ALGORITHMS
MINIMUM SPANNING TREES (MST)

SOLUTIONS

1. In a simple undirected graph with ‘n’ vertices maximum degree for each vertex is

(a) n
(b) n – 1
(c) n (n – 1)
(d) Infinite

Solution: Option (b)

Explanation:

In a simple undirected graph, self- loops and parallel edges are allowed. So, for each vertex, there can be a maximum of n – 1 edges forming a complete graph.

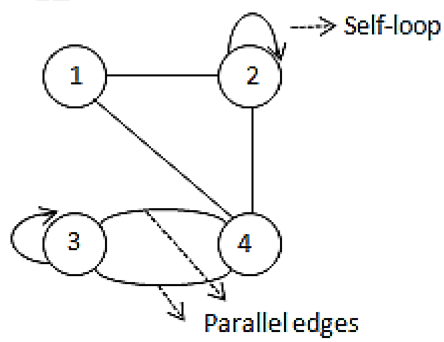
2. In a multigraph with ‘n’ vertices, maximum degree for each vertex is

(a) n
(b) n – 1
(c) n (n – 1)
(d) Infinite

Solution: Option (d)

Explanation:

Self- loops and parallel edges are allowed in a multigraph, so there can be infinite number of edges for each vertex.



3. Which of the following is/are True about Simple graphs?

(i) If the degree of each vertex is zero in a graph, then the graph is called Null graph
(ii) A 3-regular graph with 4 vertices in planar

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(iii) Number of edges in a 4-regular graph with 5 vertices is 10

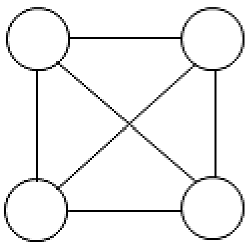
(a) only (i) & (ii)
(b) only (ii) & (iii)
(c) (i), (ii) & (iii)
(d) None

Solution: Option (c)

Explanation:

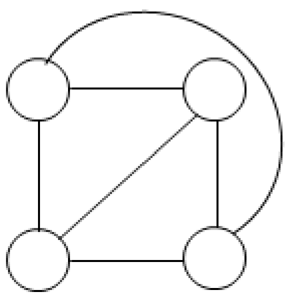
(i) is the definition of Null graph.

(ii)

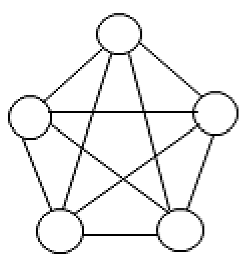


is a 3-regular graph with 4 vertices (each vertex has 3 edges).

This is planar→



(iii) A 4-regular graph with 5 vertices is K_5 (complete graph with 5 vertices).



This can be found using
No. of Edges = $\frac{n*d}{2} = \frac{5*4}{2} = 10$
where, n is the number of vertices
d is the degree of each vertex in the d-regular graph

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4. Which of the following is True about Spanning Tree?

(i) A spanning tree is a connected graph.
(ii) Number of edges in a spanning tree with n-vertices is n – 1.
(iii) Spanning tree contains cycles.
(iv) Spanning tree has directed edges.

(a) only (i) & (ii)
(b) only (i) & (iii)
(c) only (iii) & (iv)
(d) only (ii) & (iv)

Solution: Option (a)

5. Number of spanning trees for K_4 is:

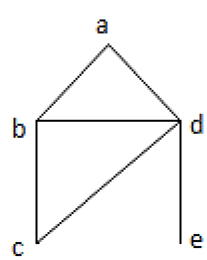
(a) 4
(b) 8
(c) 16
(d) 32

Solution: Option (c)

Explanation:

No. of spanning trees in an n-vertex complete graph is n^{n-2}.
$\therefore 4^{4-2} = 16$

6. Number of spanning trees for the following graph is



(a) 5
(b) 6

(c) 8
(d) 9

Solution: Option (c)

Explanation:

We can use Kirchoff's theorem for finding the number of spanning trees in a simple graph.

