Assignment 1

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Chapter II, Examples II

Q22 (iii) Find the conditions that the four points $\begin{pmatrix} x_1 \\ y_1 \end{pmatrix}$, $\begin{pmatrix} x_2 \\ y_2 \end{pmatrix}$, $\begin{pmatrix} x_3 \\ y_3 \end{pmatrix}$, $\begin{pmatrix} x_4 \\ y_4 \end{pmatrix}$ may be the vertices of a rhombus.

Solution : The given points are

$$\mathbf{A} = \begin{pmatrix} x_1 \\ y_1 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} x_2 \\ y_2 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} x_3 \\ y_3 \end{pmatrix}, \mathbf{D} = \begin{pmatrix} x_4 \\ y_4 \end{pmatrix},$$

Condition for the given four points be the vertices of a rhombus are ;-
1) If distances of all the four sides are equal
2) If opposite sides are parallel and
2) If diagonals are perpendicular bisectors.

Let us consider two vectors say,

$$\mathbf{U} = \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}, \mathbf{V} = \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$$

then distance can be calculated using norm of a vector, i.e.,

$$\|\mathbf{U} - \mathbf{V}\| = \sqrt{(v_1 - u_1)^2 + (v_2 - u_2)^2}$$

Here,

$$D1 = \|\mathbf{A} - \mathbf{B}\| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$D2 = \|\mathbf{B} - \mathbf{C}\| = \sqrt{(x_3 - x_2)^2 + (y_3 - y_2)^2}$$

$$D3 = \|\mathbf{C} - \mathbf{D}\| = \sqrt{(x_4 - x_3)^2 + (y_4 - y_3)^2}$$

$$D4 = \|\mathbf{D} - \mathbf{A}\| = \sqrt{(x_1 - x_4)^2 + (y_1 - y_4)^2}$$

if

$$(A - C)^T.(B - D) = 0$$

implies that AC and BD are perpendicular to each other.

and say E , F be mid points joining the lines AC and BD. Now if
AE = EC
BF = FD
implies that AC and BD bisect each other futher E and F be a same point.

The above two conditions prove that AC and BD are perpendicular bisectors.

Now if
1) D1=D2=D3=D4
2) (A-B)=$\pm(D - C) and (B - C) = \pm(A - D)$
$(implies AB//DC and BC//AD)$
3)$AC and BD are perpendicular bisectors$

Then, we can say that the given points are the vertices of a rhombus.