

Assignment-1

AI1110: Probability and Random Variables

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Question:12.13.2.15: One card is drawn at random from a well shuffled deck of 52 cards. In which of the following cases are the events E and F independent ?

- (i) E : 'the card drawn is a spade'
 F : 'the card drawn is an ace'
- (ii) E : 'the card drawn is black'
 F : 'the card drawn is a king'
- (iii) E : 'the card drawn is a king or queen'
 F : 'the card drawn is a queen or jack'.

Solution:

- (i) E denotes the event that the card drawn is spade

$$\Pr(E) = \frac{13}{52} = \frac{1}{4} \quad (1)$$

F denotes the event that card drawn is ace

$$\Pr(F) = \frac{4}{52} = \frac{1}{13} \quad (2)$$

$$\Pr(EF) = \frac{1}{52} \quad (3)$$

$$\Pr(E) \Pr(F) = \frac{1}{4} \times \frac{1}{13} = \frac{1}{52} \quad (4)$$

$$\therefore \Pr(EF) = \Pr(E) \Pr(F) \quad (5)$$

$\therefore E$ and F are independent events.

- (ii) E denotes the event that the card drawn is black

$$\Pr(E) = \frac{26}{52} = \frac{1}{2} \quad (6)$$

F denotes the event that card drawn is a king

$$\Pr(F) = \frac{4}{52} = \frac{1}{13} \quad (7)$$

$$\Pr(EF) = \frac{2}{52} = \frac{1}{26} \quad (8)$$

$$\Pr(E) \Pr(F) = \frac{1}{2} \times \frac{1}{13} = \frac{1}{26} \quad (9)$$

$$\therefore \Pr(EF) = \Pr(E) \Pr(F) \quad (10)$$

$\therefore E$ and F are independent events.

- (iii) E denotes the event that the card drawn is king or queen

$$\Pr(E) = \frac{8}{52} = \frac{2}{13} \quad (11)$$

F denotes the event that card drawn is a queen or jack

$$\Pr(F) = \frac{8}{52} = \frac{2}{13} \quad (12)$$

$$\Pr(EF) = \frac{4}{52} = \frac{1}{13} \quad (13)$$

$$\Pr(E) \Pr(F) = \frac{2}{13} \times \frac{2}{13} = \frac{4}{169} \quad (14)$$

$$\therefore \Pr(EF) \neq \Pr(E) \Pr(F) \quad (15)$$

$\therefore E$ and F are not independent events.