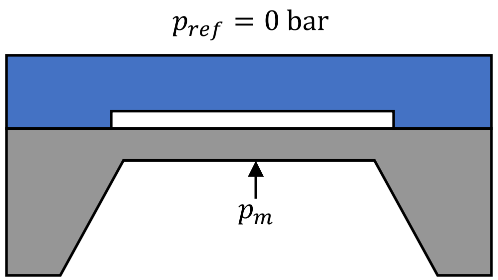
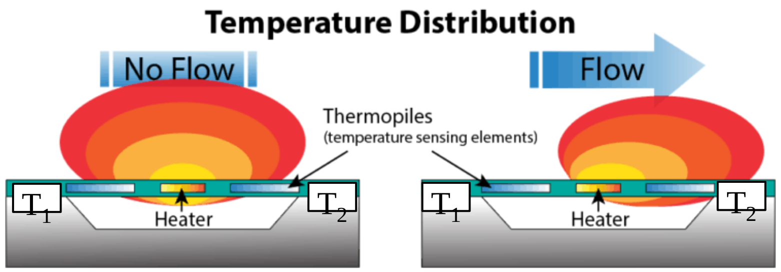
Pressure and fluid velocity measurement

IC 231 – Measurement & Instrumentation

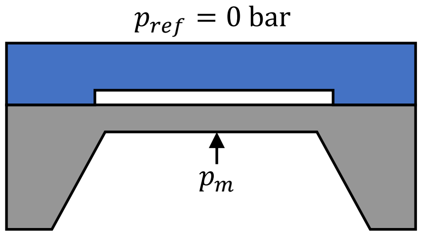


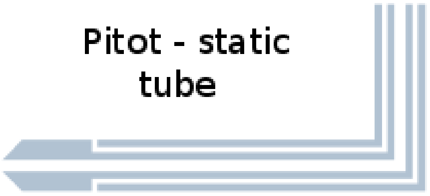


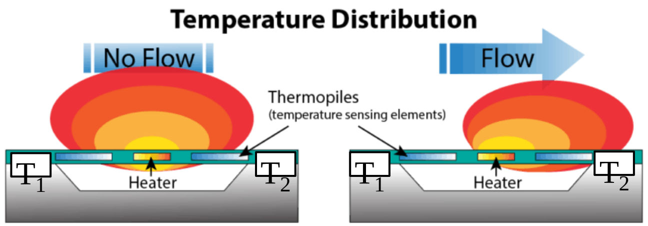
Overview

1. What is pressure?
2. What is the physics of pressure in different media?
3. What is the difference between different pressure types?
4. How can we measure pressure? Two principles!
5. What are applications of pressure measurement sensors?

Overview


$p_{ref} = 0$ bar
p_m


Pitot - static
tube

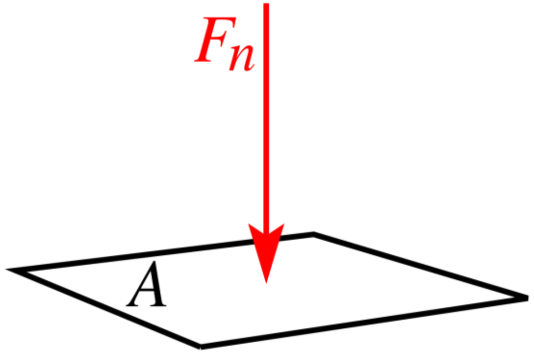

Temperature Distribution
No Flow
Flow
Thermopiles
(temperature sensing elements)
T_1
Heater
T_2
T_1
Heater
T_2

What is pressure?

Surface of snow is stable up to a certain **force per unit area.** The function of snowshoes is to **distribute the weight force over a larger area**

→ Pressure is reduced and a person will not sink

$$\frac{Force}{Area} = Pressure \qquad \frac{mg}{A} = \frac{F}{A} = p$$



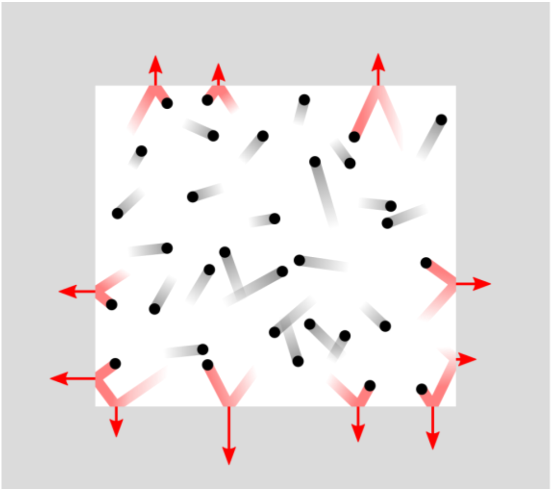
$$p = \lim_{dA \to 0} \frac{dF_n}{dA}$$

Pressure p is a:

- scalar quantity
- Proportial to force
- Inverse proportional to Area

Pressure of an ideal gas

Static pressure of an ideal gas:



$$p = \frac{nRT}{V}$$

p: pressure
n: Number of particles
R: ideal Gas constant
T: Temperature
V: Volume

Consequences for closed container:

Pressure increases with…

- increasing density ($n \uparrow$)
- Increasing temperature ($T \uparrow$)

Pressure decreases with…

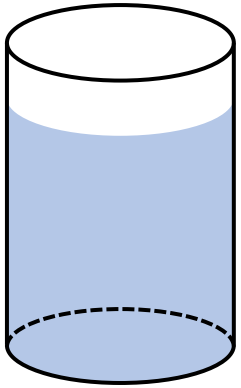
- increasing Volume ($V \uparrow$)

Assumption of an ideal gas:

- There is no force between particles
- The collision between particles is frictionless (elastic)
- Large number of molecules

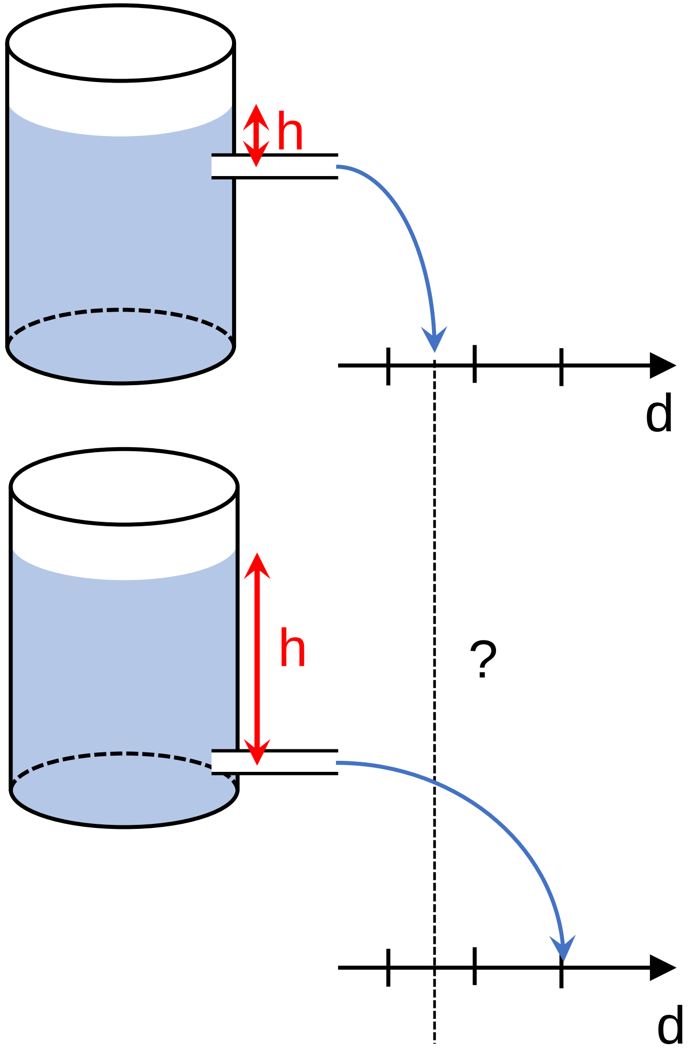
Hydrostatic pressure

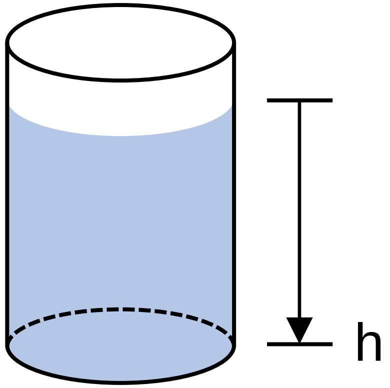
Water tank:



Case 1:

Case 2:



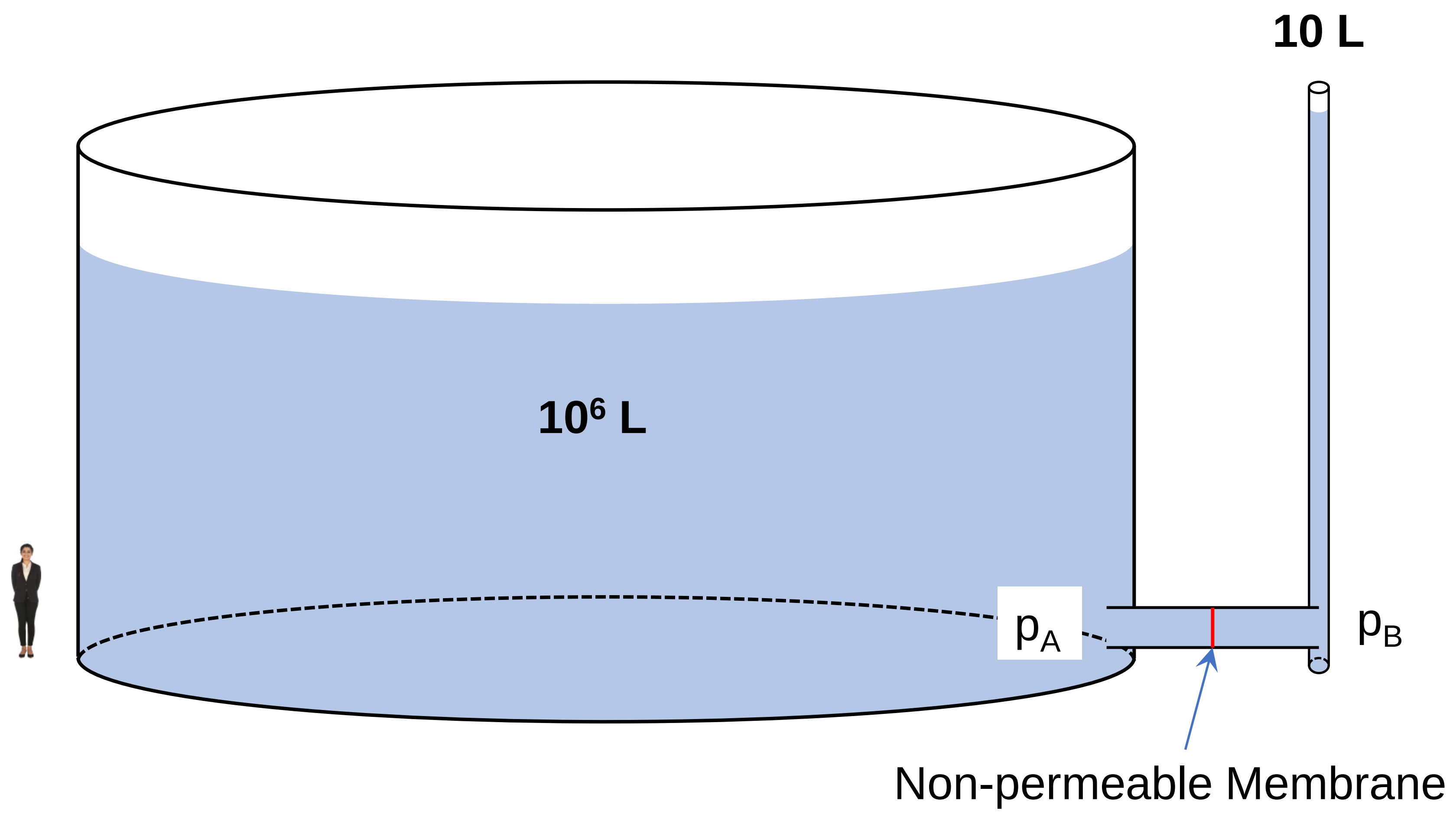


$p(h) =$

Assumptions:

- Liquid incompressible
- Cross section constant

Hydrostatic pressure



What will be the deflection of membrane?

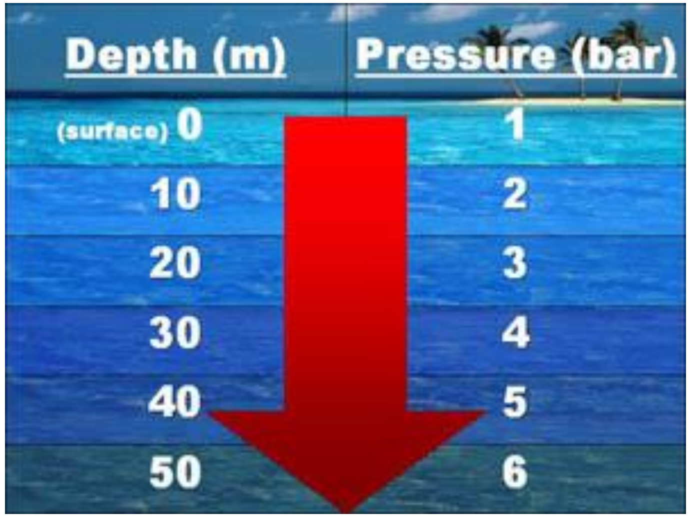
a) $p_A > p_B$

b) $p_A = p_B$

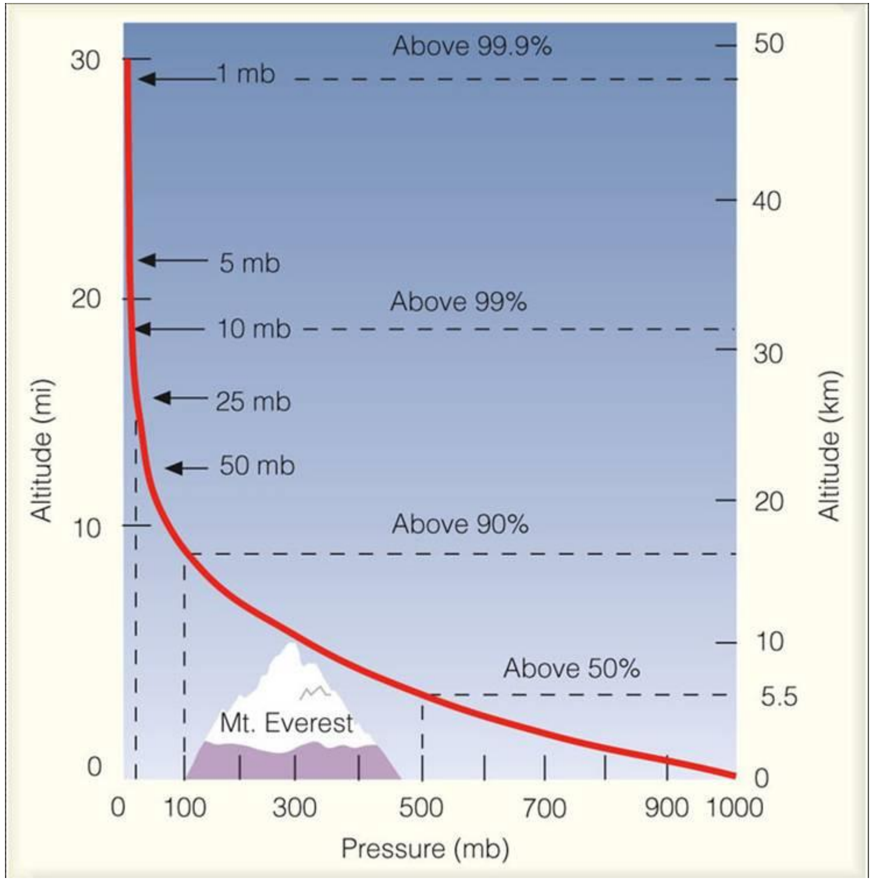
c) $p_A < p_B$

Correct answer: c)

Pressure of water and atmosphere



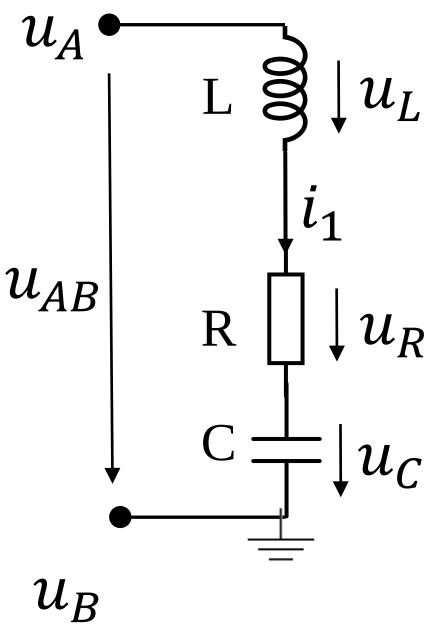
Depth (m)	Pressure (bar)
(surface) 0	1
10	2
20	3
30	4
40	5
50	6



Above ground we can assume a linear increase of atmospheric pressure with an decrease of 12 Pa per meter height.

