various countries in almost all parts of the world.

In this analysis we try to completely analyse the outbreak and further study those statistics through some of the scientific mathematical modelling techniques. "Numerous mathematical models have been developed to assess and comprehend the impact of transmissible diseases when it comes in touch with transmission medium" [2-3]. Studying this outbreak through such means makes ourselves better prepared for the next outbreak and better planning. Typical methods involved in this analysis include the Exponential growth model, Logistic growth model, SIR model and the Extended SIR model.

The Exponential model, a continuous time model often represents a population without considering other depending factors.

$$\frac{dP(t)}{dt} = rP(t)$$

$$P(t) = P(0)\exp(rt)$$

and, Logistic model, a continuous time model considered to be more realistic because of restricted resources. The logistic model exhibits an early period of exponential development followed by saturation.

$$P(t) = \frac{KP(0)\exp(rt)}{K + P(0)(\exp(rt) - 1)}$$

To further study such viral and harmful disease, "Kermack and McKendrick (1927) introduced a model based on healthy, infected, and immune individuals" [4]. It includes three states S, I and R. S: susceptible – "population prone to infection", I: infected – "population already affected and can be a catalyst to spread it" and R: removed – "population recovered or dead" and coefficients, a and b. This makes our system's transition

$$S \to I \to R$$

with,

$$I \to R : \text{coefficient } b$$

$$S + I \to 2I : \text{coefficient } a$$

The Susceptible stage can be further divided into S_{ign}- Susceptible Ignorant (with alarm), S_{res} - Susceptible Resistance, and S_{exh} - Susceptible Exhaustion because of the incorporation of the human behaviour stress analysis under the epidemic.

This results in Extended version of SIR model made up of 5 phases: Susceptible Ignorant S_{ign}, Susceptible Resistance S_{res} , Susceptible Exhaustion S_{exh}, Infected I, and removed R. This causes our system's transition $S \to I \to R$ to assess revised stages of S to present a new

$$S_{ign} \to S_{res} \to S_{exh} \to S_{ign}$$

transition, as well as construct the subsequent reactions with coefficients (constants) assigned for each.

$$I \to R : \text{coefficient } b$$

$$S_{ign} + I \to 2I : \text{coefficient } a$$

$$S_{ign} + I \to S_{res} + I : \text{coefficient } k_1$$

$$S_{res} \to S_{exh} : \text{coefficient } k_2$$

$$S_{exh} \to S_{ign} : \text{coefficient } k_3$$

$$S_{exh} + I \to 2I : \text{coefficient } a$$

Believing that, the alarming period now evolves drastically in a linear manner makes one of the above reactions $S_{ign} + I \to S_{res} + I$ having coefficient k_1 change to $S_{ign} + 2I \to S_{res} + 2I$ and coefficient q. Such change in reaction is noticed due to the consideration of Crowd Effect. Infection outbreaks in racially and ethnically diverse populations are frequently described qualitatively using SIR-type models and their extensions. [5-7]

METHODS

SIR model equations: For the components: (S)usceptible → (I)nfected → (R)emoved, the rise in the infection rate depends on the contact rate between S and I. This suggests that the flux and intensity of the $S \rightarrow I$ transition is, respectively, aI and aSI. Additionally, the flux bI is equated with the transition intensity of $I \rightarrow R$, were a and b are constants. Considering this, the SIR model equations can be written as follows:

Susceptible equation → $\frac{dS}{dt} = -aSI$

Infected equation → $\frac{dI}{dt} = aSI - bI$

Removed equation → $\frac{dR}{dt} = bI$

The parameters of SIR model can be determined by evaluating $a = r + b$, with $b = 0.1$ and value of exponential parameter r.

Extended SIR model equations: Considering the stress factors now, the additional components: (S_{ign}) Susceptible Ignorant – People those are susceptible to Covid-19 and are un-aware of the consequences and causes of infection, generally they are those with lack of exposure to the contemporary things, (S_{res})usceptible Resistance – People with some type of resistance developed or immune due to appropriate behaviour as per the issued guidelines, (S_{exh})usceptible Exhaustion – Fraction of population that is exhausted of the whole situation and are in a state of passiveness towards any suggested means of behaviour proposed to tackle with the epidemic.

Along with the (I)nfected and (R)emoved can record the increase in cases when $S(S_{ign} + S_{res} + S_{exh})$ get in touch with I. Using this knowledge and the model's reactions, the extended SIR model equations can be represented as follows:

Susceptible Ignorant equation → $\frac{dS_{ign}}{dt} = -aS_{ign}I - k_1 S_{ign}I + k_3 S_{exh}$

Susceptible Resistance equation → $\frac{dS_{res}}{dt} = k_1 S_{ign}I - k_2 S_{res}$

Susceptible Exhaustion equation → $\frac{dS_{exh}}{dt} = k_2 S_{res} - k_3 S_{exh} - aS_{exh}I$

Infected equation → $\frac{dI}{dt} = -bI + aS_{ign}I + aS_{exh}I$

Removed equation → $\frac{dR}{dt} = bI$

Here, there are five coefficients that are the reaction rates constants, $a, b, k1, k2,$ and $k3$. Each country's a and b values are derived from its initial straight-line slope ($r = a - b$), as detailed in the SIR model too, and $k_2 = 0.02$ (1/50), $k_3 = 0.01$ (1/50) and $k_1 = 1$.

Extended SIR model equations with Crowd Effect: Presuming that for $I = Ip = 0.02$ (2% of population), reactions $S_{ign} + 2I \rightarrow S_{res} + 2I$ and $S_{ign} + I \rightarrow S_{res} + I$ have the same reaction rates:

$$k_1 S_{ign} I_p = q S_{ign} I_p^2$$

and with the formulation $q = \frac{k_1}{I_p}$ the coefficient constant q can be calculated.

This reforms the extended SIR model equations as follows:

Susceptible Ignorant equation → $\frac{dS_{ign}}{dt} = -aS_{ign}I - qS_{ign}I_p^2 + k_3 S_{exh}$

Susceptible Resistance equation → $\frac{dS_{res}}{dt} = qS_{ign}I_p^2 - k_2 S_{res}$

Susceptible Exhaustion equation → $\frac{dS_{exh}}{dt} = k_2 S_{res} - k_3 S_{exh} - aS_{exh}I$

Infected equation → $\frac{dI}{dt} = -bI + aS_{ign}I + aS_{exh}I$

Removed equation → $\frac{dR}{dt} = bI$

CONTENT

The three countries taken into consideration for this study are Argentina, Australia, and Belgium. The initial data for the purpose of analysis was sourced from WHO-COVID-19-global-data.csv. The following Figure 1 illustrates the cumulative number of cases reported in all these countries with respect to the day of the count. For this purpose, the days are considered from the initial availability of data i.e., 03.01.2020 and the cases count is the cumulative number of infected cases on that day.

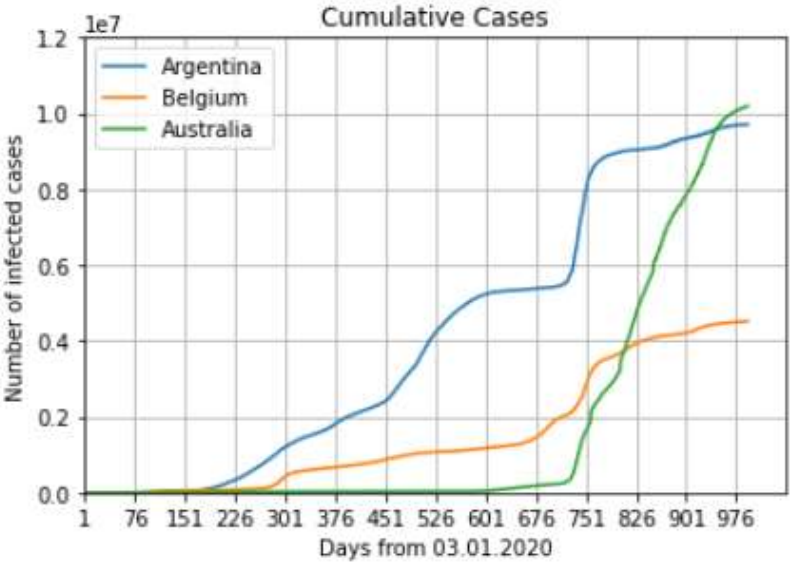
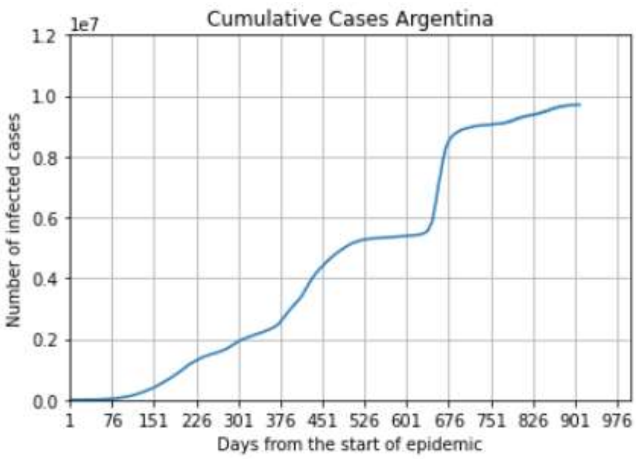
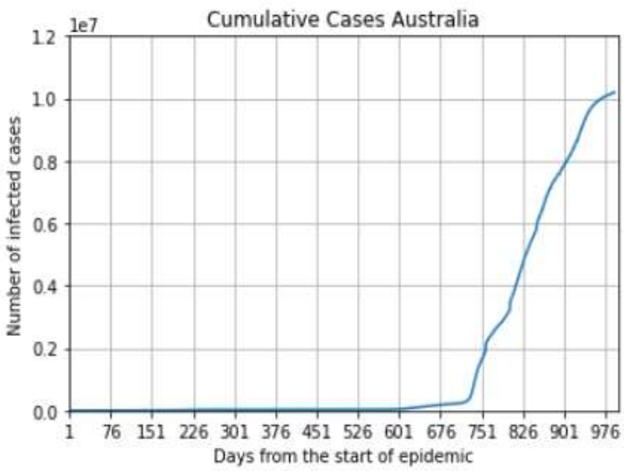


Figure 1: Cumulative cases for three countries from 30.01.2020

To come to a point of considering the start of epidemic for these countries, factors such as the imposition of restrictions for public mobility by various authorities (lockdown) is considered. According to this fact of the imposition of lockdown the start dates of epidemic in Argentina, Australia and Belgium are observed to be 27.03.2020, 22.03.2020 and 13.03.2020 respectively, which turned out to be the 1000 cases mark from the start of the cases. The cumulative case illustration is made for the three nations in the below Figure 2.





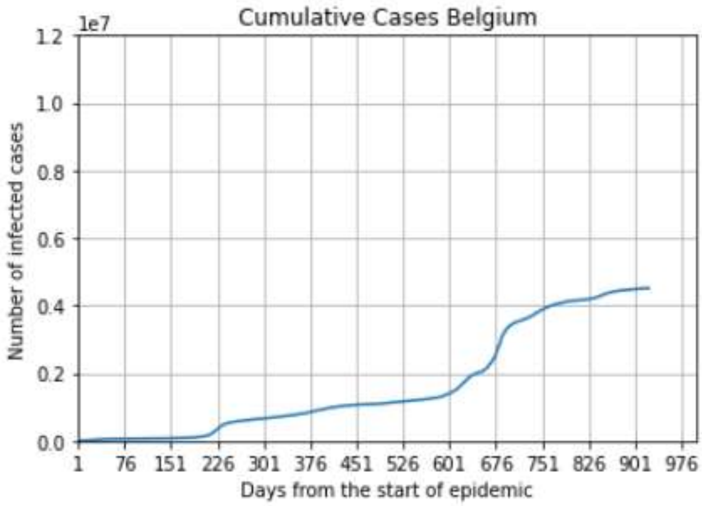


Figure 2: Cumulative cases from the start of epidemic in each country

As the above comparison of cumulative cases presented cannot be used for our further study, because of the variation in population of those countries. Here we propose a technique of normalising the number of infected cases with population of the respective country, so that comparisons can be made in fractions of populations. "We found that the respective count for population of Argentina, Australia and Belgium population in the time-period corresponding to our study are 46.15mn, 26.18mn and 11.58mn respectively" [8]. It can be observed from Figure 3 that in the countries that are in our purview, Argentina is highly populated whereas Belgium is the one with least amount of population. So, as we mentioned earlier, we will be using this fraction of normalised infected population that would be obtained by dividing the registered cases.

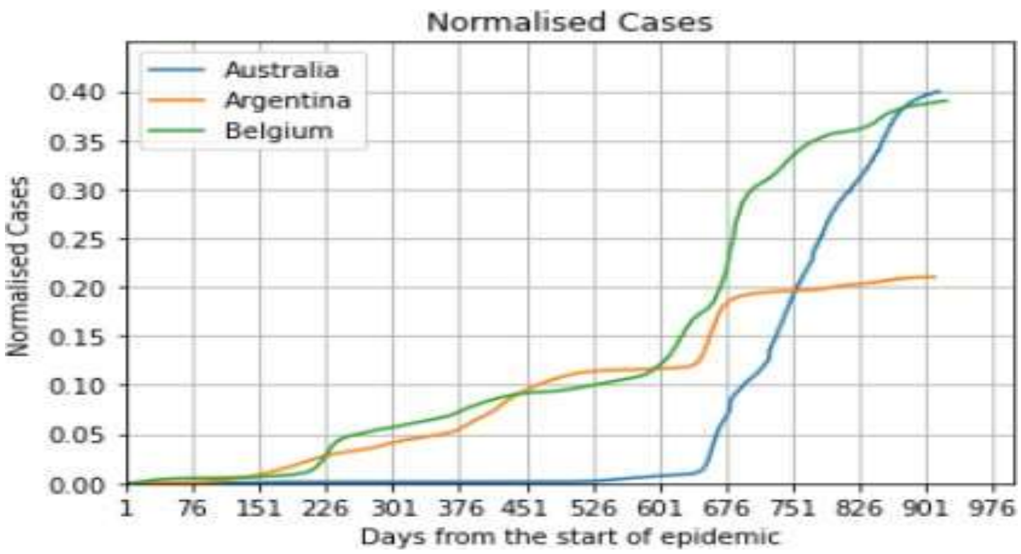


Figure 3: Normalised Cases

The number of waves observed by each country is determined by a heuristic approach based on logarithm – **Logarithmic approach**.

1. Analysis for Argentina:

The below Figure 4 represents the logarithmic graph for Argentina. When the data was approximated with a fine straight-line, four fragments were found, indicating the presence of two waves. The first wave lasted from day 1 to day 625, and the second wave lasted from day 626 to the end.

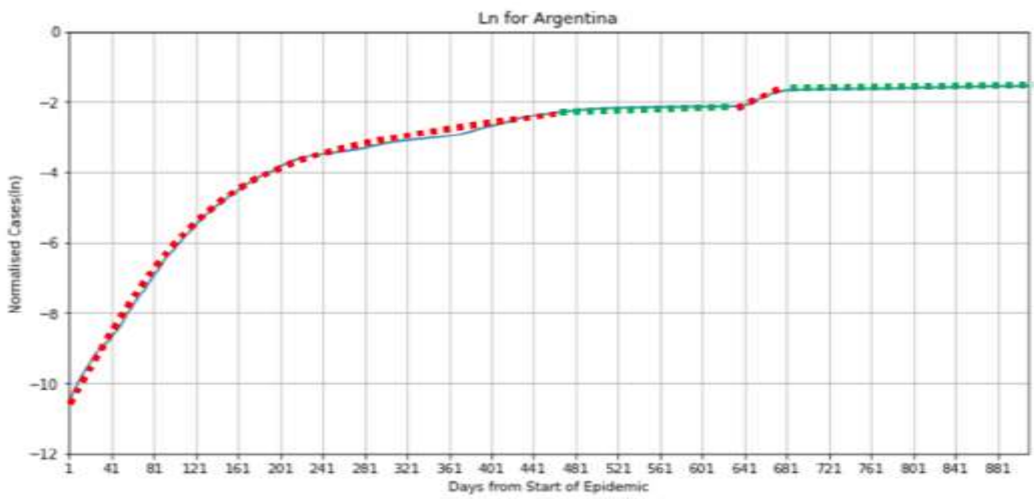


Figure 4: Logarithmic graph for Argentina

Through the Logarithmic heuristic, the graphical representations for normalised cases as well as the logarithmic value for wave 1 and wave 2 are illustrated in Figure 5 and Figure 6. To approximate the model's parameters, we will employ the formula $ln\, P(t) = a + rt$, where the slope interval in this case for wave 1 is from 1 to 130 and wave 2 from day 1 to 40.

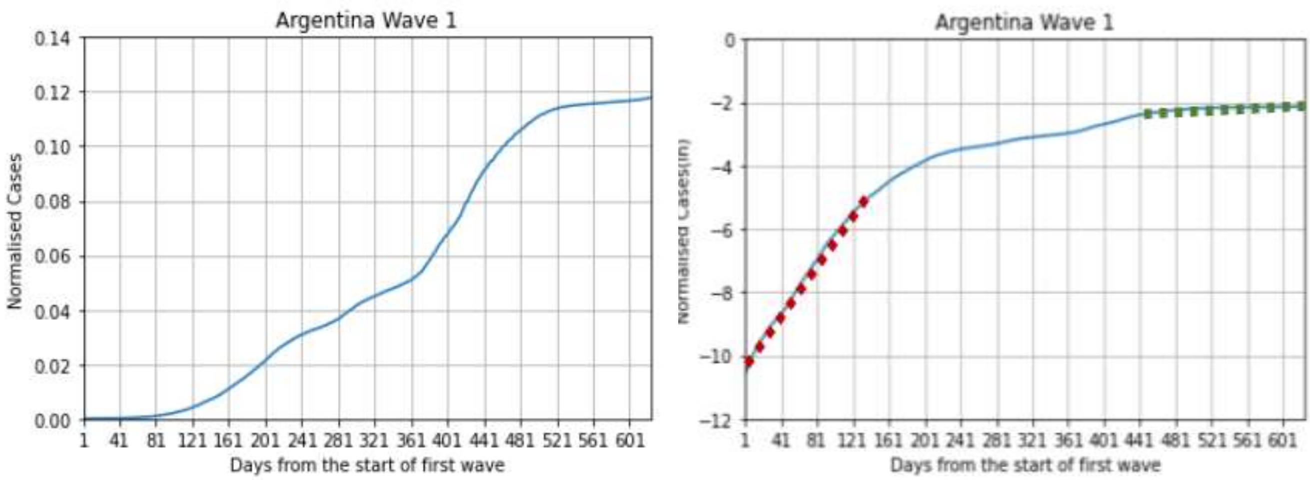


Figure 5: Normalised cases (left) and logarithmic (right) graphs for the first wave of Argentina

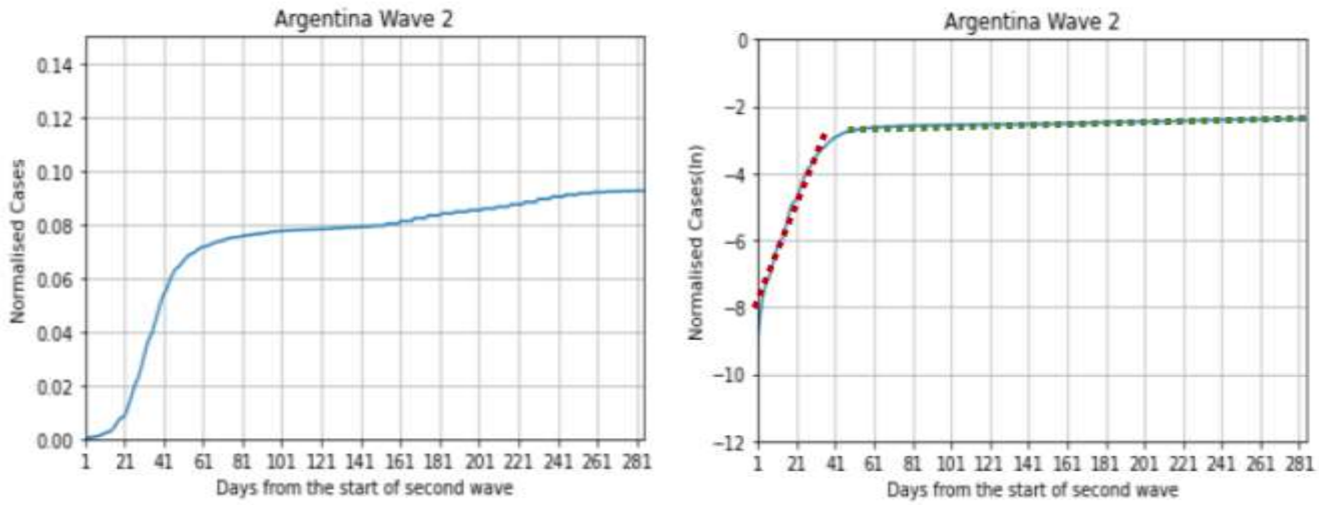


Figure 6: Normalised cases (left) and logarithmic (right) graphs for the second wave of Argentina

Exponential model prediction:

When compared to the normalised infected cases, the exponential model is predicted with parameters $a = 10.3697293$ and $r = 0.040858908$ for the first wave and $a = 8.3212$ and $r = 0.1507$ for the second wave along with its generated exponential errors, as shown in Figure 7 and Figure 8.

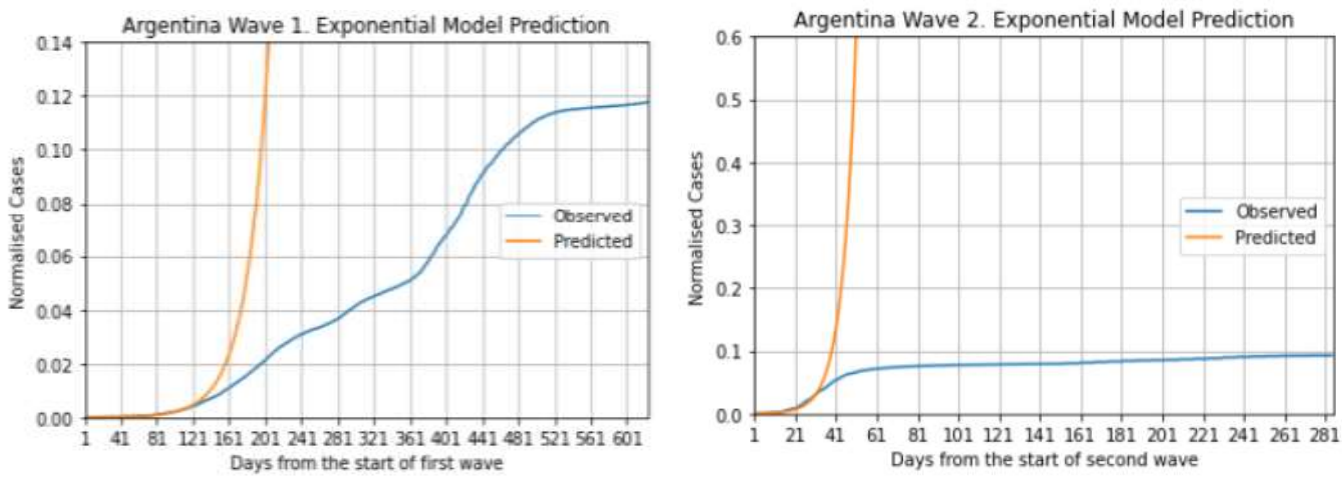


Figure 7: Exponential model prediction for Argentina

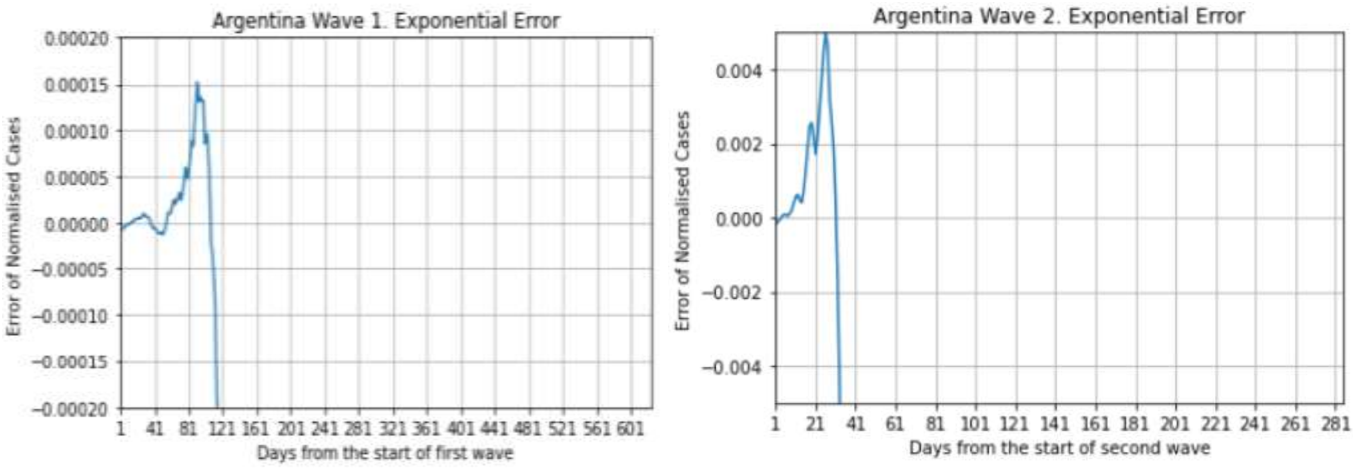


Figure 8: Exponential error for Argentina

Logistic model prediction:

For Logistic growth the carrying capacity K needs to be estimated, which can be achieved using the evaluated parameters for wave 1 and wave 2 respectively. As visible in Figure 9 and Figure 10 below, the value for K began with large numbers and progressed to constant values at the end. For Wave 1, days 14 to 37 and 56 to 107 registered a value of K greater than one, which proves useless for inferring the preliminary K value and the same was observed during the starting days of wave 2. As a result, an initial value of $K = 0.1177315$ for wave 1 and $K = 0.0925$ for wave 2 is chosen from which the model's parameters is being defined.