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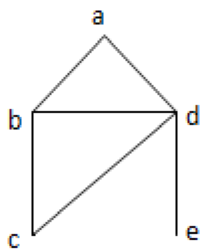
Steps in Kirchoff's Approach:

(i) Make an Adjacency matrix.
(ii) Replace all non-diagonal is by – 1.
(iii) Replace all diagonal zero's by the degree of the corresponding vertex.
(iv) Co-factors of any element will give the number of spanning trees.



$$\begin{array}{c} \\ a \\ b \\ c \\ d \\ e \end{array}\begin{array}{c} \begin{array}{ccccc} a & b & c & d & e \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

$$\begin{bmatrix} 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

(ii) Replace all non-diagonal is by – 1

(iii) Replace all diagonal zeros by the degree of corresponding vertices

$$\Downarrow$$

$$\begin{bmatrix} 2 & -1 & 0 & -1 & 0 \\ -1 & 3 & -1 & -1 & 0 \\ 0 & -1 & 2 & -1 & 0 \\ -1 & -1 & -1 & 4 & -1 \\ 0 & 0 & 0 & -1 & 1 \end{bmatrix}$$

(iv) Co-factor of any element gives the number of spanning trees for the given graph.

Co-factor of $C_{44} - (-1)^{4+4} \det(Adj(A_{44}))$

$$\left|\begin{bmatrix} 2 & -1 & 0 & 0 \\ -1 & 3 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & 0 & 1 \end{bmatrix}\right| = 8$$

∴ There are 8 spanning trees for the given graph.

7. Given a weighted undirected graph. What would be the resultant spanning tree of Kruskal's algorithm?

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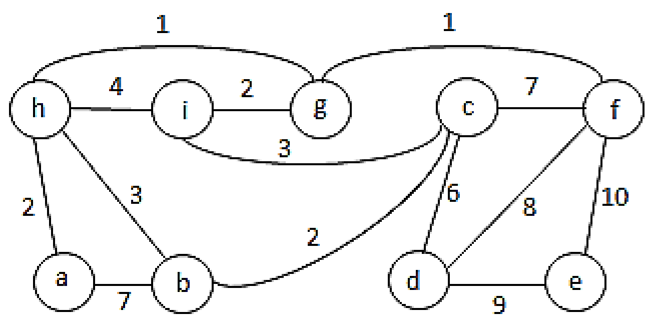
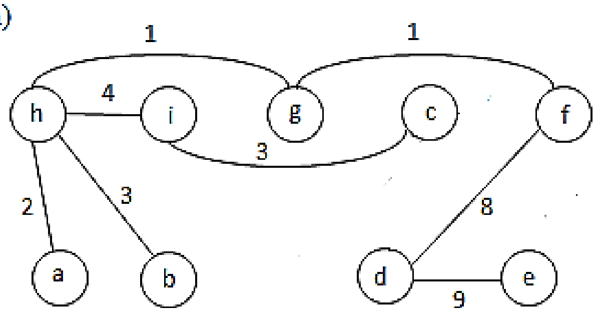
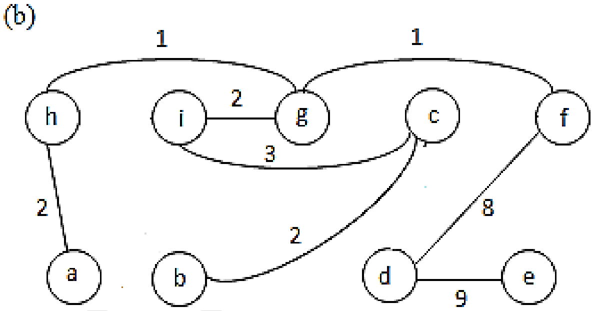


