

Assignment 1

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Chapter II, Examples II

Q22 (iii) Find the conditions that the four points $\begin{pmatrix} x_1 \\ y_1 \end{pmatrix}, \begin{pmatrix} x_2 \\ y_2 \end{pmatrix}, \begin{pmatrix} x_3 \\ y_3 \end{pmatrix}, \begin{pmatrix} x_4 \\ y_4 \end{pmatrix}$ may be the vertices of a rhombus.

Solution :

The given points are

$$\mathbf{A} = \begin{pmatrix} x_1 \\ y_1 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} x_2 \\ y_2 \end{pmatrix},$$

$$\mathbf{C} = \begin{pmatrix} x_3 \\ y_3 \end{pmatrix}, \mathbf{D} = \begin{pmatrix} x_4 \\ y_4 \end{pmatrix},$$

Conditions for the given four points to be the vertices of a rhombus are:

1. If opposite sides are parallel and
2. If diagonals are perpendicular .

if

$$(\mathbf{A} - \mathbf{B}) = k(\mathbf{D} - \mathbf{C}) \quad (1)$$

$$(\mathbf{B} - \mathbf{C}) = k(\mathbf{A} - \mathbf{D}) \quad (2)$$

1 and 2 shows AB // DC and BC // AD

if

$$(\mathbf{A} - \mathbf{C})^\top \cdot (\mathbf{B} - \mathbf{D}) = 0 \quad (3)$$

3 shows AC and BD are perpendicular to each other.

As the given four points satisfy the required conditions we can say that they are the vertices of a rhombus.

Numerical Example :

Examine whether the given points A (2,-3) and B (6,5) and C (-2,1) and D (-6,-7) forms a rhombus.

Sol: The given points are

$$\mathbf{A} = \begin{pmatrix} 2 \\ -3 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 6 \\ 5 \end{pmatrix}$$

$$\mathbf{C} = \begin{pmatrix} -2 \\ 1 \end{pmatrix}, \mathbf{D} = \begin{pmatrix} -6 \\ -7 \end{pmatrix}$$

$$(\mathbf{A} - \mathbf{B}) = \begin{pmatrix} -4 \\ -8 \end{pmatrix}, (\mathbf{D} - \mathbf{C}) = \begin{pmatrix} -4 \\ -8 \end{pmatrix}$$

$$(\mathbf{B} - \mathbf{C}) = \begin{pmatrix} 8 \\ 4 \end{pmatrix}, (\mathbf{A} - \mathbf{D}) = \begin{pmatrix} 8 \\ 4 \end{pmatrix}$$

$$(\mathbf{A} - \mathbf{B}) = (\mathbf{D} - \mathbf{C}) \quad (4)$$

$$(\mathbf{B} - \mathbf{C}) = (\mathbf{A} - \mathbf{D}) \quad (5)$$

4 and 5 shows AB//DC and BC//AD

$$(\mathbf{A} - \mathbf{C})^\top = (4 \quad -4), (\mathbf{B} - \mathbf{D}) = \begin{pmatrix} 12 \\ 12 \end{pmatrix}$$

$$(\mathbf{A} - \mathbf{C})^\top \cdot (\mathbf{B} - \mathbf{D}) = 48 - 48 = 0 \quad (6)$$

6 shows AC and BD are perpendicular to each other.

Given points A,B,C,D satisfy the required conditions hence they form a Rhombus.