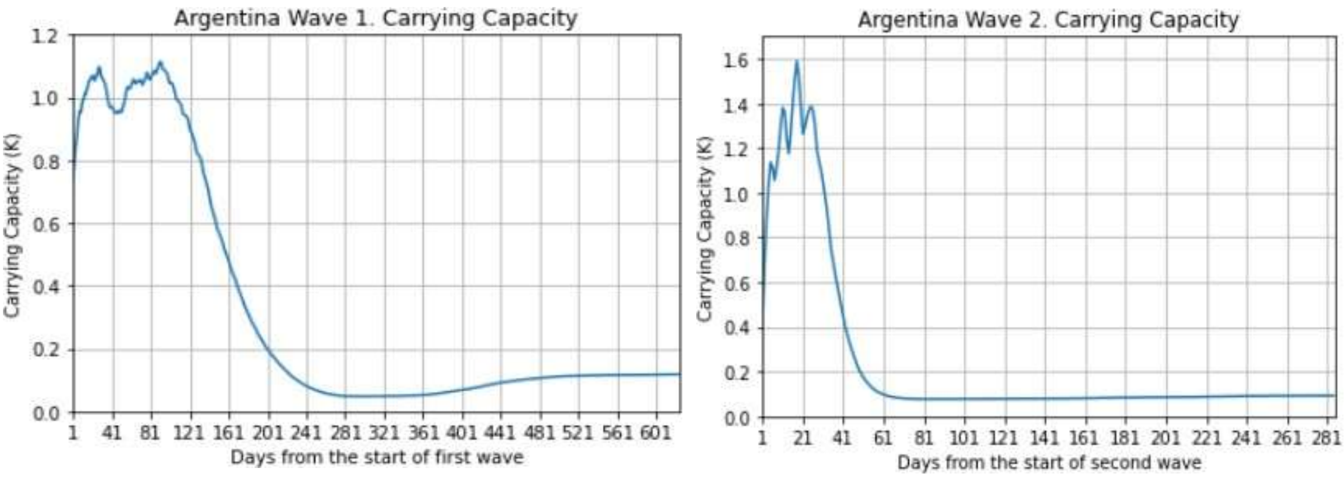


Figure 9: Carrying Capacity K for Argentina

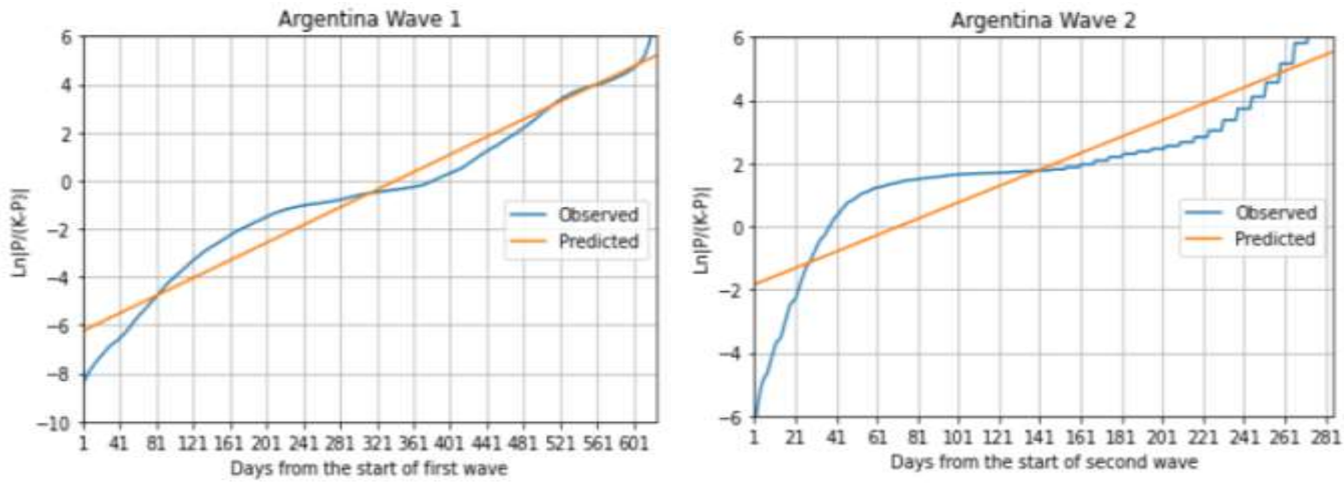


Figure 10: Estimation of logistic model parameters for Argentina

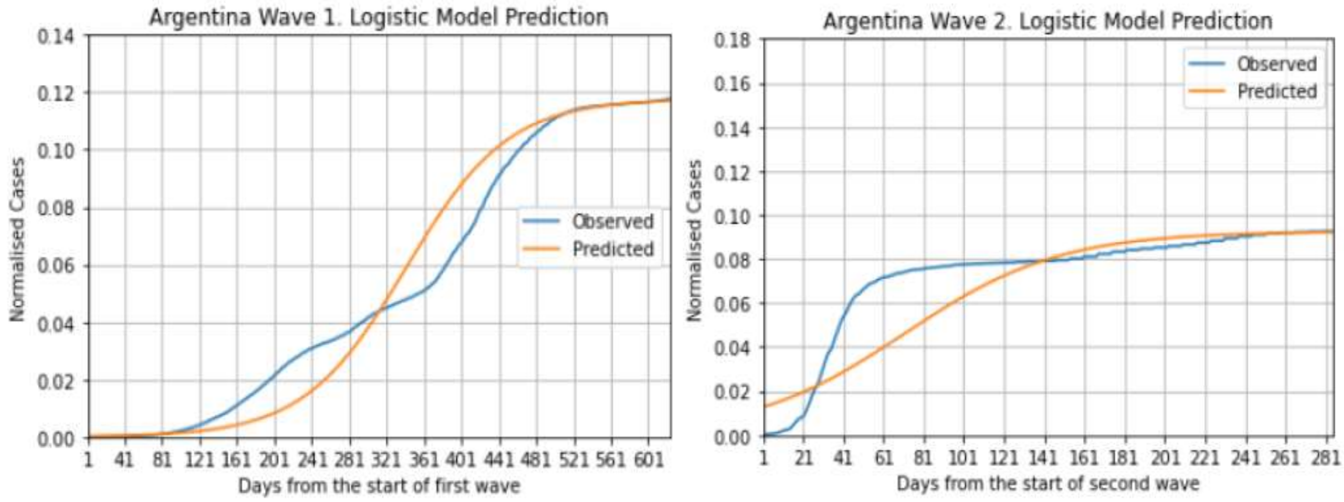


Figure 11: Logistic model prediction for initial value of K

We can conclude from Figure 11 above, Figure 12 below and Table 1 that the preliminary value of carrying capacity K chosen in Figure 10 is adequate. Regardless of whether the parameters are optimised, the S.S.E and M.S.E are only marginally affected.

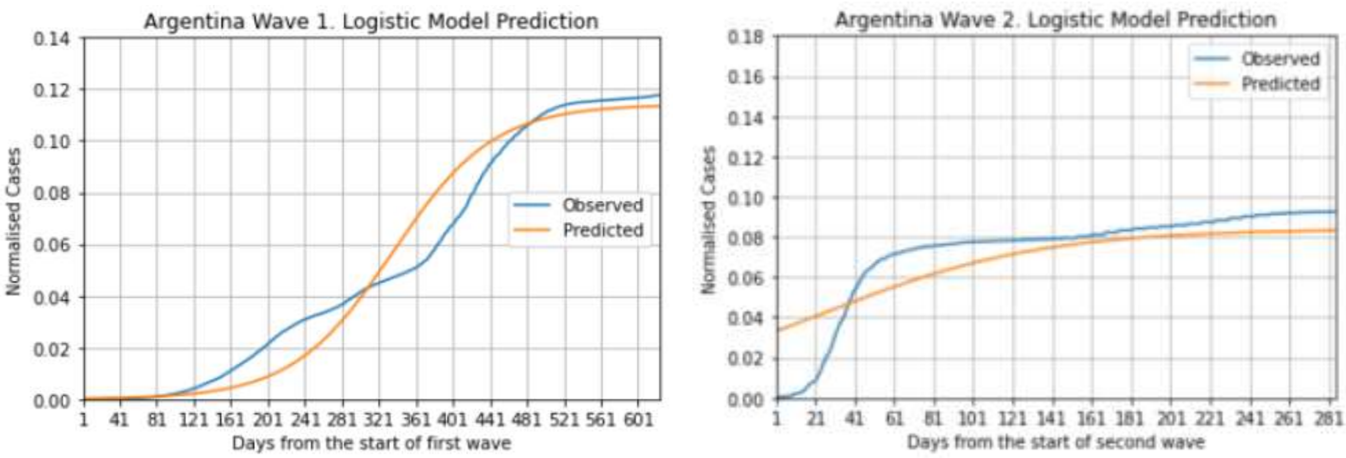


Figure 12: Logistic model prediction for optimal value of K

Wave#	Model	Parameters				
		a	r	K	S.S.E	M.S.E
Wave 1	Exponential	-10.36973	0.0408589	NA	1.902×10^{14}	3.04×10^{13}
	Logistic (initial K)	-6.277531	0.018312	0.1177315	0.054855497	0.000087769
	Logistic (optimal K)	-6.20295318	0.018399815	0.114	0.052985501	0.000084777
Wave 2	Exponential	-8.3212	0.1507	NA	4.705×10^{32}	1.651×10^{30}
	Logistic (initial K)	-1.889250242	0.026001689	0.0925	0.052818864	0.000185329
	Logistic (optimal K)	-0.468119483	0.018052593	0.084	0.050619867	0.000177614

Table 1: Exponential Model and Logistic Model Parameters for Argentina

Isolating first two waves:

When the two waves are merged as one, the exponential parameter r is assessed as $r = 0.041008$. This value is derived from the fact that the slope interval considered now is 1 - 128 days. Logarithmic time series graph considering first 2 waves of Argentina is depicted in Figure 13.

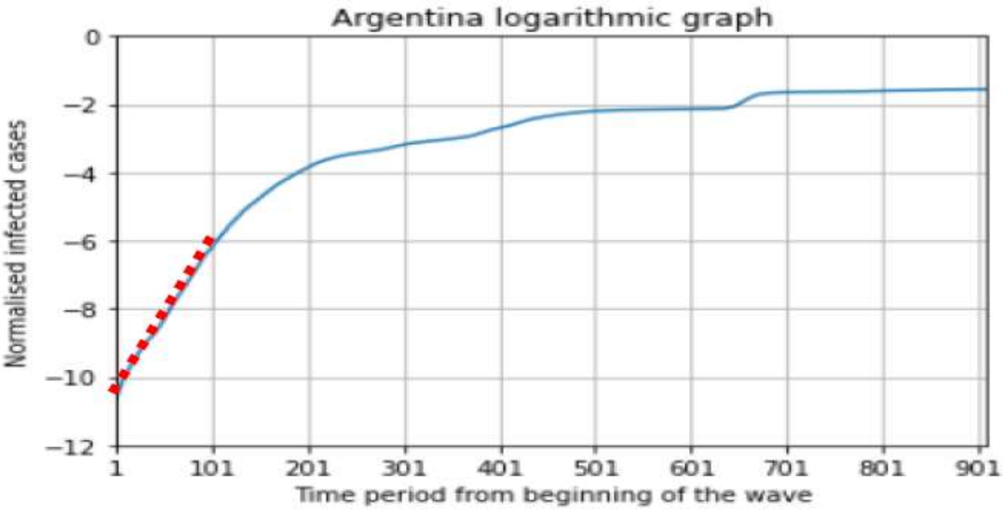


Figure 13: Argentina logarithmic graph for first two waves

Evaluating Susceptible S and exponential coefficient r:

The value for Susceptible, S, is guesstimated at the commencement and the conclusion of time-period, and the exponential coefficient r is projected by using $ln\,P(t) = c + rt$. And tabulated in the Table 2

Exponential Parameters	Start period(day)			End period(day)			r	c
	ln value	$P(t)$	S	ln value	$P(t)$	S		

Argentina	Wave 1 slope interval	-10.648	0.00002374	0.99997625	-5.3158	0.00491285	0.99508714	0.041008	-10.3762

Table 2: Estimation of Susceptible S and exponential parameters for Argentina

Defining SIR model coefficients:

SIR model's factors can be identified (Table 3) by substituting the exponential coefficient calculated above and formulae $a = r + b$.

Interval	r	c	b	a	SSE	MSE
1-128	0.041008253	-10.37622575	0.1	0.141008253	3.19661×10^{24}	3.51276×10^{21}

Table 3: Estimation of SIR coefficients for Argentina

Defining SIR Model preliminary values:

The initial value assumed for normalised cumulative data, $P\ (0)$, as listed in Table 4, forecasts the default values for $S, I,$ and R with the time - series day 1 to 128 labelled as preliminary stationarity for the first wave period.

The purpose of selecting these values is that Argentina has a huge slope from day 1 to 128 and thus we emphasise and assign R timespan to 10 days. Also, by substitution method, the exact initial values for (S)usceptible, (I)nfected, and (R)emoved components could be anticipated if $P\ (0)$ is presumed,

$$S\ (0) = 1 - P\ (0)$$

$$I\ (0) = P\ (0) - R\ (0) = P\ (0) - P\ (-10)$$

$$R\ (0) = P\ (-10)$$

$P\ (0)$	$S\ (0)$	$I\ (0)$	$R\ (0)$
0.0000481236	0.999951876	0.0000243760	0.0000237476

Table 4: Initial values of SIR model for Argentina

Defining divergent preliminary values of SIR:

With disparate initial values of $I\ (0)$ considered, as visible in Table 5, where every value found is twice the previously encountered initial value, and with this, the initial values for $R\ (0)$, $S\ (0)$, and $P\ (0)$ for each $I\ (0)$ is deduced.

Initial Values # of $I\ (0)$	$I\ (0)$	$R\ (0)$	$S\ (0)$	$P\ (0)$
$I\ (0)_1$	2.43760×10^{-5}	2.37476×10^{-5}	0.999951876	4.81236×10^{-5}
$I\ (0)_2$	$4.87520\times$	4.74953×10^{-5}	0.999903753	9.62473×10^{-5}

	10^{-5}			
$I\,(0)_3$	9.75040×10^{-5}	9.49905×10^{-5}	0.999807506	1.92495×10^{-4}
$I\,(0)_4$	1.95008×10^{-4}	1.89981×10^{-4}	0.999615011	3.84989×10^{-4}

Table 5: Divergent initial values of SIR model for Argentina

ODE System Integration for SIR:

ODE system integration for Argentina's noted steady extrapolation for $S(t), I(t), R(t)$ is conducted and summarised from their corresponding initial values as mentioned in Table 4. The generated SIR plot for the infected population fraction in relation to the time - series data, t, is displayed in Figure 14 where t is the wavelength of Argentina's first two waves = 910 days.

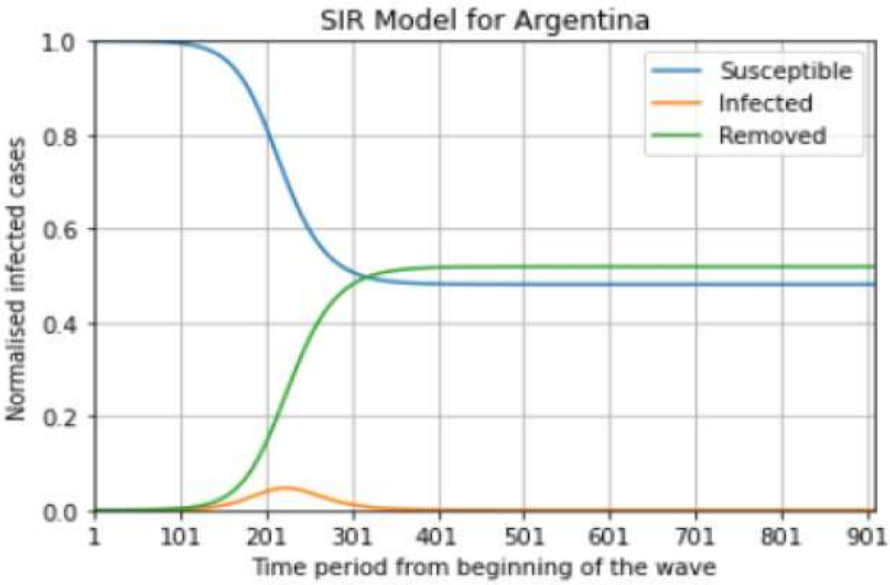


Figure 14: SIR Model for Argentina

Using the integrated SIR values, the normalised infected population integration $P(t)$ can be derived from $S(t)$, as $P(t) = (1 - S(t))$. A comparative line graph is then induced for $I(t)$ and $R(t)$ to $P(t)$. (Figure 15)

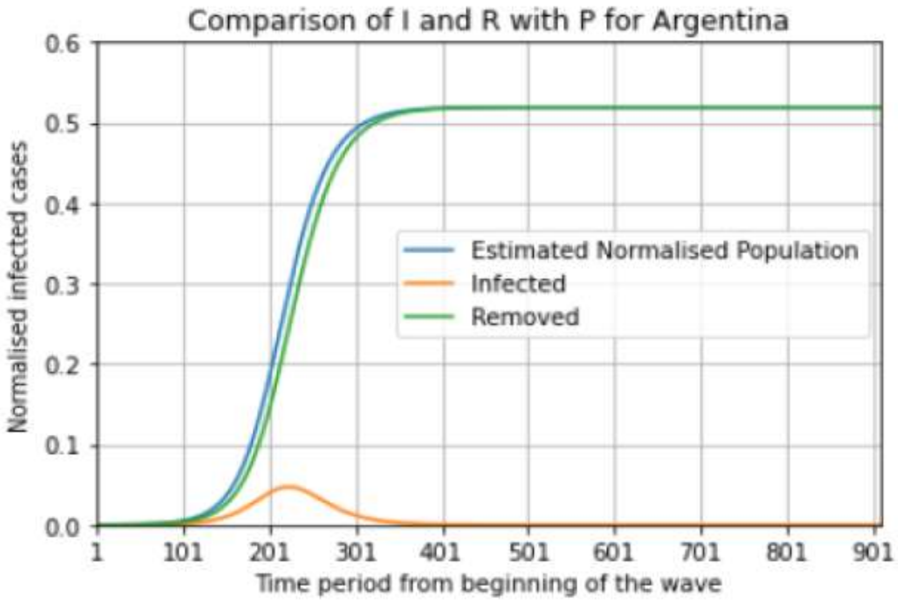


Figure 15: Comparison of I and R with P for Argentina

Determining optimal initial or preliminary value for SIR:

ODE integration performed on the divergent initial infected values $I(0)$ are discussed and explained in detail in Table 6. To determine the optimal $I(0)$, locate the one giving out the least M.S.E. "M.S.E is the difference between the actual normalised population and the approximated fraction of infected population, $P(t)$, which is obtained from $1 - S(t)$, where $S(t)$ is the ODE integration of S, over the total number of days witnessed by $I(0)$ to achieve $R(0)$."

Initial Values # of $I(0)$	$I(0)$	$R(0)$	$S(0)$	MSE
$I(0)_1$	2.43760×10^{-5}	2.37476×10^{-5}	0.999951876	1.01173×10^{-9}
$I(0)_2$	4.87520×10^{-5}	4.74953×10^{-5}	0.999903753	5.7956×10^{-9}
$I(0)_3$	9.75040×10^{-5}	9.49905×10^{-5}	0.999807506	5.3795×10^{-8}
$I(0)_4$	1.95008×10^{-4}	1.89981×10^{-4}	0.999615011	2.02381×10^{-7}

Table 6: MSE of divergent initial values of SIR model for Argentina

Figure 16 presents the graphical plot differentiating the divergent I(0)'s with the afflicted portion of population. With this Figure 16 and the Table 6 above, it is easy to conclude that the initial infected value $I(0)_1 = 2.43760 \times 10^{-5}$ is the best of the bunch, with a minimal M.S.E of 1.01173×10^{-9} and a plot that is close to the observed normalised population.

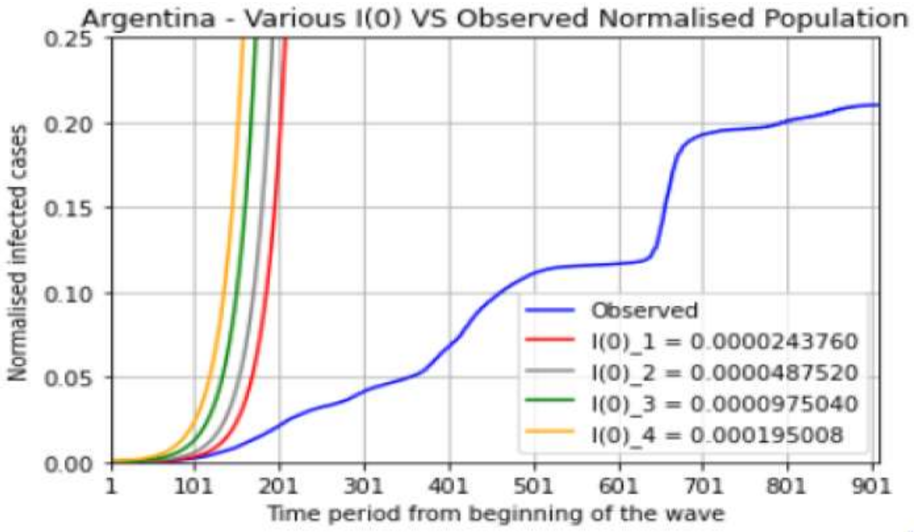


Figure 16: Comparison of divergent I(0) with observed population for Argentina

Estimating optimal SIR coefficients a and b:

As already stated, Table 3 defines the initial values of SIR coefficients a and b for Argentina. The key to determining the best value for these coefficients are by computing the M.S.E. These details are highlighted in the below Table 7, and it becomes evident that $a = 0.1$ and $b = 0.088$ can be considered as the optimum values for SIR coefficients a and b with M.S.E $= 0.000730789$

Values	a	b	SSE	MSE
Initial	0.141008253	0.1	103.851983	0.114123058
Optimal	0.1	0.088	0.665017754	0.000730789

Table 7: Initial and Optimal SIR coefficients for Argentina

With the determined optimal SIR coefficients, the optimal SIR model for Argentina and optimal comparative line graph for integrated I $(I(t)), R$ $(R(t))$, VS $P(t)$ (normalised infected population integration) is demonstrated in Figure 17.

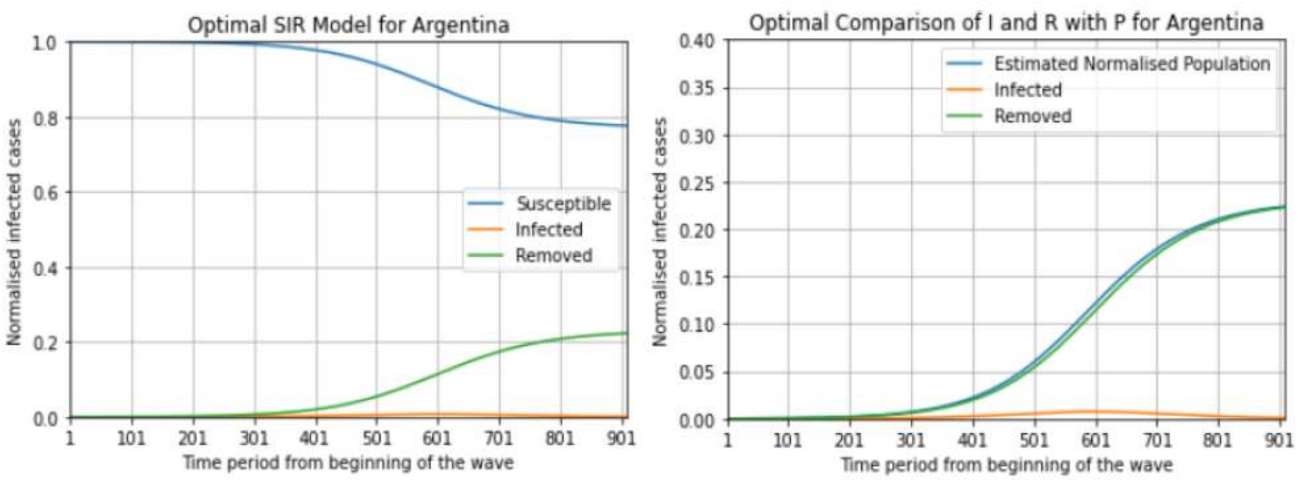


Figure 17: Optimal SIR model(left) and comparison of optimal I and R with P(right) for Argentina

Addition of the psychological analysis based on human conduct in the SIR model constructs **a new improved extended SIR model with 5 components** ($S_{ign}, S_{res}, S_{exh}, I$ and R) and **5 coefficients** (k_1, k_2, k_3, a and b).

Rapid linear increase in the extended SIR model's alarming phase is caused by the **Crowd Effect**. Considering this crowd effect, the extended SIR model can also be defined with 5 components ($S_{ign}, S_{res}, S_{exh}, I$ and R) and 5 coefficients (q, k_2, k_3, a and b)

Defining extended SIR model coefficients and preliminary values:

The parameters concerning extended SIR model without and with comprising the crowd effect are described in Table 8 and Table 9 respectively.

Slope Interval (Days)	a	b	k_1	k_2	k_3
1-128	0.141008253	0.1	1	0.02	0.01

Table 8: Extended SIR Model Parameters without considering Crowd Effect – Argentina

Slope Interval (Days)	a	b	q	k_2	k_3
1-128	0.141008253	0.1	50	0.02	0.01

Table 9: Extended SIR Model Parameters Considering Crowd Effect – Argentina

And, as explained in *METHODS* section the model's equations excluding the crowd effect can be assessed by directly substituting the initial values assumed for $S_{ign}, S_{res}, S_{exh}, I$ and R. (Table 10)

$S_{ign}(0)$	$S_{res}(0)$	$S_{exh}(0)$	$I(0)$	$R(0)$	$\frac{dS_{ign}(0)}{dt}$	$\frac{dS_{res}(0)}{dt}$	$\frac{dS_{exh}(0)}{dt}$	$\frac{dI(0)}{dt}$	$\frac{dR(0)}{dt}$
0.999951876	0	0	0.0000243759972	0.0000481236	-0.000027812	0.0000243748	0	0.000000999451647	0.00000243759972

Table 10: Extended SIR Model Initial Values without considering Crowd Effect – Argentina

With the addition of preliminary value of I_p with the rest of the components $S_{ign}, S_{res}, S_{exh}, I$ and R, the extended SIR equations including the crowd effect can be assessed.(Table 11)

$S_{ign}(0)$	$S_{res}(0)$	$S_{exh}(0)$	$I(0)$	$R(0)$	I_p	$\frac{dS_{ign}(0)}{dt}$	$\frac{dS_{res}(0)}{dt}$	$\frac{dS_{exh}(0)}{dt}$	$\frac{dI\,(0)}{dt}$	$\frac{dR(0)}{dt}$
0.999951876	0	0	0.0000243759972	0.0000481236	0.02	−0.02000247457	0.01999903752	0	0.000000999451648	0.00000243759972

Table 11: Extended SIR Model Initial Values considering Crowd Effect – Argentina

ODE System Integration for extended SIR:

In order to account for Argentina's noted steady extrapolation, the extended SIR model's ODE system integration $S(t) \rightarrow [S_{ign}(t) + S_{res}(t) + S_{exh}(t)], I(t), R(t)$ without and with crowd effect is initiated and encapsulated with its respective initial values, Table 10 for without crowd effect and Table 11 with crowd effect. Graphs for different phases of $S(t)$ and the extended SIR model for Argentina without and with crowd effect are generated in Figure 18 and Figure 19 respectively. The time series, t, which corresponds to the 910-day length of Argentina's first two waves, is the foundation for all the plots.

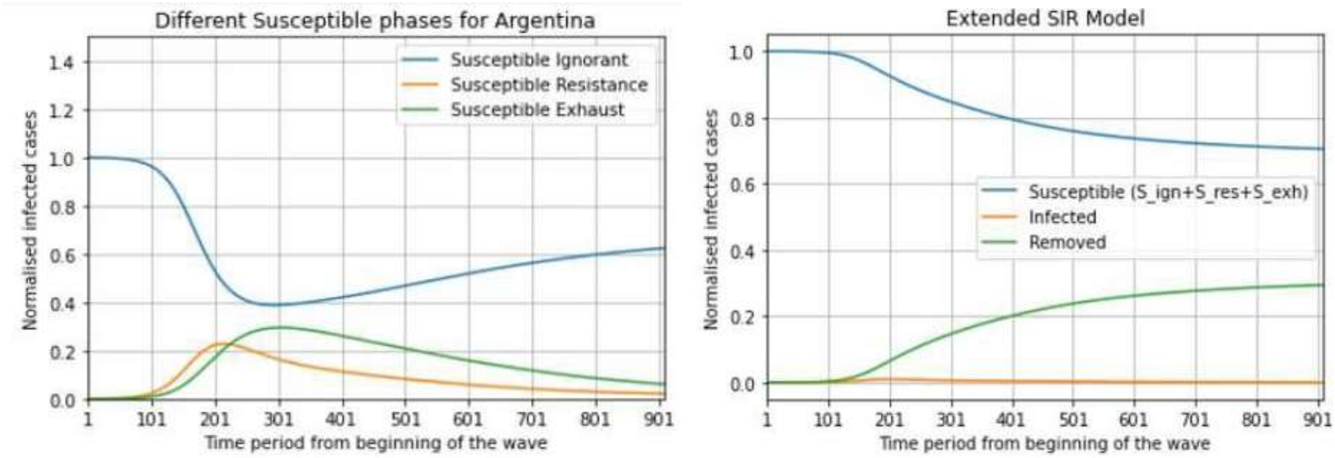


Figure 18: Different Susceptible Phases and Extended SIR Model without considering Crowd Effect -Argentina

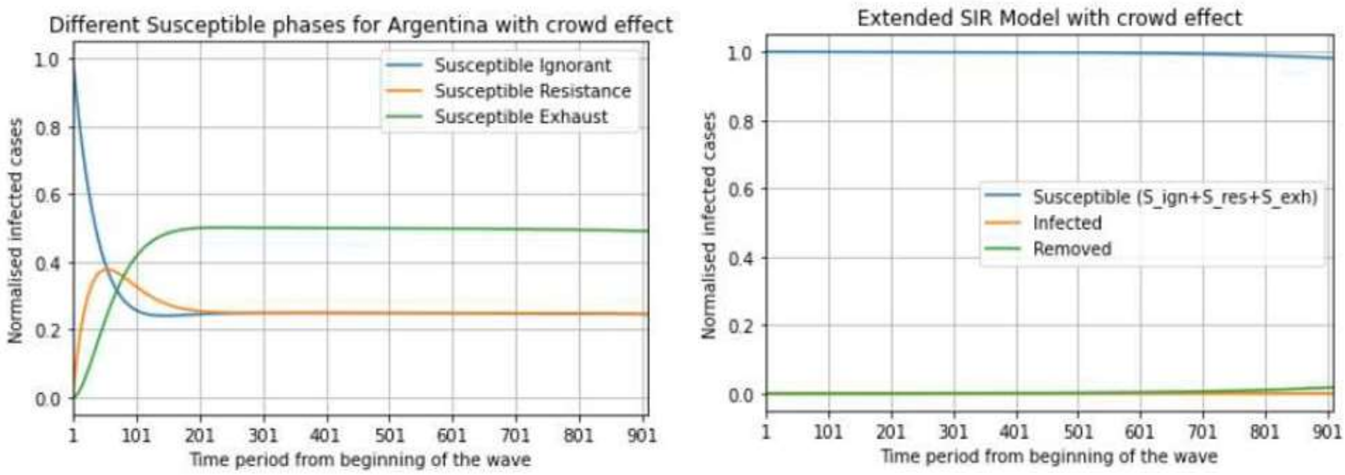


Figure 19: Different Susceptible Phases and Extended SIR Model considering Crowd Effect -Argentina

From $S(t) = S_{ign}(t) + S_{res}(t) + S_{exh}(t)$ the normalised infected population integration $P(t)$ can be determined by calculating $P(t) = 1 - (S_{ign}(t) + S_{res}(t) + S_{exh}(t))$. With this, for ODE integrated extended SIR without and with crowd effect a line graph is generated for $I(t)$ and $R(t)$ to $P(t)$ (Figure 20)

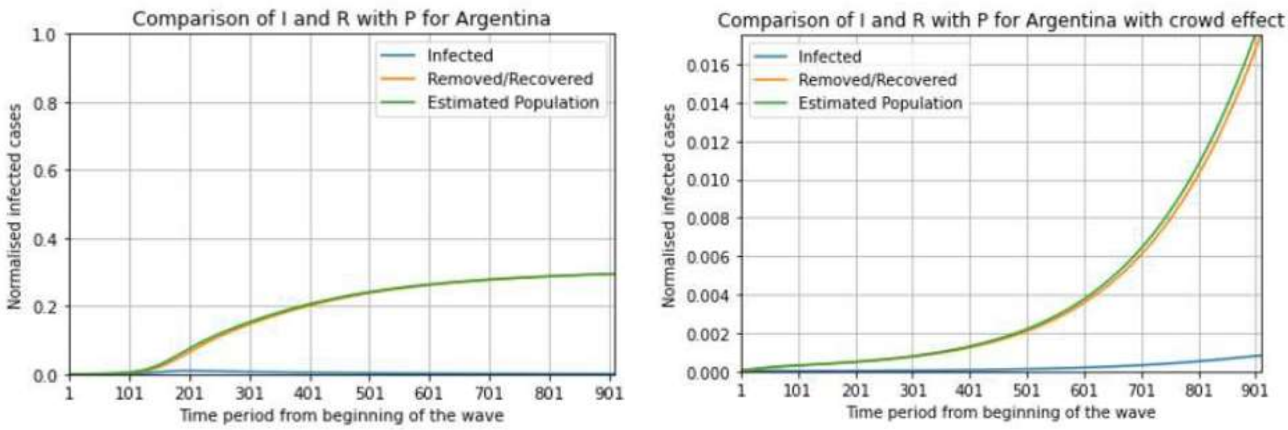


Figure 20: Comparison of I and R with P without and with considering crowd effect- Argentina

Differentiating ODE integrated values to the original observed values:

As shown in below tables the M.S.E is determined by examining the approximated ODE integration system for without (Table 12) and with (Table 13) crowd effected extended SIR preliminary data with the analysed normalised population.

