Assignment 1

H.N Srikanth - SM21Mtech12012

August 2021

Chapter II, Examples II

Q22 (iii) Find the conditions that the four points $\begin{pmatrix}x1\\y1\end{pmatrix}$, $\begin{pmatrix}x2\\y2\end{pmatrix}$, $\begin{pmatrix}x3\\y3\end{pmatrix}$, $\begin{pmatrix}x4\\y4\end{pmatrix}$ may be the vertices of a rhombus.

Solution : The given points are

$$\mathbf{A} = \begin{pmatrix}x1\\y1\end{pmatrix}, \mathbf{B} = \begin{pmatrix}x2\\y2\end{pmatrix}, \mathbf{C} = \begin{pmatrix}x3\\y3\end{pmatrix}, \mathbf{D} = \begin{pmatrix}x4\\y4\end{pmatrix},$$

Condition for the given four points be the vertices of a rhombus are ;-
1) If distances of all the four sides are equal and
2) If diagonals are perpendicular bisectors.

Let us consider two vectors say,

$$\mathbf{U} = \begin{pmatrix}u1\\u2\end{pmatrix}, \mathbf{V} = \begin{pmatrix}v1\\v2\end{pmatrix}$$

then distance can be calculated using norm of a vector, i.e.,

$$\|\mathbf{U} - \mathbf{V}\| = \sqrt{(u1 - v1)^2 + (u2 - v2)^2}$$

Here,

$$D1 = \|\mathbf{A} - \mathbf{B}\| = \sqrt{(x2 - x1)^2 + (y2 - y1)^2}$$

$$D2 = \|\mathbf{B} - \mathbf{C}\| = \sqrt{(x3 - x2)^2 + (y3 - y2)^2}$$

$$D3 = \|\mathbf{C} - \mathbf{D}\| = \sqrt{(x4 - x3)^2 + (y4 - y3)^2}$$

$$D4 = \|\mathbf{D} - \mathbf{A}\| = \sqrt{(x1 - x4)^2 + (y1 - y4)^2}$$

Let m1 and m2 be the slopes of the lines AC and BD
if m1*m2 = -1
implies that AC and BD are perpendicular to each other.

and say E , F be mid points joining the lines AC and BD. Now if
AE = EC
BF = FD
implies that AC and BD bisect each other futher E and F be a same point.

The above two conditions prove that AC and BD are perpendicular bisectors.

Now if
1) D1=D2=D3=D4
2) AC and BD are perpendicular bisectors

Then, we can say that the given points are the vertices of a rhombus.