$$1\ \text{Pa} = 1\ \frac{\text{N}}{\text{m}^2}$$

Pressure and flow – Lumped models

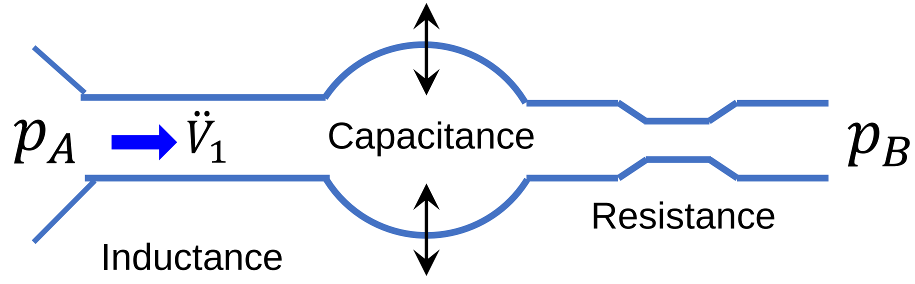


ODE:

$L\ddot{q} + R\dot{q} + \frac{1}{C}q = u_1$ with charge q

Using $i = \dot{q}$

$$L\frac{di}{dt} + Ri + \frac{1}{C}\int i\,dt = u_1$$



Fluid Inductance → Kinetic Energy of Mass flow
Fluid Resistance → Pressure drop in pipes
Fluid capacitance → Elastic elements that store fluid
Fluidic current → Volume flow rate

→ **A pressure difference between two points leads to a volume flow**

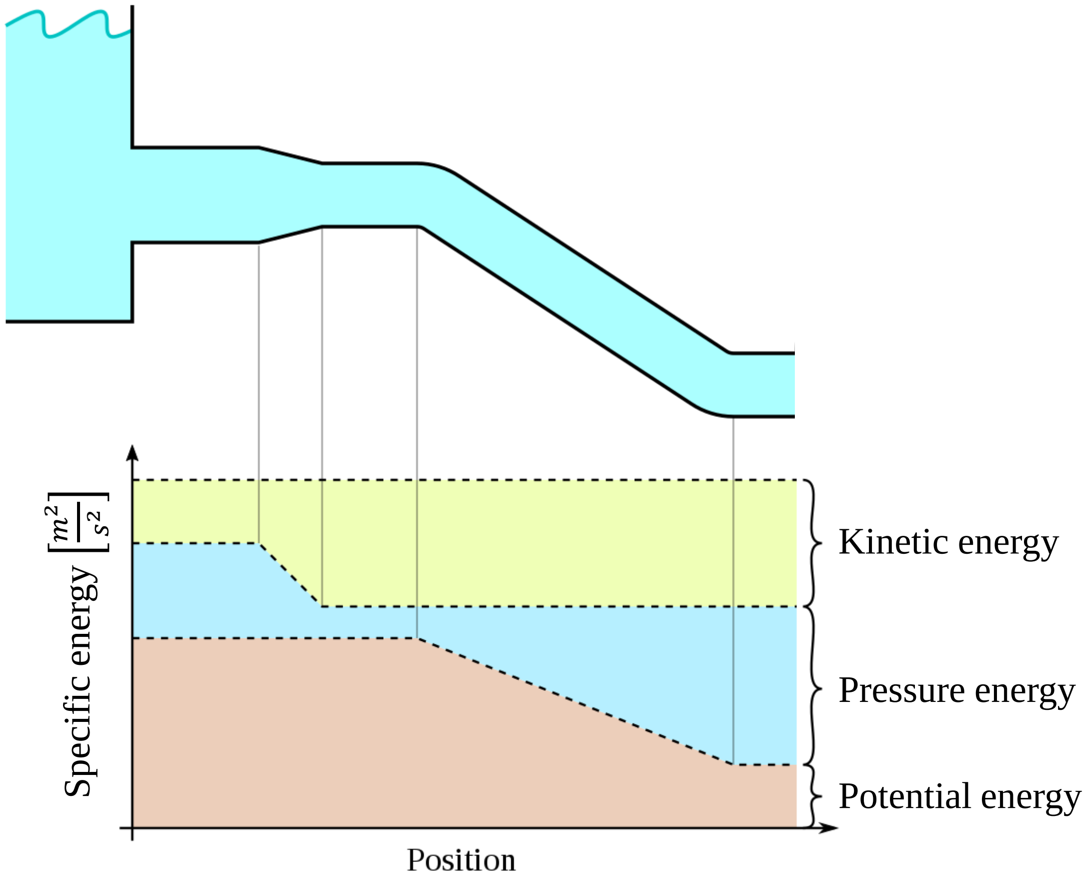
Discipline	Effort	Flow	Inertance (Kinetic)			Resistance (Dissipative)			Compliance (Potential)		
			Symbol	Element	Equation	Symbol	Element	Equation	Symbol	Element	Equation
Generalized model	Ψ	$\dot{\zeta}$			$\Psi_I = M\ddot{\zeta}$			$\Psi_R = D\dot{\zeta}$			$\Psi_C = K\zeta$
Electrical	Voltage u $[V]$	q [C] → Charge $\dot{q} = i$ [C/s = A] → current		Inductance $L\ [H]$	$u = L\ddot{q}$		Resistor $R[\Omega]$	$u = R\dot{q}$		Capacitance C [F]	$u = \frac{1}{C}q$
Fluid	Pressure p [bar]	V $[\mathrm{m}^3]$ → Volume $\dot{V}[\mathrm{m}^3/s]$ Volumetric flow rate*		Fluidic Inductance $L[\mathrm{bar}\cdot(m^3/s^2))^{-1}]$	$\Delta p = L_h\ddot{V}$		Fluidic resistance $R[\mathrm{bar}\cdot(m^3/s))^{-1}]$	$\Delta p = R_f\dot{V}$		Fluidic Cap. $C\ [\mathrm{m}^3\cdot\mathrm{bar}^{-1}]$	$\Delta p = \frac{1}{C}V$

Pressure and flow – Bernoulli equation

Balance equation for specific energy. Specific energy is the energy per unit mass $\left(\left[\frac{J}{kg}\right] = \left[\frac{m^2}{s^2}\right]\right)$

$$\frac{u^2}{2} \qquad + \qquad gh \qquad + \qquad \frac{p}{\rho} \qquad = const.$$

$$\text{kinetic energy} + \text{gravitational energy} + \text{pressure energy} = \text{const}$$



Assumptions:

- Incompressible fluid
- No friction

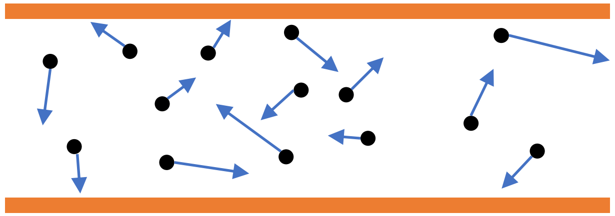
Multiplication with ρ yields a pressure equation:

$$\rho\frac{u^2}{2} + \rho gh + p = \rho * const$$

$$\boxed{\text{kinetic pressure} + \text{hydrostatic pressure} + \text{internal static pressure} = \text{total pressure}}$$