In terms of definition, "M.S.E. is the difference between the original normalised population and the estimated proportion of people infected, $P(t)$, which is measured as $1 - S(t)$, for which $S(t)$ is the ODE integration of $S_{ign(t)} + S_{res}(t) + S_{exh}(t)$, over the total days observed by $I(0)$ to attain $R(0)$."

$I(0)$	$R(0)$	$S_{ign}(0)$	$S_{res}(0)$	$S_{exh}(0)$	MSE
0.0000243759972	0.0000481236	0.999951876	0	0	1.02154E-02

Table 12: Calculation of M.S.E for initial data O.D.E without considering Crowd Effect- Argentina

$S_{ign}(0)$	$S_{res}(0)$	$S_{exh}(0)$	$I(0)$	$R(0)$	I_p	M.S.E
0.999951876	0	0	0.0000243759972	0.0000481236	0.02	0.0133502385

Table 13: Calculation of M.S.E for initial data O.D.E considering Crowd Effect- Argentina

Figure 21 displays the data exploration graph for the actual normalised inhabitants versus the predicted normalised infected population, $P(t)$, without considering and with considering the crowd effect.

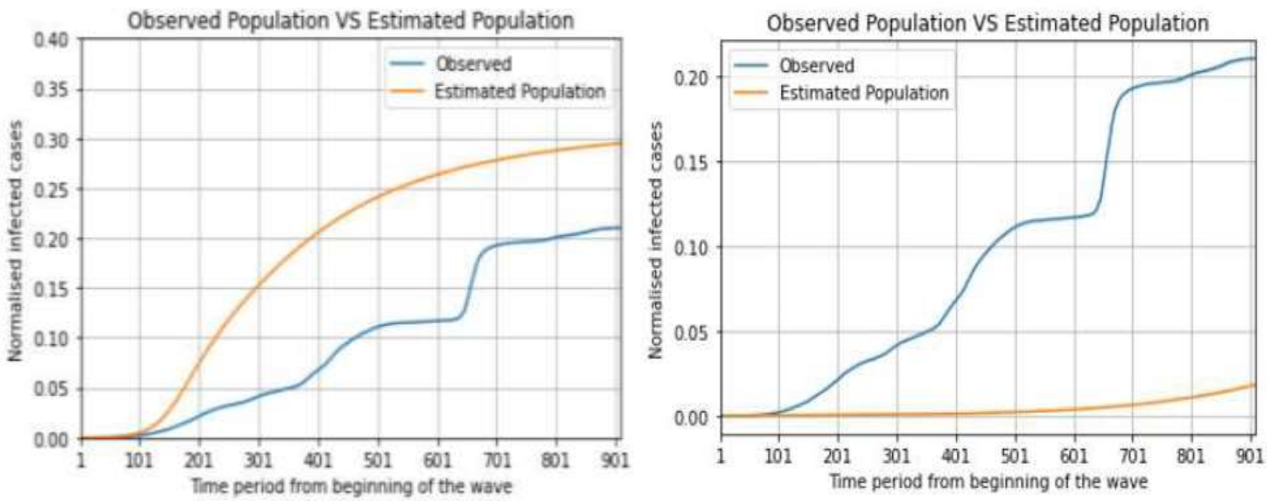


Figure 21: Observed VS Estimated Population without and with considering Crowd Effect -Argentina

Estimating optimal extended SIR coefficients:

To determine the optimum value for extended SIR model without crowd effect coefficients a, b, k_1, k_2 and k_3 and with crowd effect coefficients a, b, q, k_2 and k_3, it is critical to recognise their consequent M.S.E value. The investigated data for each parameter are shown in Table 14 without considering the crowd effect and Table 15 with considering the crowd effect, in-order to ascertain its best value.

a	b	k_1	k_2	k_3	M.S.E	
0.141008253	0.1	1	0.02	0.01	0.010215448	**Initial value**
0.07	0.05	0.5	0.01	0.005	0.000186166	**Half of Initial value**
0.071	0.051	0.51	0.011	0.0051	0.000234545	
0.069	0.049	0.49	0.0099	0.0049	0.000197559	
0.069	0.049	0.53	0.01	0.0049	0.000175031	**Optimal value**

Table 14: Extended SIR Model Optimal Coefficients Estimation for Argentina without crowd effect

a	b	q	k_2	k_3	M.S.E	
0.141008253	0.1	50	0.02	0.01	0.0133502385	**Initial value**
0.069	0.049	15	0.0099	0.0049	0.0137090271	
0.06	0.04	10	0.009	0.004	0.0075572199	
0.06	0.037	10	0.009	0.004	0.0032467969	
0.06	0.037	10	0.0085	0.0043	0.0030032550	**Optimal value**
0.06	0.037	10	0.0085	0.004	0.0030280141	

Table 15: Extended SIR Model Optimal Coefficients Estimation for Argentina with crowd effect

As it can be clearly seen in the above tables, the most desirable value for the improved SIR model coefficients a, b, k_1, k_2 and k_3 without considering the crowd effect are $0.069, 0.049, 0.53, 0.01$ and 0.0049 respectively and the optimal coefficients for the extended SIR model with crowd effect are $a = 0.06, b = 0.037, q = 10, k2 = 0.0085$ and $k3 = 0.004$, as it gives out the least M.S.E.

After examining the best coefficients for Argentina including and excluding the crowd effect, the expanded SIR model and the integrated I $(I(t)), R$ $(R(t))$ VS $P(t)$ (estimated population) graphs for the same are provided in Figure 22 and Figure 23 below.

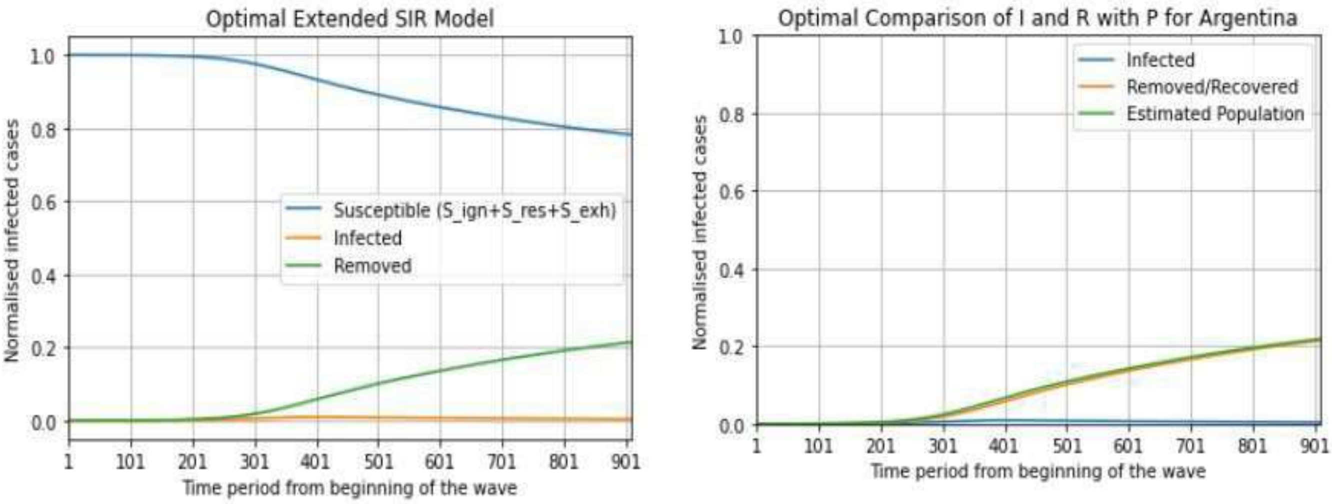


Figure 22: The optimal SIR Model and Comparison of I and R with P without considering Crowd Effect -Argentina

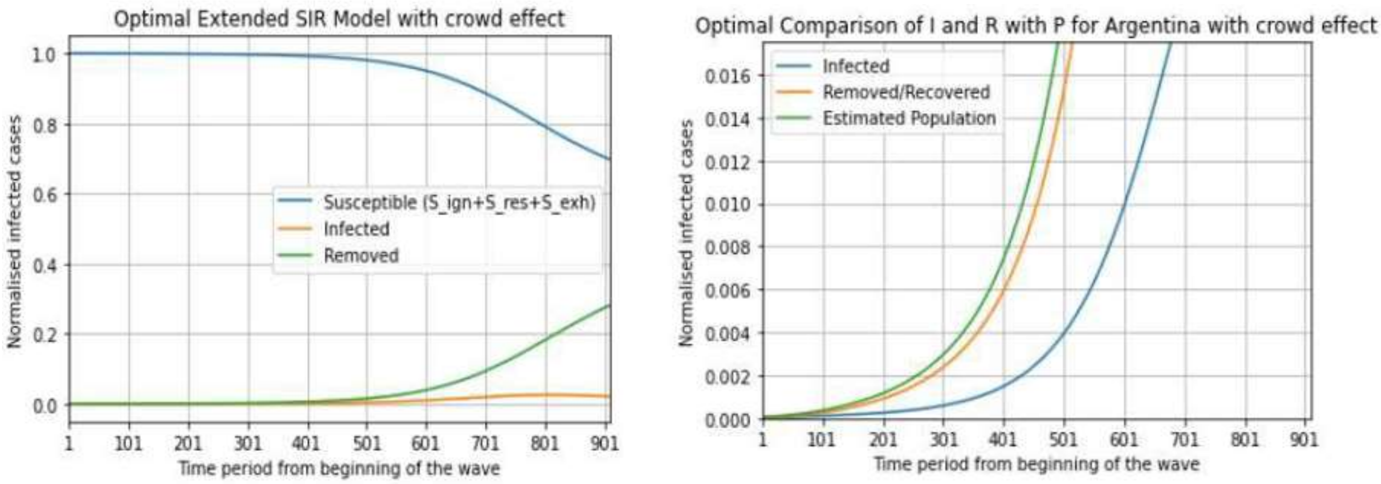


Figure 23: The optimal SIR Model and Comparison of I and R with P considering Crowd Effect -Argentina

Distinct Trials with extended SIR coefficients excluding crowd effect:

Table 16 analyses several possibilities for the extended SIR coefficients, based on which, for each experiment a revised SIR model graph is created after performing the ODE integration and shown in Figure 24.

Set of Trials	a	b	k_1	k_2	k_3
Trial 1	0.11111	0.05555	0.55555	0.01555	0.00555
Trial 2	0.13555	0.08888	0.05555	0.01111	0.00222
Trial 3	0.1	0.05	0.9	0.005	0.01

Table 16. Various Trails for Coefficients of Extended SIR Model - Argentina

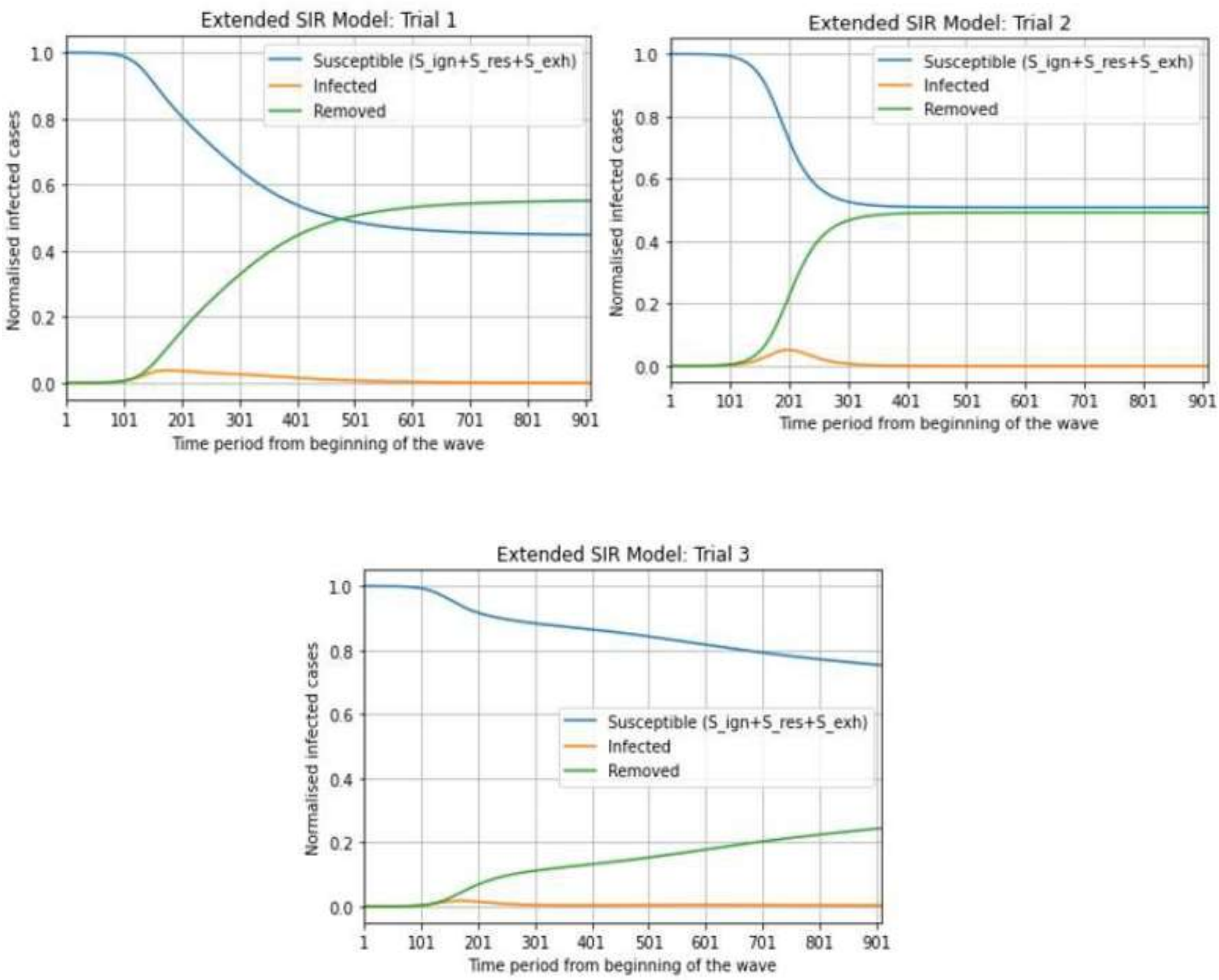


Figure 24. Extended SIR Model for respective trails -Argentina

Additionally, as demonstrated in Table 17 below, for the performed trails, the ODE integrated system and the observed populace are collated to compute the M.S.E, as defined many a times earlier, "M.S.E. is the difference between the original normalised population and the estimated proportion of people infected, $P(t)$, which is measured as $1 - S(t)$, for which $S(t)$ is the ODE integration of $S_{ign(t)} + S_{res}(t) + S_{exh}(t)$, over the total days observed by $I(0)$ to attain $R(0)$."

Set of Trials	a	b	k_1	k_2	k_3	MSE
Trial 1	0.11111	0.05555	0.55555	0.01555	0.00555	0.102799929
Trial 2	0.13555	0.08888	0.05555	0.01111	0.00222	0.103524181
Trial 3	0.1	0.05	0.9	0.005	0.01	0.002471713

Table 17. Different Trails for calculating M.S.E by varying all the coefficients-Argentina

The data analysis graph for the examined different $a, b, k1, k2,$ and $k3$ trails is shown in Figure 25 for the observed normalised residents contrasted with the estimated normalised infected population, $P(t)$.

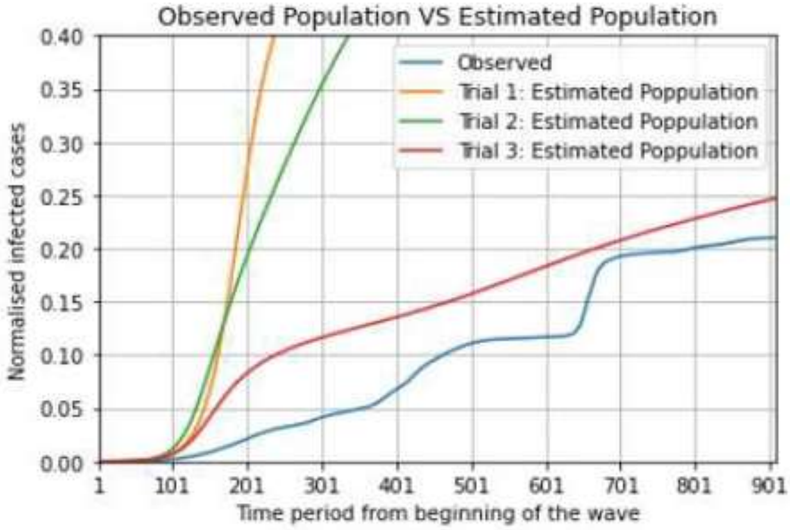


Figure 25.Observed Population VS Estimated Population for different Trails -Argentina

Table 17 and Figure 25 reveal that among the many choices for $a, b, k1, k2,$ and $k3$ studied, the ideal solution with minimal M.S.E is $a = 0.1, b = 0.05, k1 = 0.9, k2 = 0.005,$ and $k3 = 0.01$. Also, its examined proportion of people infected, $P(t)$, falls near the observed proportion of Argentina population.

Distinct Trials with I_p for crowd effected extended SIR model:

Several values for I_p (considered population percentage) which refers to the epidemic's "visibility" and news media reporting, are accounted for the ODE integrated revised SIR model incorporating the crowd effect. M.S.E is subsequently determined for each I_p trial to select the best option from the collection. Table 18 describes this procedure in full.

$S_{ign}(0)$	$S_{res}(0)$	$S_{exh}(0)$	$I(0)$	$R(0)$	I_p	M.S.E	
0.999951876	0	0	0.0000243759972	0.0000481236	0.01	0.0008751427	
0.999951876	0	0	0.0000243759972	0.0000481236	0.015	0.0087965647	
0.999951876	0	0	0.0000243759972	0.0000481236	0.02	0.0133502385	← Initial value
0.999951876	0	0	0.0000243759972	0.0000481236	0.04	0.0145949869	

Table 18. Calculation of M.S.E for distinct I_p after considering Crowd Effect- Argentina

The estimated percentage of the infected population, $P(t)$, is contrasted with the observed infected population in Figure 26 for the diverse expectations of I_p as described in Table 18 above.

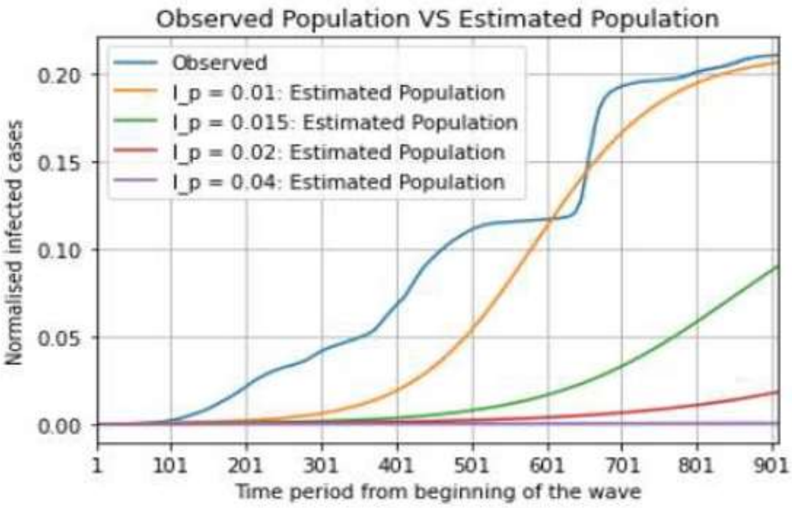


Figure 26. Observed VS Estimated Population for different IP trails after Considering Crowd Effect-Argentina

From Table 18 and Figure 26, it can be confirmed that $I_p = 0.01$ produces the least M.S.E value and hence, making it the best option out of all the experimented options for I_p. Additionally, $P(t)$, the analysed fraction of sick people, is close to the normalised population of Argentina as a whole.

2. Analysis For Australia:

The number of waves affected by Australia is determined using a logarithmic heuristic. We can see from the logarithmic Figure 27 below that Australia suffered three waves. The first wave can be observed between days 1 and 98, wave 2 from days 99 to 472, and wave 3 from the 473rd day till data availability.

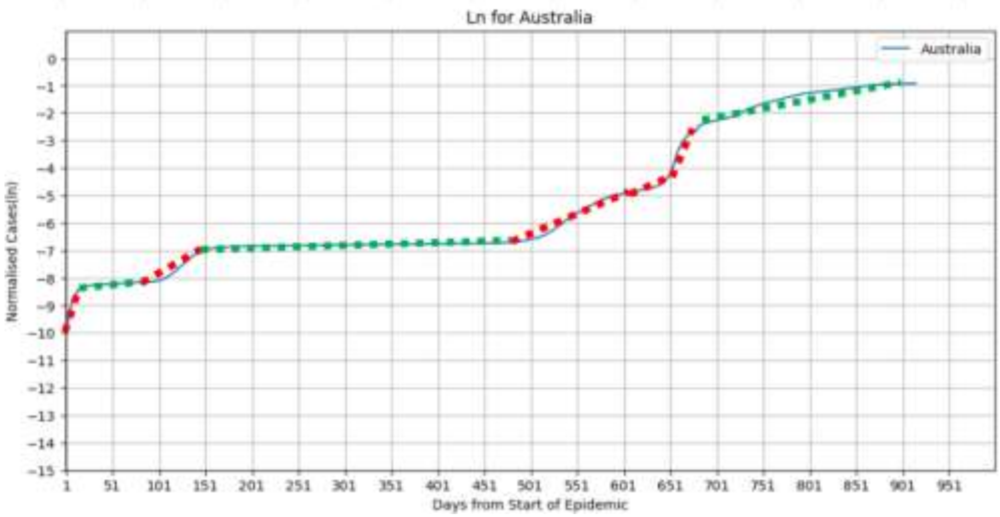


Figure 27: Logarithmic graph for Australia

To determine the parameters of exponential growth model, the slope interval is ascertained for each wave independently and the intervals are 14, 48, and 273 days, respectively. Figure 28, Figure 29, and Figure 30 show the normalised cases and logarithmic graphs for each wave that Australia experienced.

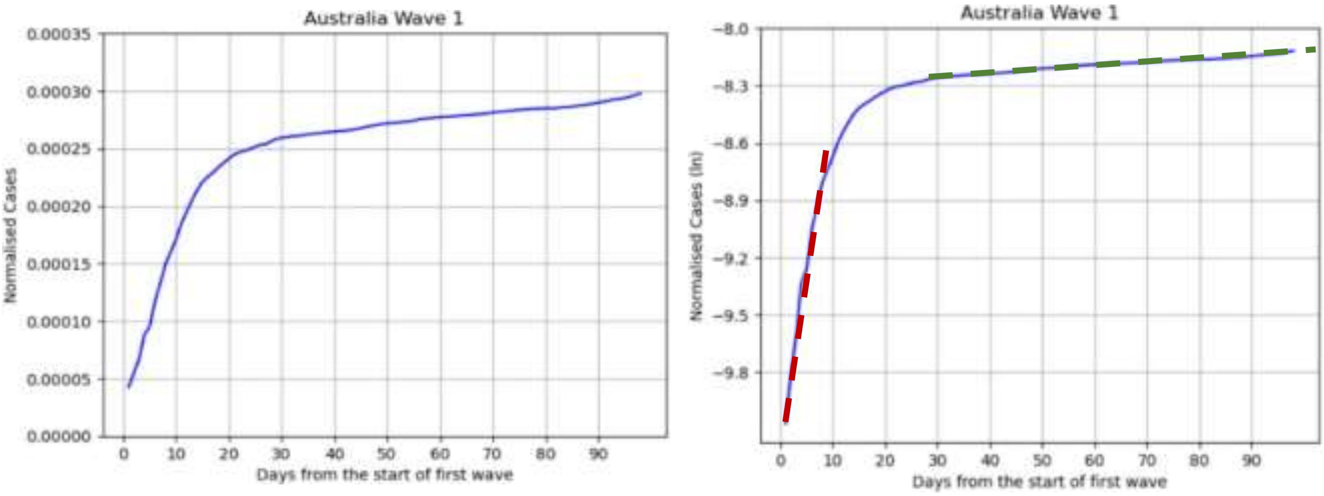


Figure 28: Normalised cases (left) and logarithmic (right) graphs for the first wave of Australia

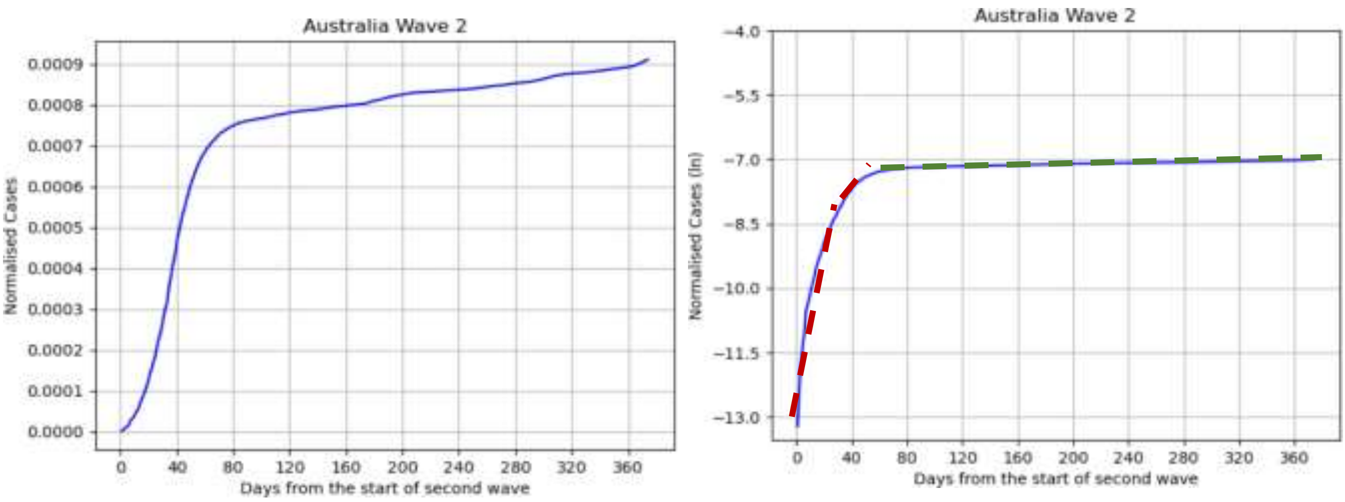


Figure 29: Normalised cases (left) and logarithmic (right) graphs for the second wave of Australia

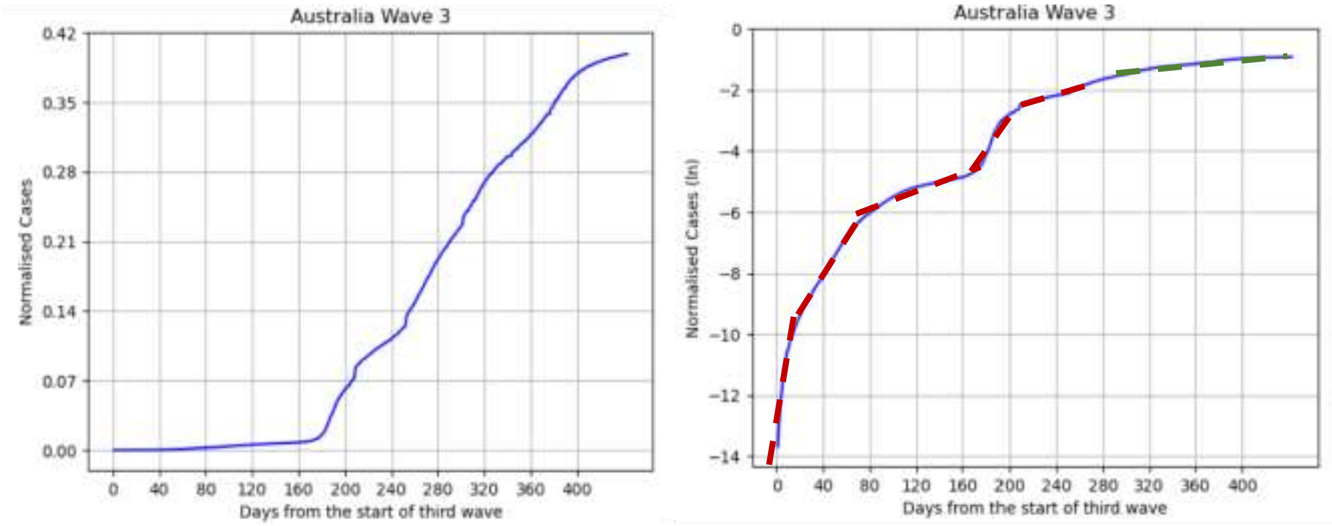


Figure 30: Normalised cases (left) and logarithmic (right) graphs for the third wave of Australia

Exponential model prediction:

The exponential model is frequently regarded as a simple and faster-achieving paradigm for explaining the behaviour of transmissible diseases. To analyse the covid-19 behavioural patterns, we incorporated exponential model on all waves of Australia's normalised cases. The parameters predicted by the exponential model are $a = -9.92231$ and $r = 0.119425$ for wave 1, $a = -11.3593$ and $r = 0.09602$ for wave 2, and $a = -9.35994$ and $r = 0.030631$ for wave 3. Below Figure 31, Figure 32, and Figure 33 depicts exponential growth and associated error for Australia's three waves.

