

# MATHEMATICS

## SECTION A

January 31, 2024

### 1 Vector Algebra

1. Find the magnitude of each of the vectors  $\vec{a}$  and  $\vec{b}$ , having the same magnitude such that the angle between them is  $60^\circ$  and their scalar product is  $\frac{9}{2}$ .
2. If  $\theta$  is the angle between the two vectors  $\hat{i} - 2\hat{j} + 3\hat{k}$  and  $3\hat{i} - 2\hat{j} + \hat{k}$ , find  $\sin \theta$ .

### 2 Trigonometry

3. Find the value of  $\tan^{-1} \sqrt{3} - \cot^{-1} (\sqrt{-3})$ .
4. Prove that:  $3 \sin^{-1} x = \sin^{-1} (3x - 4x^3)$ ,  $x \in \left(-\frac{1}{2}, \frac{1}{2}\right)$

### 3 Functions

5. If  $a * b$  denotes the larger of 'a' and 'b' and if  $aob = (a * b) + 3$ , then write the value of  $(5) o (10)$ , where  $*$  and  $o$  are binary operations.

### 4 Matrices

6. If the matrix  $A = \begin{pmatrix} 0 & a & -3 \\ 2 & 0 & -1 \\ b & 1 & 0 \end{pmatrix}$  is skew symmetric, find the values of 'a' and 'b'.

### 5 Probability

7. A black and a red die are rolled together. Find the conditional probability of obtaining the sum 8, given that the red die resulted in a number less than 4.

### 6 Differential Equation

8. Find the differential equation representing the family of curves  $y = ae^{bx+5}$ , where  $a$  and  $b$  are arbitrary constants.

## 7 Integration

9. Evaluate:

$$\int \frac{\cos 2x + 2 \sin^2 x}{\cos^2 x} dx$$

10. Using the integration, find the area of the region in the first quadrant enclosed by the X-axis, the line  $y = x$  and the circle  $x^2 + y^2 = 32$ .

## 8 Calculus

11. The total cost  $C(x)$  associated with the production of  $x$  units of an item is given by  $C(x) = 0.005x^3 - 0.02x^2 + 30x + 5000$ . Find the material cost when 3 units are produced, where by marginal cost we mean the instantaneous rate of change of total cost at any level of output.

## 9 Differentiation

12. Differentiate  $\tan^{-1}\left(\frac{1+\cos x}{\sin x}\right)$  with respect to  $x$ .
13. If  $(x^2 + y^2)^2 = xy$ , find  $\frac{dy}{dx}$

## 10 Geometry

14. An open tank with a square base and vertical sides is to be constructed from a metal sheet so as to hold a given quantity of water. Show that the cost of material will be least when depth of the tank is half of its width. If the cost is to be borne by nearby settled lower income families, for whom water will be provided, what kind of value is hidden in this question ?

## 11 Vectors

15. Find the shortest distance between the lines  $\vec{r} = (4\hat{i} - \hat{j}) + \lambda(\hat{i} + 2\hat{j} - 3\hat{k})$  and  $\vec{r} = (\hat{i} - \hat{j} + 2\hat{k}) + \mu(2\hat{i} + 4\hat{j} - 5\hat{k})$ .

## 12 Conics

16. Find the equations of the tangent and the normal, to the curve  $16x^2 + 9y^2 = 145$  at the point  $(x_1, y_1)$ , where  $x_1 = 2$  and  $y_1 > 0$

## 13 Linear Programming

17. A factory manufactures two types of screws  $A$  and  $B$ , each type requiring the use of two machines, an automatic and a hand-operated. It takes 4 minutes on the automatic and 6 minutes on the hand-operated machines to manufacture a packet of screws ' $A$ ' while it takes 6 minutes on the automatic and 3 minutes on the hand-operated machine to manufacture a packet of screws ' $B$ '. Each machine is available for at most 4 hours on any day. The manufacturer can sell a packet of screws ' $A$ ' at a profit of 70 paise and screws ' $B$ ' at a profit of 1. Assuming that he can sell all the screws he manufactures, how many packets of each type should the factory owner produce in a day in order to maximize his profit.