Pressure and flow

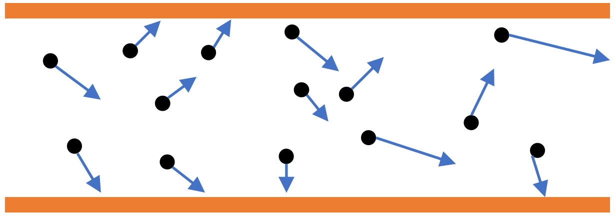
Resting fluid



→ Pressure is isotropic

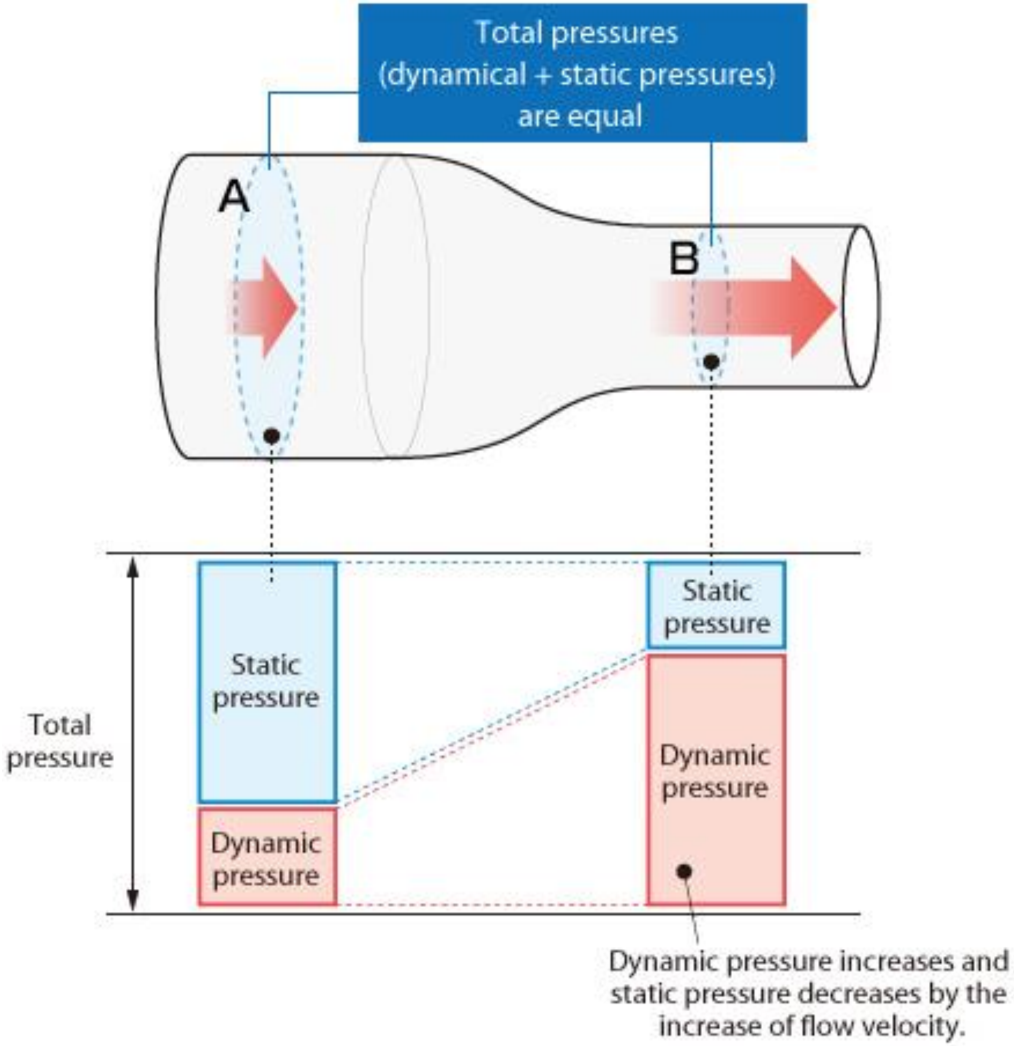
$\underbrace{\text{Vertical pressure} \uparrow \;=\; \text{horizontal pressure} \rightarrow}_{\text{Static pressure}}$

Moving fluid



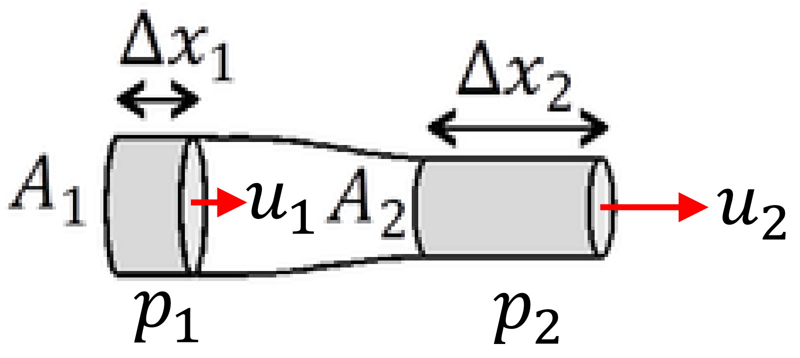
→ Pressure is anisotropic

$\underbrace{\text{Vertical pressure} \uparrow}_{\text{Static pressure}} \neq \underbrace{\text{horizontal pressure}}_{\text{Static + dynamic pressure}} \rightarrow$



Adapted from: https://www.cradle-cfd.com/media/column/a71

Pressure and flow



Incompressible fluid: Density is constant

$$\frac{\Delta m_1}{\Delta t} = \frac{\Delta m_2}{\Delta t} \rightarrow \frac{\rho \Delta V_1}{\Delta t} = \frac{\rho \Delta V_2}{\Delta t} \rightarrow \frac{\rho A_1 \Delta x_1}{\Delta t} = \frac{\rho A_2 \Delta x_2}{\Delta t}$$

$$\Rightarrow A_1 u_1 = A_2 u_2 \qquad \boxed{\frac{A_1}{A_2} = \frac{u_2}{u_1}}$$

u_i: Fluid velocity

From Bernoulli's equation:

$$\rho \frac{u_1^2}{2} + \cancel{\rho g z} + p_1 = \rho \frac{u_2^2}{2} + \cancel{\rho g z} + p_2$$

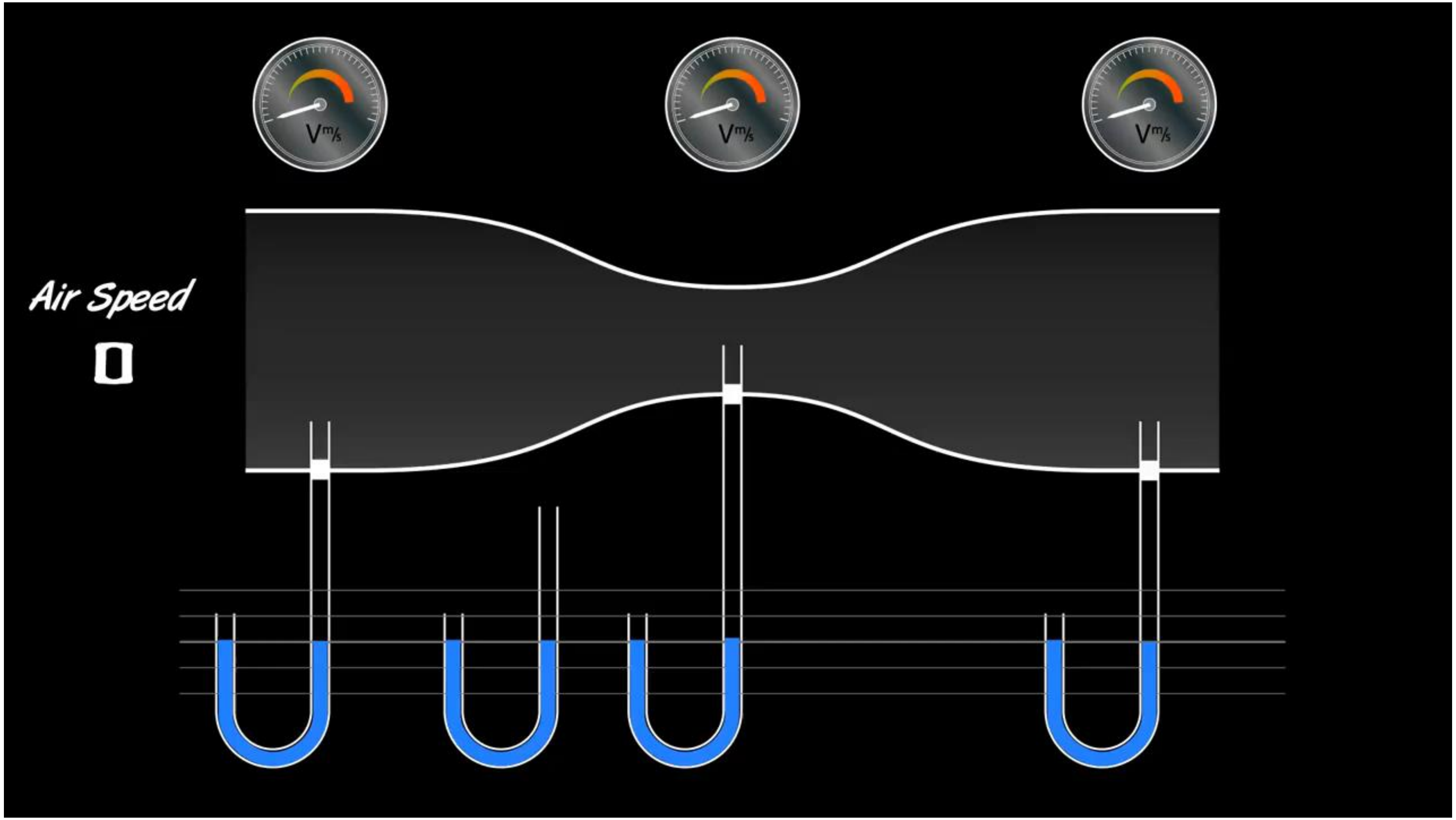
$$p_1 - p_2 = \Delta p_{12} = \frac{\rho}{2}(u_2^2 - u_1^2)$$

$$\Delta p_{12} = \frac{\rho}{2}\left(\left(\frac{A_1}{A_2}\right)^2 u_1^2 - u_1^2\right)$$