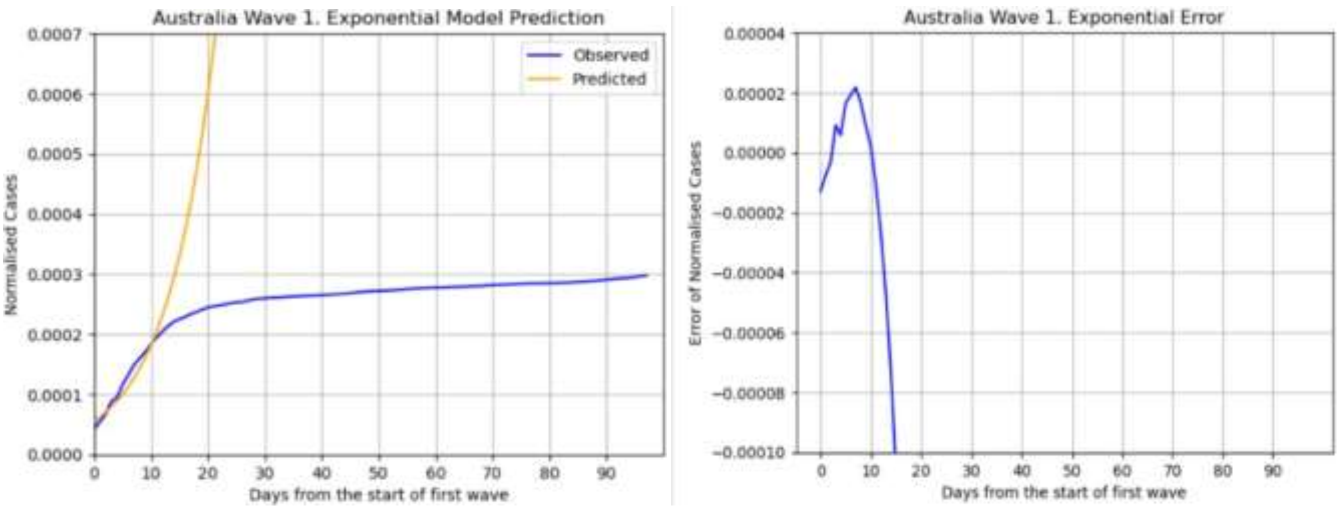


Figure 31: Exponential growth and associated error for Australia's First wave

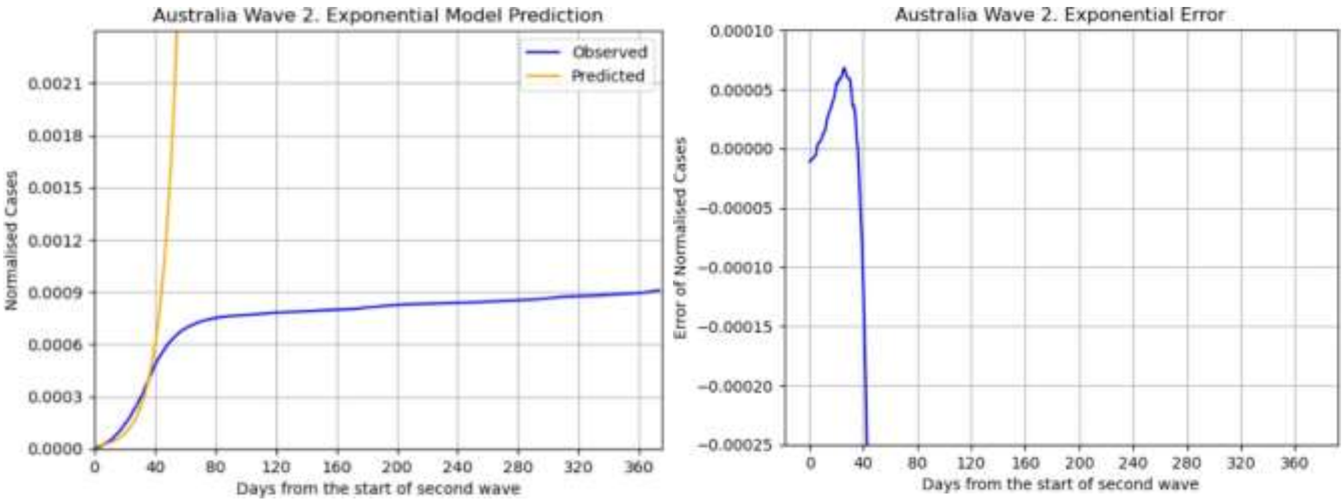


Figure 32: Exponential growth and associated error for Australia's Second wave

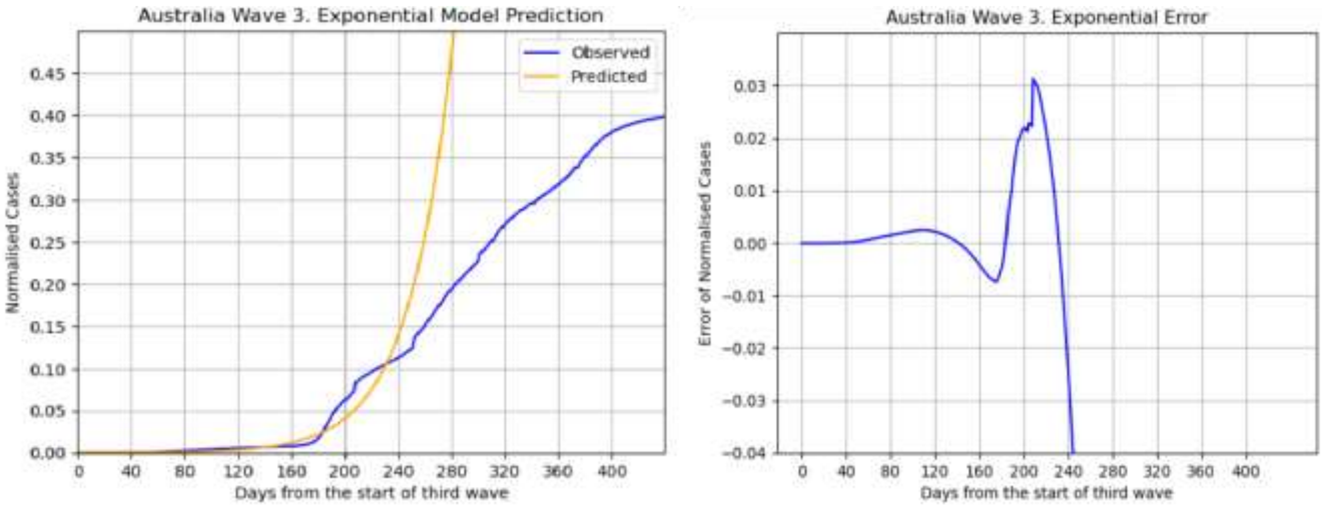


Figure 33: Exponential growth and associated error for Australia's Third wave

Logistic model prediction:

The logistic model is more realistic, preventing exponential growth after the population has reached it carrying capacity. The carrying capacity K for all the three waves is computed using the parameters a and r determined above. Days 4-11 for wave 1, days 6-35 for wave 2, and days 39-142 and 185-237 for wave 3 all reported a value $K > 1$, which was irrelevant for calculating the initial value of K as portrayed in the Figure 34, Figure 35 and Figure 36. As a result, an initial value of $K = 0.000382$, $K = 0.000902$, and $K = 0.404101$ are chosen for waves 1, 2, and 3.

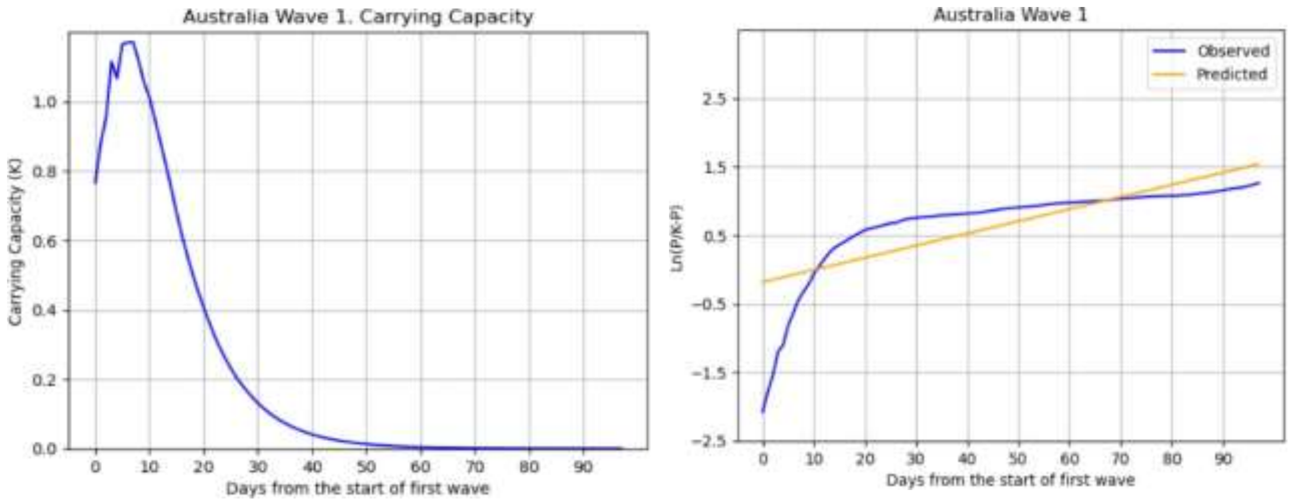


Figure 34: Carrying Capacity K (left) and estimation of logistic model parameters (right) for Australia's first wave

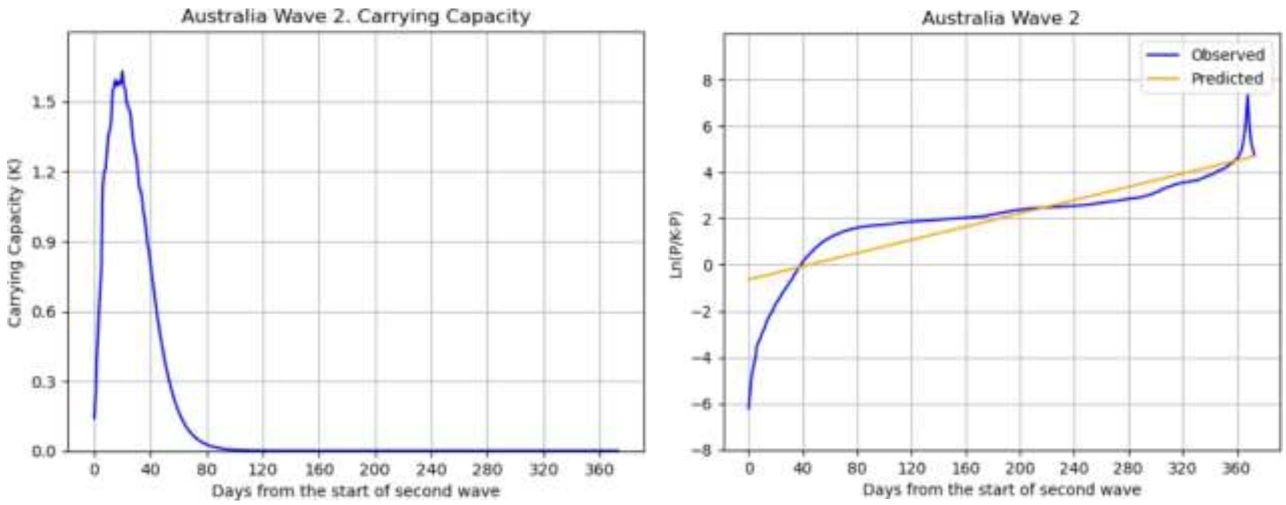


Figure 35: Carrying Capacity K (left) and estimation of logistic model parameters (right) for Australia's second wave

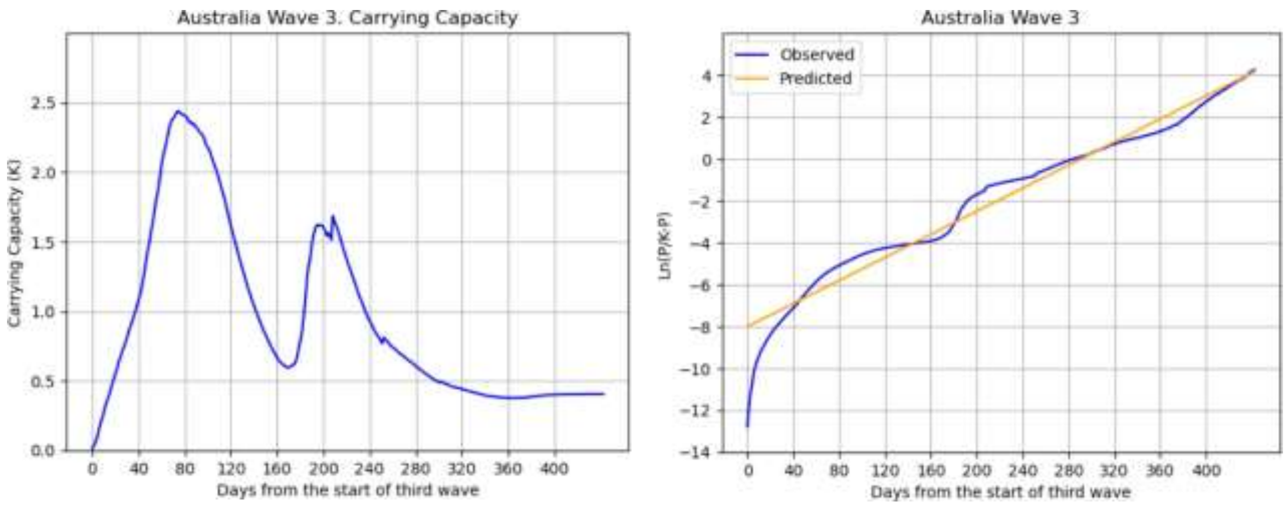


Figure 36: Carrying Capacity K (left) and estimation of logistic model parameters (right) for Australia's third wave

As per Figure 37, Figure 38 ,and Figure 39**,** as well as Table 19, the initial carrying capacity value chosen above is adequate, as parameter optimisation has no influence on the SSE and MSE.

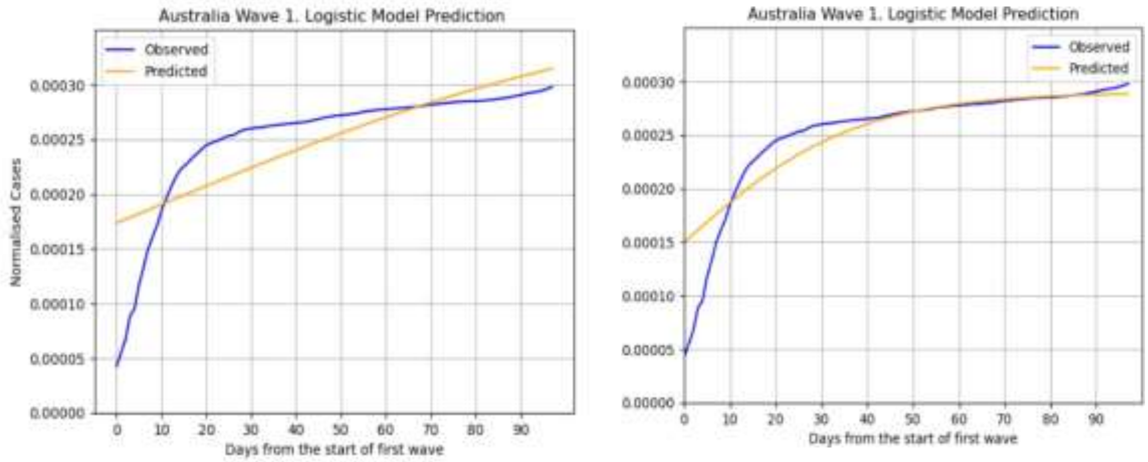


Figure 37: Logistic model prediction for initial value of K (left) and for optimal value of K (right) for Australia's First wave

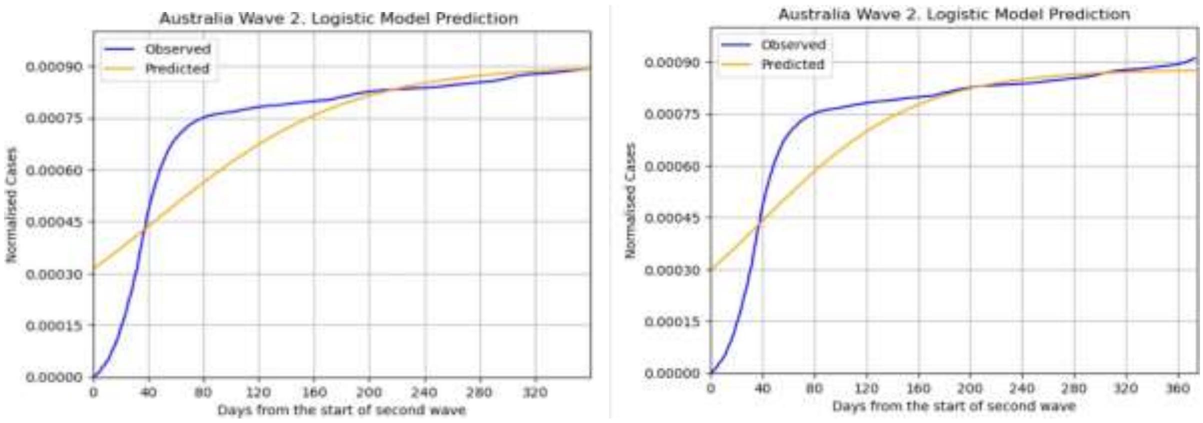


Figure 38: Logistic model prediction for initial value of K (left) and for optimal value of K (right) for Australia's Second wave

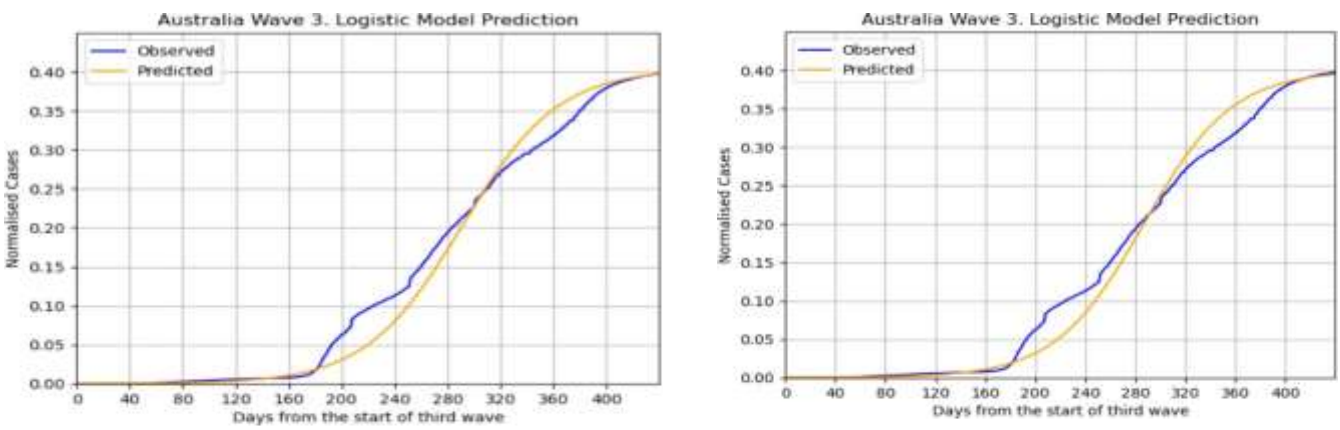


Figure 39: Logistic model prediction for initial value of K (left) and for optimal value of K (right) for Australia's Third wave

Wave #	Model	Parameters				
		a	r	K	S.S.E	M.S.E
Wave 1	Exponential	-9.92231	0.119425	NA	1.66×10^{02}	1.69
	Logistic (initial K)	-0.201349	0.01777	0.000382	0.000000108	0.000000001
	Logistic (optimal K)	-0.011588	0.052721	0.000290	0.000000055	0.000000000
Wave 2	Exponential	-11.3593	0.09602	NA	1.21×10^{22}	3.24×10^{19}
	Logistic (initial K)	-0.65081	0.014302	0.000902	0.000004426	0.000000011
	Logistic (optimal K)	-0.688314	0.016881	0.000878	0.000003732	0.000000009
Wave 3	Exponential	-9.35994	0.030631	NA	7.46×10^{04}	1.68×10^{02}
	Logistic (initial K)	-8.04004	0.027551	0.404101	0.16071	0.000363
	Logistic (optimal K)	-8.124456	0.02824	0.400090	0.15412	0.000348

Table 19: Exponential Model and Logistic Model Parameters for Australia

Isolating first two waves:

We integrated the first two waves of Australia being researched for future scientific investigations. Figure 40 shows the gradient of Australia's logarithmic value. The slope interval in this case spans from 1 to 34 days. The exponential interval for Australia's first wave was changed from 1-14 to 1-34. Australia experienced the shortest exponential growth in its first wave. We determined that extending the exponential period is preferable after plotting the diverging infection fraction graph. Changing the exponential period caused the exponential parameter r to change.

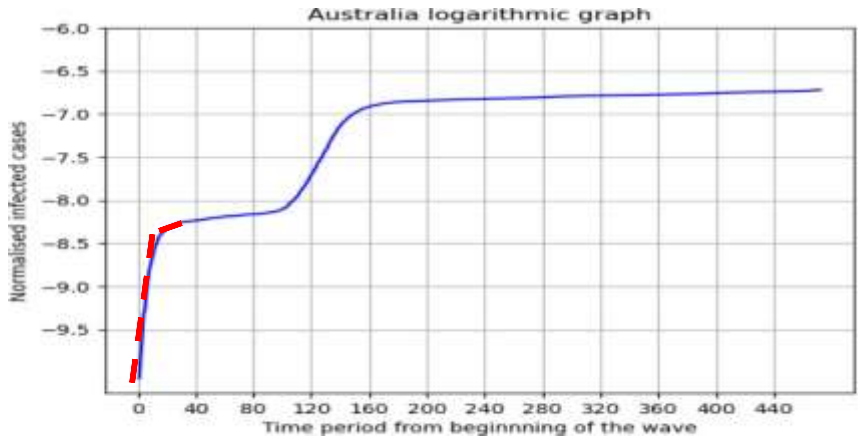


Figure 40:Australia's logarithmic graph for first two waves

Evaluating Susceptible S and exponential coefficient r:

The value for Susceptible, S is estimated at the start and end of the period and the exponential model parameter r is projected by using $ln\ P(t) = c + rt$. And portrayed in Table 20.

Exponential Parameters		Start period(day)			End period(day)			r	c
		ln value	$P(t)$	S	ln value	$P(t)$	S		
Australia	Wave 1 Slope Interval	-10.0685	0.00004239	0.99995761	-8.45	0.00021390	0.9997861	0.040873	-9.31744

Table 20:Estimation of Susceptible S and exponential parameters for Australia

Defining SIR model coefficients:

For the first 34 days, Australia experiences exponential growth. The SIR coefficients of the model are calculated by taking $b = 0.1$ into consideration. Table 21 illustrate the SIR model's parameters. In this instance $a = r + b$.

Interval	r	c	b	a	SSE	MSE
1-34	0.040873	-9.31744	0.1	0.140873	5.87×10^{09}	1.24×10^{07}

Table 21: Estimation of SIR coefficients for Australia

Defining SIR Model preliminary values:

$P(0) = 0.0001709$ and $R(0) = 0.0000423$ were used to determine $S(0)$ and $I(0)$. The prior values were used because Australia has a short exponential growth period of 1-34 days. As a result, we established a 10-day recovery time for afflicted individuals.

$$S\ (0) = 1 - P\ (0)$$

$$I\ (0) = P\ (0) - R\ (0) = P\ (0) - P\ (-10)$$

$$R\ (0) = P\ (-10)$$

Table 22 shows the calculated initial values of S, I, R, and P.

$P(0)$	$S(0)$	$I(0)$	$R(0)$
0.000170942	0.999829058	0.00012855	0.00004239

Table 22:Initial values of SIR model for Australia

Defining divergent preliminary values of SIR:

After defining the initial values, we calculated varied values of $I(0)$ by multiplying the preceding value by 1.3. The values obtained are presented in Table 23 below.

Initial Values of $I(0)$	$I(0)$	$R(0)$	$S(0)$	$P(0)$
$I(0)_1$	0.00012855	0.00004239	0.99982905	0.00017094
$I(0)_2$	0.00016711	0.00005511	0.99977777	0.00022098
$I(0)_3$	0.00018383	0.00006062	0.99975555	0.00024126

Table 23: Divergent initial values of SIR model for Australia

ODE System Integration for SIR:

The ODE integration is calculated for Australia's first wave exponential period. Figure 41 illustrates the SIR plot for the infected population fraction with respect to the time series, t, which reflects the whole period of Australia's first two waves, 472 days.

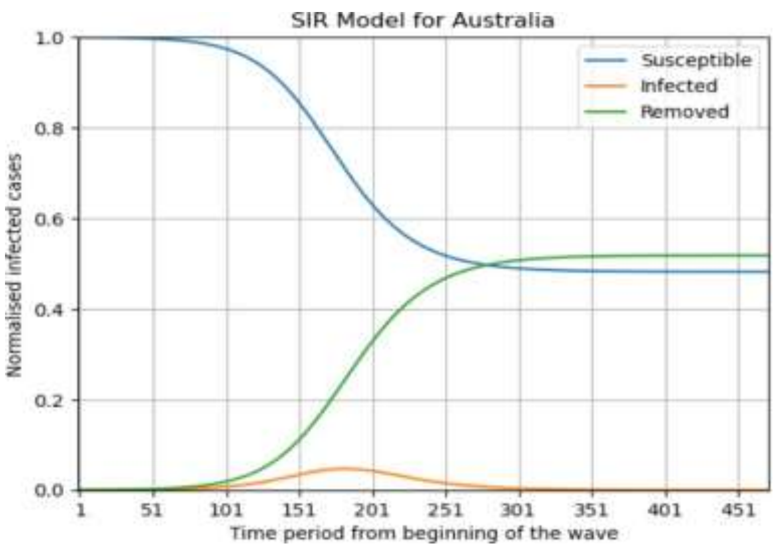


Figure 41: SIR Model for Australia

We estimated the normalised infected cases integration $P(t)$ from $1 - S(t)$ using integrated SIR values. After being calculated, a comparison graph between I, R, and P with respect to time t is shown in Figure 42.

