

Problem 1.4 *Suppose that the precondition (of Algorithm 1.1) is changed to say:*

$$x \geq 0 \wedge y > 0 \wedge x, y \in \mathbb{Z}$$

where $\mathbb{Z} = \{\dots, -2, -1, 0, 1, 2, \dots\}$. Is the Algorithm still correct in this case?

Solution:

The original precondition (with which the algorithm is correct) is:

$$x \geq 0 \wedge y > 0 \wedge x, y \in \mathbb{N}$$

where $\mathbb{N} = \{0, 1, 2, \dots\}$. So our work has already been done for us; any member of \mathbb{Z} which is ≥ 0 is also in \mathbb{N} (and any member of \mathbb{N} is in \mathbb{Z}), so these preconditions are equivalent. Given that the algorithm was correct under the original precondition, it is also correct under the new one.