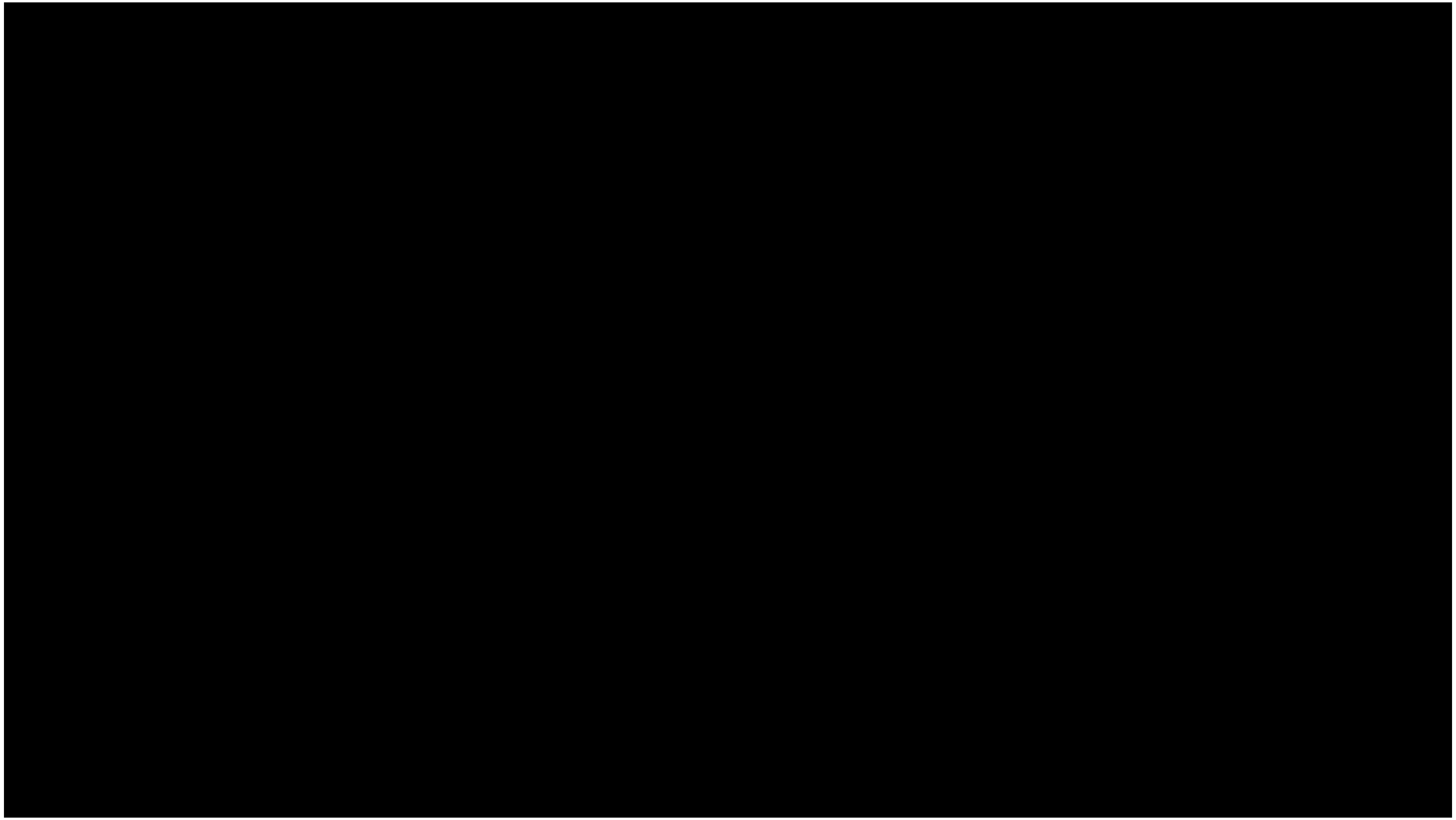
$$u_1 = \sqrt{\frac{2\Delta p_{12}}{\rho\left(\left(\frac{A_1}{A_2}\right)^2 - 1\right)}}$$

→ If A_1, A_2 and ρ is known we can measure the flow velocity u_1 by measuring the differential static pressure.

Applications: Venturi-Meter



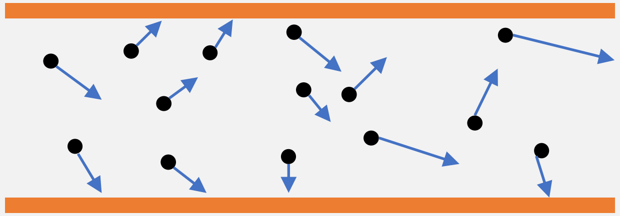
Applications: Venturi-Meter



Applications: Pitot-Tube

How else can we measure flow velocity, if we cannot have two cross sections area?

Moving fluid



→ Pressure is anisotropic

Vertical pressure ↑ ≠ horizontal pressure →

Static pressure | Static + dynamic pressure

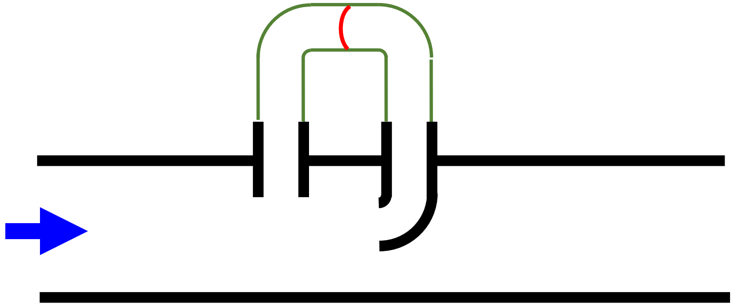
→ Simultaneous measurement of horizontal and vertical pressure

$$p_{stagnation} = p_{static} + p_{dynamic}$$

$$p_{stagnation} = p_{stat} + \frac{1}{2}\rho u^2$$

$$u = \sqrt{\frac{2}{\rho}\left(p_{stagnation} - p_{static}\right)}$$

$p_{static} < p_{stagnation}$



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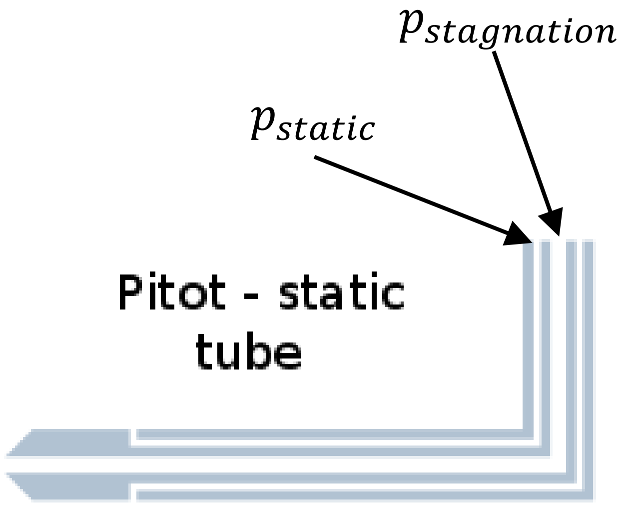
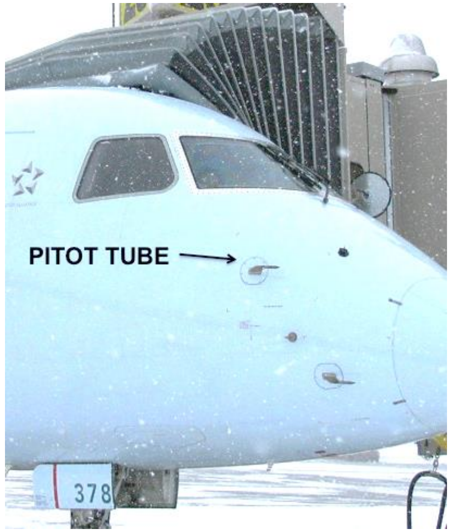
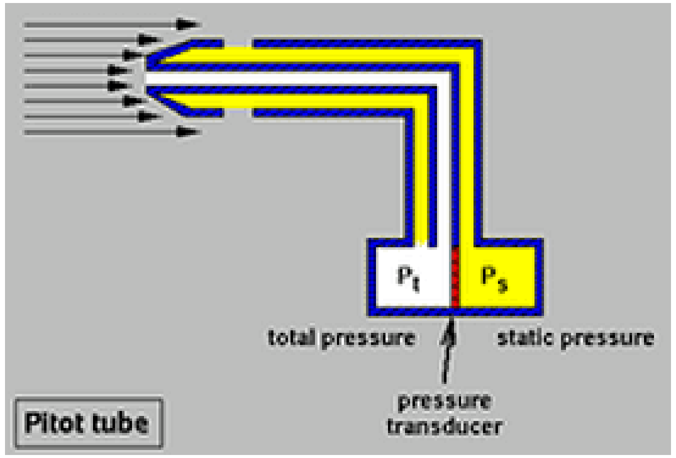
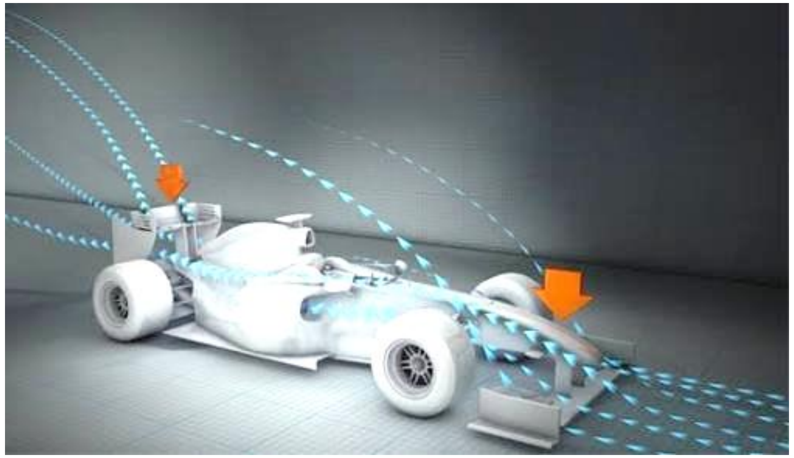








