AIP Assignment 3

Name: B Srinath Achary SR NO: 21441

Problem 1

Implementation:

- From the paper we know that the optimal coefficients are given by the equation:

$$\alpha = \frac{H^{-1}e}{e^T H^{-1} e}$$

  where H : NxN correlation matrix of the random vector ($n_{(1)}, n_{(2)}, \ldots, n_{(N)})^T$

  e : N dimensional vector of ones

- For computing H matrix we have used the following equations

  Marginal of $n_{(i)}$:

$$g_{n_{(i)}}(x) = K_i F_n^{i-1}(x)[1 - F_n^{N-i}(x)] f_n(x)$$
$$where\ K_i = \frac{N!}{(i-1)!(N-i)!}$$

  Joint density of $n_{(i)}$ and $n_{(i)}$:

$$g_{n_{(i)}n_{(j)}}(x,y) = K_{i,j} F_n^{i-1}(x)[F_n(y) - F_n(x)]^{j-i-1}[1 - F_n^{N-j}(y)] f_n(x) f_n(y)$$
$$where\ K_{i,j} = \frac{N!}{(i-1)!(j-i-1)!(N-j)!}$$

  For H:

$$H_{ij} = \int_{-1}^{1}\int_{-1}^{y} xy g_{n_{(i)}n_{(j)}} dxdy \quad (i < j)$$
$$H_{ii} = \int_{-1}^{1} x^2 g_{n_{(i)}} dx$$

Results and Analysis:

- alpha = **[0.5 0 0 0 0.5]**^{T}

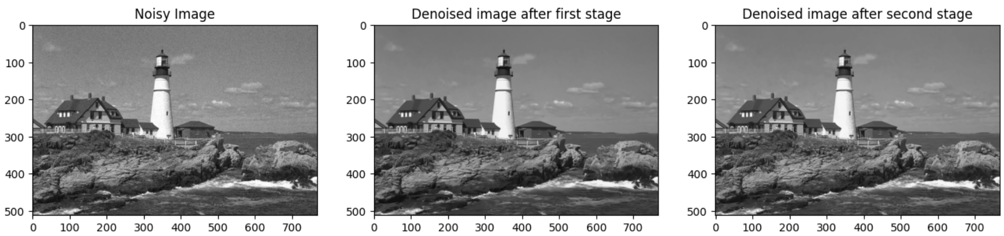
- This means that the first and last order statistics (i.e., the minimum and maximum values) are given equal weight in the filtering process, just like the median.
- Intuitively, this makes sense because for a uniform distribution, all values between the minimum and maximum values are equally likely to occur. Therefore, giving equal weight to the minimum and maximum values in the filtering process ensures that the filter is able to preserve the overall shape of the signal, while removing the noise.

Problem 2

Question 1:

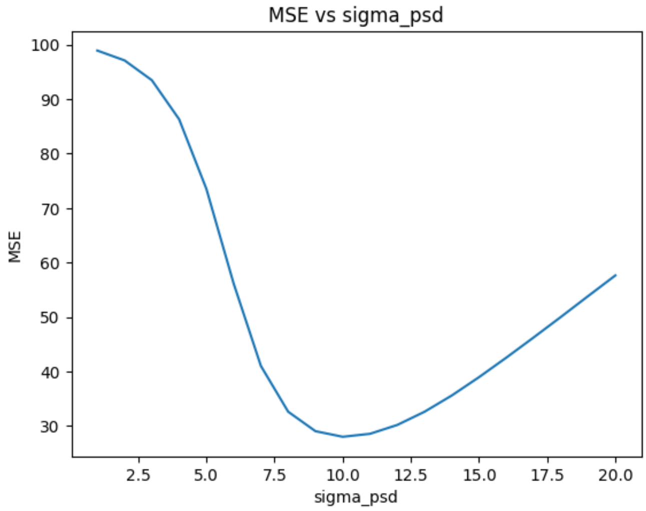
Results and analysis:

- MSE for first stage output: **32.9445599606207**
- MSE for second stage output: **27.991241997061405**



- The second stage of the BM3D algorithm improves the denoising performance by exploiting the collaborative filtering of similar 2D image patches. This is done by first grouping similar patches from the noisy image and then applying 3D transform-domain collaborative filtering to each group. The resulting filtered patches are then aggregated to form the final denoised image.
- The collaborative filtering in the second stage is more effective than the first stage as it considers a larger number of similar patches and their inter-correlations. This results in a more accurate and consistent estimation of the underlying image and hence, a lower MSE compared to the first stage. Additionally, the second stage also benefits from the initial denoising performed in the first stage, which helps in removing some of the noise and making the patches more similar for better grouping.

Question 2:



- The curve is expected to be U-shaped, with the MSE decreasing as the input noise variance increases to a certain point and then increasing again as the noise becomes too high. This is because at low levels of input noise variance, the algorithm does not have much noise to remove, so the MSE is low. As the input noise variance increases, the algorithm can remove more noise, leading to a decrease in MSE. However, as the noise becomes too high, the algorithm can no longer remove all of it, leading to an increase in MSE.
- We can see that we get the minima where sigma_psd matches with the true sigma of the added noise i.e 10.

Question 3:

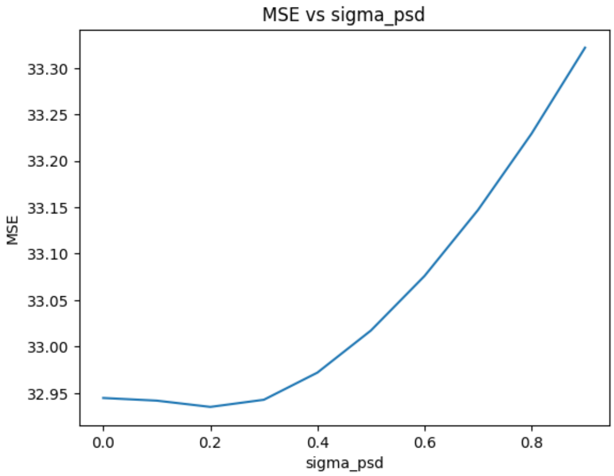
Implementation:

- The initial step in the experiment involved applying the BM3D algorithm with a noise variance of 10 to the noisy image. The output from the first stage was then selected and fed back into the first stage of the BM3D algorithm using different noise variances.

Results and Analysis:

- MSE loss with Wiener filter: **27.991241997061405**
- MSE loss with twice Hardthresholding Estimate: **32.93494881042101**

- The Wiener filter in the second stage with a hard thresholding estimate and compared the performance with the former in terms of MSE. It was found that the hard thresholding estimate resulted in a higher MSE compared to the Wiener filter. This is expected as the Wiener filter is a better estimator when the noise is Gaussian, which was the case in our experiments. Hard thresholding works better when the noise is sparse, which was not the case in our experiments.



- The plot illustrates the relationship between MSE and sigma for the output of the first stage after being processed for the second time. It is observed that the minimum MSE is obtained at a sigma value of 0.2. This can be attributed to the fact that after the image was passed to the first stage for the first time, a significant amount of denoising had already occurred and only low levels of noise were present. Therefore, when the image was passed through the first stage for the second time, a very low sigma value was required compared to the previous value