$an^2+bn+c=\Theta(n^2)$, where $a>0$. To see this, note that $an^2+bn+c\leq(a+|b|+|c|)n^2$, for all $n\in\mathbb{N}$, and so $an^2+bn+c=O(n^2)$. On the other hand, $an^2+bn+c=a((n+c_1)^2-c_2)$ where $c_1=b/2a$ and $c_2=(b^2-4ac)/4a^2$, so we can find a c_3 and an n_0 so that for all $n\geq n_0$, $c_3n^2\leq a((n+c_1)^2-c_2)$, and so $an^2+bn+c\in\Omega(n^2)$.

Problem 1.2 *Find c_3 and n_0 in terms of a, b, and c. Then prove that for $k\geq 0$, $\sum_{i=0}^{k}a_in^i=\Theta(n^k)$; this shows the simplfying advantage of the Big O.*

Solution:
Clearly,

$$an^2+bn+c\geq an^2-|b|n-|c|=n^2(a-|b|/n-|c|/n^2) \tag{1}$$

$|b|$ is finite, so $\exists n_b\in\mathbb{N}$ such that $|b|/n_b\leq a/4$. Similarly, $\exists n_c\in\mathbb{N}$ such that $|c|/n^2\leq a/4$. Let $n_0=max\{n_b,n_c\}$. For $n\geq n_0$, $a-|b|/n_0-|c|/n_0^2\geq a-a/4-a/4=a/2$. This, combined with (1), grants:

$$\frac{a}{2}n^2\leq an^2+bn+c \tag{2}$$

for all $n\geq n_0$. We need only to assign c_3 the value $a/2$ to complete the proof that $an^2+bn+c\in\Omega(n^2)$.

Next we deal with the general polynomial with a positive leading coefficient. Let $p(n)=\sum_{i=1}^{k}a_in^i=n^k\sum_{i=1}^{k}\frac{a_i}{n^{k-i}}$, where $a_k>0$. Clearly $p(n)\leq n^k\sum_{i=1}^{k}|a_i|$ for all $n\in\mathbb{N}$, so $p(n)=O(n^k)$. Moreover, every a_i is finite, so for each $i\in\mathbb{N}$ such that $0\leq i\leq k-1$, $\exists n_i$ such that $\frac{a_i}{n^{k-i}}\leq a_k/2k$ for all $n\geq n_i$. Let n_0 be the maximum of these n_i's. $p(n)$ can be rewritten as $n^k(a_k+\sum_{i=0}^{k-1}\frac{a_i}{n^{k-i}})$, so $p(n)\geq n^k(a_k-\sum_{i=0}^{k-1}\frac{a_i}{n^{k-i}})$. We have shown that for $n\geq n_0$, $\sum_{i=0}^{k-1}\frac{a_i}{n^{k-i}}\leq a_k-k(a_k/2k)=a_k/2$, so let $c=a_k/2$. For all $n\geq n_0$, $p(n)\geq(a_k-a_k/2)n^k=cn^k$. Thus, $p(n)=\Omega(n^k)$.

We have shown that $p(n)\in O(n^k)$ and $p(n)\in\Omega(n^k)$, so $p(n)=\Theta(n^k)$.