Straight Lines

11^{th} Maths - Chapter 10

This is Problem-10 from Exercise 10.4

1. If three lines whose equations are $y = m_1x+c_1$, $y = m_2x+c_2$ and $y = m_3x+c_3$ are concurrent, then show that $m_1(c_2 - c_3) + m_2(c_3 - c_1) + m_3(c_1 - c_2) = 0$.

   Solution: Given lines can be written as

$$m_1x - y + c_1 = 0 \tag{1}$$

$$m_2x - y + c_2 = 0 \tag{2}$$

$$m_3x - y + c_3 = 0 \tag{3}$$

   The above lines can be written in the form of

$$\mathbf{n}^\top \mathbf{x} = c \tag{4}$$

Therefore,

$$\begin{pmatrix} m_1 & -1 \end{pmatrix} \mathbf{x} = c_1 \tag{5}$$

$$\begin{pmatrix} m_2 & -1 \end{pmatrix} \mathbf{x} = c_2 \tag{6}$$

$$\begin{pmatrix} m_3 & -1 \end{pmatrix} \mathbf{x} = c_3 \tag{7}$$

Solving equations (5), (6)and (7) augumented matrix is

$$\begin{pmatrix} m_1 & -1 & c_1 \\ m_2 & -1 & c_2 \\ m_3 & -1 & c_3 \end{pmatrix} \tag{8}$$

$$\xleftrightarrow{R_2 \leftarrow m_1 R_2 - m_2 R_1} \begin{pmatrix} m_1 & -1 & c_1 \\ 0 & m_2 - m_1 & m_1 c_2 - m_2 c_1 \\ m_3 & -1 & c_3 \end{pmatrix} \tag{9}$$

$$\xleftrightarrow{R_3 \leftarrow m_1 R_3 - m_3 R_1} \begin{pmatrix} m_1 & -1 & c_1 \\ 0 & m_2 - m_1 & m_1 c_2 - m_2 c_1 \\ 0 & m_3 - m_1 & m_1 c_3 - m_3 c_1 \end{pmatrix} \tag{10}$$

$$\xleftrightarrow{R_3 \leftarrow R_3 \frac{m_2 - m_1}{m_3 - m_1} - R_2} \begin{pmatrix} m_1 & -1 & c_1 \\ 0 & m_2 - m_1 & m_1 c_2 - m_2 c_1 \\ 0 & 0 & (m_1 c_3 - m_3 c_1)(\frac{m_2 - m_1}{m_3 - m_1}) - m_1 c_2 - m_2 c_1 \end{pmatrix} \tag{11}$$

Now, for lines to be concurrent, then the third row should be equal to zero.

Therefore,

$$(m_1c_3 - m_3c_1)(\frac{m_2 - m_1}{m_3 - m_1}) - m_1c_2 - m_2c_1 = 0 \tag{12}$$

$$\frac{(m_1c_3 - m_3c_1)(m_2 - m_1) - ((m_1c_2 - m_2c_1)(m_3 - m_1))}{m_3 - m_1} = 0 \tag{13}$$

$$m_2c_3 - m_1c_3 + m_3c_1 - m_3c_2 + m_1c_2 - m_2c_1 = 0 \tag{14}$$

$$m_1(c_2 - c_3) + m_2(c_3 - c_1) + m_3(c_1 - c_2) = 0 \tag{15}$$

Hence proved

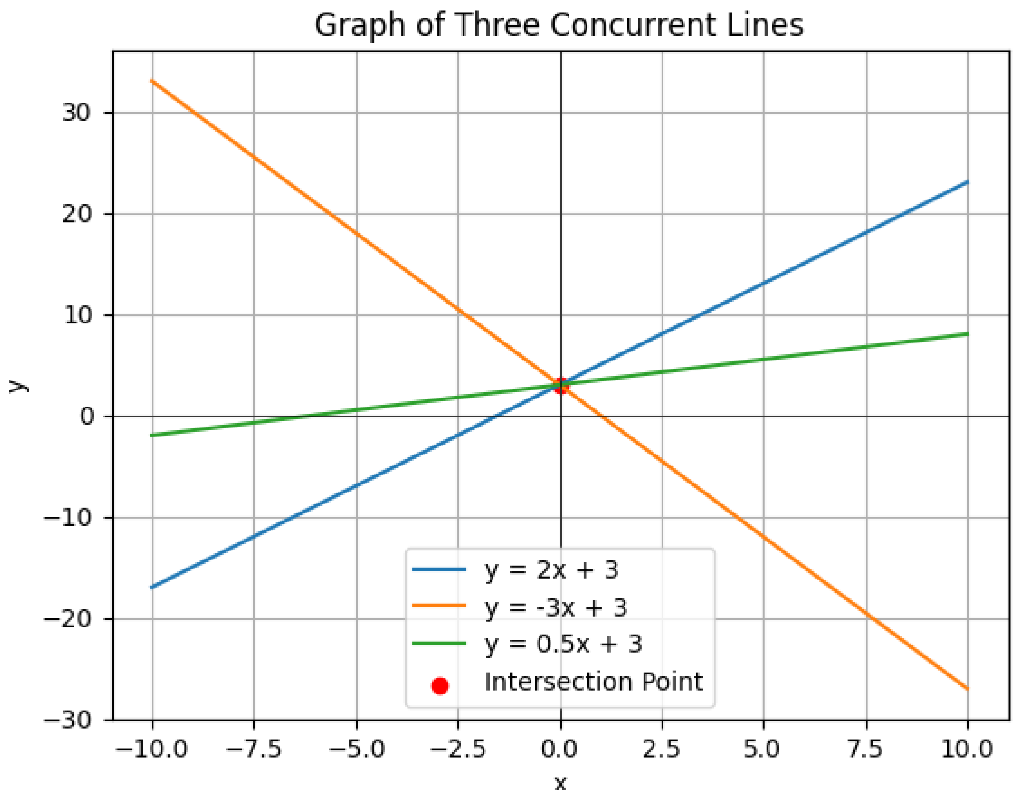

Figure 1: Straight Lines