Agenda :-

- Addition and Multiplication Rule
- Permutation basics
- Combination basics and properties
- Pascal Triangle
- Find N-th column title

Given 10 girls and 7 boys, How many different pairs can be formed?

Note: pair = 1 boy + 1 girl

Boys

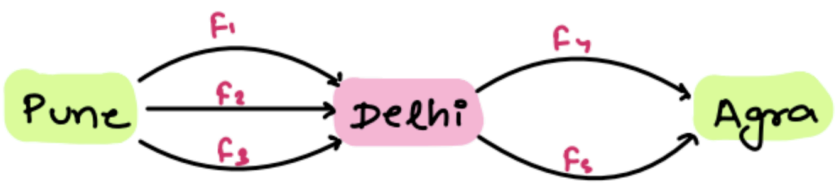
B_1
B_2
B_3
⋮
B_7

Girls

G_1
G_2
G_3
⋮
G_{10}

=> 70 pairs.

Ques

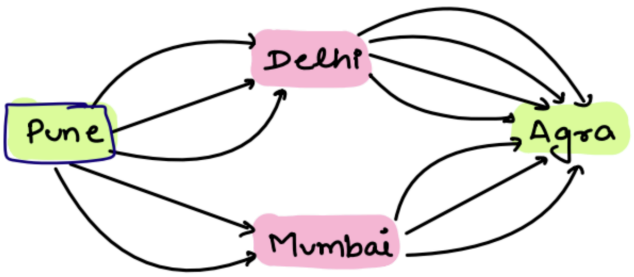


No. of ways Pune to Agra?.

no. of ways to reach from Pune to Delhi * no. of ways to reach from Delhi to Agra.

3 * 2 = 6.

Ques 7.



Pune to Agra.

Pune to Agra via delhi or Pune to Agra via Mumbai

12 + 6 = 18 ways,

And → *

OR → +

-: **Permutation** :-

↳ arrangement of objects.

↳ here order matters.

$(i,j) \; != \; (j,i)$

RB, BR

RBG, BGR, GRB...

Ques

Given 3 distinct characters. In how many ways, we can arrange them?

s = "abc"

→ 3!.

3 2 1 = 6 arrangements.

a → b → c
a → c → b

b → c → a
b → a → c

c → a → b
c → b → a

Ques) s = 'abcd'

4 * 3 * 2 * 1 ⇒ 24 ⇒ 4!.

In how many ways n distinct characters can be arranged?

$n * (n-1) * (n-2) * (n-3) \ldots$

$\Rightarrow n!$

Ques how many ways you can arrange 2 out of 4 characters?

${}^4P_2 \Rightarrow \frac{4!}{2!}$ a b c d $\rightarrow$ 4 3 $\Rightarrow 12$

a b
b a
a c
c a
a d
d a ...

Ques) 3 distinct characters, arrange 2 characters out of this, (a, b, c).

${}^3P_2 = \frac{3!}{1!}$ $3 * 2 =$ 6 arrangements

Ques) Given 5 distinct characters in how many ways we can arrange them in 2 places.

$^5P_2 \Rightarrow \frac{5!}{3!}$ $\quad \underline{5} * \underline{4} = 20$ arrangements.

Ques) N distinct characters, we need to arrange 3 characters.

$\underline{N} \quad \underline{(N-1)} \quad \underline{(N-2)} \Rightarrow N * (N-1) * (N-2)$.

Ques) N distinct characters, we need to arrange r characters.

$\underline{(N-0)} \quad \underline{(N-1)} \quad \underline{(N-2)} \quad \underline{(N-3)} \cdots \quad \underline{(N-(r-1))}$

1 2 3 4 5 ... r

r places

$N * (N-1) * (N-2) * \cdots * (N-(r-1))$

$\Rightarrow \frac{N * (N-1) * (N-2) \cdots * (N-r+1) * (N-r)(N-r-1) \cdots 1}{(N-r)(N-r-1) \cdots 1}$

$\Rightarrow$ $\frac{N!}{(N-r)!} = {}^nP_r$

↓

no. of ways to arrange r places from n distinct characters.

RB, BR

Combination :- no. of ways to select something.

↳ Order doesn't matter.

RB = BR (i, j) = (j, i)

RGB = GBR

Given 4 players, count the number of ways of selecting 3 players.

$\{ P_1 \quad P_2 \quad P_3 \quad P_4 \}$

$P_1 \quad P_2 \quad P_3$
$P_1 \quad P_2 \quad P_4$
$P_2 \quad P_3 \quad P_4$
$P_1 \quad P_3 \quad P_4$

} 4 ways.

