

Assignment 1

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Chapter II, Examples II

Q22 (iii) Find the conditions that the four points $\begin{pmatrix} x1 \\ y1 \end{pmatrix}, \begin{pmatrix} x2 \\ y2 \end{pmatrix}, \begin{pmatrix} x3 \\ y3 \end{pmatrix}, \begin{pmatrix} x4 \\ y4 \end{pmatrix}$ may be the vertices of a rhombus.

Solution : The given points are

$$\mathbf{A} = \begin{pmatrix} x1 \\ y1 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} x2 \\ y2 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} x3 \\ y3 \end{pmatrix}, \mathbf{D} = \begin{pmatrix} x4 \\ y4 \end{pmatrix},$$

Condition for the given four points be the vertices of a rhombus are :-

- 1) If distances of all the four sides are equal and
- 2) If diagonals are perpendicular bisectors.

Let us consider two vectors say,

$$\mathbf{U} = \begin{pmatrix} u1 \\ u2 \end{pmatrix}, \mathbf{V} = \begin{pmatrix} v1 \\ v2 \end{pmatrix}$$

then distance can be calculated using norm of a vector, i.e.,

$$\|\mathbf{U} - \mathbf{V}\| = \sqrt{(v1 - u1)^2 + (v2 - u2)^2}$$

Here,

$$D1 = \|\mathbf{A} - \mathbf{B}\| = \sqrt{(x2 - x1)^2 + (y2 - y1)^2}$$

$$D2 = \|\mathbf{B} - \mathbf{C}\| = \sqrt{(x3 - x2)^2 + (y3 - y2)^2}$$

$$D3 = \|\mathbf{C} - \mathbf{D}\| = \sqrt{(x4 - x3)^2 + (y4 - y3)^2}$$

$$D4 = \|\mathbf{D} - \mathbf{A}\| = \sqrt{(x1 - x4)^2 + (y1 - y4)^2}$$

Let $m1$ and $m2$ be the slopes of the lines AC and BD

if $m1 * m2 = -1$

implies that AC and BD are perpendicular to each other.

and say E, F be mid points joining the lines AC and BD. Now if

$$AE = EC$$

$$BF = FD$$

implies that AC and BD bisect each other further E and F be a same point.

The above two conditions prove that AC and BD are perpendicular bisectors.

Now if

$$1) D1 = D2 = D3 = D4$$

$$2) AC \text{ and } BD \text{ are perpendicular bisectors}$$

Then, we can say that the given points are the vertices of a rhombus.