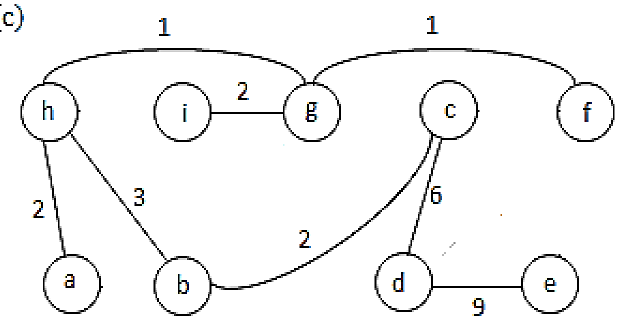
(a)



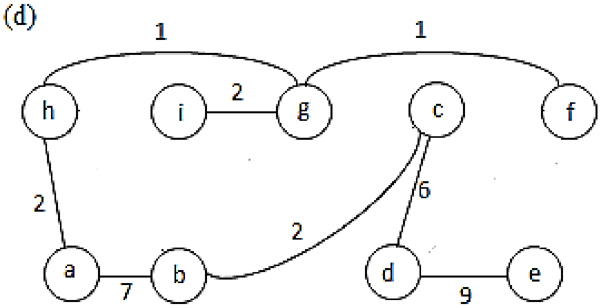
(b)



(c)

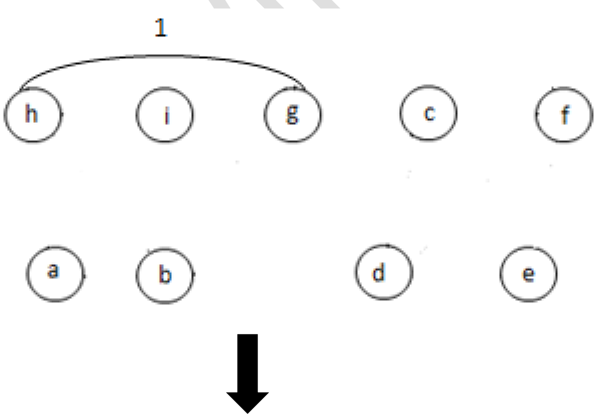


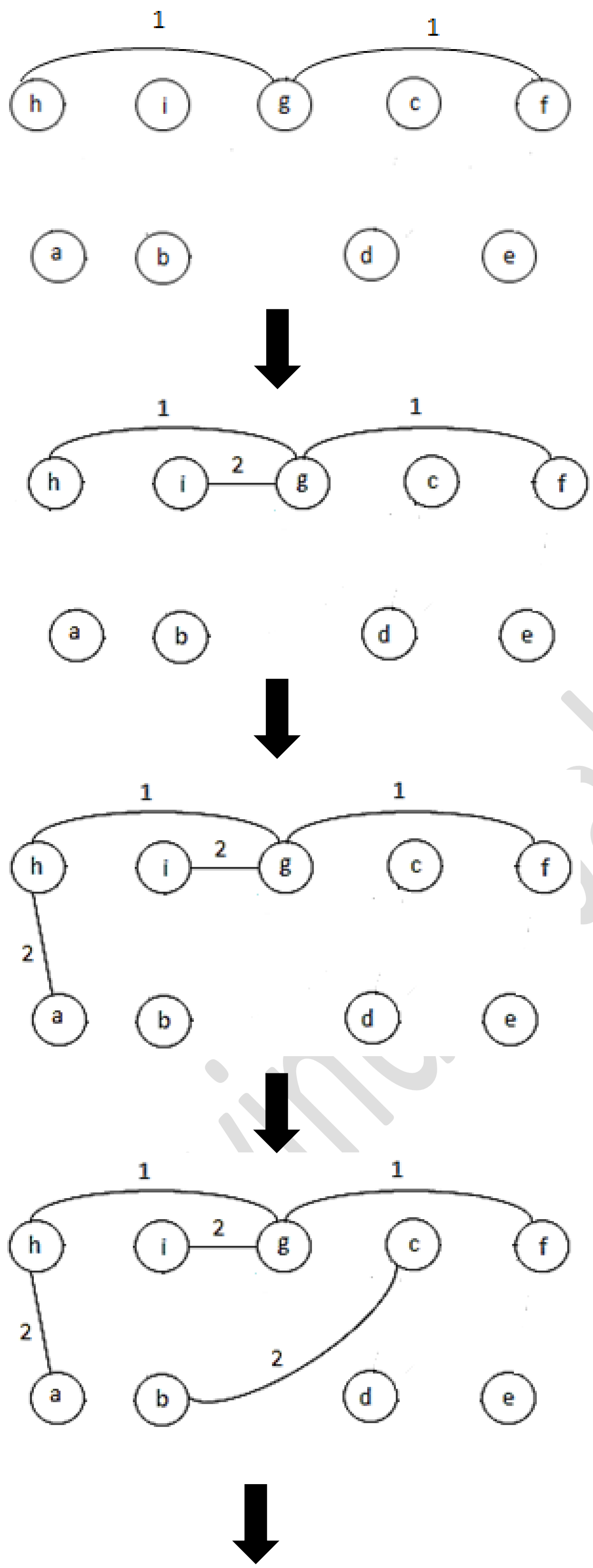
(d)



