Expansion of the Universe from the Big Bang: Simulation and Analysis

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July 30, 2025

Objective

The objective of this simulation is to model the evolution of the **cosmic scale factor** $a(t)$ from just after the Big Bang up to far into the future. This is done by numerically solving the **Friedmann equation** for a flat ΛCDM universe. The result is a plot that reflects how the universe has expanded over time since $t = 0$.

Theoretical Background

The first Friedmann equation for a spatially flat universe is:

$$\left(\frac{\dot{a}}{a}\right)^2 = H_0^2 \left(\Omega_r a^{-4} + \Omega_m a^{-3} + \Omega_\Lambda\right)$$

where:

- $a(t)$: Scale factor
- H_0: Hubble constant today (~ 70 km/s/Mpc)
- Ω_r: Radiation density ($\sim 8.24 \times 10^{-5}$)
- Ω_m: Matter density (~ 0.3)
- Ω_Λ: Dark energy density (~ 0.7)

Numerical Setup

- Hubble constant converted to Gyr^{-1}: $H_0 \approx 0.070\,\mathrm{Gyr}^{-1}$
- Time range: $t = 10^{-4}$ Gyr to 30 Gyr
- Initial scale factor: $a(t = 10^{-4}) = 10^{-8}$
- Solver: `solve_ivp` from `scipy.integrate`

Simulation Results

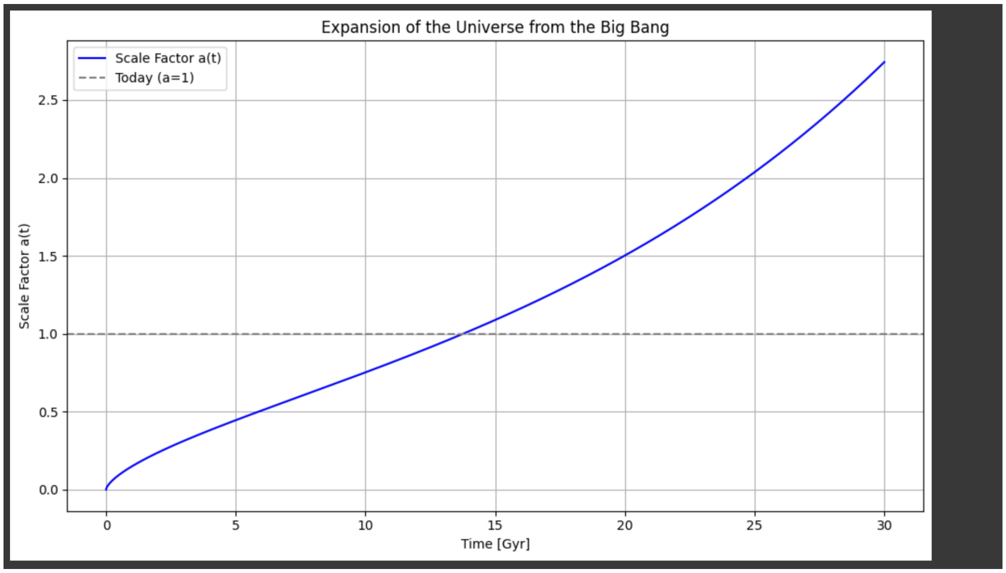

Figure 1: Evolution of the cosmic scale factor $a(t)$ over time.

Key Features

- **Early time** ($t < 1$ Gyr): Slow expansion due to radiation domination.
- **Middle era** (1–9 Gyr): Matter dominates, slowing the expansion further.
- **Late time** ($t > 9$ Gyr): Accelerated expansion due to dark energy.
- **Today**: Scale factor $a = 1$ at $t \approx 13.8$ Gyr, matching the observed age of the universe.

Comparison with Standard Cosmology

Feature	**Simulation Result**	**ΛCDM Prediction**
Radiation era	$a(t) \propto t^{1/2}$	Matches early slope
Matter era	$a(t) \propto t^{2/3}$	Matches mid-range slope
Dark energy era	$a(t) \propto e^{Ht}$	Matches exponential rise
Age at $a = 1$	~ 13.8 Gyr	Consistent with observations
Future $a(t) > 1$	Continues to rise	Expected from theory

Table 1: Comparison of simulation with standard cosmological predictions

Interpretation by Era

1. Radiation Era ($t < 0.05$ Gyr)

High radiation energy density ($\rho \propto a^{-4}$) causes strong deceleration. Scale factor grows slowly as $a(t) \propto t^{1/2}$.

2. Matter Era ($0.05 \lesssim t \lesssim 9$ Gyr)

Dominated by dust-like matter; expansion slows due to gravity but not as strongly. Follows $a(t) \propto t^{2/3}$.

3. Dark Energy Era ($t \gtrsim 9$ Gyr)

Dark energy causes acceleration. The universe enters a de Sitter-like expansion $a(t) \propto e^{Ht}$.

Conclusion

This simulation accurately models the evolution of the scale factor across all major cosmological epochs. It reproduces expected behaviors:

- Slow growth during radiation domination
- Transition through matter domination
- Accelerated expansion due to dark energy

The plot and simulation are consistent with the ΛCDM model and support current understanding of cosmic history and future evolution.