(i) $\left.\begin{array}{l} w^T x + b = 1 \\ w^T x + b = -1 \end{array}\right\}$ Margin Boundaries

$w^T x + b = 0$ Maximum margin

In this situation of problem we don't know value of w and b, so we can normaliz equation

$w^T x + b = +\nu$ $\quad\quad$ $w^T x + b = -\nu$

$\frac{w^T}{\nu} x + \frac{b}{\nu} = 1$ $\quad\quad$ $\frac{w^T x}{\nu} + \frac{b}{\nu} = -1$

$\frac{w^T}{\nu} = W^T \quad \frac{b}{\nu} = B.$

$$\underbrace{W^T x + B = +1 \quad\quad W^T x + B = -1}$$

$\Downarrow$

By solving $\quad W^T x + B = 0$

(W^T and B are known)

There by by changing $(+1, -1)$ to $(+\nu, -\nu)$ the solution of Maximum margin hyperplane in unchanged.

② $L_p = \frac{1}{2}||\omega||^2 - \sum_i \left(\alpha_i \left(\omega^T x_i + b\right) y_i - 1\right)$

$\rho = \frac{1}{||\omega||}$

from KKT conditions.

$\frac{\partial L}{\partial \omega} \Rightarrow \omega = \frac{1}{2}\sum_i \alpha_i y_i x_i \rightarrow ①$

$\frac{\partial L}{\partial b} \Rightarrow \sum_i \alpha_i y_i = 0. \rightarrow ②$

By substituting ① & ② L_p we get L_D.

$L_D = \frac{-1}{2}||\omega||^2 - \omega^T \sum_i \alpha_i y_i x_i + b\sum_i \alpha_i y_i + \sum_i \alpha_i$

$L_D = \frac{||\omega||^2}{2} - ||\omega||^2 + b(0) + \sum_i \alpha_i$

$\frac{\partial L_p}{\partial \alpha} = 0 \Rightarrow \left(\omega^T x_i + b\right) y_i - 1 = 0 - ③$

(③ in L_p).

In SVM $L_p = L_D$.

$\frac{1}{2}||\omega||^2 - \sum_i \alpha_i (0) = \frac{1}{2}||\omega||^2$

$\frac{1}{2}||\omega||^2 = \frac{-1}{2}||\omega||^2 + \sum_i \alpha_i$

$\sum_i \alpha_i = ||\omega||^2$ $\quad ||\omega|| = 1/\rho.$

$$\boxed{\sum_i \alpha_i = \frac{1}{\rho^2}}$$

(3.) 1.) $k(x,z) = k_1(x,z) + k_2(x,z)$ k_1 & k_2 are valid kernel funcs

let $\phi^1(x) = (\phi_1^1(x)\ \phi_2^1(x) \ldots \phi_{N_1}^1(x))$

$\phi^2(x) = (\phi_1^2(x)\ \phi_2^2(x) \ldots \phi_{N_2}^2(x))$

be the feature maps of x_1 & x_2

$\phi(x) = ((\phi_1^1(x)\ \phi_2^1(x) \ldots \phi_{N_1}^1(x)), (\phi_1^2(x) \ldots \phi_{N_2}^2(x))$

Mapping show $\phi(x) \cdot \phi(y) = \phi^1(x) \cdot \phi^1(y) + \phi^2(x) \phi^2(y)$

This shows it follows kernel trick.

(2.) $k(x,z) = k_1(x,z)\ k_2(x,z)$

$\phi^1(x) = (\phi_1^1(x) \cdot \phi_2^1(x) \ldots \phi_{N_1}^1(x))$

$\phi^2(x) = (\phi_1^2(x)\ \phi_2^2(x) \ldots \phi_{N_2}^2(x))$

Here we are multiplying expression for k_1 & k_2,

to see that kernels with that space products of

feautures from ϕ^1 & ϕ^2

Obeys Kernel Trick.

(3.) $k(x,z) = h(k_1(x,z))$ h is a polynomial function

Since each polynomial fun. Term is production of kernels and

& we can say that $h(k_1(x,z))$ is a positive

definite function.

(4.) $k(x,z) = \exp(k_1(x,z))$

$\exp(x) = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \ldots$ from (1)

$k(x,z) = \lim_{i \to \infty} \frac{k_1(x,z)}{i!}$ $\Rightarrow$ $k(x,z)$ is a kernel function

(5.) $k(x,z) = \exp\left(\frac{-\|x-z\|^2}{2\sigma^2}\right)$.

$$= \exp\left(\frac{-\|x-z\|^2}{2\sigma^2}\right)$$

$$= \exp\left(\frac{-\|x\|^2 - \|z\|^2 + 2x^T z}{2\sigma^2}\right)$$

$$= \exp\left(\frac{-\|x\|^2}{2\sigma^2}\right)\exp\left(\frac{-\|z\|^2}{2\sigma^2}\right)\exp\left(\frac{2x^T \cdot z}{2\sigma^2}\right).$$

$$= h(x) \cdot h(z) \cdot \exp(k_1(x,z)).$$

from (1) & (3) The following follows to be a kernel function.