

ANALYSIS OF ALGORITHM

TYPES OF ALGORITHM

Solutions

1. If $T(n) = 2T(n/2) + \log n$, for $n \geq 2$ and $T(1) = 0$, then $T(n)$ is :

- (a) $O(n)$ (b) $O(n \log n)$
(c) $O(n (\log n)^2)$ (d) $O(n^2)$

Solution: Option (c)

Apply Master's theorem

2. Let $T(n) = T(n-1) + 2^n$ for $n > 1$ and $T(1) = 1$. Then $T(n)$ is _____ [choose tightest upper bound]

- (a) $O(n 2^n)$ (b) $O(2^{2n})$
(c) $O(2^n)$ (d) None of these

Solution: Option (c)

$$\begin{aligned} T(n) &= T(n-1) + 2^n \\ &= T(n-2) + 2^{n-1} + 2^n \\ &= T(n-3) + 2^{n-2} + 2^{n-1} + 2^n \\ &= T(1) + 2^2 + 2^3 + \dots + 2^{n-1} + 2^n \end{aligned}$$

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$$\begin{aligned} &= 1 + \sum_{i=2}^n 2^i \\ &= 1 + \frac{2^{n+1} - 1}{2 - 1} - 3 \\ &= 2^{n+1} - 3 \\ &= O(2^n) \end{aligned}$$

3. Find the length of the longest common subsequence of "algorithmsexam" and "cseonlinetest".

- (a) 2
(c) 4

- (b) 3
(d) 5

Solution: Option (c)

4. The number of comparisons required to find maximum and minimum in the given array of n-element using divide and conquer:

(a) $\left\lfloor \frac{3n}{2} \right\rfloor$

(b) $\left\lceil \frac{3n}{2} \right\rceil$

(c) $\left\lfloor \frac{3n}{2} \right\rfloor + 2$

(d) $\left\lceil \frac{3n}{2} \right\rceil - 2$

Solution: Option (d)

The recurrence relation is:

$$T(n) = 2T\left(\frac{n}{2}\right) + 2$$

5. Assume an array $A[1 \dots n]$ has n-elements, and every element of an array is less than or equal to n. An element is said to be majority element, if it occurred in more than $n/2$ positions of an array. What is the best time complexity to check whether the majority of elements exist or not?

- (a) $O(\log n)$
(c) $O(n \log n)$

- (b) $O(n)$
(d) $O(n^2)$

Solution: Option (b)

It can be done by Moore's Voting Algorithm.

6. Suppose the given array of size n is sorted other than finding 10 elements and last 50 elements. Find the sorting algorithm which can run faster than other algorithms.

- (a) Merge sort
(c) Bubble sort

- (b) Quick sort
(d) Insertion sort

Solution: Option (d)

7. Given an array of distinct integers $A[1, 2, \dots, n]$. Find the tightest upper bound to check the existence of any index i for which $A[i] = i$.

- (a) $O(1)$
- (b) $O(\log n)$
- (c) $O(n)$
- (d) None of these

Solution: Option (c)

Linear search.

8. Which of the following problem may not give optimal solution by greedy strategy?

- (a) 0/1 Knapsack problem
- (b) Single source shortest path
- (c) Huffman coding
- (d) None of these

Solution: Option (a)

Fractional Knapsack can be solved by greedy method.

9. Let $f(n) = \Omega(n)$, $g(n) = O(n)$ and $h(n) = \Theta(n)$. Then $[f(n) \cdot g(n)] + h(n)$ is:

- (a) $\Omega(n)$
- (b) $O(n)$
- (c) $\Theta(n)$
- (d) None of these

Solution: Option (a)

10. What is the time complexity of travelling salesman problem with n vertices using dynamic programming?

- (a) Linear time
- (b) Logarithmic time
- (c) Exponential time
- (d) Quadratic time

Solution: Option (c)

11. Running time of 0/1 Knapsack problem using dynamic programming is _____. Assume n is the no. of items and m is the capacity of Knapsack.

- (a) $O(n^2)$
- (b) $O(n^w)$
- (c) $O(w^n)$
- (d) $O(nw)$

Solution: Option (d)

12. If graph contains negative weight edges then which of the following is correct when we run dijkstra's algorithm?

- (a) It may not terminate
- (b) It terminates but may produce incorrect result
- (c) It never terminates due to cycle in graph
- (d) None of these

Solution: Option (b)

It terminates after $|E|$ relations and $|V| + |E|$ priority queue operations, but may produce incorrect results.

13. An array has elements which have been partitioned by first pass of quicksort. If the following elements are in the array, then find the pivot element chosen by quicksort during first pass.

15 11 12 10 13 8 7 19

Solution: 19

14. Consider the following function

```
find (int n)
{
    if (n < 2 ) then return;
    else
    {
        sum= 0;
        for (i= 1; i ≤ 4; i++)
            find ( $\frac{n}{2}$ );
        for (i=1; i≤ n*n; i++)
            sum= sum + 1;
    }
}
```

Assume that the division operation takes constant time and “sum” is global variable. What is the time complexity of “find (n)” ?

- (a) $\Theta(n^2)$
 (c) $\Theta(n^3)$

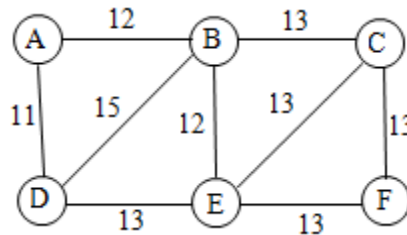
- (b) $\Theta(n^2 \log n)$
 (d) None of these

Solution: Option (b)

$$T(n) = 4T\left(\frac{n}{2}\right) + n^2 + 1$$

$$\therefore \Theta(n^2 \log n)$$

15. Consider the following graph G:



Find the no. of minimum cost spanning tree using Kruskal's or Primus algorithm.

- (a) 3
 (c) 7
 (b) 5
 (d) 4

Solution: Option (b)

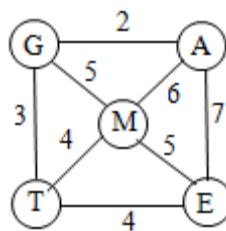
16. Finding Hamiltonian cycle in a graph requires _____ running time.

- (a) polynomial
 (c) both (a) and (b)
 (b) non-polynomial
 (d) none of these

Solution: Option (b)

Hamiltonian problem is NF-complete.

17. Assume Dijkstra's algorithm to find the shortest paths from node 'a' in following graph:



Find the no. of edges not included in any of the shortest paths from G.

Solution: 4

Edge (T, M), (M, E), (M, A) and (A, E) are not included.

18. Consider the following message:

aabbbbabccddccccbbdd

Find the number of bits required for Huffman encoding of the above message:

- (a) 30
- (b) 38
- (c) 42
- (d) 46

Solution: Option (c)

19. If Huffman tree coded as left child with '0' and right child with '1' from every node then what is the decoded message for 110100?

- (a) abc
- (b) bcd
- (c) acb
- (d) bda

Solution: Option (d)

20. Let $T(n) = T(n-1) + 1/n$.

Then $T(n)$ is:

- (a) $\Theta(1)$
- (b) $\Theta(\log n)$
- (c) $\Theta(\log \log n)$
- (d) $\Theta(n)$

Solution: Option (b)