→ $^4C_3$

Ques) no. of ways to arrange 4 players in 3 slots.

_ _ _ {P1, P2, P3, P4}

$^4P_3 \Rightarrow \frac{4!}{(4-3)!} = 24.$

P1 P2 P3
P1 P2 P4
P2 P3 P4
P1 P3 P4
} Combinations

Permutations

$^4P_3$

P1 P2 P3	P1 P2 P4	P1 P3 P4	P2 P3 P4
P1 P3 P2	P1 P4 P2	P1 P4 P3	P2 P4 P3
P2 P1 P3	P2 P1 P4	P3 P1 P4	P3 P2 P4
P2 P3 P1	P2 P4 P1	P3 P4 P1	P3 P4 P2
P3 P1 P2	P4 P1 P2	P4 P1 P3	P4 P2 P3
P3 P2 P1	P4 P2 P1	P4 P3 P1	P4 P3 P2
{P1 P2 P3}	{P1 P2 P4}	{P1 P3 P4}	{P2 P3 P4}

for every selection = 6 arrangements

Total no. of selections * no. of arrangement of each selection = Total no of arrangements

$n * 6 = 24$

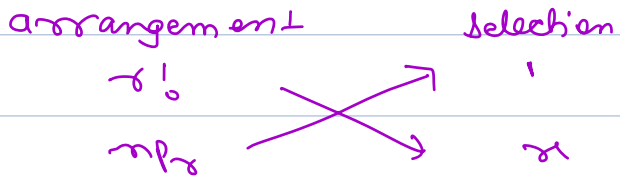
$n = 4$

Ques) Given n elements how many ways we can arrange r items out of that?

$\downarrow$

nP_r

arrange r items $= r!$



$\Rightarrow r! * r = nP_r$

$\Rightarrow r = \frac{n!}{(n-r)! * r!} \Rightarrow {}^nC_r$

$\Rightarrow r = {}^nC_r$

no. of ways of selecting r thing out of n things

${}^nC_r * r! = {}^nP_r$

$r \rightarrow {}^rC_2 \rightarrow$

$\frac{r!}{2! * (r-2)!}$

$\frac{r \times (r-1) \times (r-2)!}{2! * (r-2)!}$

Property 1 :- $n \to n$

$nC_n \to 1$

a b c d

↓

a, b, c, d .

Selecting 0 items from n items :- 1

$nC_0 = 1$

Property 2 :- 4 boys (B1 B2 B3 B4)

Is 3 boys .

$nC_r = nC_{n-r}$

$4C_3 \Rightarrow \frac{4!}{3! \times 1!} \Rightarrow 4$.

Selecting 3 players out of 4		not selecting
$B_1\ B_2\ B_3$	→	B_4
$B_1\ B_3\ B_4$	→	B_2
$B_2\ B_3\ B_4$	→	B_1
$B_1\ B_3\ B_4$	→	B_2

Property :- given n distinct elements, select r items.

$$^{n}C_{r} = {}^{n-1}C_{r-1} + {}^{n-1}C_{r}$$

$B_1\ B_2\ B_3\ B_4\ B_5$

B_5 comes or not to come

$$^{5}C_{3} = {}^{4}C_{2} + {}^{4}C_{3}$$

$$\frac{(n-1)!}{(n-r)! \times (r-1)!} + \frac{(n-1)!}{r! \times (n-1-r)!}$$

$$\frac{(N-1)!}{(R-1)! \times (N-R) \times (N-R-1)!} + \frac{(N-1)!}{R \times (R-1)! \times (N-R-1)!}$$

$$\Rightarrow \frac{(N-1)!}{(R-1)! \times (N-R-1)!} \left[\frac{1}{N-R} + \frac{1}{R} \right]$$

$$\frac{(N-1)!}{(R-1)! \times (N-R-1)!} \left[\frac{R + N - R}{R \times (N-R)} \right]$$

$$\Rightarrow \frac{N \times (N-1)!}{R \times (R-1)! \times (N-R)(N-R-1)!}$$

$$\Rightarrow \frac{N!}{R! \times (N-R)!}$$

$$= {}^{N}C_{R}.$$

Ques Pascal's Δ.