Solution: Option (c)

Explanation:




1 1
h i g c f
a b d e
1 1
h i 2 g c f
a b d e
1 1
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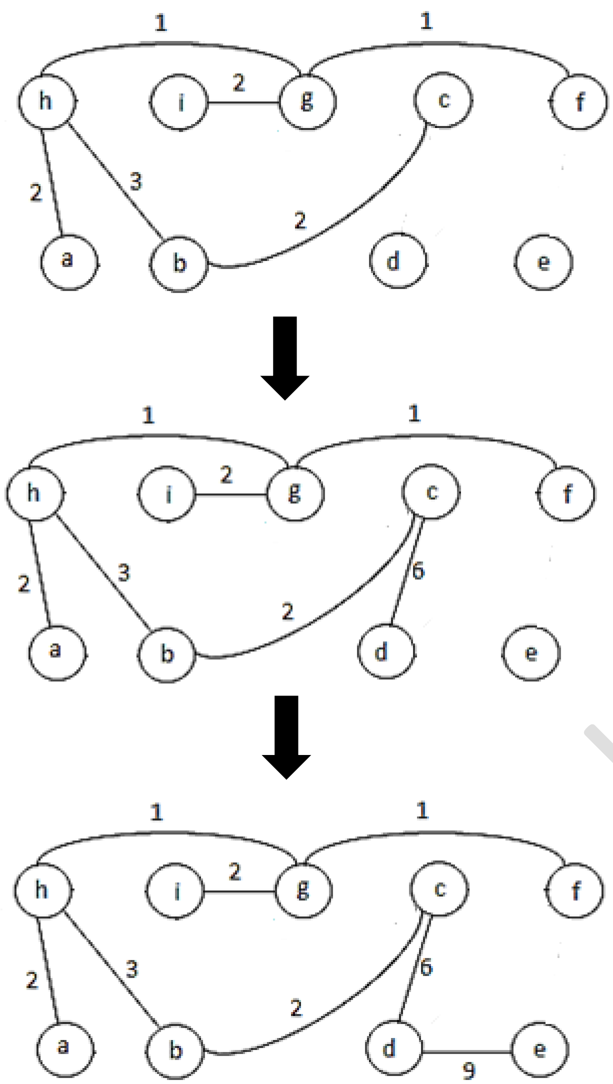
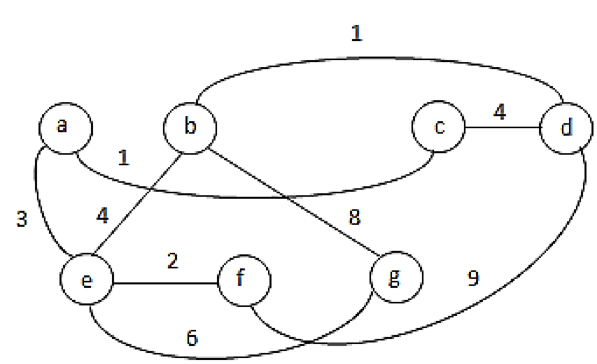
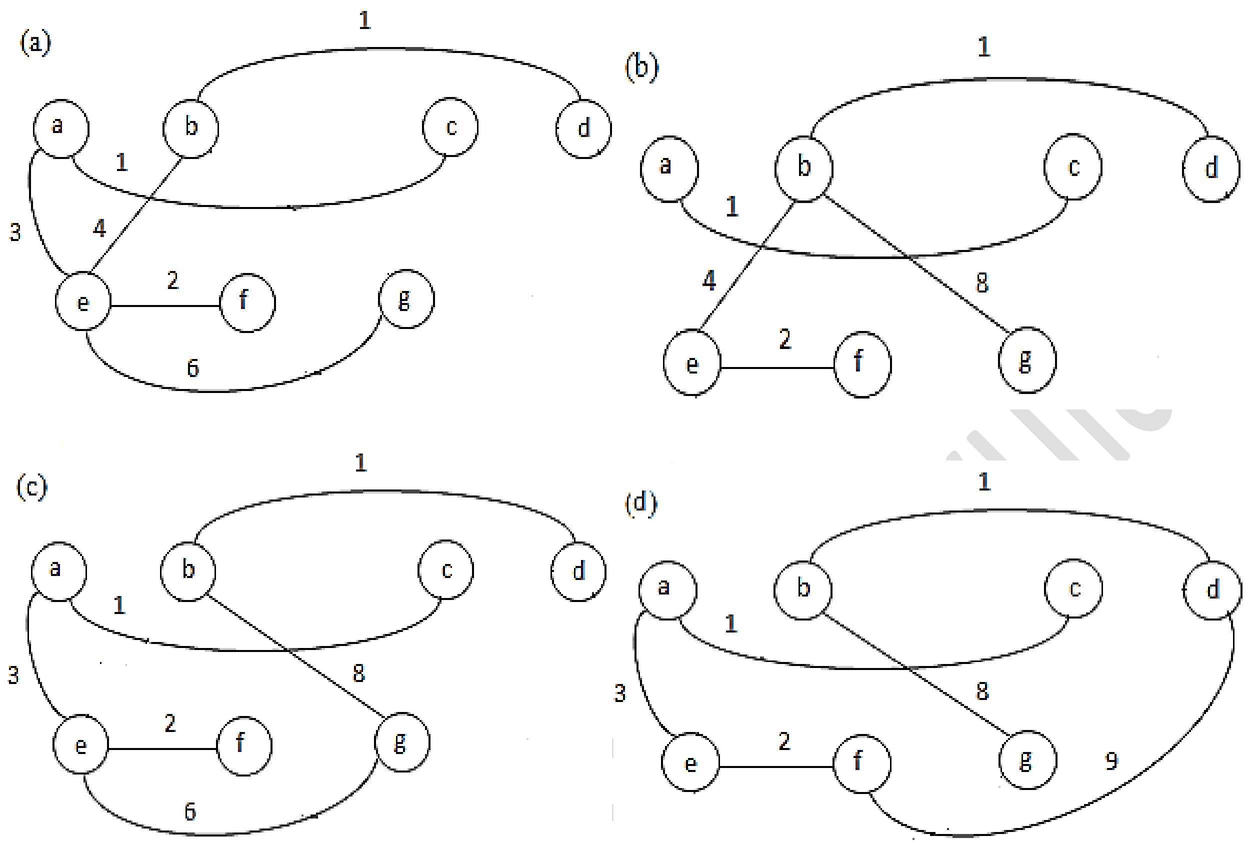
2
a b d e
1 1
h i 2 g c f
2 2
a b d e