http://www.formula1-dictionary.net/pitot_tube.html

Differential pressure MEMS sensors - Principles

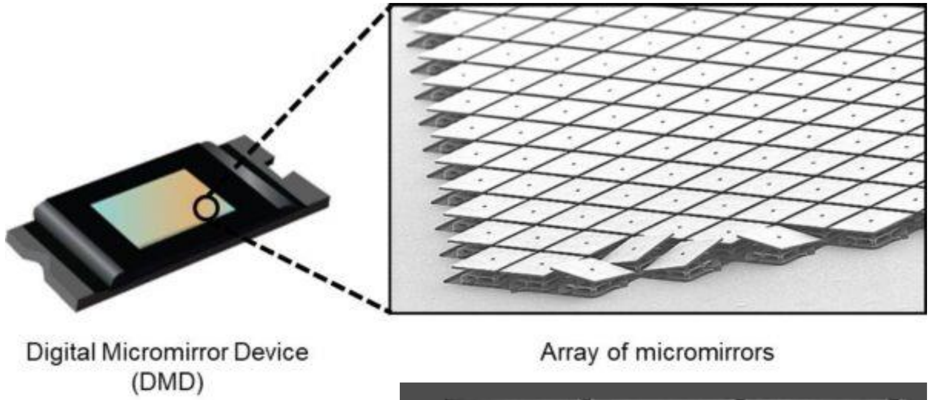
MEMS: Micro-**E**lectro-**M**echanical **S**ystem

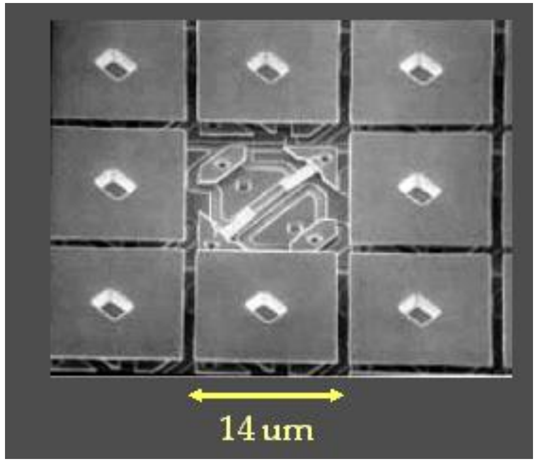
What means **Micro**?
→ In most cases components range from ~ 1 µm – 100 µm
→ Devices range from 10 µm – some mm

What means **Electro**?
→ Electronics are inbuild…
- as transducers
- for signal processing (e.g., amplifier, ADC, mixer,….)

What means **Mechanical**?
→ The system has mechanical components either as
- Actuator
- Sensor

What means **System?**
→ All components are fully embedded, interfaced and optimised





https://en.wikipedia.org/wiki/Digital_Light_Processing

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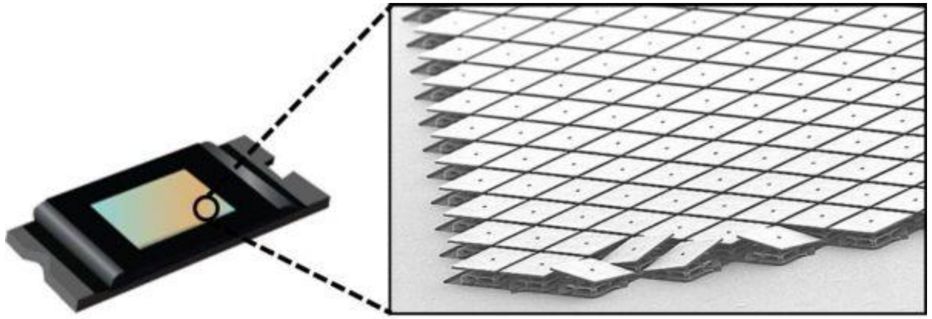
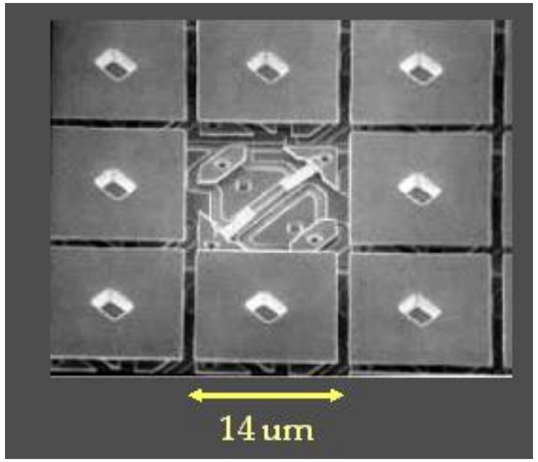


Digital Micromirror Device (DMD)

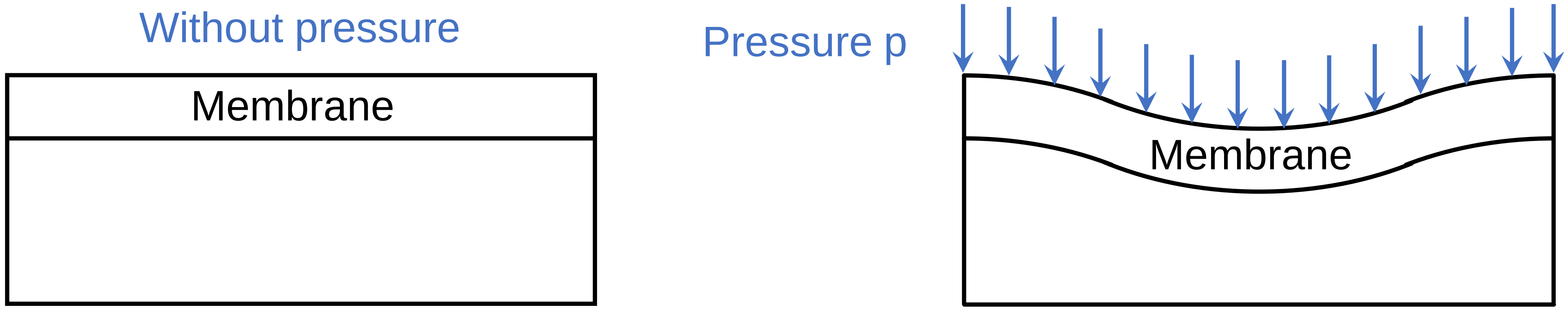
Array of micromirrors



Differential pressure MEMS sensors - Principles

Goal of pressure sensor:
Transformation of a pressure **p** into an electrical signal (mostly a voltage **U**)

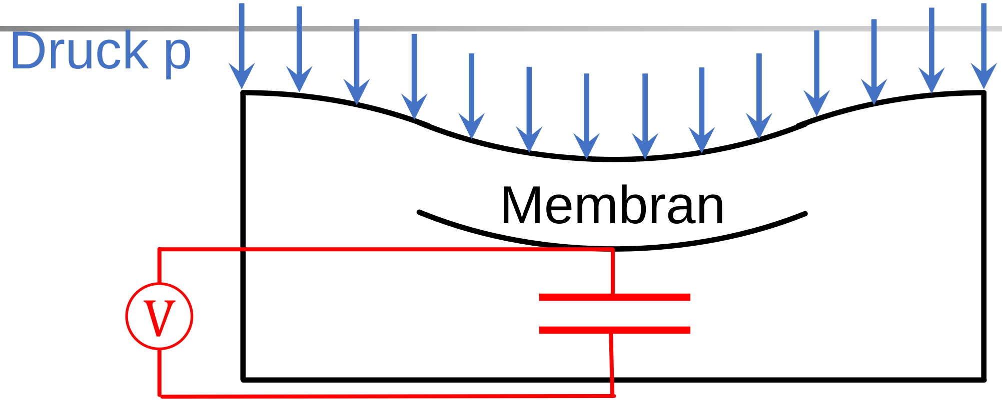
Option 1:
Measurement of an elastic deformation of a gas-tight (hermetic seal) or liquid tight membrane