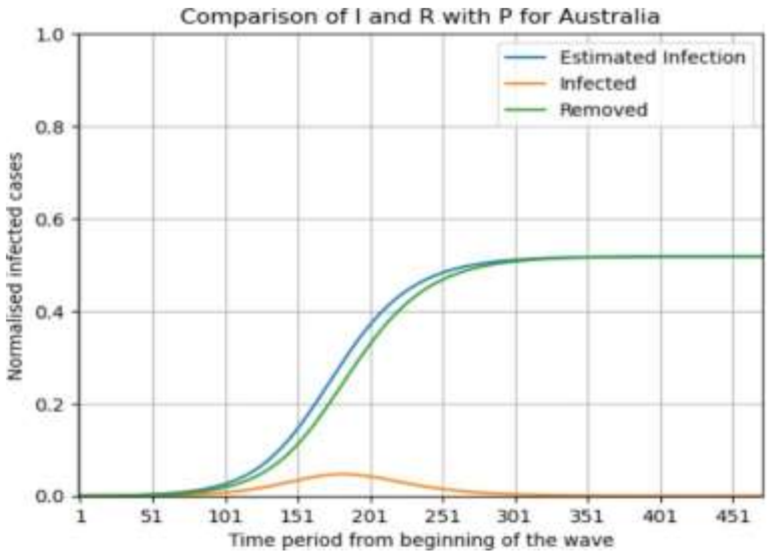


Figure 42:Comparison of I and R with P for Australia

Determining optimal initial or preliminary value for SIR:

With the Ordinary Differential Equation integration of the different initial infection values $I(0)$ summarised and specified in Table 24, we have figured out that the optimal value of $I(0)$ by finding the lowest M.S.E which is computed from the observed infected population P and the estimated normalised infected population $P(t)$.

Initial Values of $I(0)$	$I(0)$	$R(0)$	$S(0)$	MSE
$I(0)_1$	0.00012855	0.00004239	0.99982905	1.24×10^{-8}
$I(0)_2$	0.00016711	0.00005511	0.99977777	1.87×10^{-8}
$I(0)_3$	0.00018383	0.00006062	0.99975555	2.81×10^{-8}

Table 24: MSE of divergent initial values of SIR model for Australia

Figure 43 compares the various $I(0)$ graphs to the observed normalised instances. We may infer from this and Table 24 that $I(0)_1 = 0.00012855$ is the best ideal value since it has the lowest M.S.E and is the most closely related to the observed normalised cases curve.

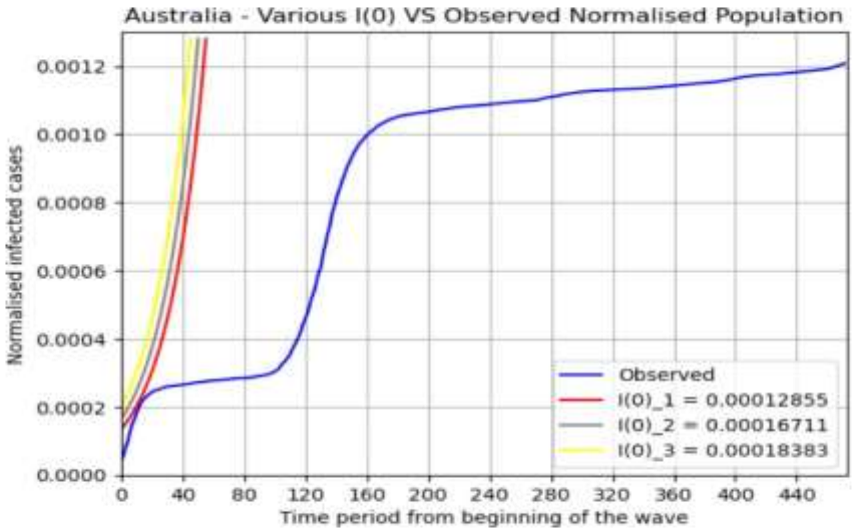


Figure 43: Comparison of divergent I(0) with observed population for Australia

Estimating optimal SIR coefficients a and b:

The Table 25 below shows the SIR coefficients initial and ideal values, a and b. The ideal values are obtained by calculating the least M.S.E value (mean of the observed infected population P and the estimated normalised infected population $P(t)$. The SIR coefficients for Australia that provide the lowest M.S.E are $a = 0.11$ and $b = 0.09$, respectively.

Values	a	b	SSE	MSE
Initial	0.140873	0.1	71.261	0.1507
Optimal	0.11	0.09	12.6461	0.0267

Table 25: Initial and Optimal SIR coefficients for Australia

Australia's ideal SIR plot for the infected population percentage with respect to time series, t, and optimal comparison of I, R with P is displayed from the recorded optimal value of a and b Figure 44

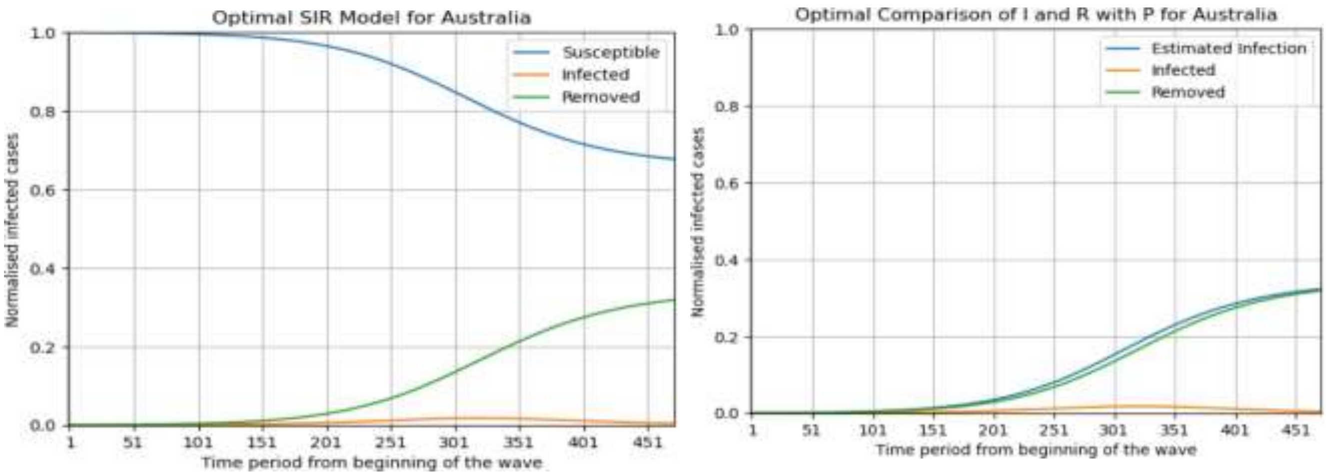


Figure 44: Optimal SIR model(left) and comparison of optimal I and R with P(right) for Australia

Social Psychology addition to SIR Model:

By including social psychological characteristics, we intended to improve the accuracy of an SIR model. In the extended SIR model, the Susceptible stage S is further divided into Susceptible-Ignorant, Resistance, and Exhaustion. The Crowd Effect could be employed to improve the extended SIR model even further. We presented extended SIR model graphs with and without crowd effect in series to show more comparable findings.

Defining extended SIR model coefficients and preliminary values:

The slope interval and extended SIR model parameters are initially assumed to be equivalent for both scenarios without and with crowd effect. The Extended SIR Model parameters for both scenarios are summarised in the below Table 26 and Table 27**.**

Slope Interval (Days)	a	b	k_1	k_2	k_3
1-34	0.140873	0.1	1	0.02	0.01

Table 26: Extended SIR Model Parameters without considering Crowd Effect – Australia

Slope Interval (Days)	a	b	q	k_2	k_3
1-34	0.140873	0.1	50	0.02	0.01

Table 27: Extended SIR Model Parameters considering Crowd Effect – Australia

The initial values substituted in the extended SIR model without crowd effect kinetic equations, as well as the subsequent values from the derivations are shown in the Table 28.

$S_{ign}(0)$	$S_{res}(0)$	$S_{exh}(0)$	$I(0)$	$R(0)$	$\frac{dS_{ign}(0)}{dt}$	$\frac{dS_{res}(0)}{dt}$	$\frac{dS_{exh}(0)}{dt}$	$\frac{dI(0)}{dt}$	$\frac{dR(0)}{dt}$
0.9998290	0	0	0.0001285	0.0000423	-0.00014663	0.00012853	0	0.00000525	0.000012

Table 28: Extended SIR Model Initial Values without considering Crowd Effect – Australia

The extended SIR equations with the crowd effect can be analysed by combining the preliminary value of I_p with the other components $S_{ign}, S_{res}, S_{exh}, I$ and R and shown in Table 29.

$S_{ign}(0)$	$S_{res}(0)$	$S_{exh}(0)$	$I(0)$	$R(0)$	I_p	$\frac{dS_{ign}(0)}{dt}$	$\frac{dS_{res}(0)}{dt}$	$\frac{dS_{exh}(0)}{dt}$	$\frac{dI(0)}{dt}$	$\frac{dR(0)}{dt}$
0.9998290	0	0	0.0001285	0.0000423	0.02	-0.0200184	0.0199965	0	0.000005251	0.000012

Table 29: Extended SIR Model Initial Values considering Crowd Effect – Australia

ODE System Integration for extended SIR:

For Australia, the ordinary differential equation integration is determined for extended SIR – $S(t) \rightarrow [S_{ign}(t) + S_{res}(t) + S_{exh}(t)], I(t), R(t)$ from its corresponding initial values. Figure 45 and Figure 46 depicts different susceptible phases and extended SIR model for Australia without and with taking crowd effect into account.

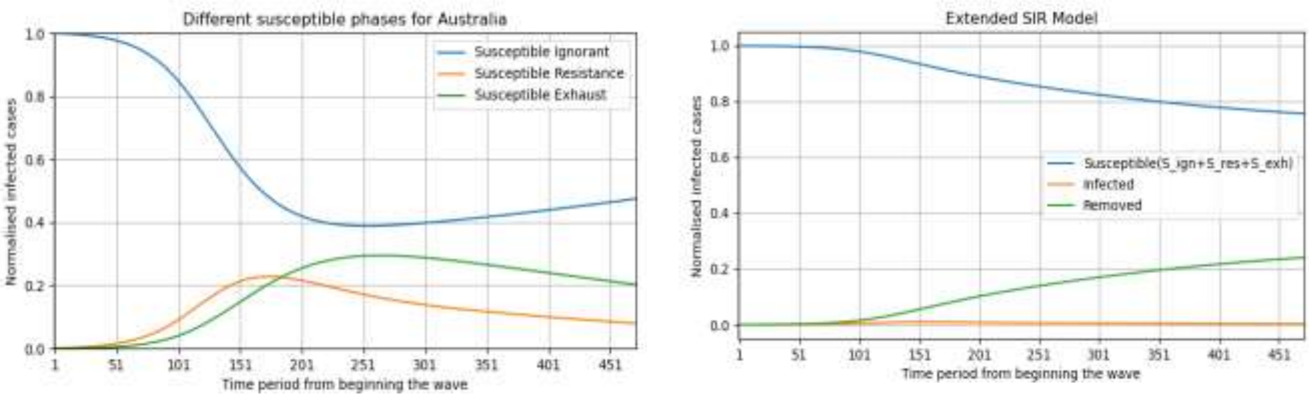


Figure 45: Different Susceptible Phases and Extended SIR Model without considering Crowd Effect -Australia

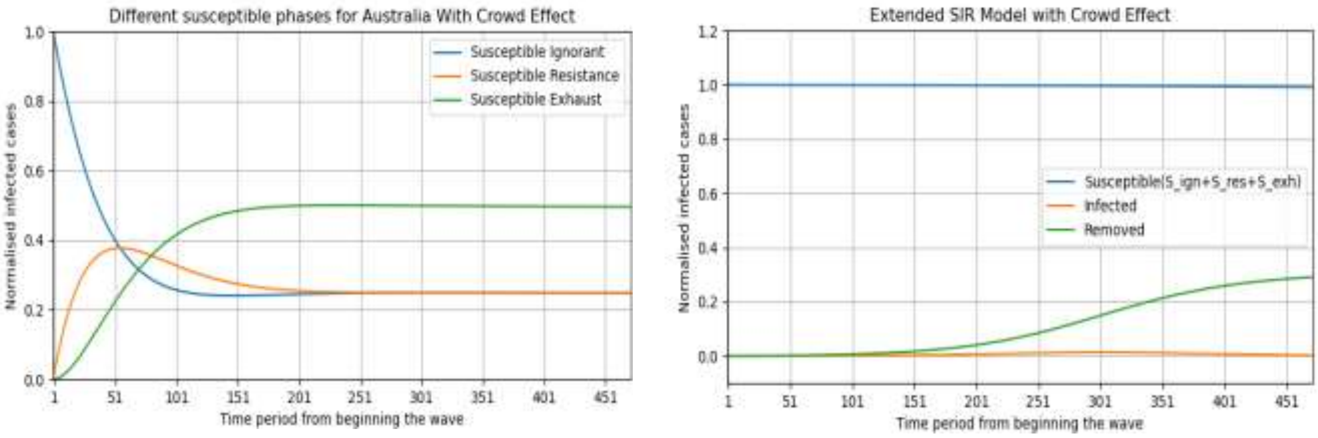


Figure 46: Different Susceptible Phases and Extended SIR Model considering Crowd Effect -Australia

Using above integrated SIR values, we determined the infected population integration $P(t)$ from $(1 - [S_{ign}(t) + S_{res}(t) + S_{exh}(t)])$. Once determined, a comparative plot is generated. Figure 47 depicts the relationship between I, R, P with respect to time t for without and with crowd effect.

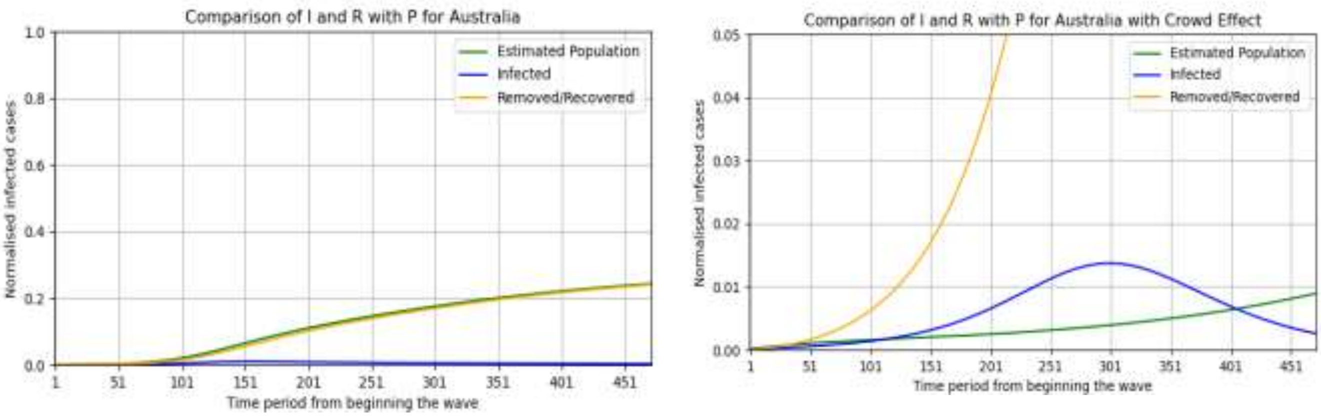


Figure 47: Comparison of I and R with P without and with considering crowd effect- Australia

Differentiating ODE integrated values to the original observed values:

The model's error is calculated by subtracting the exponential growth exp $\wedge(c + r * t)$ from observed normalised cases. Mean square error value is calculated by sum squaring and dividing error resulting value by the time. The Table 30 and Table 31 below shows the initial values considered and their mean square error of the model without and with crowd effect.

$I(0)$	$R(0)$	$S_{ign}(0)$	$S_{res}(0)$	$S_{exh}(0)$	MSE
0.00012855	0.00004239	0.999829058	0	0	2.20×10^{-2}

Table 30 : Calculation of M.S.E for initial data O.D.E without considering Crowd Effect- Australia

$S_{ign}(0)$	$S_{res}(0)$	$S_{exh}(0)$	$I(0)$	$R(0)$	I_p	MSE

0.999829058	0	0	0.00012855	0.00004239	0.02	0.000010913

Table 31: Calculation of M.S.E for initial data O.D.E considering Crowd Effect- Australia

The Figure 48 below shows the observed normalised population vs the estimated fraction of infected people when crowd effect is not taken into consideration and vice versa.

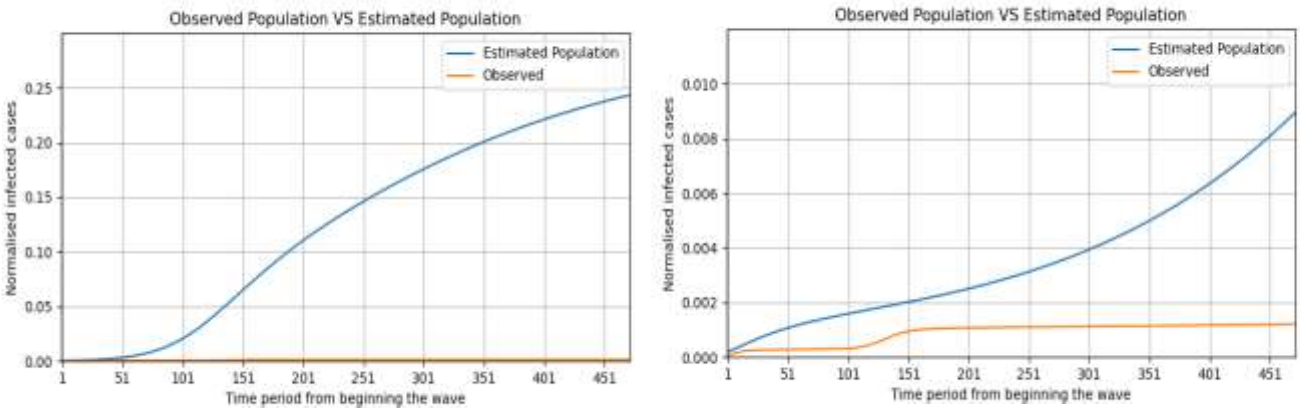


Figure 48: Observed VS Estimated Population without and with considering Crowd Effect -Australia

Estimating optimal extended SIR coefficients:

To determine the optimal value of the extended SIR model coefficients, the coefficient constants $a, b, k_1, k_2, and\ k_3$ should all be modified by considering and experimenting with different values.

We determined the optimal SIR coefficients a, b, k_1, k_2 and k_3 by computing the lowest M.S.E. from the observed infected population and the estimated normalised infected population. Table 32 shows the various extended SIR coefficients that are used to calculate the lowest M.S.E value without taking crowd effect into consideration.

Table 32 below shows that $a = 0.16, b = 0.14, k_1 = 0.5, k_2 = 0.05\ and\ k_3 = 0.03$ are the optimal values for the extended SIR model coefficients, with an M.S.E. of 0.00910375.

a	b	k_1	k_2	k_3	MSE
0.140873	0.1	1	0.02	0.01	0.02196110 (Initial)
0.18	0.16	0.4	0.07	0.02	0.01035597
0.15	0.13	0.3	0.03	0.025	0.00979278
0.16	0.14	0.5	0.05	0.03	0.00910375 (Optimal)

Table 32: Extended SIR Model Optimal Coefficients Estimation for Australia without crowd effect

We next assumed various crowd impact SIR model coefficients $a, b, q, k_2\ and\ k_3$ and calculated the best coefficients with the lowest MSE. Even after considering multiple extended SIR coefficients, it is evident that $a = 0.13, b = 0.099,\ q = 49,\ k_1 = 0.53, k_2 = 0.019, and\ k_3 = 0.0099$ are the best choices for the improved SIR model coefficients, with a minimum M.S.E. of 0.000000297. The experimental coefficient values and their mean square error values are shown in the Table 33 below.

a	b	q	k_2	k_3	MSE
0.140873	0.1	50	0.02	0.01	0.000010913 (Initial)
0.13	0.099	49	0.019	0.0099	0.000000297 (Optimal)
0.084	0.04	40	0.009	0.004	0.025812659
0.077	0.037	30	0.008	0.003	0.000959337

Table 33: Extended SIR Model Optimal Coefficients Estimation for Australia with crowd effect

Figure 49 and Figure 50 below shows Australia's ideal extended SIR plot for the infected population fraction with respect to time series, t, and optimal comparison of I, R with P, without and with taking crowd effect into account.

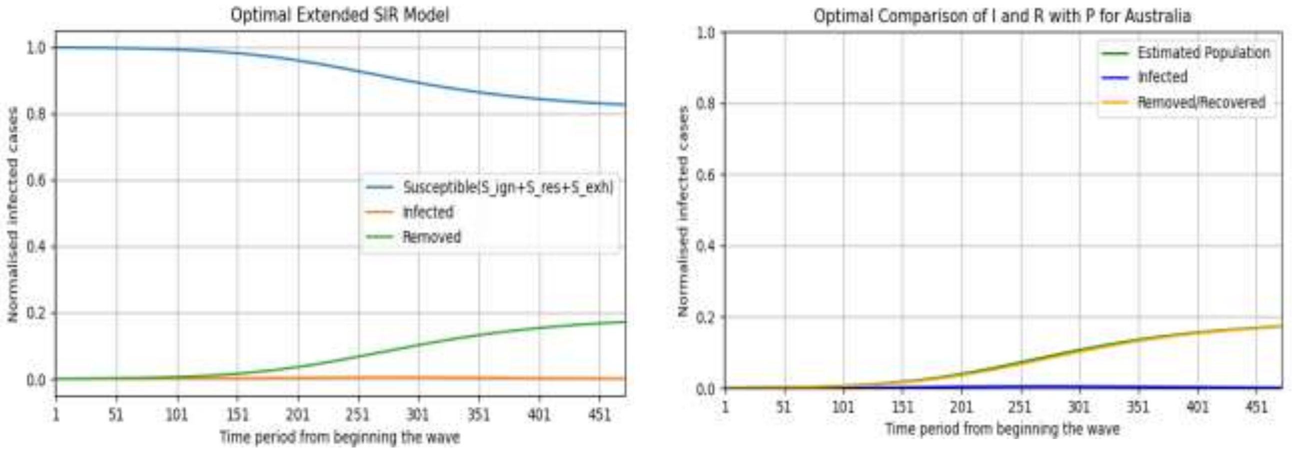


Figure 49: The optimal SIR Model and Comparison of I and R with P without considering Crowd Effect -Australia

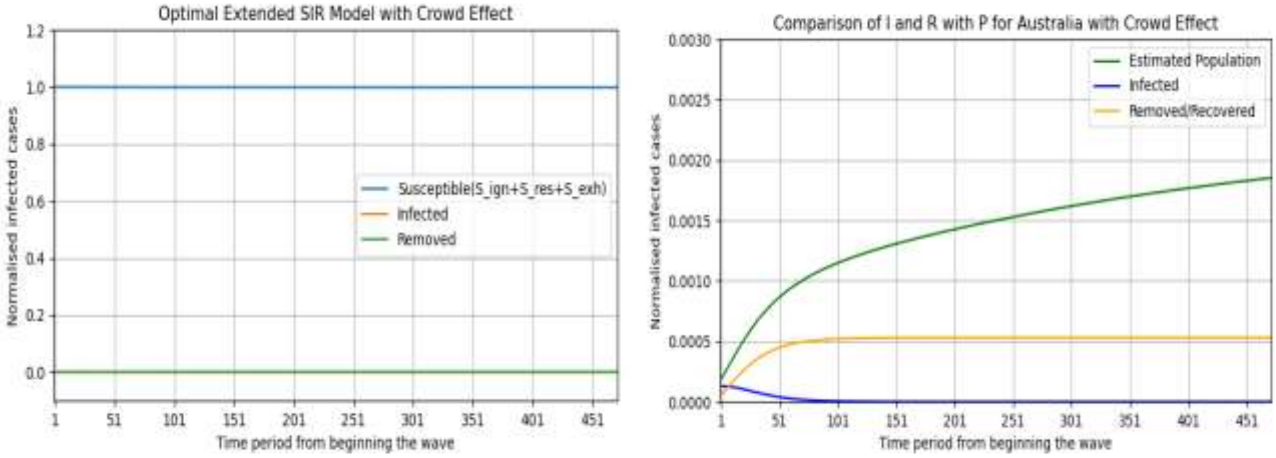


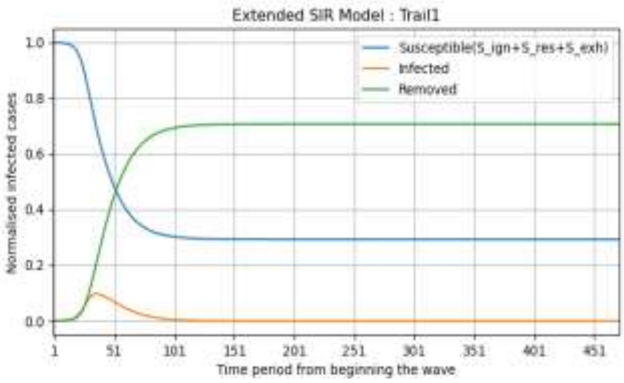
Figure 50: The optimal SIR Model and Comparison of I and R with P considering Crowd Effect -Australia

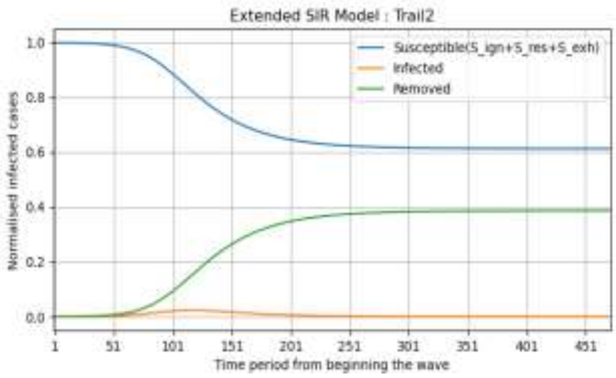
Distinct Trials with extended SIR coefficients excluding crowd effect:

For each trail, we investigated multiple coefficients and produced an extended SIR model for the infected population with respect to the epidemic period. The tabular and graphical representations of all the trails are shown in Table 34 and Figure 51.

Set of Trials	a	b	k_1	k_2	k_3
Trial 1	0.444	0.189	0.999	0.095	0.025
Trial 2	0.222	0.160	0.690	0.088	0.044
Trial 3	0.310	0.170	0.799	0.099	0.034

Table 34: Various Trails for Coefficients of Extended SIR Model - Australia





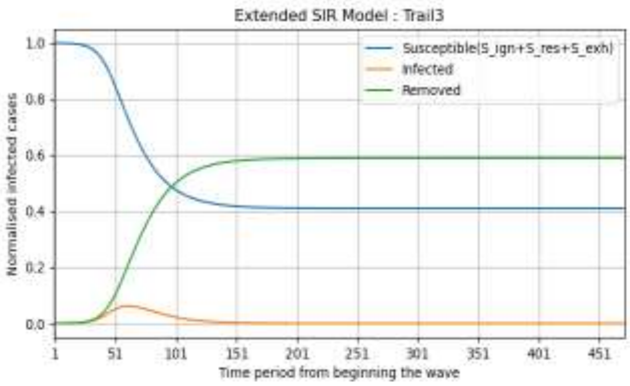


Figure 51:Extended SIR Model for respective trails -Australia

Additionally, we calculated the M.S.E. from the observed infected population and the predicted normalised infected population to find the best SIR coefficients $a, b, k_1, k_2, and\ k_3$ for each trail and shown in Table 35.

Set of Trials	a	b	k_1	k_2	k_3	MSE
Trial 1	0.444	0.189	0.999	0.095	0.025	0.44260143
Trial 2	0.222	0.160	0.690	0.088	0.044	0.09913602
Trial 3	0.310	0.170	0.799	0.099	0.034	0.28525954

Table 35: Different Trails for calculating M.S.E by varying all the coefficients- Australia

Figure 52Figure 52 depicts the comparison of observed normalised population to the estimated infected population for the different a, b, k_1, k_2 and k_3 estimated SIR coefficients.

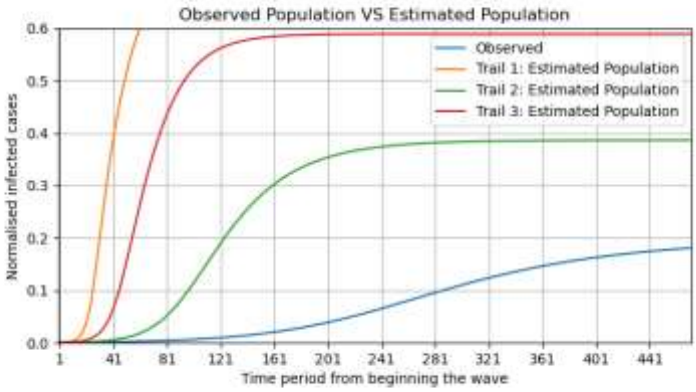


Figure 52: Observed Population VS Estimated Population for different Trails -Australia

Even after taking into account several extended SIR coefficients, it is clear that "$a = 0.069$, $b = 0.049$, $k_1 = 0.53$, $k_2 = 0.01$ and $k_3 = 0.0049$ are considered to be the best values for the revised SIR model coefficients, with a minimal M.S.E. of 0.000175031."