8. Given a weighted undirected graph. What would be the resultant spanning tree of Prim's Algorithm?

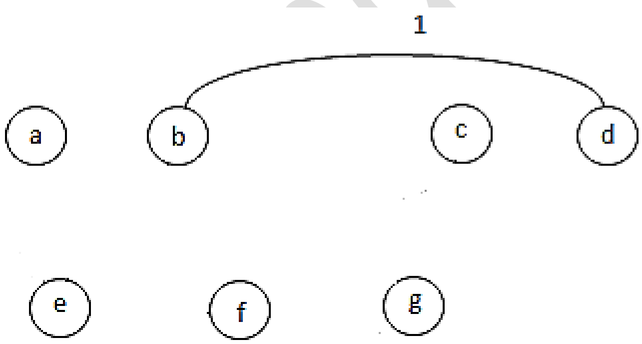


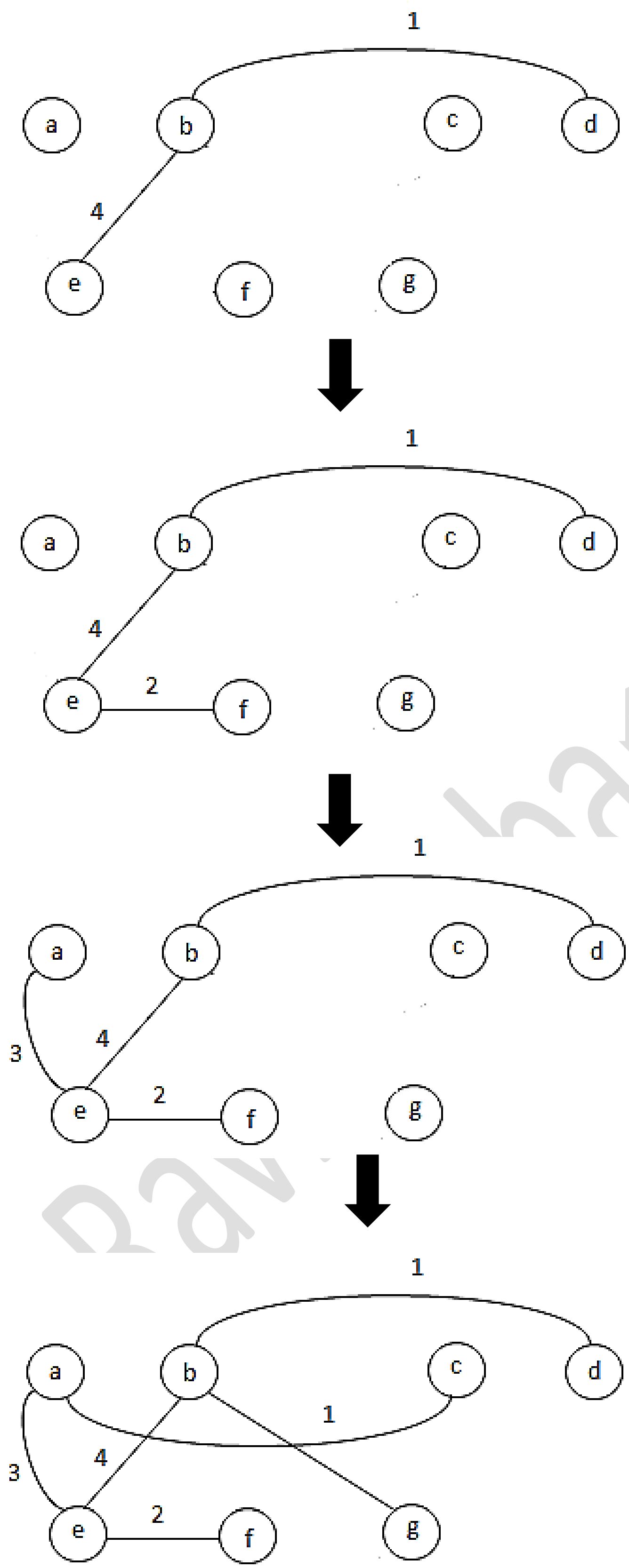


Solution: Option (a)