Differential pressure MEMS sensors - Principles

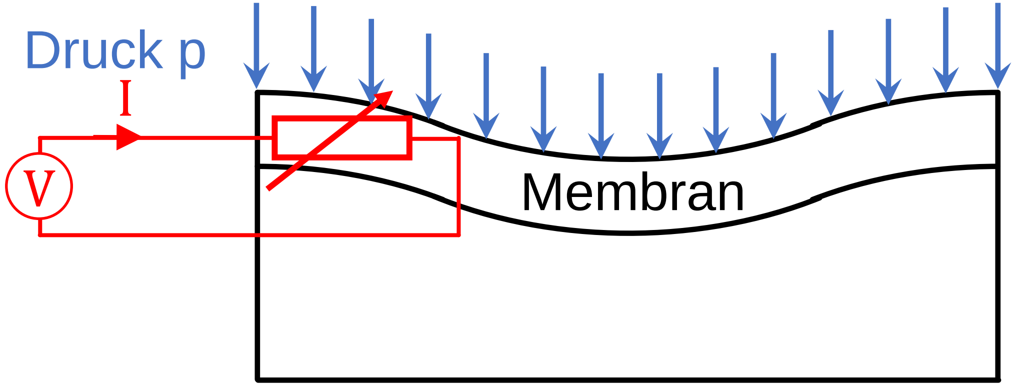
1. Capacitive

→ Measurement of the distance of two capacitor electrode plates.



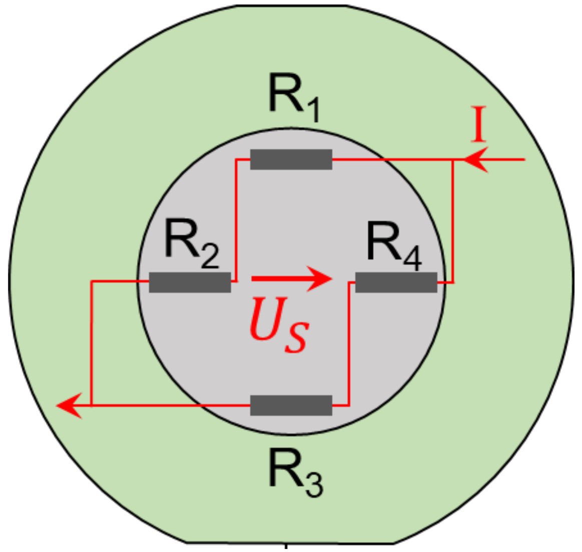
2. **Piezoresistive measurement**

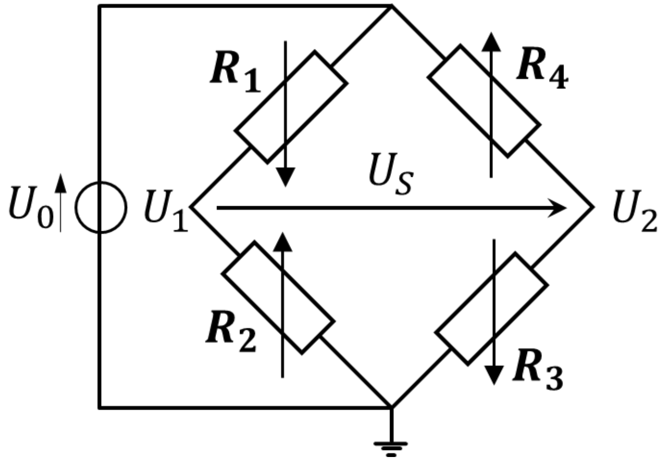
→ Mechanical deformation leads to a change in the resistance of a material. This can be detected using an electrical circuit.



Differential pressure MEMS sensors - Principles

Top View membrane



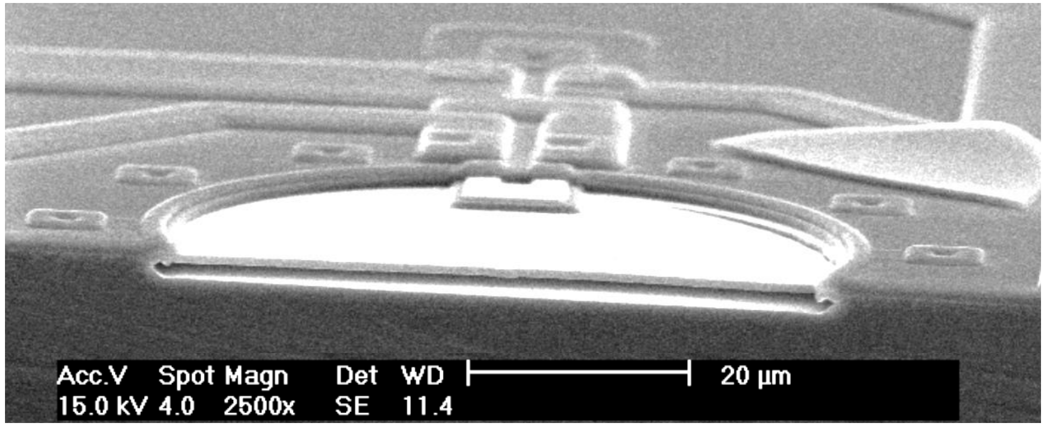


Fixed Frame

Membrane

$$S = \frac{U_S}{\Delta p} = U_0 \bar{\pi} \left[\frac{3{R_M}^2(1-\nu)}{4t^2} \right]$$

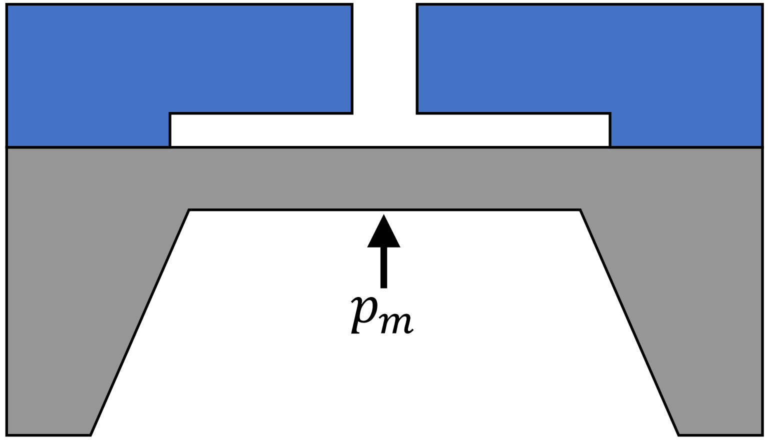
S	Sensitivity
$\bar{\pi}$	Piezoresistive coeffcient
Δp	Pressure difference
R_m	Radius Membrane
ν	Poisson ratio (ratio of transverse strain to axial strain)
t	Thickness membrane



Differential pressure MEMS sensors - Principles

Sensor for relative pressure

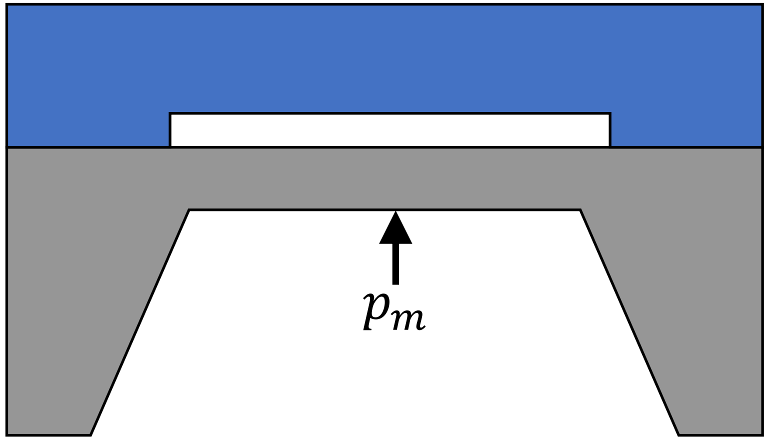
p_{ref} = environmental pressure



- Tyre pressure
- Measurement of flow (Pitot-tube)