Distinct Trials with I_p for crowd effected extended SIR model:

We calculated the lowest M.S.E. from the observed infected population and the predicted normalised infected population to establish the optimum SIR coefficient I_p. The extended SIR coefficients used to determine the lowest M.S.E value are shown in Table 36.

$S_{ign}(0)$	$S_{res}(0)$	$S_{exh}(0)$	$I(0)$	$R(0)$	I_p	MSE
0.999829058	0	0	0.00012855	0.00004239	0.015	0.024653565
0.999829058	0	0	0.00012855	0.00004239	0.010	0.000761902
0.999829058	0	0	0.00012855	0.00004239	0.020	0.000010913
0.999829058	0	0	0.00012855	0.00004239	0.025	0.000000448

Table 36: Calculation of M.S.E for distinct I_p after considering Crowd Effect- Australia

Figure 53 displays the comparison of the observed normalised population to the predicted infected population for the various estimated SIR coefficient I_P.

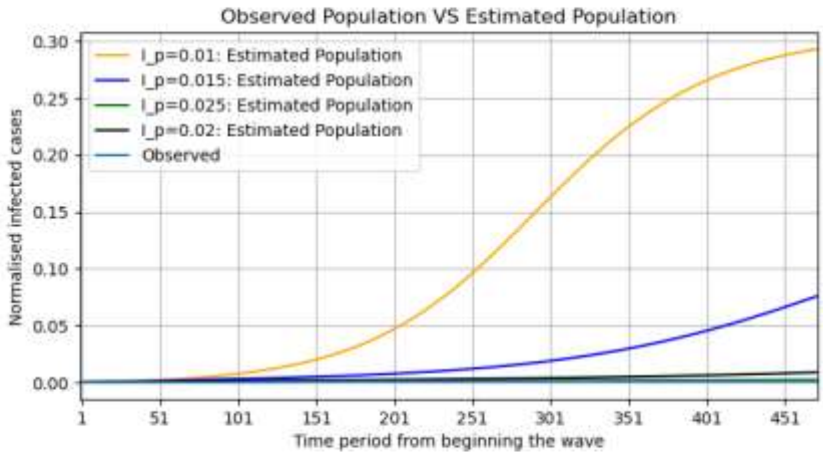


Figure 53: Observed VS Estimated Population for different I_P trails after Considering Crowd Effect-Australia

Table 36 and Figure 53 show that, in the four I_p values considered, $I_p = 0.025$ is the best value with least mean square error when compared with the remaining proportion of infected considerations.

3. Belgium

In this detailed analysis on Belgium, we have used logarithmic heuristic to divide the total stretch of available data for COVID cases into several waves to make a sensible inference from the study. where we found that the cases can be divided into three waves which range from days 1 to 131 ,132 to 573 and the final wave of cases start from the 574^{th} day and continue till the end of data. Which has been conveyed through the below Figure 54.

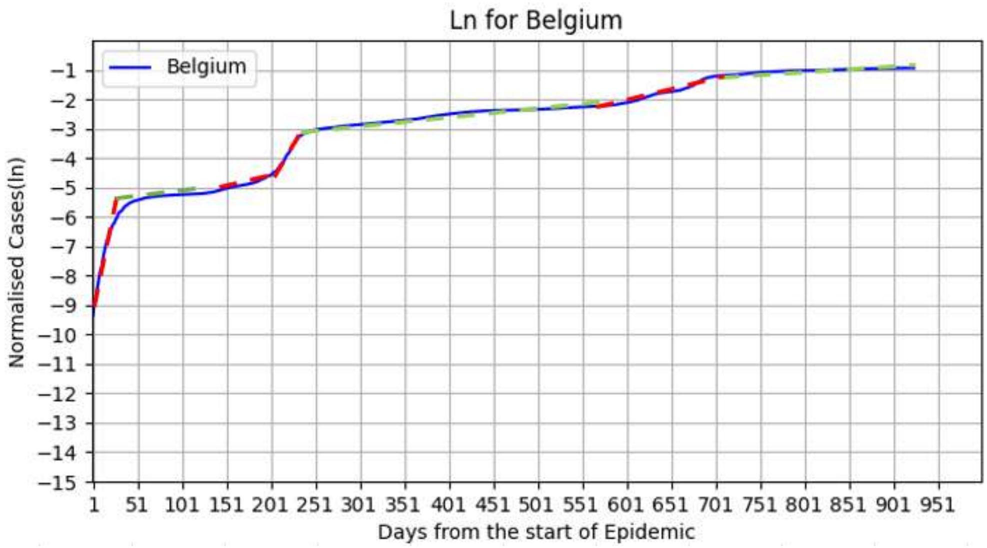


Figure 54: Logarithmic graph for Belgium.

After segregating the whole length of COVID cases into several waves using Logarithmic heuristic the respective individual waves are portrayed in the below Figure 55, Figure 56 and Figure 57, the linear slope interval used to do so is 20 for wave 1 i.e., day 1 to 20 and 43 for wave 2 i.e., from day 55 to 98 in wave 2 respectively.

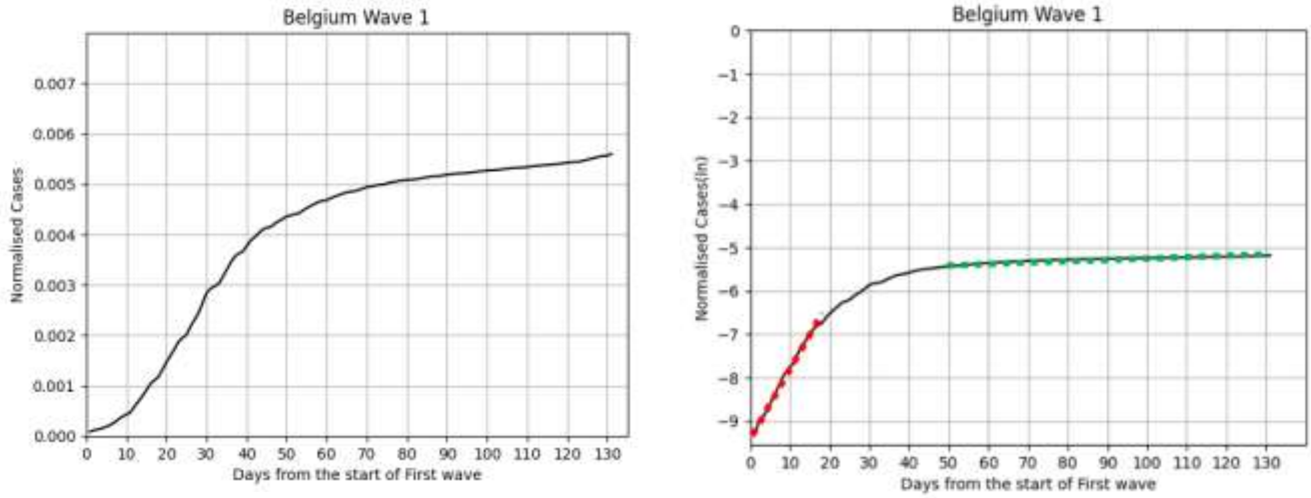


Figure 55: Normalised cases (left) and logarithmic (right) graphs for the first wave of Belgium.

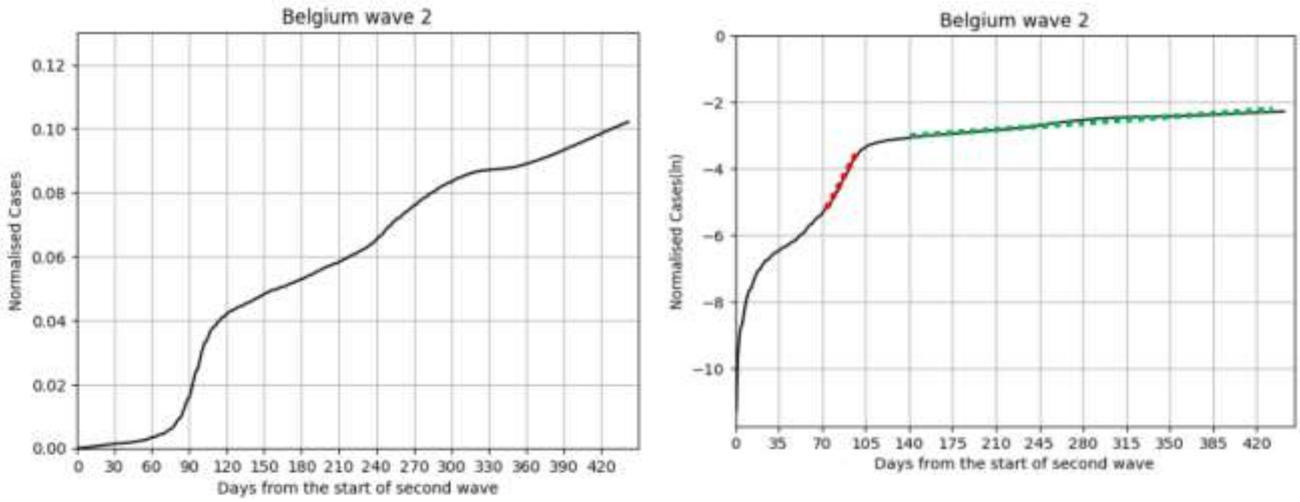


Figure 56: Normalised cases (left) and logarithmic (right) graphs for the second wave of Belgium.

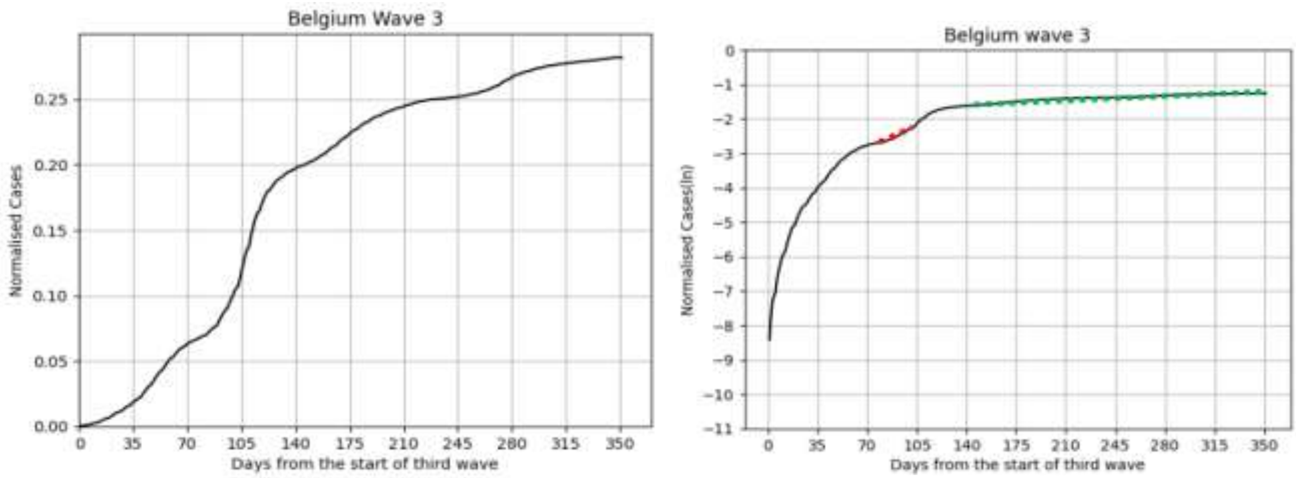


Figure 57: Normalised cases (left) and logarithmic (right) graphs for the Third wave of Belgium.

Exponential model Prediction:

The cases predicted by incorporation of exponential model to Belgium's data and the observed cases (Normalised cases) in Belgium for the respective waves along with the errors associated with the prediction are shown in the below Figure 58, Figure 59 and Figure 60. The parameters resulting in prediction through exponential model are $a = -9.33498$ and $r = 0.15147$ for wave 1, $a = -8.84981$ and $r = 0.07643$ for wave 2 and $a = -7.6222$, $r = 0.13242$ for wave 3 respectively

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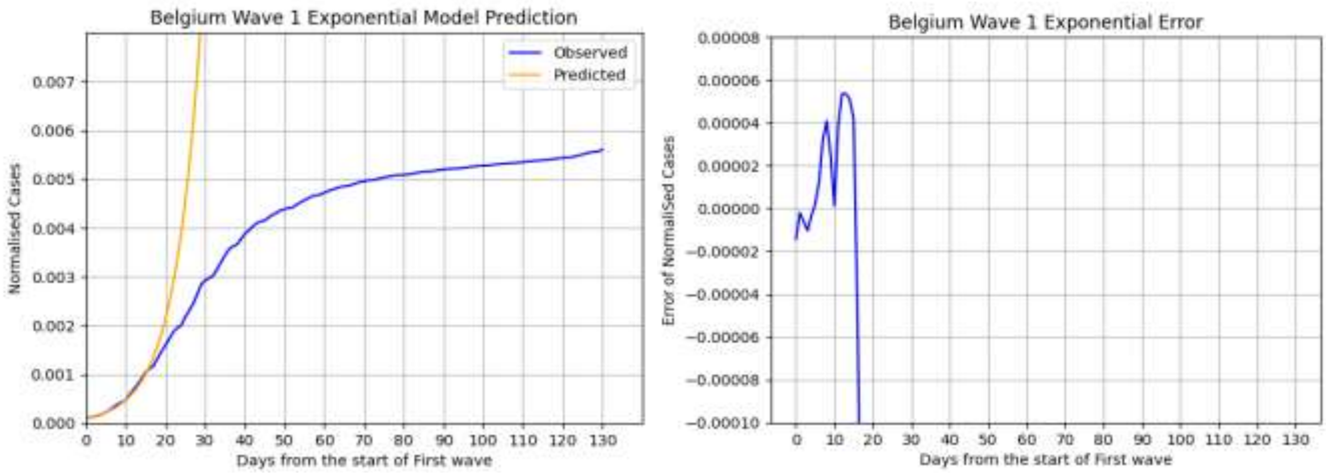

Figure 58:Exponential growth and associated error for Belgium's First wave.

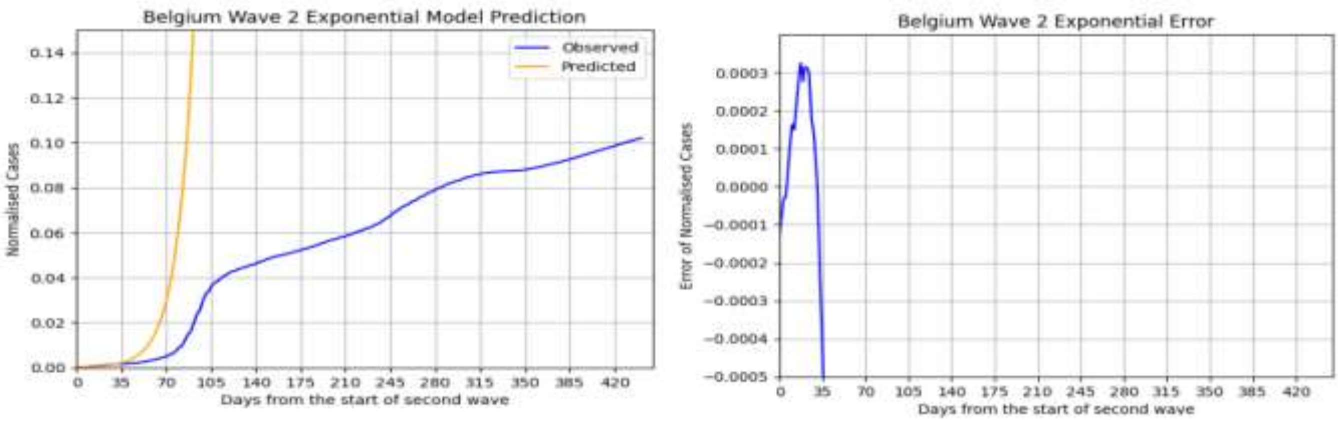

Figure 59: Exponential growth and associated error for Belgium's Second wave.

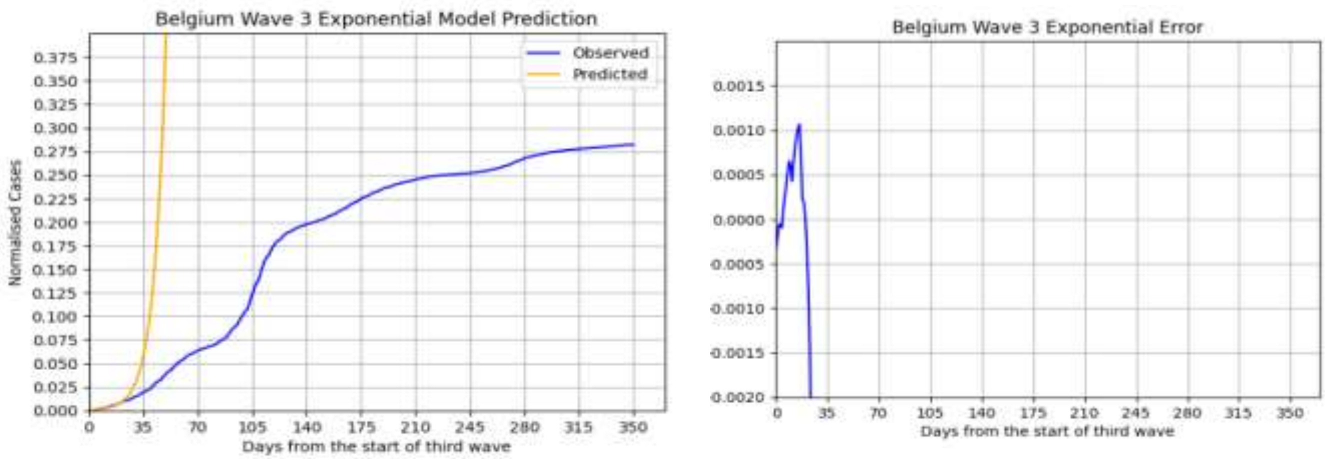

Figure 60: Exponential growth and associated error for Belgium's Third wave.

Logistic Model Prediction:

While considering for the logistic growth in the cases the parameter 'carrying capacity' K, is essential for the study. Which is here calculated from the parameters of the respective waves. As shown in the below Figure 61 the K value for the wave 1 goes beyond the possible maximum threshold which is 1 between the days 6 and 16 for wave, days 7 to 31 for wave 2 as can be seen in

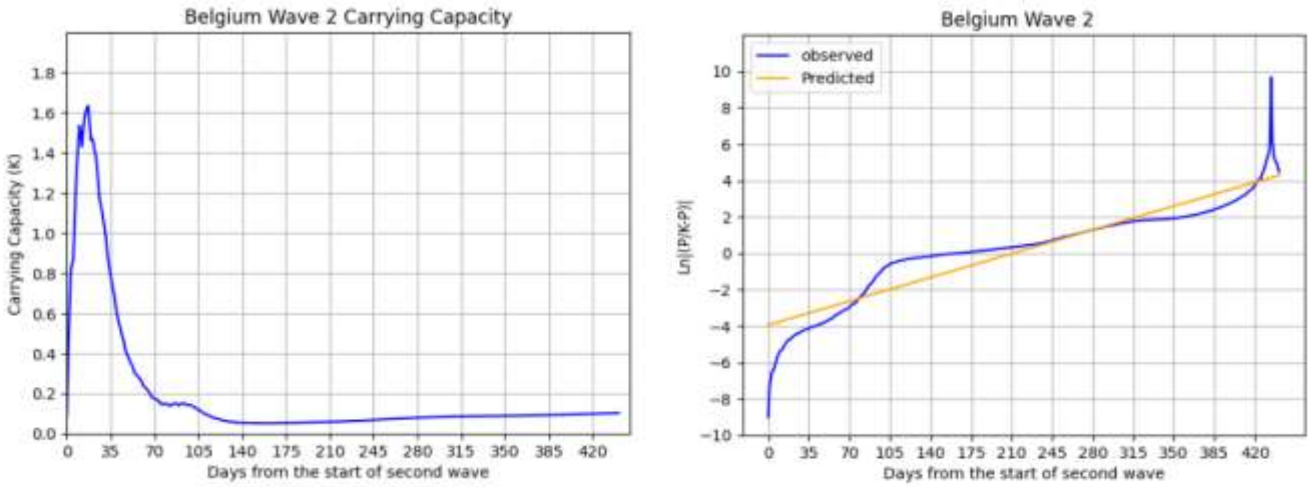

Figure 62 and days 6 to 21 for wave 3, portrayed in Figure 63, which hardly makes sense. So we have considered K value as 0.005464112 for wave 1, 0.101019 for wave 2 and 0.28140 for wave 3. The required model parameters are calculated using these values of K and illustrated in the respective previously stated figures.

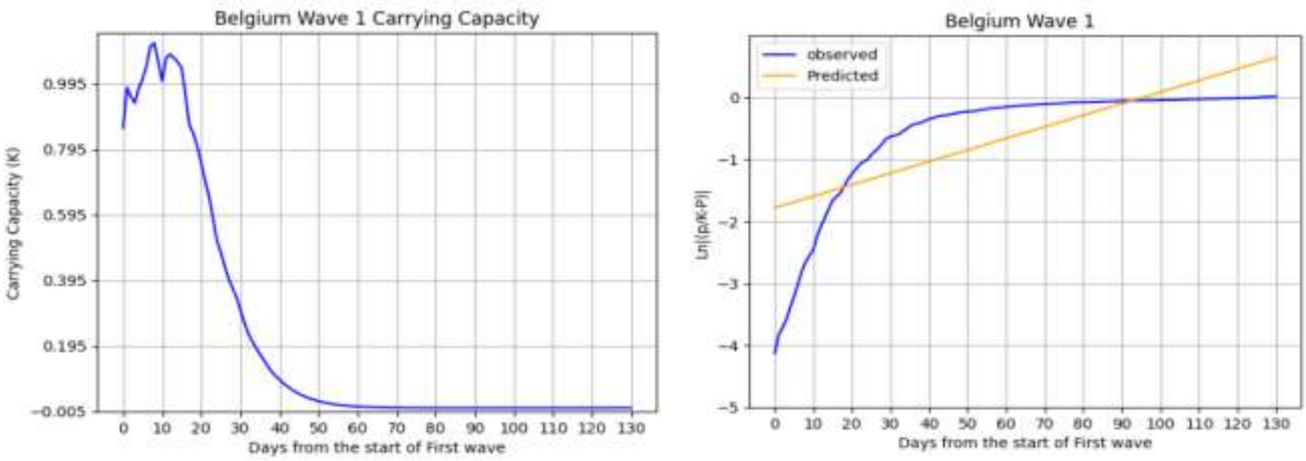

Figure 61: Carrying Capacity K (left) and estimation of logistic model parameters (right) for Belgium's first wave.

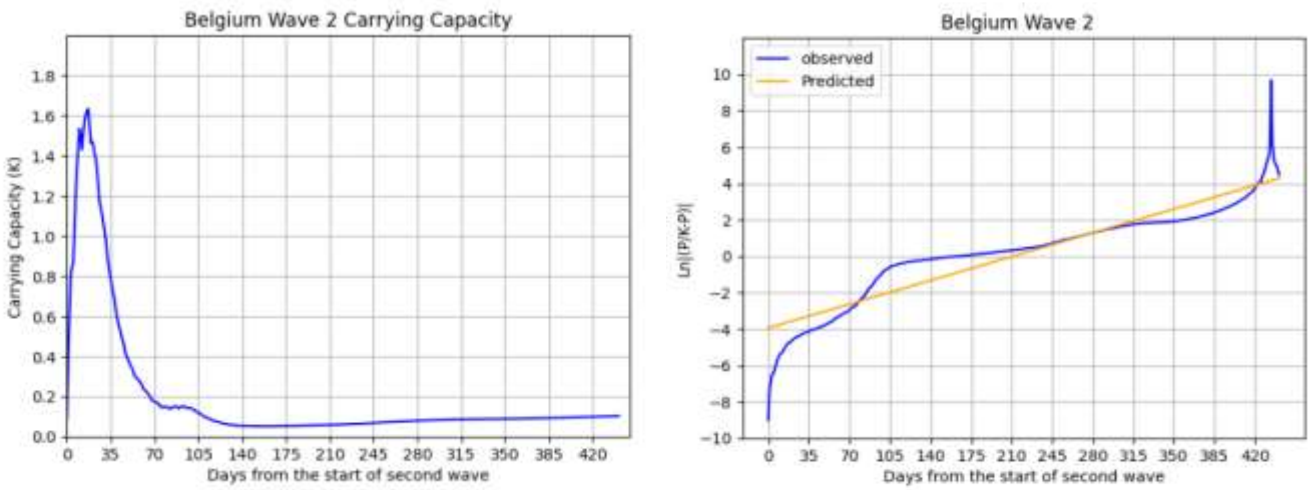

Figure 62: Carrying Capacity K (left) and estimation of logistic model parameters (right) for Belgium's second wave.

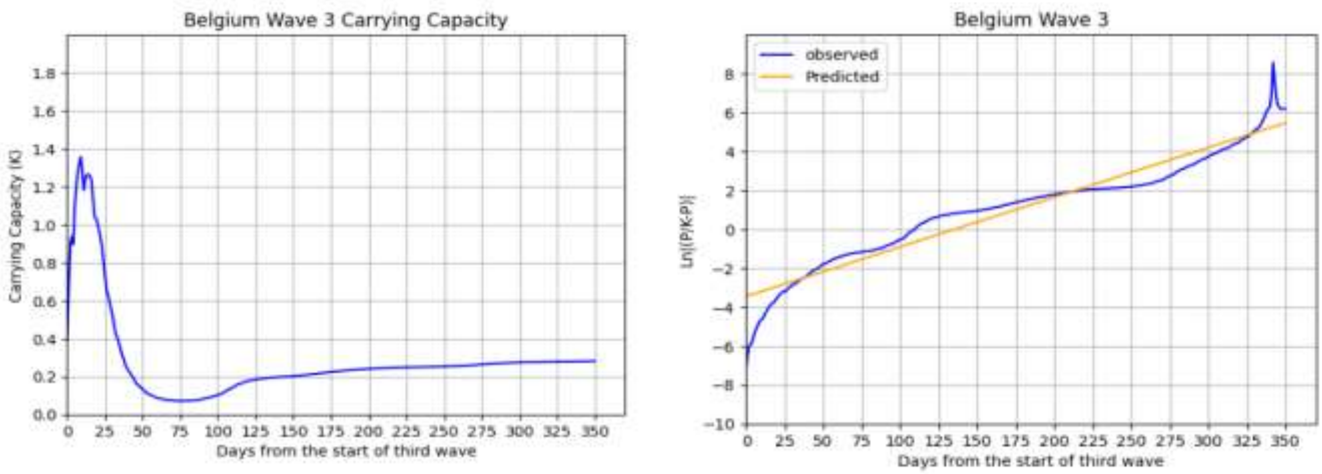

Figure 63: Carrying Capacity K (left) and estimation of logistic model parameters (right) for Belgium's third wave.

Further, in our study we have identified the optimal parameter K for carrying capacity such that the resulting Mean Squared Error diminishes, it can be observed from the Table 37 that the used value of K is efficient and even after optimisation the resultant error in the M.S.E value is negligible.

The below Table 37 provides a summarised picture of the calculated parameters for all the three waves experienced by Belgium and the respective models used to study those registered cases.

Wave #	Model	Parameters				
		a	r	K	SSE	MSE
Wave 1	Exponential	-9.33498	0.15147	NA	5.125×10^{9}	3.9×10^{7}
	Logistic (Initial K)	-1.799763726	0.018688	0.005464	0.0006476	4.94×10^{-6}
	Logistic (optimal K)	-6.20295318	0.0184	0.0052	2.235×10^{-5}	1.70655×10^{-7}
Wave 2	Exponential	-8.94981	0.07643	NA	2.609×10^{22}	5.903×10^{19}
	Logistic (Initial K)	-3.95921	0.01866	0.10102	0.055687	0.000125989
	Logistic (optimal K)	-3.97659	0.01887	0.100519	0.05475	0.00012387
Wave 3	Exponential	-7.6222	0.13242	NA	6.049×10^{17}	1.723×10^{15}
	Logistic (Initial K)	-3.4789	0.0255	0.2814	3.38897	0.17968
	Logistic (optimal K)	-3.6461	0.0288	0.269	3.24455	0.10953

Table 37: Exponential Model and Logistic Model Parameters for Belgium

Isolating First two waves:

For our further analysis on some complex models to be precise and simple we have isolated the data corresponding to the first two waves. We have explained this through the following plot Figure 64. The regression coefficient r obtained was 0.15147 when considering the separate entities and the obtained value for the same after considering it to be single entity is 0.16186 and the change in this value is a s a result of combining two waves together and observing the linear growth.