Explanation:

Prim's Algorithm





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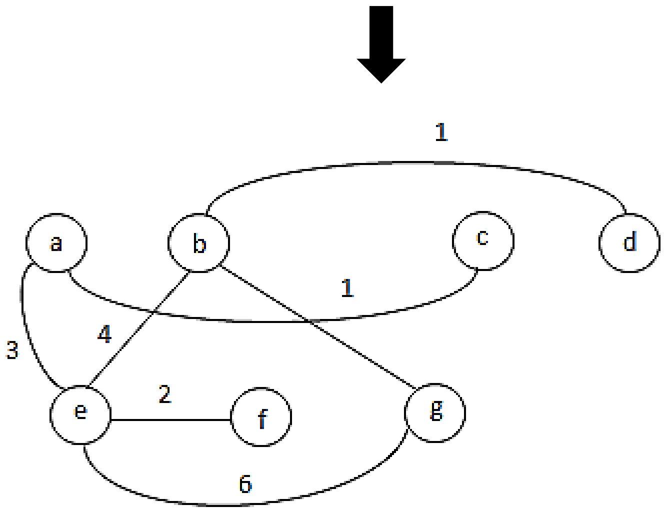
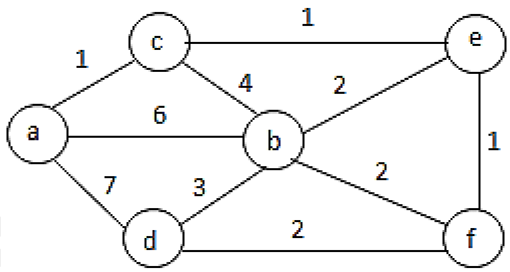


9. Consider the following graph:



Which one of the following cannot be the sequence of edges added, in that order, to a minimum spanning tree using Kruskal's algorithm?

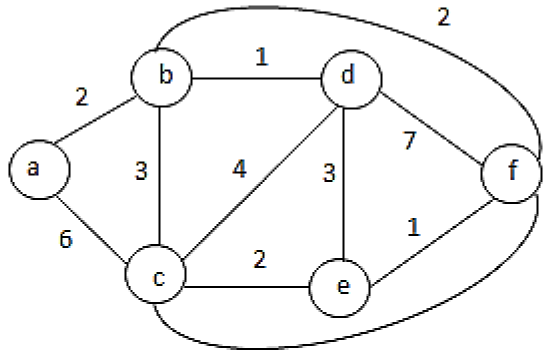
(a) (a – c), (c– e), (e – f), (e – b), (f – d)
(b) (a – c), (e – f), (e – c), (f – b), (f – d)
(c) (c – e), (a – c), (e – f), (b – d), (f – b)
(d) (c – e), (e – f), (a – c), (f – d), (b – f)

Solution: Option (c)

Explanation:

In the given graph, edge (b – d) has more weight than edges (d – f), (b – f), (e – b). So, edge (b – d) is not in the Minimum Spanning Tree.

10. Consider the following graph:



Which one of the following cannot be the sequence of edges added in that order, to a minimum spanning tree using Prim's algorithm? Assume that the algorithm starts either at vertex b or f.

(a) (b – d), (b – f), (f – e), (c – e), (b – a)
(b) (b – d), (b – a), (b – f), (f – e), (e – c)
(c) (f – e), (f – b), (b – d), (a – b), (e – c)
(d) (f – e), (e – c), (b – d), (b – a), (b – f)

Solution: Option (d)

Explanation:

Prim's algorithm always gives a tree during any intermediate step. But option (d) forms a forest.

13. Let G be an undirected connected graph with distinct edge weight. Let e_{max} be the edge with maximum weight and e_{min} be the edge with minimum weight. Which of the following statement is/are True?

