Assignment 1

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Chapter II, Examples II

Q22 (iii) Find the conditions that the four points $\begin{pmatrix}x_1\\y_1\end{pmatrix}, \begin{pmatrix}x_2\\y_2\end{pmatrix}, \begin{pmatrix}x_3\\y_3\end{pmatrix}, \begin{pmatrix}x_4\\y_4\end{pmatrix}$ may be the vertices of a rhombus.

Solution : The given points are

$$\mathbf{A} = \begin{pmatrix}x_1\\y_1\end{pmatrix}, \mathbf{B} = \begin{pmatrix}x_2\\y_2\end{pmatrix}, \mathbf{C} = \begin{pmatrix}x_3\\y_3\end{pmatrix}, \mathbf{D} = \begin{pmatrix}x_4\\y_4\end{pmatrix},$$

Conditions for the given four points to be the vertices of a rhombus are ;-
1) If opposite sides are parallel and
2) If diagonals are perpendicular .

if

$$(\mathbf{A}-\mathbf{B}) = k*(\mathbf{D}-\mathbf{C}) \quad (1)$$
$$(\mathbf{B}-\mathbf{C}) = k*(\mathbf{A}-\mathbf{D}) \quad (2)$$

1 and 2 shows AB // DC and BC // AD

if

$$(\mathbf{A}-\mathbf{C})^\top.(\mathbf{B}-\mathbf{D}) = 0 \quad (3)$$

3 shows AC and BD are perpendicular to each other.

As the given four points satisfy the required conditions we can say that they are the vertices of a rhombus.

Numerical Example :

Examine whether the given points A (2,-3) and B (6,5) and C (-2,1) and D (-6,-7) forms a rhombus.

Sol: The given points are

$$\mathbf{A} = \begin{pmatrix}2\\-3\end{pmatrix}, \mathbf{B} = \begin{pmatrix}6\\5\end{pmatrix}, \mathbf{C} = \begin{pmatrix}-2\\1\end{pmatrix}, \mathbf{D} = \begin{pmatrix}-6\\-7\end{pmatrix},$$

$$(\mathbf{A}-\mathbf{B}) = \begin{pmatrix}-4\\-8\end{pmatrix}, (\mathbf{D}-\mathbf{C}) = \begin{pmatrix}4\\8\end{pmatrix}$$

$$(\mathbf{B}-\mathbf{C}) = \begin{pmatrix}8\\4\end{pmatrix}, (\mathbf{A}-\mathbf{D}) = \begin{pmatrix}8\\4\end{pmatrix}$$

$$(\mathbf{A}-\mathbf{B}) = -(\mathbf{D}-\mathbf{C}) \quad (4)$$
$$(\mathbf{B}-\mathbf{C}) = (\mathbf{A}-\mathbf{D}) \quad (5)$$

4 and 5 shows AB//DC and BC//AD

$$(\mathbf{A}-\mathbf{C})^\top = (4 \quad -4), (\mathbf{B}-\mathbf{D}) = \begin{pmatrix}12\\12\end{pmatrix}$$

$$(\mathbf{A}-\mathbf{C})^\top.(\mathbf{B}-\mathbf{D}) = 48-48 = 0 \quad (6)$$

6 shows AC and BD are perpendicular to each other.

Given points A,B,C,D satisfy the required conditions hence they form a Rhombus.