

Assignment 1

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Chapter II, Examples II

Q22 (iii) Find the conditions that the four points $\begin{pmatrix} x_1 \\ y_1 \end{pmatrix}, \begin{pmatrix} x_2 \\ y_2 \end{pmatrix}, \begin{pmatrix} x_3 \\ y_3 \end{pmatrix}, \begin{pmatrix} x_4 \\ y_4 \end{pmatrix}$ may be the vertices of a rhombus.

Solution : The given points are

$$\mathbf{A} = \begin{pmatrix} x_1 \\ y_1 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} x_2 \\ y_2 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} x_3 \\ y_3 \end{pmatrix}, \mathbf{D} = \begin{pmatrix} x_4 \\ y_4 \end{pmatrix},$$

Conditions for the given four points to be the vertices of a rhombus are :-

- 1) If opposite sides are parallel and
- 2) If diagonals are perpendicular .

if

$$(\mathbf{A} - \mathbf{B}) = k.(\mathbf{D} - \mathbf{C})$$

$$(\mathbf{B} - \mathbf{C}) = k.(\mathbf{A} - \mathbf{D})$$

shows AB // DC and BC // AD

if

$$(\mathbf{A} - \mathbf{C})^\top . (\mathbf{B} - \mathbf{D}) = 0$$

implies that AC and BD are perpendicular to each other.

Numerical Example :

Examine whether the given points A (2,-3) and B (6,5) and C (-2,1) and D (-6,-7) forms a rhombus.

Sol: The given points are

$$\mathbf{A} = \begin{pmatrix} 2 \\ -3 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 6 \\ 5 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} -2 \\ 1 \end{pmatrix}, \mathbf{D} = \begin{pmatrix} -6 \\ -7 \end{pmatrix},$$

$$(\mathbf{A} - \mathbf{B}) = \begin{pmatrix} -4 \\ -8 \end{pmatrix}, (\mathbf{D} - \mathbf{C}) = \begin{pmatrix} 4 \\ -8 \end{pmatrix}$$

$$(\mathbf{B} - \mathbf{C}) = \begin{pmatrix} 8 \\ 4 \end{pmatrix}, (\mathbf{A} - \mathbf{D}) = \begin{pmatrix} 8 \\ 4 \end{pmatrix}$$

$$(\mathbf{A} - \mathbf{B}) = -(\mathbf{D} - \mathbf{C})$$

$$(\mathbf{B} - \mathbf{C}) = (\mathbf{A} - \mathbf{D})$$

This shows AB//DC and BC//AD

$$(\mathbf{A} - \mathbf{C})^\top = (4 \quad -4), (\mathbf{B} - \mathbf{D}) = \begin{pmatrix} 12 \\ 12 \end{pmatrix}$$

$$(\mathbf{A} - \mathbf{C})^\top . (\mathbf{B} - \mathbf{D}) = 48 - 48 = 0$$

This shows AC and BD are perpendicular.

Given points A,B,C,D satisfy both the conditions hence they form a Rhombus

Now if given four points satisfy the above conditions then, we can say that the given points are the vertices of a rhombus.