(i) Every minimum spanning tree at G must contain e_{min}.
(ii) G has a unique minimum spanning tree.
(iii) No minimum spanning tree contains e_{max}.
(iv) If e_{max} is in a minimum spanning tree, then its removal leads to a disconnected graph.

(a) only (i), (ii) & (iii)
(b) only (i), (ii) & (iv)
(c) only (ii) & (iii)
(d) only (ii) & (iv)

Solution: Option (b)

11. Best case time complexity of Kruskal's algorithm is:

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(a) Ω (E log v)
(b) Ω (v log v)
(c) Ω (E + v log v)
(d) Ω (v^2)

Solution: Option (c)

Explanation:

<u>Algorithm-</u>

(1) Create Min heap $\rightarrow$ O (E)
(2) Delete Min from Min heap and Add to MST if no cycle formed until (v – 1) edges are added to MST. Delete Min takes O ($\log_2$ E)

Best case is when MST is formed by first(v – 1) delete means without getting a cycle at all.

E + (v – 1) $\log_2$ E
= E + v $\log_2$ E
but log E = Θ(log v)
= E + v log v
= Ω (E + v log v)

12. Time complexity of prim's algorithm using binary min heap

(a) O [v log v]
(b) O [(v + E) log v]
(c) O [E log E]
(d) O [v^2]

Solution: Option (b)

Explanation:

Algorithm using binary Min heap:

Step 1- Build Min heap procedure takes O (v)
Step 2- Extract Min takes O (log v) time, the total time for all calls to extract min is O (v log v)
Step 3- Now, the decrease key operation have to be repeated till only one vertex is left in min-heap.

$\therefore$ The total no. of states = v
$\therefore$ Total decrease key operation takes time :- v $\times$ v $\log_2$ v
Total time complexity = v + v log v + v^2 log v (v + v^2) log v
But v^2 = E (in complete graph)
$\therefore$ O [(v + E) log v]

14. Let T and T' be 2-spanning trees of a connected graph G. Suppose that an edge e is in T but not in T' and edge e' in T' but not in T, and remaining edges are assumed to be same. Then which of the following is True after performing the following operations?

(i) (T – {e}) ∪ {e'}
(ii) (T' – {e'}) ∪ {e}

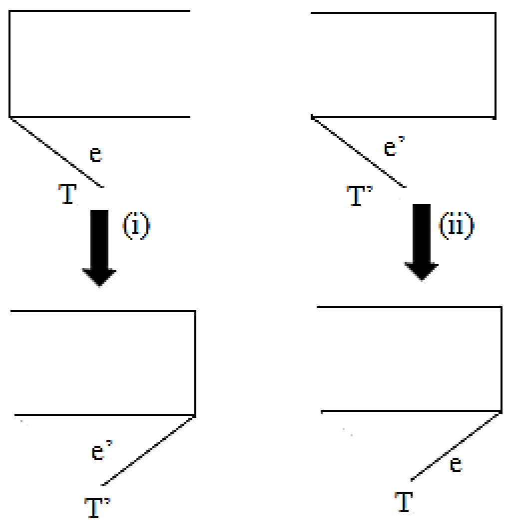
(a) Both T and T' are spanning trees
(b) Only T is a spanning tree but not T'
(c) Only T' is a spanning tree but not T
(d) Neither of them are spanning trees

Solution: Option (a)

Explanation:

(i) If e deleted from T and e' is added then the resultant spanning tree is indeed T'.
(ii) If e' is deleted from T' and e is added then the resultant spanning tree is indeed T.

For example:



15. Which of the following is / are True about Minimum spanning tree?

(i) For a given graph, more than one MST may be possible.
(ii) For a given graph, if there are more than one MST, then at least one of the edge weight will be repeated.
(iii) If the given graph is connected and no edge weight is repeated, then exactly one MST is possible.
(iv) There will be no MST if graph is disconnected.

(a) only (i) & (ii)

(b) only (ii) & (iii)

(c) only (i), (ii) & (iii)

(d) All

Solution: Option (d)

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