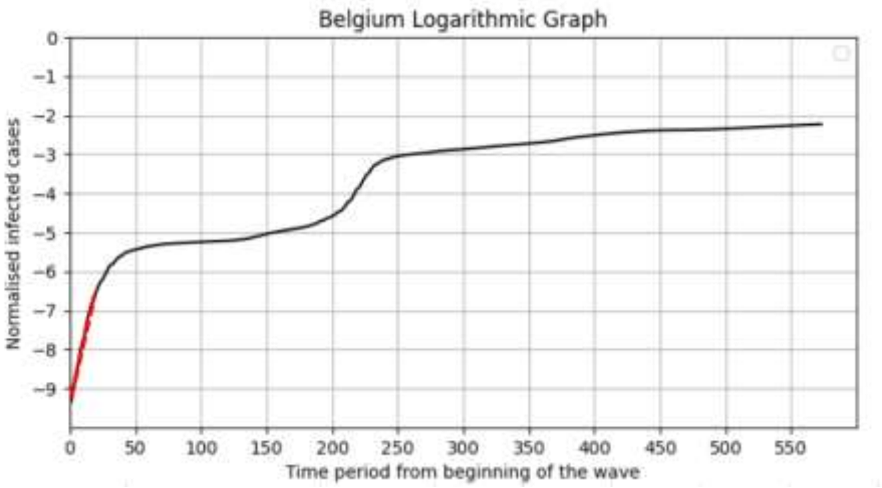

Figure 64: Belgium's logarithmic graph for first two waves

Evaluating Susceptible S and exponential parameter r:

The value of Susceptible component S of the SIR Model specified earlier, is calculated once for the beginning of the cases and last available date each. Here the exponential parameter r is derived using the equation $\ln p(t) = c + rt$ and shown in the below Table 38.

Exponential Parameters		Start period(day)			End period(day)			r	c
		ln value	$P(t)$	S	ln value	$P(t)$	S		
Belgium	Wave 1 slope interval	-9.334	0.00000883	0.99991164	-6.8025	0.001038	0.99896190	0.16186	-9.40093

Table 38: Estimation of Susceptible S and exponential parameters for Belgium

Defining SIR model parameters:

The SIR model parameters are populated in the Table 39 below, by employing the exponential parameter r specified in the above Table 38 in the empirical $a = r + b$.

Interval	r	c	b	a	SSE	MSE
1-25	0.132008	-9.18208	0.1	0.232008	2.289×10^{58}	3.99599×10^{55}

Table 39: Estimation of SIR coefficients for Belgium

Defining SIR Model initial values:

The assumed value $P(0)$ for the cumulative data is populated in the below Table 40, and it is used to calculate the values of S, I and R using the initial P value as explained from the below equations.

$$S(0) = 1 - P(0)$$

$$I(0) = P(0) - R(0) = P(0) - P(-10)$$

$$R(0) = P(-10)$$

P (0)	S (0)	I (0)	R (0)
0.00046835	0.99953165	0.000379995	0.000088354

Table 40: Initial values of SIR model for Belgium

Defining multiple initial values of SIR:

After considering various initial values for $I(0)$ as stated in the below Table 41, the initial group of values $S(0), I(0), R(0) and\ P(0)$ are calculated basing the equations elaborated above. The remaining values for $I(0)$ and $R(0)$ are taken at random by doubling the previous value on contrary the above equations are used to calculate $S(0) and\ R(0)$

Initial Values # of I (0)	I (0)	R (0)	S (0)	P (0)
$I(0)_1$	3.80×10^{-4}	8.84×10^{-5}	9.9953165×10^{-1}	4.68×10^{-4}
$I(0)_2$	7.60×10^{-4}	1.77×10^{-4}	9.990633×10^{-1}	9.37×10^{-4}
$I(0)_3$	1.52×10^{-3}	3.53×10^{-4}	9.981266×10^{-1}	1.87×10^{-3}
$I(0)_4$	1.82×10^{-3}	4.42×10^{-4}	9.97751×10^{-1}	2.25×10^{-3}

Table 41: Divergent initial values of SIR model for Belgium

ODE System Integration:

Now, we have integrated the ODE system of equations $I(t), R(t) and\ P(t)$ and the results are plotted against the respective number of the day in the time series considered for integration in the below Figure 65.

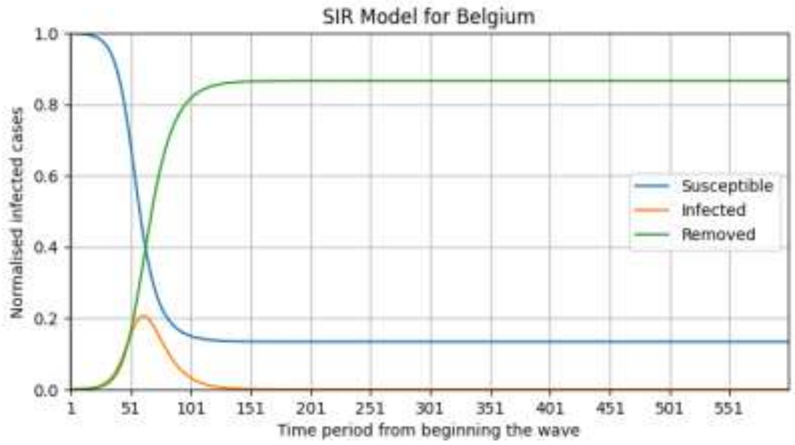

Figure 65:SIR Model for Belgium.

Here by using the obtained results after integration the fraction of infected population $P(t)$ can be estimated as $1 - S(t)$, the below Figure 66 shows the comparison of $I(t)$ and $R(t)$with the estimated fraction of infected population $P(t)$

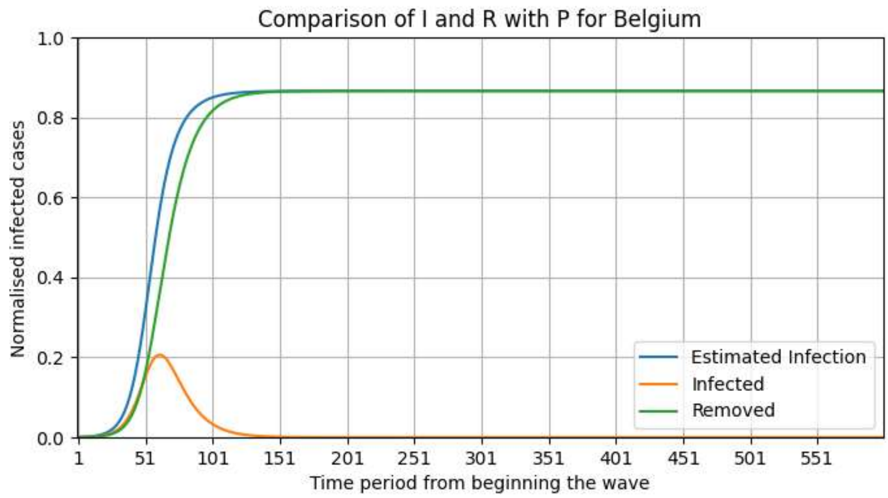

Figure 66: Comparison of I and R with P for Belgium.

Determining optimal initial value:

The below Table 42 shows us the values calculated for $I(t), R(t), S(t)$ and the error associated between the observed normalised cases and estimated cases $P(t)$, the error has been calculated as $P - P(t)$ and the resulting Mean Squared Error is represented in the below Table 42 for respective iterations.

Initial Values # of $I(0)$	$I(0)$	$R(0)$	$S(0)$	MSE
$I(0)_1$	$3.79995\text{x}10^{-4}$	$8.8354\text{x}10^{-5}$	0.9995316	8.9152×10^{-7}
$I(0)_2$	7.59990×10^{-4}	1.7671×10^{-3}	0.9990632	$2.48085\text{x}10^{-6}$
$I(0)_3$	$1.51998\text{x}10^{-3}$	$1.51998\text{x}10^{-3}$	$1.51998\text{x}10^{-3}$	$1.51998\text{x}10^{-3}$
$I(0)_4$	$1.82397\text{x}10^{-3}$	$4.24103\text{x}10^{-4}$	0.9977519	$6.67703\text{x}10^{-5}$

Table 42: MSE of divergent initial values of SIR model for Belgium

The below Figure 67 shows the exponential curves obtained from different $I(0)$ values with the observed normalised cases corresponding to distinct $I(0)$ values and from the above Table 42 it can be concluded that the error for $I(0)_1$ =$3.7995X10^{-4}$ is less and the curve is closer to the normal cases and can be observed in the below Figure 67, The error is 8.9152×10^{-7}.

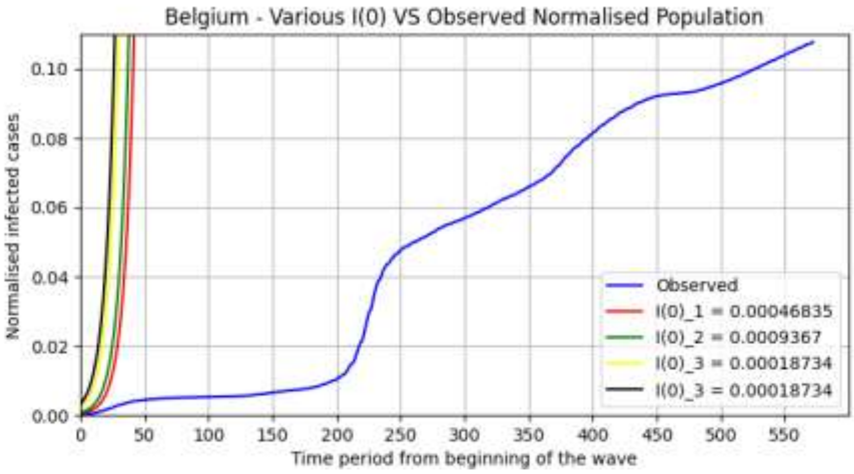

Figure 67: Comparison of divergent I(0) with observed population for Belgium.

Estimating optimal SIR coefficients a and b:

The initial SIR coefficients a and b along with the associated M.S.E is shown in Table 43. Now, to find a better or optimal alternatives for these coefficients the M.S.E values shows us the way, in our case the optimal coefficients resulting in least error are found to be $a = 0.18$ and $b = 0.14$ as presented in the below Table 43.

Values	a	b	SSE	MSE
Initial	0.232008	0.1	333.048	0.581237
Optimal	0.18	0.14	47.6668	0.083188

Table 43: Initial and Optimal SIR coefficients for Belgium

The below Figure 68 illustrates the optimal SIR model for Belgium after completing the calculations with the newly obtained optimal parameters a and b also the optimal comparison of I and R with $P(t)$.

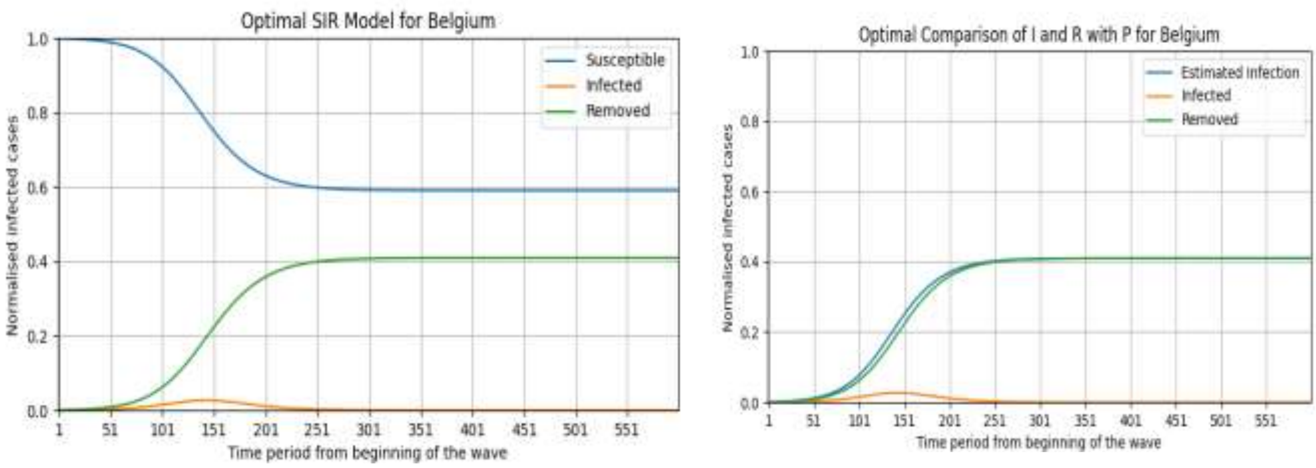

Figure 68:Optimal SIR model(left) and comparison of optimal I and R with P(right) for Belgium.

Under the analysis of SIR model, other external factors such as Socio-Psychological parameters of the susceptible people in the society could be included in the model. This new technique of inclusion leaves us with a new model and opportunity to study the cases in more depth and this new model is termed as Extended-SIR model, Further breakdown in this model can be done by taking the crowd effect into consideration. The respective coefficients involved in those models are studies thoroughly going forward in the study.

Extended-SIR Model (Socio-Psychological Inclusions):

The parameters involved in the Extended SIR model are k_1, k_2, k_3,a and b obtained from the equations under the holding conditions of this model. When Crowd effect is concerned in the model then the coefficients involved are q, k_2, k_3 , a and b. The below Table 44 and Table 45 represents the same.

Slope Interval (Days)	a	b	k_1	k_2	k_3
1-25	0.232008	0.1	1	0.02	0.01

Table 44: Extended SIR Model Parameters without considering Crowd Effect – Belgium

Slope Interval (Days)	a	b	q	k_2	k_3
1-25	0.232008	0.1	50	0.02	0.01

Table 45: Extended SIR Model Parameters considering Crowd Effect – Belgium

Considering the equations specified in the *Methods* section of the initial description, we have calculated the equations based on the assumed initial values of S_{ign}.S_{res}, S_{exh}. I and R which are as populated in the below Table 46.

$S_{ign}(0)$	$S_{res}(0)$	$S_{exh}(0)$	$I(0)$	$R(0)$	$\frac{dS_{ign}(0)}{dt}$	$\frac{dS_{res}(0)}{dt}$	$\frac{dS_{exh}(0)}{dt}$	$\frac{dI(0)}{dt}$	$\frac{dR(0)}{dt}$
0.99953165	0	0	0.0003799	0.00008835	-0.00000004121	0.000379817	0	-0.000050121	0.00003799

Table 46: Extended SIR Model Initial Values without considering Crowd Effect – Belgium

Now the below Table 47 explains the same results as above but, after considering crowd effect in the model.

$S_{ign}(0)$	$S_{res}(0)$	$S_{exh}(0)$	$I(0)$	I_P	$R(0)$	$\frac{dS_{ign}(0)}{dt}$	$\frac{dS_{res}(0)}{dt}$	$\frac{dS_{exh}(0)}{dt}$	$\frac{dI(0)}{dt}$	$\frac{dR(0)}{dt}$
0.99953165	0	0	0.0003799	0.02	0.00008835	-0.01907	0.018986	0	-0.000050121	0.000002437

Table 47: Extended SIR Model Initial Values considering Crowd Effect – Belgium

O.D.E System Integration for extended SIR:

In the above Table 47 and the Table 46 we have presented the evaluated O.D.E system of equations for Extended SIR model with and without considering Crowd Effect respectively. Various Susceptible phases $S(t)$ and the extended SIR model and the same model after considering crowd effect for the country Belgium is shown in Figure 69 and Figure 70.

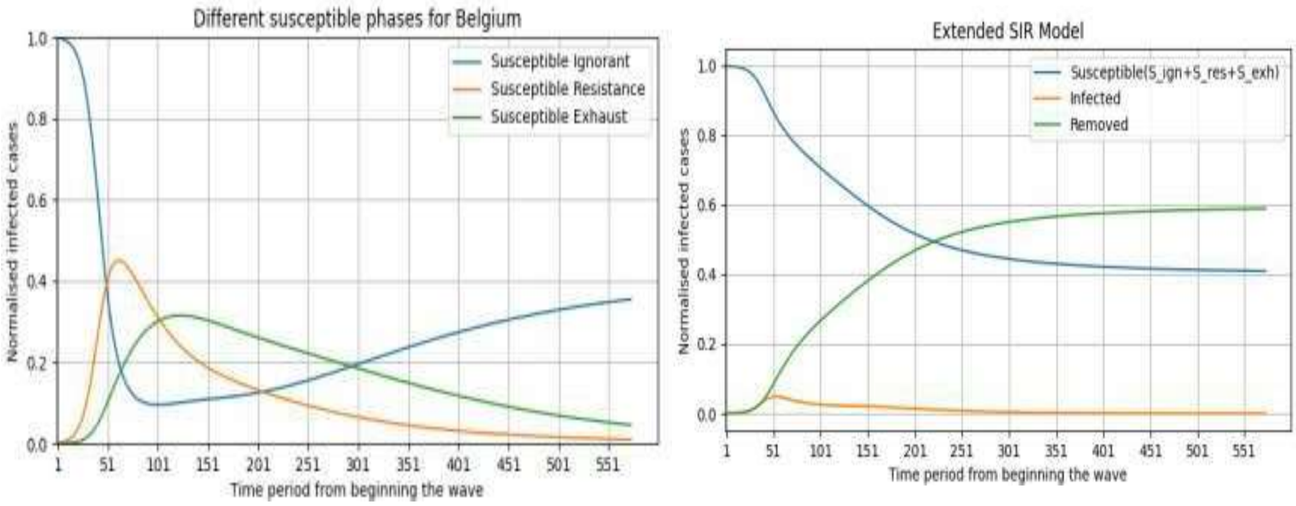

Figure 69:Different Susceptible Phases and Extended SIR Model without considering Crowd Effect -Belgium

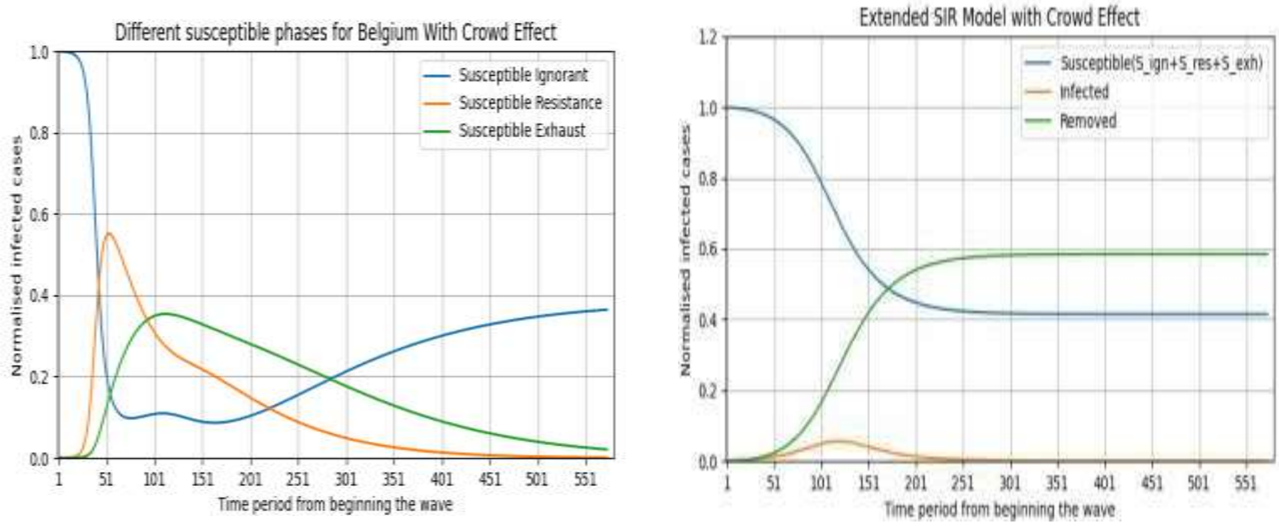

Figure 70:Different Susceptible Phases and Extended SIR Model considering Crowd Effect -Belgium

Now we have the susceptible component $S(t)$ obtained from O.D.E integration which is $(S_{ign} + S_{res} + S_{exh})$ and from this we have estimated the total fraction of infected population $P(t)$ which is $1 - S(t)$. Having calculated this, we have compared the $I(t)$ and $R(t)$ with respect to $P(t)$ with and without taking crowd effect into account. The same is conveyed from the below Figure 71.

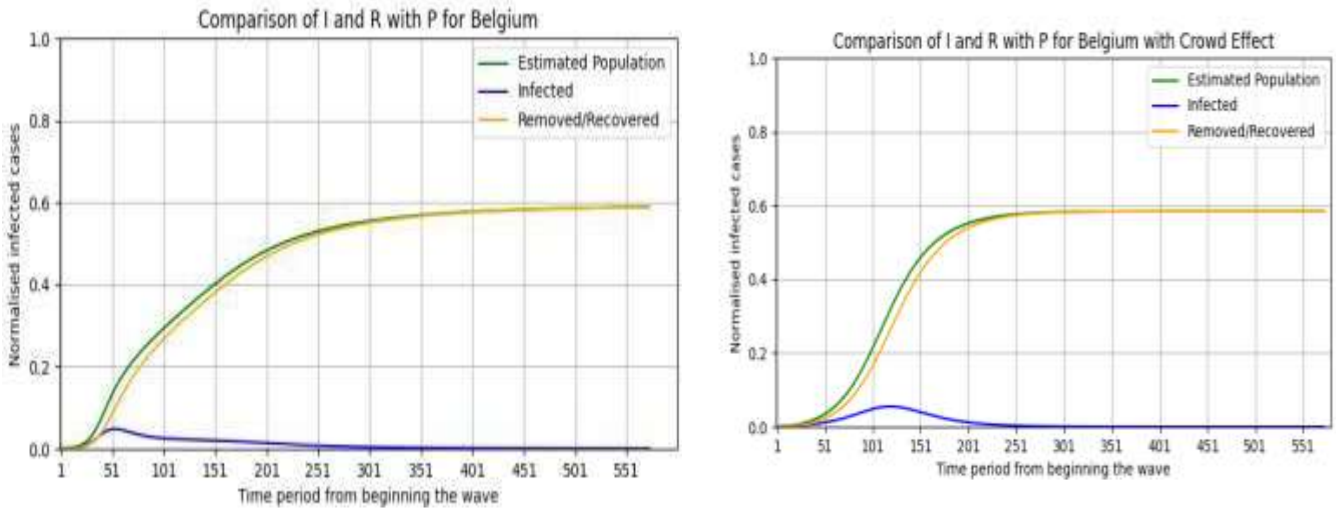

Figure 71: Comparison of I and R with P without and with considering crowd effect- Belgium

Comparing ODE integrated values to the original observed values:

The M.S.E has been calculated for extended SIR model without (Table 48) and with (Table 49) considering Crowd Effect.

$I(0)$	$R(0)$	$S_{ign}(0)$	$S_{res}(0)$	$S_{exh}(0)$	M.S.E
0.0003799	0.00008835	0.99953165	0	0	0.18934

Table 48: Calculation of M.S.E for initial data O.D.E without considering Crowd Effect- Belgium

$S_{ign}(0)$	$S_{res}(0)$	$S_{exh}(0)$	$I(0)$	$R(0)$	I_p	M.S.E
0.99953165	0	0	0.0003799	0.00008835	0.02	0.202309

Table 49: Calculation of M.S.E for initial data O.D.E considering Crowd Effect- Belgium

The M.S.E here has been calculated from the difference observed between the actual observed normalised cases and estimated cases $P(t)$, which can be calculated from the susceptible parameter $S(t)$ taking $P(t) = 1 - S(t)$ where $S(t)$ is the sum of each component of overall susceptible cases($S(t) = S_{ign}(t) + S_{res}(t) + S_{exh}(t)$).The below Figure 72 gives us a comparison of observed population and estimated population in both the cases before and after considering crowd effect.

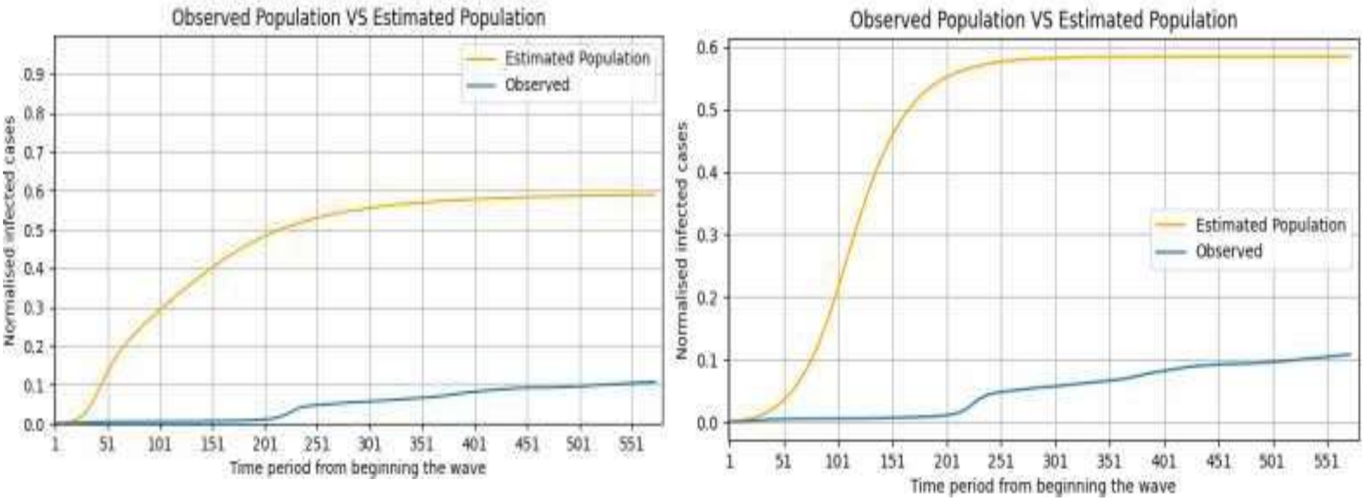

Figure 72: Observed VS Estimated Population without and with considering Crowd Effect -Belgium.

Estimating optimal extended SIR coefficients:

In-order to arrive at an optimal conclusion regarding the extended SIR coefficients with and without considering crowd effect the respective errors associated play a vital role, for our study we have considered M.S.E as a representation of the error. Table 50 and Table 51 shows the respective data considered for optimisation in both the cases of SIR models. The trail data leading to minimal M.S.E had the optimal coefficients.

a	b	k_1	k_2	k_3	M.S.E	
0.232008	0.1	1	0.02	0.01	0.18934	Initial Value
0.24	0.18	0.06	0.06	0.002	0.1099	
0.18	0.14	0.01	0.04	0.005	0.08087	
0.2	0.16	0.05	0.1	0.04	0.06349	**Optimal Value**

Table 50: Extended SIR Model Optimal Coefficients Estimation for Belgium without crowd effect

a	b	q	k_2	k_3	M.S.E	
0.232008	0.1	50	0.02	0.01	0.202309	Initial Value
0.225	0.008	65	0.04	0.005	0.790568	
0.18	0.005	45	0.01	0.02	0.626326	
0.16	0.08	75	0.05	0.04	0.155402	**Optimal Value**
0.25	0.15	80	0.06	0.005	0.214002	

0.30	0.01	25	0.02	0.08	0.809368

Table 51: Extended SIR Model Optimal Coefficients Estimation for Belgium with crowd effect

It can be clearly observed from the above tables, Table 50 and Table 51, that without considering crowd effect, the optimal parameters are $a = 0.2, b = 0.16, k_1 = 0.05, k_2 = 0.1$ and $k_3 = 0.04$, whereas for the model considering the crowd effect the optimal parameters are $a = 0.16, b = 0.08, q = 75, k_2 = 0.05$ and $k_3 = 0.04$.

As, we have arrived at our required optimal systems, from the iterations mentioned in the above tables, Table 50 and Table 51, we have now plotted the optimal extended SIR model and the optimal comparison of $I(t), R(t)$ values with the estimated infected population $P(t)$ and without and considering Crowd Effect as shown in Figure 73 and Figure 74.

