Assignment-1

AI1110: Probability and Random Variables

Indian Institute of Technology Hyderabad

Arugonda Srikar
CS22BTECH11008

Question:12.13.3.4: In answering a question on a multiple choice test, a student either knows the answer or guesses. Let $\frac{3}{4}$ be the probability that he knows the answer and $\frac{1}{4}$ be the probability that he guesses. Assuming that a student who guesses at the answer will be correct with probability $\frac{1}{4}$. What is the probability that the student knows the answer given that he answered it correctly?

Solution:

let $X \in \{0, 1\}$ $X = 0$ means he guesses the answer
$X = 1$ means he knows the answer
let $Y \in \{0, 1\}$ $Y = 0$ means the answer is incorrect
$Y = 1$ means the answer is correct
We need to find $\Pr(X = 1 \mid Y = 1)$ From given information,

$$\Pr(X = 0) = \frac{1}{4} \tag{1}$$

$$\Pr(X = 1) = \frac{3}{4} \tag{2}$$

$$\Pr(Y = 1 \mid X = 0) = \frac{\frac{1}{4} \times \frac{1}{4}}{\frac{1}{4}} = \frac{1}{4} \tag{3}$$

$$\Pr(Y = 1 \mid X = 1) = \frac{1 \times \frac{3}{4}}{\frac{3}{4}} = 1 \tag{4}$$

$$\Pr(X = 1 \mid Y = 1) = \frac{\Pr(Y = 1 \mid X = 1)\Pr(X = 1)}{\Pr(Y = 1 \mid X = 1)\Pr(X = 1) + \Pr(Y = 1 \mid X = 0)\Pr(X = 0)} \tag{5}$$

$$= \frac{1 \times \frac{3}{4}}{1 \times \frac{3}{4} + \frac{1}{4} \times \frac{1}{4}} \tag{6}$$

$$= \frac{12}{13} \tag{7}$$

$$\therefore \quad \Pr(X = 1 \mid Y = 1) = \frac{12}{13} \tag{8}$$