

# Straight Lines

## 11<sup>th</sup> Maths - Chapter 10

This is Problem-10 from Exercise 10.4

1. If three lines whose equations are  $y = m_1x + c_1$ ,  $y = m_2x + c_2$  and  $y = m_3x + c_3$  are concurrent, then show that  $m_1(c_2 - c_3) + m_2(c_3 - c_1) + m_3(c_1 - c_2) = 0$ .

**Solution:** Given lines can be written as

$$-m_1x + y = c_1 \quad (1)$$

$$-m_2x + y = c_2 \quad (2)$$

$$-m_3x + y = c_3 \quad (3)$$

The above lines can be written in the form of

$$\mathbf{n}^\top \mathbf{x} = c \quad (4)$$

Therefore,

$$\begin{pmatrix} -m_1 & 1 \end{pmatrix} \mathbf{x} = -c_1 \quad (5)$$

$$\begin{pmatrix} -m_2 & 1 \end{pmatrix} \mathbf{x} = c_2 \quad (6)$$

Solving equations (5) and (6) augmented matrix is

$$\begin{pmatrix} -m_1 & 1 & c_1 \\ -m_2 & 1 & c_2 \end{pmatrix} \quad (7)$$

$$\xleftrightarrow{R_1 \leftarrow R_1 - R_2} \begin{pmatrix} -m_1 + m_2 & 0 & c_1 - c_2 \\ -m_2 & 1 & c_2 \end{pmatrix} \quad (8)$$

$$\xleftrightarrow{R_1 \leftarrow \frac{1}{-m_1 + m_2} R_1} \begin{pmatrix} 1 & 0 & \frac{c_1 - c_2}{m_2 - m_1} \\ -m_2 & 1 & c_2 \end{pmatrix} \quad (9)$$

$$\xleftrightarrow{R_2 \leftarrow R_2 + m_2 R_1} \begin{pmatrix} 1 & 0 & \frac{c_1 - c_2}{m_2 - m_1} \\ 0 & 1 & c_2 + \frac{(c_1 - c_2)m_2}{m_2 - m_1} \end{pmatrix} \quad (10)$$

$$\Rightarrow \begin{pmatrix} 1 & 0 & \frac{c_1 - c_2}{m_2 - m_1} \\ 0 & 1 & \frac{c_1 m_2 - m_1 c_2}{m_2 - m_1} \end{pmatrix} \quad (11)$$

Therefore,

$$\mathbf{x} = \begin{pmatrix} \frac{c_1 - c_2}{m_2 - m_1} \\ \frac{c_1 m_2 - m_1 c_2}{m_2 - m_1} \end{pmatrix} \quad (12)$$

As the three lines are concurrent, equation(3) passes through the point  $\mathbf{x}$ , so substitute the above point in equation (3)

$$-m_3 \frac{(c_1 - c_2)}{m_2 - m_1} + \frac{c_1 m_2 - c_2 m_1}{m_2 - m_1} = c_3 \quad (13)$$

$$-m_3 c_1 + m_3 c_2 + c_1 m_2 - m_1 c_2 = c_3 m_2 - c_3 m_1 \quad (14)$$

$$-m_3 c_1 + m_3 c_2 + c_1 m_2 - m_1 c_2 - c_3 m_2 + c_3 m_1 = 0 \quad (15)$$

$$m_1(c_2 - c_3) + m_2(c_3 - c_1) + m_3(c_1 - c_2) = 0 \quad (16)$$

Therefore hence proved

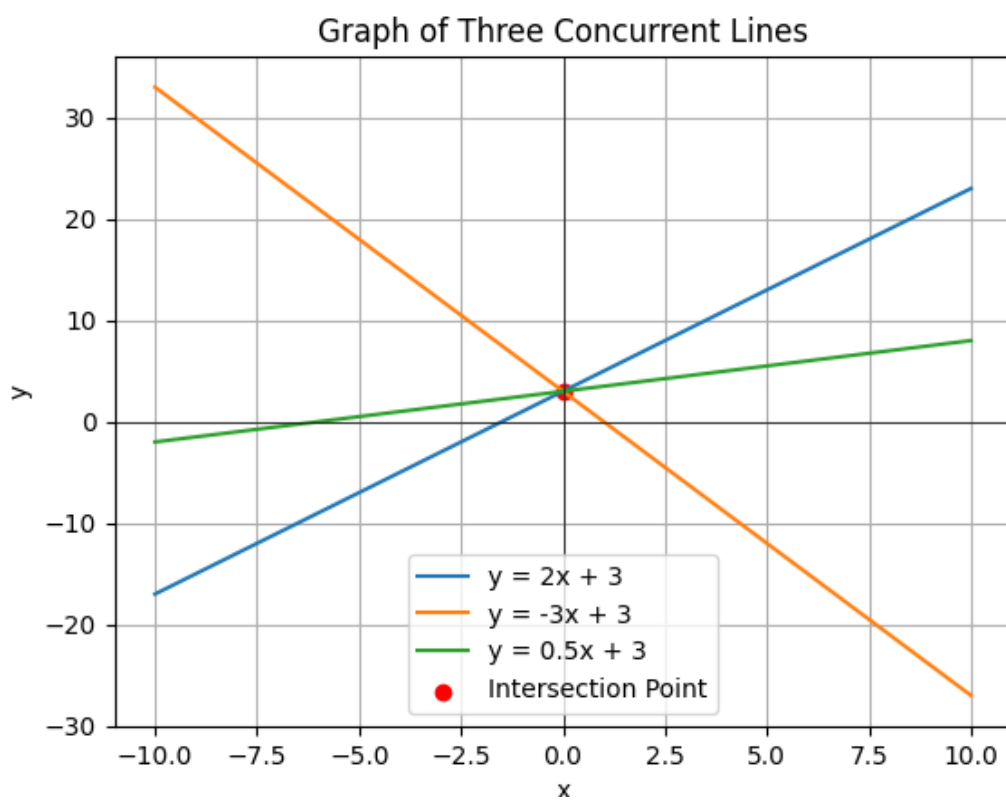


Figure 1: Straight Lines