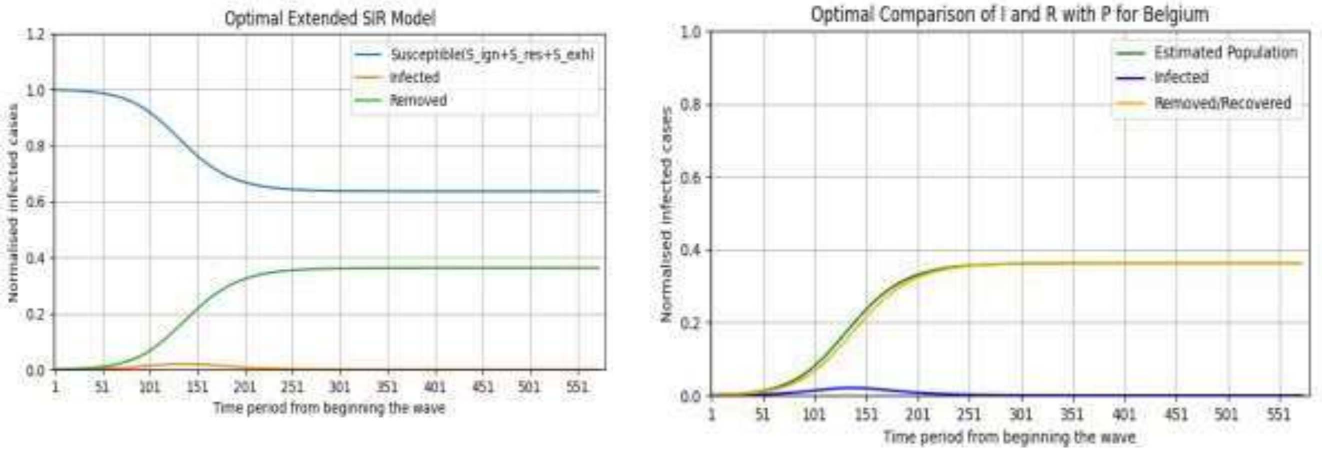

Figure 73: The optimal SIR Model and Comparison of I and R with P without considering Crowd Effect -Belgium

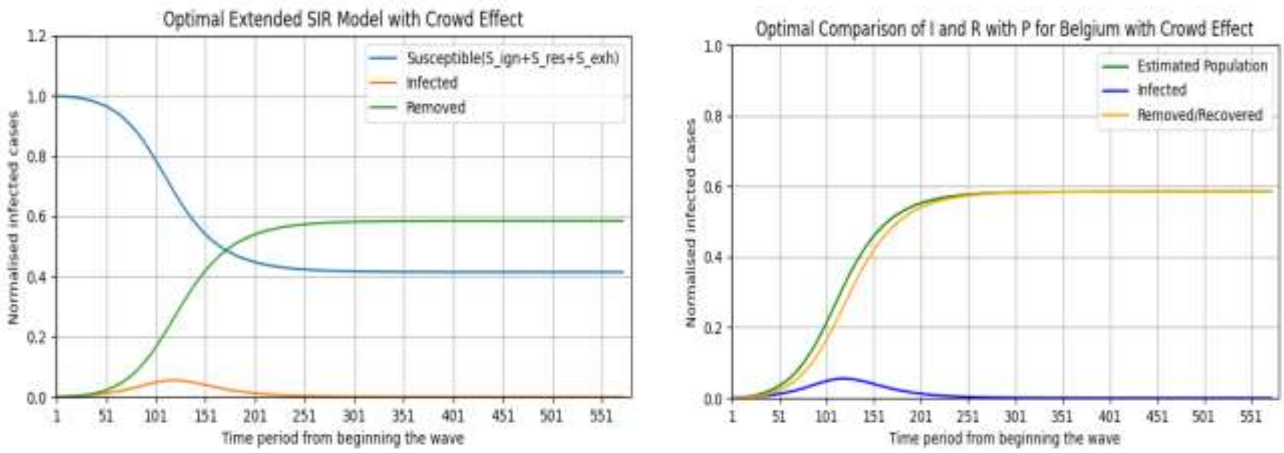

Figure 74: The optimal SIR Model and Comparison of I and R with P considering Crowd Effect -Belgium.

Distinct Trials with extended SIR coefficients excluding crowd effect:

Here in this section of analysis we have populated the various possible random values for the extended SIR coefficients as you can see in Table 52 and the respective Extended SIR plots corresponding to each iteration is shown below in the below Figure 75.

Set of Trials	a	b	k_1	k_2	k_3
Trial 1	0.116004	0.05	0.5	0.01	0.005
Trial 2	0.1005	0.045	0.42	0.008	0.004
Trial 3	0.098	0.04	0.38	0.006	0.003

Table 52: Various Trails for Coefficients of Extended SIR Model - Belgium

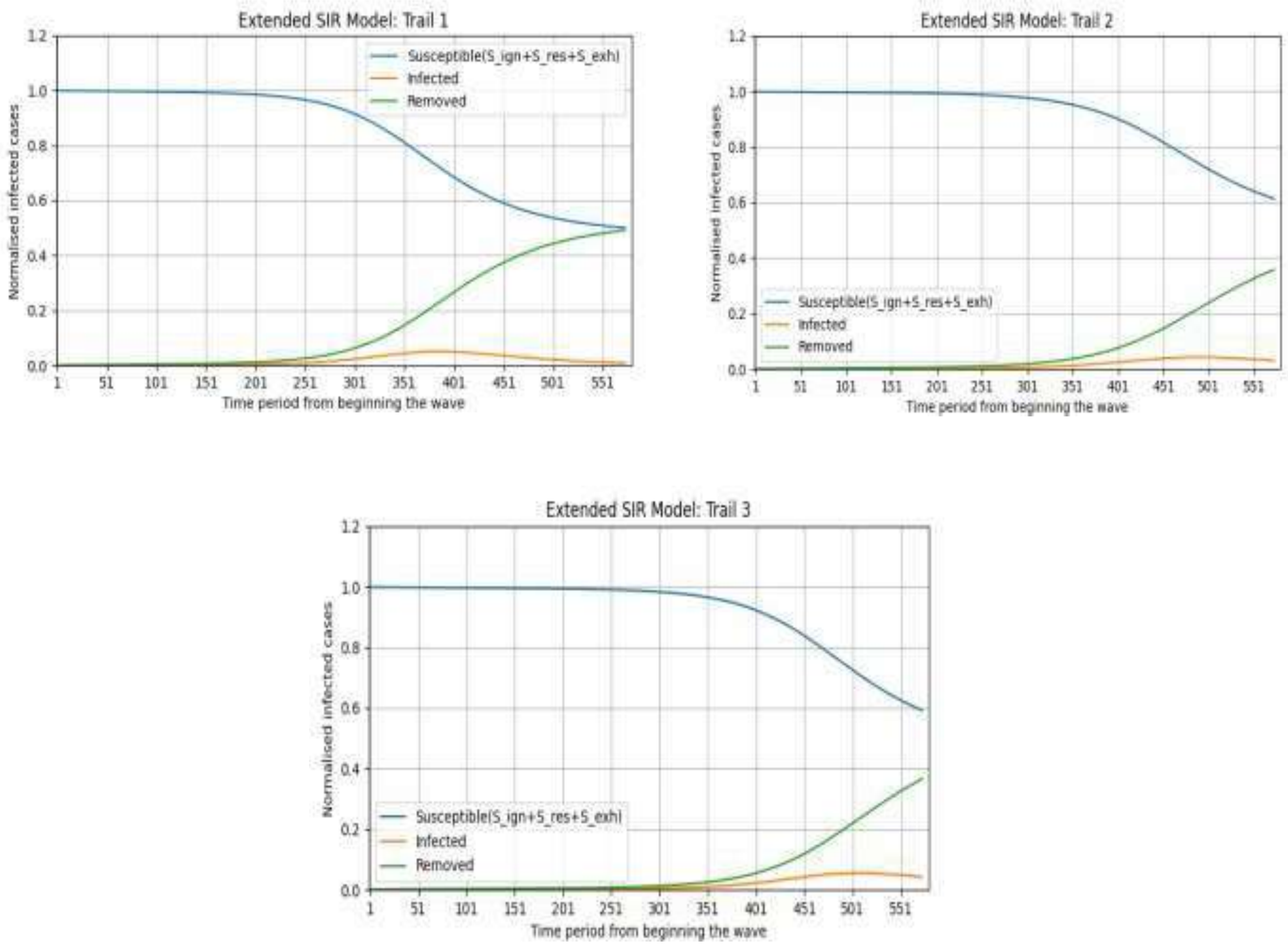

Figure 75: Extended SIR Model for respective trails -Belgium.

In the below Table 53 the M.S.E corresponding to each trail has been populated and the error has been calculated using $S(t)$ values obtained after the integration of O.D.Es with the respective coefficients in the trail. Finally, the population $P(t)$ has been calculated using the empirical $P(t) = 1 - S(t)$.The observed error is difference between the observed population fraction and estimated fraction of infected population.

Set of Trails	a	b	k_1	k_2	k_3	M.S.E
Trail 1	0.116004	0.05	0.5	0.01	0.005	0.039431
Trail 2	0.1005	0.045	0.42	0.008	0.004	0.009587
Trail 3	0.098	0.04	0.38	0.006	0.003	0.009793

Table 53: Different Trails for calculating M.S.E by varying all the coefficients- Belgium

After the above analysis, a plot Figure 76 has been generated comparing the observed infected population and the estimated infected population $P(t)$, for each trail as discussed earlier.

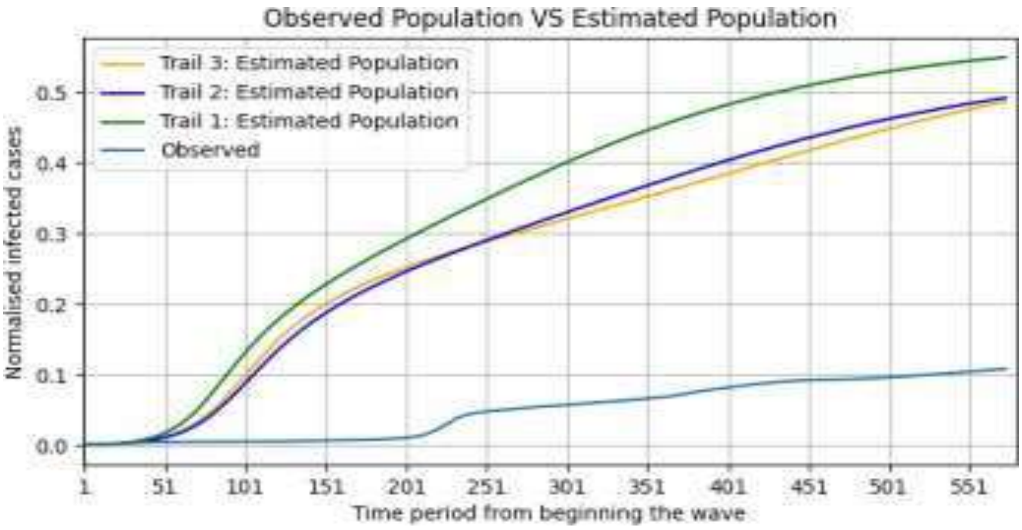

Figure 76: Observed Population VS Estimated Population for different Trails -Belgium.

It is very clear from the above Table 53 that the parameters corresponding to trail 3 are ideal as the error associated with that set is minimum and the Figure 76 seconds this hypothesis as the estimated population line corresponding to trail 3 is passing closer to the originally observed infected population.

Distinct Trials with I_p for crowd effected extended SIR model:

In this section of analysis, we have considered various values for I_P, which is fraction of population considered to be infected. Later the M.S.E for different iterations of I_p is calculated, after performing the integration on O.D.Es and presented in Table 54. M.S.E is calculated from the error identified between the observed normalised cases and the estimated fraction of infected population

$S_{ign}(0)$	$S_{res}(0)$	$S_{exh}(0)$	$I(0)$	$R(0)$	I_p	M.S.E	
0.99953165	0	0	0.0003799	0.00008835	0.025	0.185498	
0.99953165	0	0	0.0003799	0.00008835	0.02	0.202309	Initial Value
0.99953165	0	0	0.0003799	0.00008835	0.015	0.257352	
0.99953165	0	0	0.0003799	0.00008835	0.010	0.388271	

Table 54:Calculation of M.S.E for distinct Ip after considering Crowd Effect- Belgium

In the below Figure 77 various $I_p$values and the estimated population calculated for every I_p value and the registered observed normalised cases.

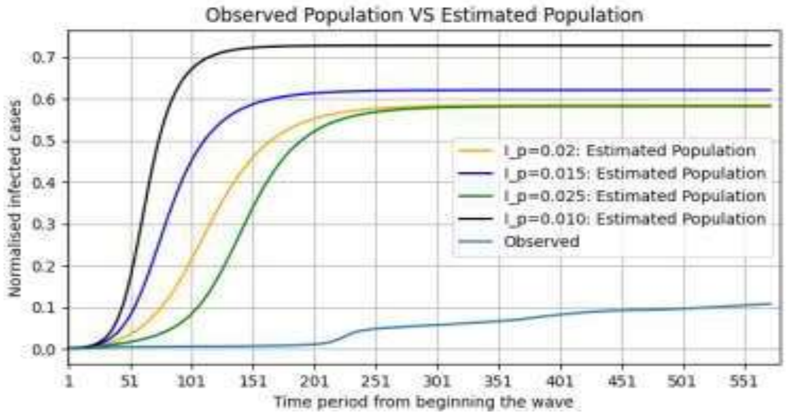

Figure 77: Observed VS Estimated Population for different IP trails after Considering Crowd Effect-Belgium.

The above Table 54 clearly states that the parameters corresponding to I_p value 0.025 are ideal as the associated error for those values of parameters is minimum and the Figure 77 confirms this notion as the estimated population line corresponding to I_p value 0.025 lies closer to the initially observed population fraction curve.

SIR model and Extended SIR model comparison:

Similarities	Differences
➢ In both situations, the Infected I and Removed (alive or dead) R stages are handled equally. ➢ The initial values for the SIR components' coefficients a and b, as well as $I(0)$, $R(0)$, and $S_{ign}(0)$, are the same.	➢ Susceptible S is subpopulated into three distinct phases in the extended SIR model. Susceptible Exhaustion S_{exh}, Susceptible Ignorance S_{ign}, and Susceptible Resistance S_{res}. ➢ Due to the addition of a new coefficient q to the kinetic equation of the extended SIR model, the mean square errors of the SIR and extended SIR models differ. ➢ When the estimated population of the infected fraction is compared to the observed normalised population, it is seen that the curves generated for SIR model and Extended SIR model differ substantially.

The distinction between the Extended SIR Model with and without Crowd Effect:

The accuracy of any model can be significantly improved by including other dependent factors. In this chapter, to demonstrate improved accuracy, we extended the SIR model by incorporating the crowd effect. To examine the predictive accuracy of this extended SIR model we determined Mean Square Error(Table 55) of the model with and without consideration of crowd effect.

	Argentina	Belgium	Australia
Optimal MSE	0.00017503	0.063490	0.00910375
Optimal MSE with Crowd Effect	0.00300326	0.155402	0.00000297

Table 55: Error comparison for Argentina, Belgium, and Australia with and without crowd effect

It's evident from Table 55 the ideal MSE with crowd effect is greater in Argentina and Belgium than the optimal MSE without crowd effect. This might be due to the reaction rate or population fraction I_p

used to calculate the kinetic equations. However, in the instance of Australia, the optimal MSE with crowd effect is smaller than the latter.

There is little variance in the SIR model and comparison of I and R with P plots with and without Crowd Effect. Considering the additional dependent parameters Diffusion Process, Population Density, Age Parameter, and Immunisation may improve the model's accuracy to a greater extent.

FURTHER IMPROVEMENTS

We may increase the accuracy of the SIR Model and their modifications by taking the following parameters into account while discussing the next hike in COVID cases:

- When differentially integrating the SIR system of transitions, the diffusion process should be considered. “Individuals disperse by a diffusion process, with diffusion constant D” [9].
- Incorporating an age parameter into the model will improve the model's accuracy. “Susceptibility to infection in individuals under 20 years of age is approximately half that of adults aged over 20 years [10]”.
- For the first wave, we assumed a stable population. When population birth, mortality, and immigration are considered, observed normalised occurrences will be accurate.[11]
- With a lockdown in place and everyone immunised, an infected person's recovery period $R(0)$ can be reduced to one week.
- Vaccinations have increased throughout time. When comparing the second wave to the first, the number of vulnerable to infection drops.
- When lockout was applied, the rate of infected dissemination reduced, resulting in fewer S→I system transitions.

CONCLUSION

The study of COVID modelling here involved a high concentration on Argentina, Belgium, and Australia. The proportion of cases reported per population in Argentina is lower, however the cumulative occurrences for the same are deceptive. When it comes to the number of waves, Argentina had a couple, as a rare stagnation can be seen, but Australia had three waves, the first of which was very sluggish, and Belgium had three waves that were faster than the other two. The statistics show that all the waves are asynchronous, owing to the country's economic reliance on tourism [12-13], as well as other variables such as the enforcement of restrictions and the availability of health facilities. The population size of each nation was first used to calculate the percentage of infected cases, from which the population growth models for each observed wave in each of the country were generated. The population growth models include the Exponential growth model and Logistic growth model. Figure 78 below represents the exponential and logistic population growth for one of the analysed country's observed waves (Argentina) during the COVID outbreak.

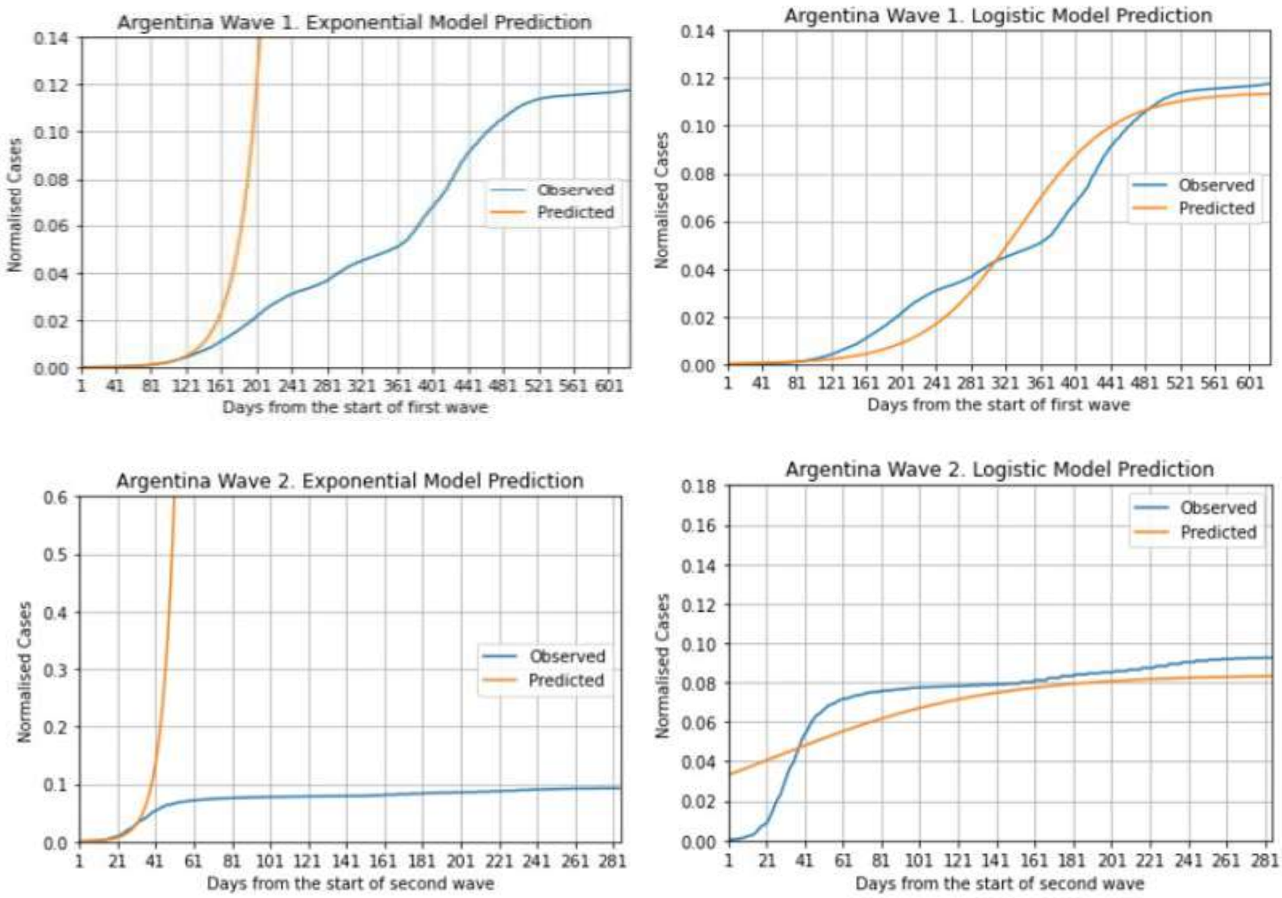

Figure 78: Exponential and logistical population growth for wave 1 and wave 2 for one of the analysed country

To swiftly dive and used to analyse the impact that the epidemic had on these 3 nations, a proposed model broadly termed as the SIR model was introduced consisting of three phases of elements: (S)usceptible - population that is vulnerable to infection (I)nfected - population that has the disease and can transmit it (R)emoved - population that has either recovered or died. For reference, the below Figure 79 displays the SIR model and the optimal SIR model for an analysed country (Argentina).

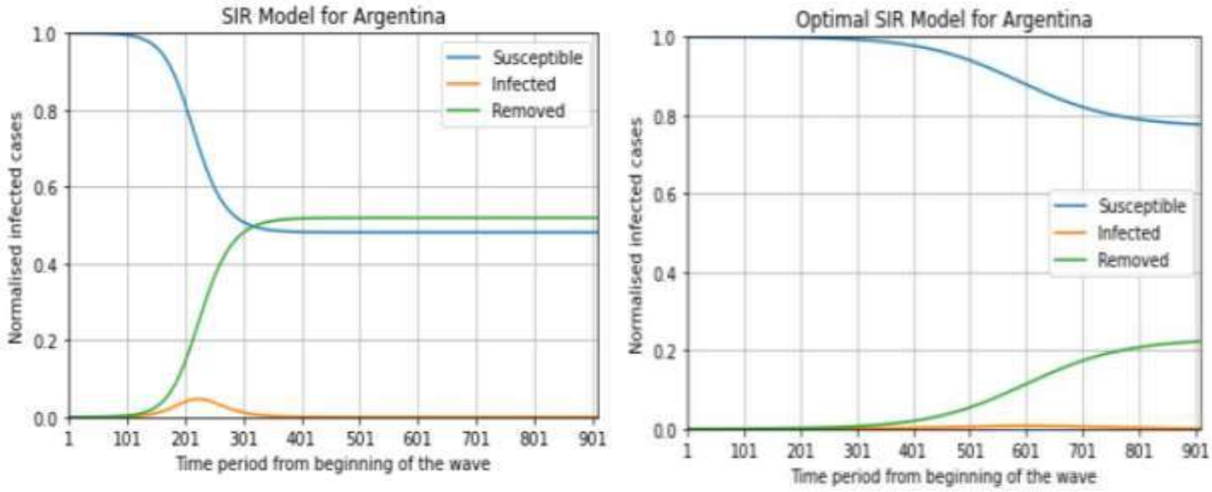

Figure 79: SIR model(left) and Optimal SIR model(right) for the analysed country.

We were able to perform and estimate for each country the rate and integrated value for each component's derived ODE using the SIR model, as well as assess its normalised infected population estimation.

Following the addition of human social psychology factors, the SIR model's components were increased from three to five, with the susceptible population being divided into three phases or subpopulations based on human behavior: $S_{ign}, S_{exh},$ and S_{res} (Figure 80 illustrates the different phases of S). The extended SIR model is calculated and integrated using the same methodology as the SIR model. Refer Figure 81 for the extended SIR model and its optimal plot for an analysed country (Argentina).

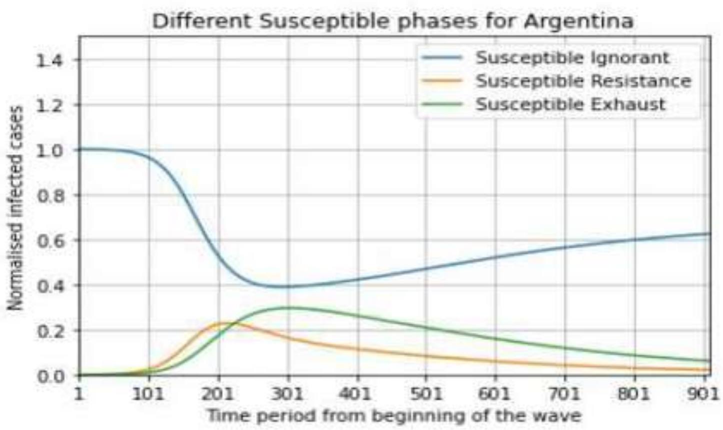

Figure 80: Different Susceptible Phases for Analysed country

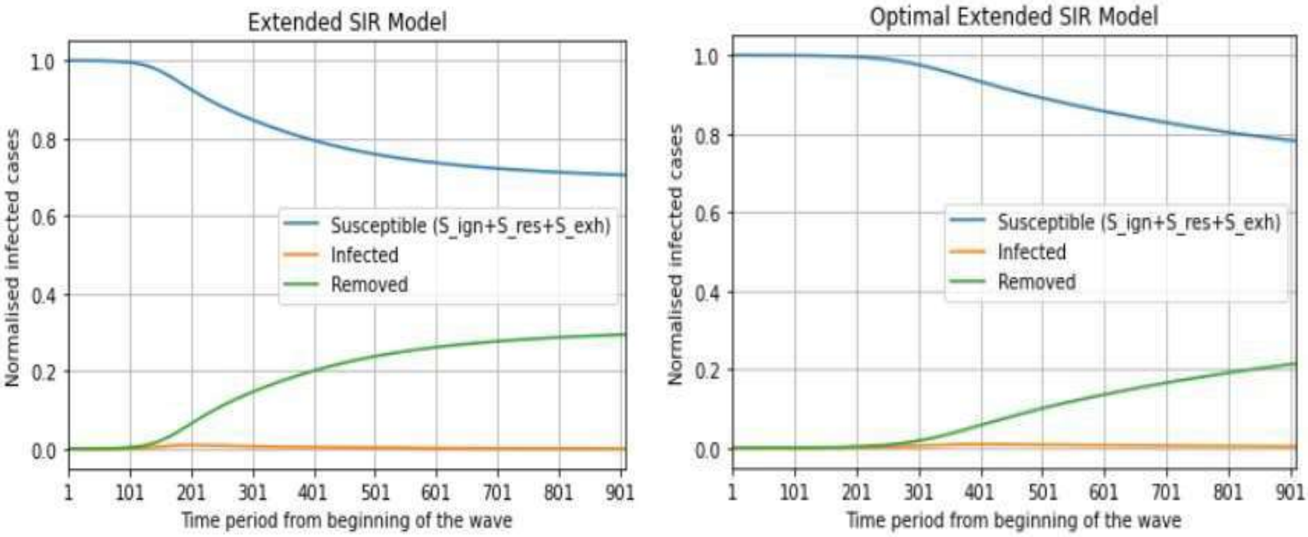

Figure 81: Extended SIR model and Optimal SIR model for analysed country

A comparison of the models used for predicting the COVID scare in the three nations that are being highlighted—Argentina, Belgium, and Australia—is provided in the Table 56 below.

Model	Coefficients considered	Argentina M.S.E	Belgium M.S.E	Australia M.S.E
Exponential	a, r	3.04×10^{13}	3.9×10^{7}	1.69
Logistic (Optimal)	a, r, K	0.000084777	1.70655×10^{-7}	0.0000000006
SIR (Optimal)	a, b	1.01173×10^{-9}	8.9152×10^{-7}	1.24×10^{-8}
Extended SIR	a, b, k_1, k_2, k_3	1.021542×10^{-2}	0.18934	2.20×10^{-2}

Table 56: Comparison of models considered for modelling of COVID in Argentina, Belgium, and Australia

REFERENCES

Anon., 2020. *Worldometer.* [Online]
Available at: https://www.worldometers.info/coronavirus/
[Accessed 23 November 2022].

Anon., September 2020. Journal of Business Research. *Tourism and COVID-19: Impacts and implications for advancing and resetting industry and research,* Volume Volume 117, pp. Pages 312-321.

Barua, S., 3 April 2020. *Understanding Coronanomics: The Economic Implications of the Coronavirus (COVID-19) Pandemic.* [Online]
Available at: https://papers.ssrn.com/sol3/papers.cfm?abstract_id=3566477

D.Okuonghaea, A., 2020. *Analysis of a mathematical model for COVID-19 population dynamics in Lagos, Nigeria,* 139(110032).

Davies, N. G., 2020. *Nature.* [Online]
Available at: https://www.nature.com/articles/s41591-020-0962-9
[Accessed 11 November 2022].

Grassly, N. F. C., 2008. Mathematical models of infectious disease transmission. *Nat Rev Microbiol,* Volume 6, p. 477–487.

Hadeel AlQadi, M. B.-Y., 2022. *journals plos org.* [Online]
Available at: https://journals.plos.org/plosone/article?id=10.1371/journal.pone.0265815
[Accessed 23 November 2022].

K. Dietz, D. S., 1985. Mathematical Models for Infectious Disease Statistics. *A Celebration of Statistics,* Issue ISBN : 978-1-4613-8562-2.

Medlock, j., 22 & 24 May 2022. *Mathematical Modeling of Epidemics,* pp. 15-18.

OusmaneKoutou, A. B. D. B. S., March 2022. Mathematics and Computers in Simulation. *Mathematical analysis of the impact of the media coverage in mitigating the outbreak of COVID-19,* Volume Volume 205, pp. Pages 600-618.

Shabir Ahmad, A. U., October 2020. Fractional order mathematical modeling of COVID-19 transmission. *Chaos, Solitons & Fractals,* 139(110256).

Titus Rotich, R. L., August 2020. Mathematical Modeling of Covid-19 Disease Dynamics and Analysis of Intervention Strategies. *Mathematical Modelling and Applications,* Volume 5(3), pp. 176-182.

Vaishali Deshwal, V. K., 2021. Study of Coronavirus Disease (COVID-19) Outbreak in India. Volume 15, pp. 62-73.

Weisstein, E. W., 2019. *"Kermack-McKendrick Model." From MathWorld--A Wolfram Web Resource.* [Online]
Available at: https://mathworld.wolfram.com/Kermack-McKendrickModel.html
[Accessed 10 December 2022].