$n = 6$,

$^0C_0$ $\Rightarrow$ 1

$^1C_0$ $^1C_1$ $\Rightarrow$ 1 1

$^2C_0$ $^2C_1$ $^2C_2$ $\Rightarrow$ 1 2 1

$^3C_0$ $^3C_1$ $^3C_2$ $^3C_3$ $\Rightarrow$ 1 3 3 1

$^4C_0$ $^4C_1$ $^4C_2$ $^4C_3$ $^4C_4$

$^5C_0$ $^5C_1$ $^5C_2$ $^5C_3$ $^5C_4$ $^5C_5$

$^6C_0$ $^6C_1$ $^6C_2$ $^6C_3$ $^6C_4$ $^6C_5$ $^6C_6$

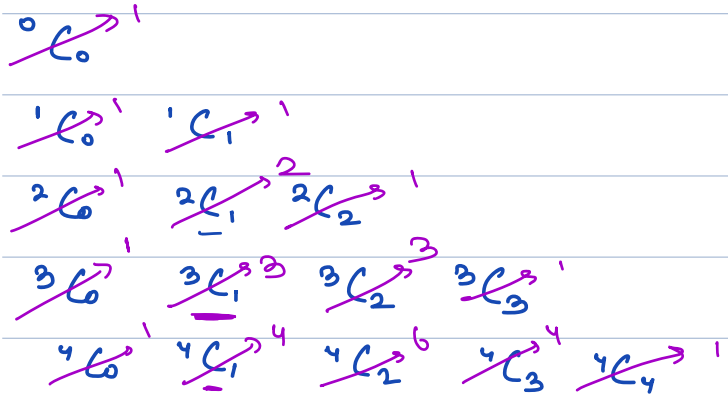
Brute force :-

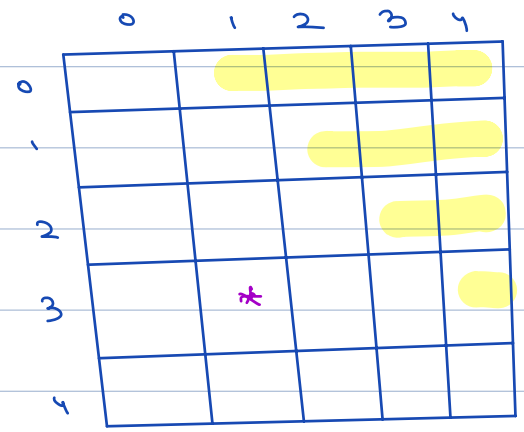
run 2 for loops, calculate the value of, ^nC_r for every place and print it.

$^nC_0$ & $^nC_n = 1$

$$^{n}C_{r} = {}^{n-1}C_{r-1} + {}^{n-1}C_{r}$$

$n = 4$





T.C → $O(n^2)$

S.C → $O(n^2)$

$n = 4$

```
Pascals Triangle (n) {
    ncR[n+1][n+1]
    for (i=0; i<=n; i++) {
        ncR[i][0] = 1;
        ncR[i][i] = 1;
        for (j=1; j<i; j++) {
            ncR[i][j] = ncR[i-1][j] +
                        ncR[i-1][j-1];
        }
    }
}
```

Ques Excel Column Title :-

1	2	3	4	...	26	27	28	...	52	53	...
A	B	C	D		Z	AA	AB	...	AZ	BA	...

N = 30 , N = 50 , N = 78

N = 30 → AD

N = 50 → AX

(1 - 26) — A Z

(27 - 52) — AA AZ

(53 - 78) — BA BZ

(79 - 104) — (CA - CZ)

Base 2

0
1
10
11
100

Base - 8

0
1
2
3
4
5
6
7
10
11
12
:
17
20
21
:

Base - 26

A → 1
B → 2
C → 3
:
Z → 26
AA
AB
:
AZ
BA
:
BZ

idea:

N = 1000

26	1000
26	38
26	1
	0

12 → L
12 → W
1 → A

→ AWL

N = 78

26	78
26	3
	0

0
3 → C

~~X~~

whenever we get a remainder 0, divide by 1 less Quotient, so you don't get 0 remainder, since nothing is mapped to it.

26	78
26	2
	0

26 → Z
2 → B

=> BZ

Way - 2

(A - Z) → (1 to 26)

(A - Z) → (0 to 25)

26	78 - 1 = 77	25 → Z
26	2 - 1 = 1	1 → B
	0	

→ BZ

26	1000 - 1 = 999	11 → L
26	38 - 1 = 37	11 → L
26	1 - 1 = 0	0 → A
	0	

→ ALL

```
void columnTitle(int n) {
    ans = "";
    while(n > 0) {
        ans = (char) ((n - 1) % 26 + 'A') + ans; // char + string
        n = (n - 1) / 26
    }
    return ans
}
```