Sensor for aboslute pressure

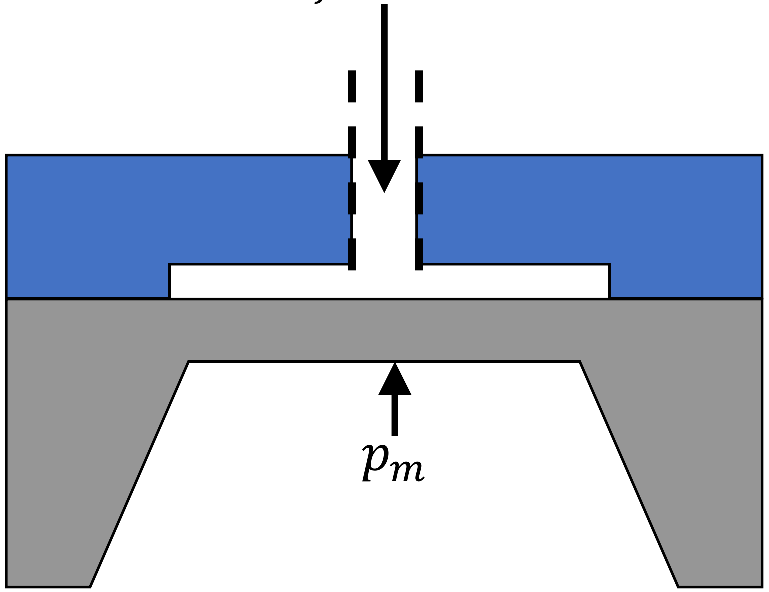
p_{ref} = 0 bar



- Barometric measurements
- Altitude meter
- Vaccum technology (e.g., packaging technology)

Sensor for difference pressure

p_{ref} = x bar

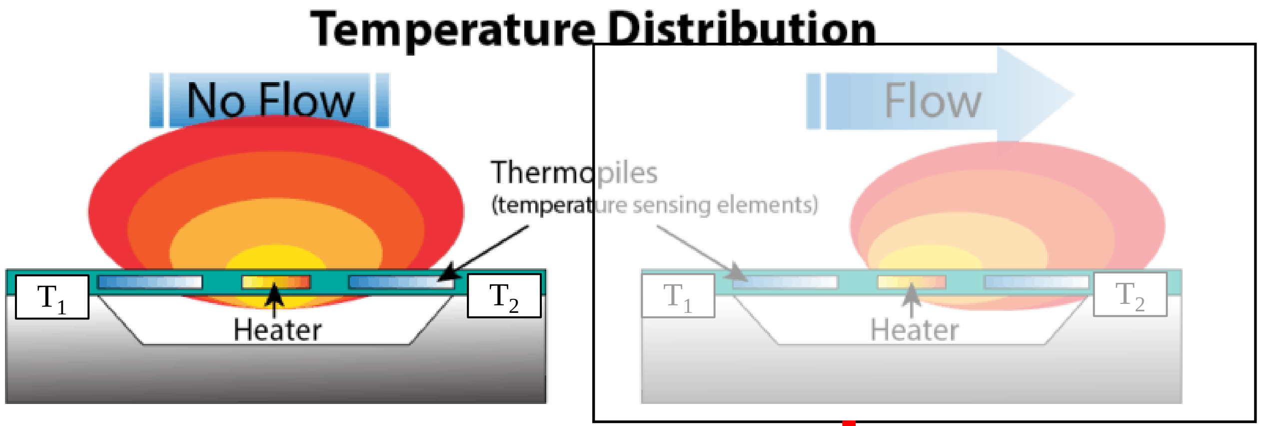


- Medical systems
- Cooling and heating technology
- Venturi measurement

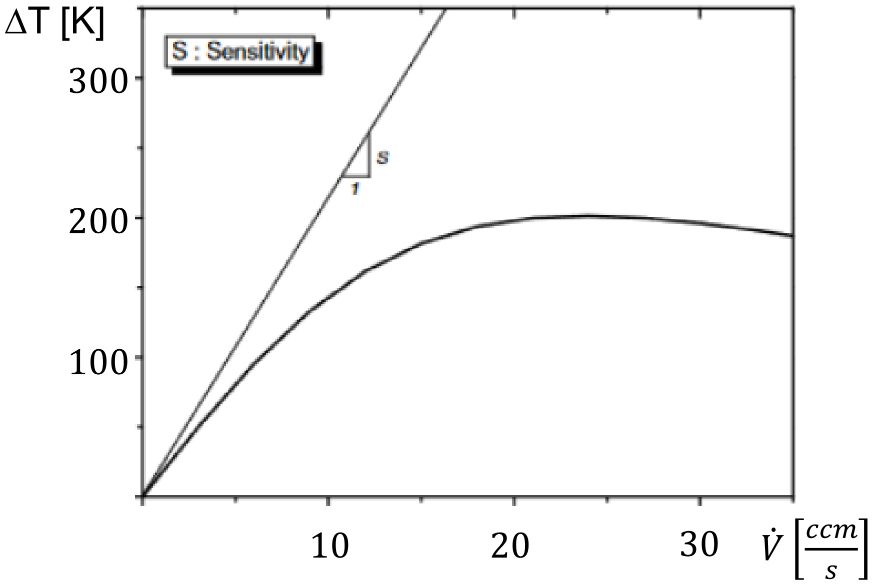
Differential pressure MEMS sensors - Principles

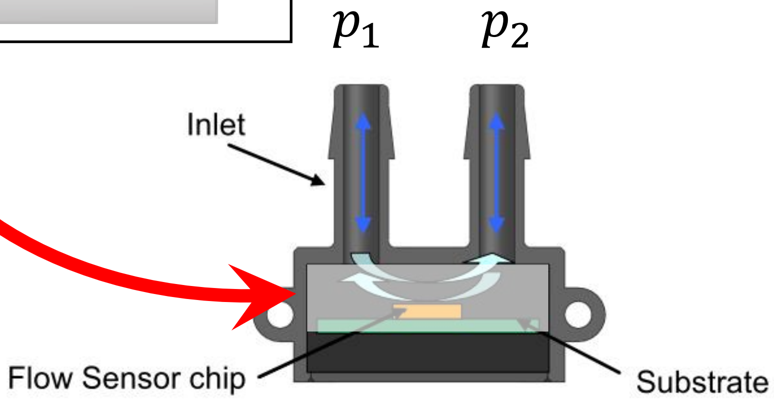
Goal of pressure sensor:
Transformation of a pressure **p** into an electrical signal

Option 2:
Thermal mass flow meter



https://www.alicat.com/knowledge-base/types-of-gas-flow-meters/





From slide 8:

$$\Delta p = p_2 - p_1 = R_f \dot{V}(\Delta T)$$

Differential pressure MEMS sensors - Principles

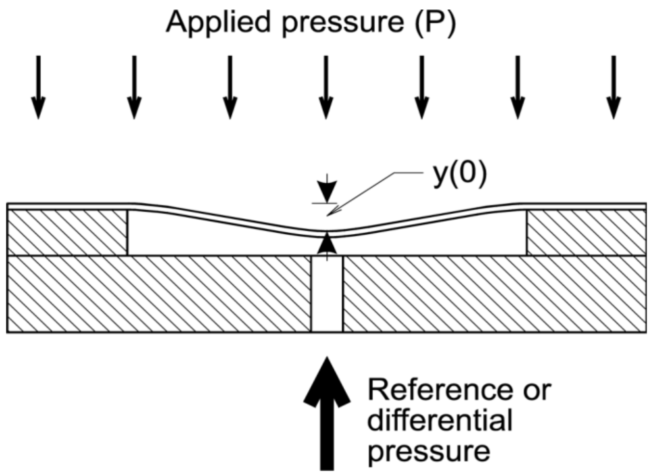
Advantages:

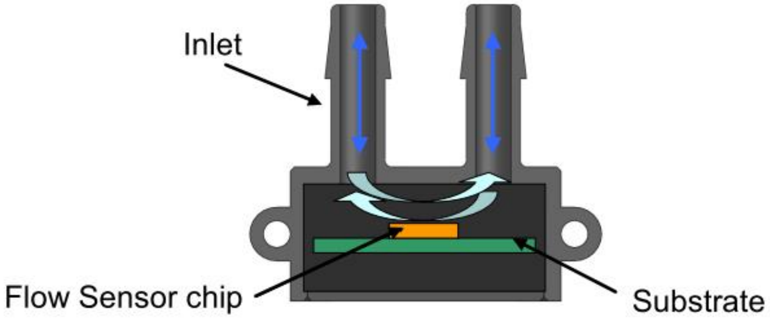
Membrane base pressure sensor

- Highly sensitive
- Absolute/Relative/Difference pressure sensor
- Measure static pressure from small reservoirs

Thermal mass flow pressure sensor

- Highly sensitive
- Mechanically stable against overpressure
- No moving parts





Differential pressure MEMS sensors - Principles

https://www.apogeeinstruments.com/content/SB-100-manual.pdf

https://omronfs.omron.com/en_US/ecb/products/pdf/en-D6F-PH_users_manual.pdf