

**Problem 1.4** *Suppose that the precondition (of Algorithm 1.1) is changed to say:*

$$x \geq 0 \wedge y > 0 \wedge x, y \in \mathbb{Z}$$

*where  $\mathbb{Z} = \{\dots, -2, -1, 0, 1, 2, \dots\}$ . Is the Algorithm still correct in this case?*

**Solution:** The original precondition (with which the algorithm is correct) is:

$$x \geq 0 \wedge y > 0 \wedge x, y \in \mathbb{N}$$

where  $\mathbb{N} = \{0, 1, 2, \dots\}$ . So our work has already been done for us; any member of  $\mathbb{Z}$  which is  $\geq 0$  is also in  $\mathbb{N}$  (and any member of  $\mathbb{N}$  is in  $\mathbb{Z}$ ), so these preconditions are equivalent. Given that the algorithm was correct under the original precondition, it is also correct under the new one.  $\square$